SIOG 231: GEOMAGNETISM AND ELECTROMAGNETISM

Chapter 14: MT and GDS in Practice

Introduction

Because the skin depth generates a natural scale length for time varying fields, the response of Earth to time variations in the externally generated geomagnetic field can be interpreted in terms of electrical conductivity with depth and location. The estimation of purely geomagnetic response functions and their interpretation in terms of mantle electrical conductivity structure dates from the end of the 19th century, but the additional use of electric field measurements only started with Cagniard's work in the 1950s.

Recap of Theory

Continuing with our half-space formulation, we can define an impedance by taking the ratio of orthogonal field components

$$Z = \frac{E_x(\omega)}{H_y(\omega)} = \frac{\mu\omega}{k}$$

where we recall that the complex wavenumber is

$$k = (i\sigma\mu\omega)^{\frac{1}{2}} = \sqrt{\frac{\sigma\mu\omega}{2}} + i\sqrt{\frac{\sigma\mu\omega}{2}}$$

and so we can see that E_x leads H_y by 45° and that the half-space resistivity is given by

$$\rho = \frac{1}{\sigma} = \frac{1}{2\pi f\mu}|Z|^2 = \frac{T}{2\pi\mu}\left|\frac{E_x}{H_y}\right|^2$$

for fields with period T. This is the magnetotelluric formula made famous by Cagniard (1953). Perhaps more usefully (since measurements tend to be made in **B** rather than **H**),

$$\rho = \frac{\mu}{\omega}\left|\frac{E_x}{B_y}\right|^2 \quad .$$

We can also compute a phase difference between E and H:

$$\Phi = \arg(Z)$$

Another quantity called the c-response or inductive scale length (Weidelt, 1972) (also called admittance by Parker, 1994) is also used, given by

$$c = -\frac{E_x}{i\omega B_y}$$

(note that c is complex) from which it can be seen that

$$\rho = \omega\mu|c|^2 \quad .$$

If Earth really were a homogeneous half-space, the resistivity obtained at all periods would be the same and equal to the true resistivity of the earth, and the phase between **B** and **E** would be constant at 45°. Of course,

this is never the case, but resistivities are computed nevertheless and described as 'apparent resistivities'. Solutions for computing the apparent resistivity response of layered, 2D and even 3D models exist and are used to interpret the MT data. The phase relationship between E and B is also estimated, and is useful because it is independent of the amplitudes of E and B. Complicated near-surface structure can change the amplitude of the electric fields very easily, but because these distortions are not inductive, the phase relationships are not altered. We discuss this a little more below.

The Magnetotelluric Method

Figure 6.1(a) shows a schematic layout of an MT measurement; the magnetic and electric fields are measured orthogonal to each other. To a first approximation, the magnetic field is the EM source and the electric field is the EM response. For a 1D earth, this would be the only measurement required; for a 2D or 3D earth, measurements are made in both directions, and perhaps even the vertical magnetic field would be measured. The length of the electric field dipoles would typically be of order 50–200 m.

In Figure 6.1(b) we provide a quantitative example of this behavior, plotting magnetic field, electric field, and current density for 0.5 Hz energy propagating into a model containing a buried conductive layer. All the fields decay quasi-exponentially within the ground, but inside the conductor the magnetic fields can be seen to attenuate more rapidly due to induced electric currents, which peak inside this layer. At a higher frequency (5 Hz), magnetic fields attenuate before the conductive layer is reached because of the shorter skin depth. Without the conductive layer, fields propagate more deeply into the earth, but importantly the electric field at the surface depends on structure at depth. This is the principle of the MT method.

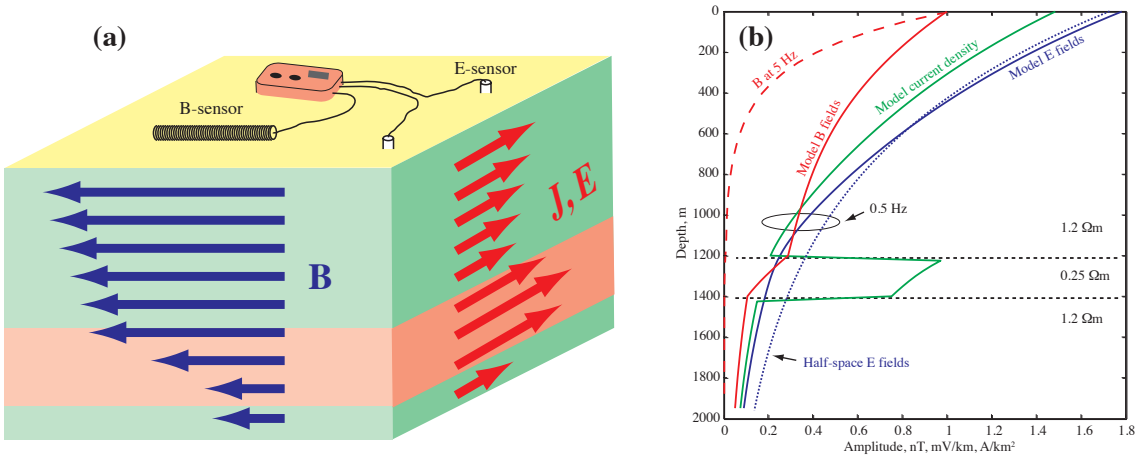

Figure 6.1 (a) Schematic of an MT measurement. Orthogonal components of the magnetic and electric field are measured at the surface of the earth. In conductive layers (red), electric fields and currents are induced whose secondary magnetic fields attenuate the primary magnetic field more rapidly than in the background material. (b) In a quantitative example of this behavior, magnetic field, electric field, and current density are plotted for 0.5 Hz energy propagating into a model containing a buried conductive layer (solid lines). For comparison, magnetic fields for a 5 Hz source-field are shown (broken red line). The dotted blue line shows the electric fields without the conductive layer.

MT measurements can be made from the low end of the radio frequency spectrum, using radio stations and

lightning as sources of energy, to periods of several days. The table of skin depths gives some indication of the relationship between conductivity, period, and penetration scale. Structure can profoundly affect the way MT fields propagate, but skin-effect attenuation provides an indication of the limits to deepest sensitivity (see Figure 6.1b again). The type of instrumentation used to record the E and B fields will depend on the periods required, and the time over which data are collected will determine the longest period available. Horizontal spacing of MT sites can vary from continuous electric field dipoles for shallow crustal studies, individual sites spaced tens of kilometers apart for regional profiles, to single long period soundings for mantle conductivity studies.

MT Data Processing

The raw measurements made in the field are time series of magnetic $B(t)$ and electric $E(t)$ fields, usually in two orthogonal directions. An example is given in Figure 6.2. The objective of MT data processing is to obtain the frequency domain response of the coherent part of these signals.

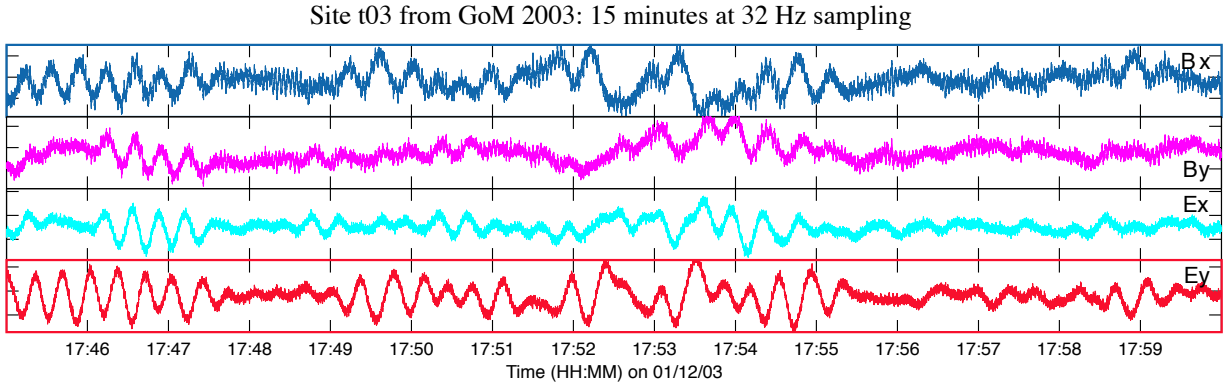

Figure 6.2 Sample of MT timeseries measurements, in arbitrary units. Note that B_x and E_y are anti-correlated, and B_y and E_x are correlated.

We need to remind ourselves of some simple statistical terms. The *expectation value* of a random variable X which is described by probability $\Phi(x)$ on the real numbers x is

$$E[X] = \int_{-\infty}^{\infty} x\Phi(x)dx$$

One common probability function is the normal, or Gaussian, distribution with mean $\bar{x}$ and standard deviation σ:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\bar{x})^2/2\sigma}$$

This concept can be extended to functions of x

$$E[f(X)] = \int_{-\infty}^{\infty} f(x)\Phi(x)dx$$

The mean and variance of X are then defined as

$$\bar{X} = \mathrm{E}[X] \qquad \mathrm{var}[X] = \mathrm{E}[(X - \mathrm{E}[X])^2] = \mathrm{E}[(X - \bar{X})^2]$$

which for two random variables X and Y can be extended to the covariance

$$\text{cov}[X, Y] = \text{E}[(X - \bar{X})(Y - \bar{Y})]$$

which reduces to

$$\text{cov}[X, Y] = \text{E}[(XY)]$$

if X and Y are zero-mean.

All this depends on having an infinite number of data, which of course is impossible, so we instead define a *sample mean*,

$$\bar{x} = \frac{1}{N} \sum_{n=1}^{N} X_n$$

sample variance,

$$\sigma_x^2 = \frac{1}{N} \sum_{n=1}^{N} (X_n - \bar{X})^2 = \frac{1}{N-1} \sum_{n=1}^{N} (X_n - \bar{x})^2$$

and *sample covariance*

$$\text{cov}_{xy} = \frac{1}{N-1} \sum_{n=1}^{N} (X_n - \bar{x})(Y_n - \bar{y})$$

which can be generalized to the sample covariance matrix where you have multiple random variables

$$\text{cov}_{ik} = \frac{1}{N-1} \sum_{n=1}^{N} (x_{ni} - \bar{x}_i)(x_{nk} - \bar{x}_k)$$

In the analysis below, we will assume everything is zero mean. Then

$$\text{cov}_{ik} = \frac{1}{N} \sum_{n=1}^{N} (x_{ni})(x_{nk})$$

Back to MT: The frequency domain transfer function between the horizontal magnetic and electric field is called the magnetotelluric impedance tensor Z:

$$\begin{bmatrix} E_x(\omega) \\ E_y(\omega) \end{bmatrix} = \begin{bmatrix} Z_{xx}(\omega) & Z_{xy}(\omega) \\ Z_{yx}(\omega) & Z_{yy}(\omega) \end{bmatrix} \begin{bmatrix} H_x(\omega) \\ H_y(\omega) \end{bmatrix}$$

The first thing we need to do is convert the time series of E and B into the frequency domain. For this we use the discrete Fourier transform. For N samples of a measurement X spaced Δt apart in time, the discrete Fourier transform is

$$\tilde{X}(m\Delta f) = \Delta t \sum_{n=0}^{N-1} X_{n+1} e^{-2\pi i m n/N} \quad , \qquad m = 1, 2, \ldots N/2 - 1$$

where $\Delta f = (N\Delta t)^{-1}$ is the frequency bandwidth, or width of the Fourier bins. Note that $\tilde{X}$ is complex – we have mapped N time samples to $N/2 - 1$ complex numbers (which could be thought of as amplitude and phase). There is a $m = 0$ term, which is just the mean of the timeseries.

The value $|\tilde{X}(m\Delta f)|^2$ is the periodogram, which has a variance of 100% which cannot be reduced by increasing N. In order to increase the reliability of the Fourier transform, we need to do some averaging. There are three approaches:

- frequency averaging
- window averaging
- multi-taper averaging

Tapering a time series means

$$\tilde{X}(m\Delta f) = \Delta t \sum_{n=0}^{N-1} w_n X_{n+1} e^{-2\pi i m n/N}$$

where the taper w_n is a set of weights over the sample interval N. It is common to taper the timeseries with a single taper such the beginning and end go to zero, to stop a trend in the timeseries introducing spurious frequencies, but in multitaper averaging a number of orthogonal taper functions generate a number of independent estimates of $\tilde{X}(m\Delta f)$ which can be averaged. But, by and large MT processing doesn't use multi-taper methods, but just uses window and frequency averaging. In frequency averaging one averages the $\tilde{X}(m\Delta f)$ over a number of adjacent frequencies, trading off frequency resolution for reliability. In window averaging, one breaks up the timeseries into smaller segments and then averages the resulting $\tilde{X}(m\Delta f)$. It is common to combine both.

From here we will drop the ω dependence for clarity, and although most people define Z in terms of H, we will use B since these are the timeseries we actually measure. The difference is just a factor of μ_o. With sufficient averaging, one could compute a 1D MT response

$$Z = \frac{\tilde{E}_y(\omega)}{\tilde{B}_x(\omega)}$$

but the full impedance matrix

$$\begin{bmatrix} \tilde{E}_x \\ \tilde{E}_y \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} \tilde{B}_x \\ \tilde{B}_y \end{bmatrix}$$

has only two equations in up to 8 unknowns (4 complex numbers), so we are going to have to divide our timeseries into a number of segments (here M) anyway

$$\begin{bmatrix} \tilde{E}_x^1 \\ \tilde{E}_x^2 \\ \vdots \\ \tilde{E}_x^M \\ \tilde{E}_y^1 \\ \tilde{E}_y^2 \\ \vdots \\ \tilde{E}_y^M \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} \tilde{B}_x^1 \\ \tilde{B}_x^2 \\ \vdots \\ \tilde{B}_x^M \\ \tilde{B}_y^1 \\ \tilde{B}_y^2 \\ \vdots \\ \tilde{B}_y^M \end{bmatrix}$$

Another way to look at this is that we need more than one source field (magnetic field) polarization (or direction). Remember that all these are Fourier estimates from the time series, and there is a system of equations like this for every frequency (or an average over some band of frequencies).

This would work, but not well. To see why, let us go back to the simplification of a 1D response

$$Z = \frac{\tilde{E}_y}{\tilde{B}_x}$$

The standard treatment of a transfer function would consider B the input and E the output with noise ϵ in the output measurement only:

$$\tilde{E} = Z\tilde{B} + \epsilon$$

where we have M samples of $\tilde{E}$ and $\tilde{B}$. The least squares solution is obtained by

$$\min\left(\sum_{k=1}^{M} |\tilde{E}_k - Z\tilde{B}_k|^2\right)^{\frac{1}{2}}$$

which in the classic least squares approach is estimated by

$$\hat{Z} = \frac{\sum_k \tilde{E}_k(\tilde{B}_k)^*/M}{\sum_k |\tilde{B}_k|^2/M}$$

or the cross spectrum between B and E divided by the power spectrum of B. (We will drop the normalizing M going forward, for obvious reasons.) Note that least squares assumes that the noise ϵ is well behaved over the M samples (stationary, independent, zero mean, Gaussian). Let's say that there is a burst of noise during some part of the data collection (radio transmission, train going by, high winds, etc.). Then the noise distribution will depart from the ideal Gaussian. This can be handled by a *robust* averaging of the Fourier coefficients, using a set of weights **w** to down-weight the outliers and bring the distribution back to normal:

$$\hat{Z} = \frac{\sum_k w_k\tilde{E}_k(\tilde{B}_k)^*}{\sum_k w_k|\tilde{B}_k|^2}$$

Before we go further, it is useful to generalize this and show how things can be viewed in terms of covariance matrices. If we multiply both sides of

$$\tilde{E} = Z\tilde{B} + \epsilon$$

by $\tilde{B}^*$ and take expectation values

$$\mathrm{E}[\tilde{E}\tilde{B}^*] = Z\mathrm{E}[\tilde{B}\tilde{B}^*] + \mathrm{E}[\epsilon\tilde{B}^*]$$

we note that if ϵ is zero-mean and independent of B the last term goes to zero so

$$\hat{Z} = \frac{\mathrm{E}[\tilde{E}\tilde{B}^*]}{\mathrm{E}[\tilde{B}\tilde{B}^*]} = \frac{\mathrm{cov}[\tilde{E}, \tilde{B}^*]}{\mathrm{var}[\tilde{B}]}$$

That is, Z is described by two terms of a covariance matrix formed from a vector **X** of random variables which are just our Fourier estimates

$$\mathbf{X} = \begin{bmatrix} \tilde{E} \\ \tilde{B} \end{bmatrix}$$

with a covariance matrix

$$C = \begin{bmatrix} \mathrm{E}[\tilde{E}\tilde{E}^*] & \mathrm{E}[\tilde{E}\tilde{B}^*] \\ \mathrm{E}[\tilde{B}\tilde{E}^*] & \mathrm{E}[\tilde{B}\tilde{B}^*] \end{bmatrix} = \begin{bmatrix} Z^2\sigma_{\tilde{b}}^2 + \sigma_\epsilon^2 & Z\sigma_{\tilde{b}}^2 \\ Z^*\sigma_{\tilde{b}}^2 & \sigma_{\tilde{b}}^2 \end{bmatrix}$$

where

$$\mathrm{E}[\tilde{B}\tilde{B}^*] = \sigma_b^2$$

$$\mathrm{E}[\tilde{B}\tilde{E}^*] = Z^*\sigma_b^2$$

$$\mathrm{E}[\tilde{E}\tilde{E}^*] = Z^2\sigma_b^2 + \sigma_\epsilon^2$$

This is soluble, because the covariance matrix provides 4 constraints (the variances of E and B, and the complex covariance of E and B) for 4 unknowns (the two complex elements of Z, the variance of $\tilde{b}$, and the noise variance σ_ϵ^2).

The trouble is, there is noise in B as well as E. This problem was evident to Gamble et al. (1979), who noted that the power spectrum of B included the noise in B, but it is clearer when we use the covariance approach. Consider now noise in both fields

$$\tilde{B} = \tilde{b} + \beta \qquad \tilde{E} = Z\tilde{b} + \epsilon$$

Now our covariance matrix is

$$C = \begin{bmatrix} \mathrm{E}[\tilde{E}\tilde{E}^*] & \mathrm{E}[\tilde{E}\tilde{B}^*] \\ \mathrm{E}[\tilde{B}\tilde{E}^*] & \mathrm{E}[\tilde{B}\tilde{B}^*] \end{bmatrix} = \begin{bmatrix} Z^2\sigma_{\tilde{b}}^2 + \sigma_\epsilon^2 & Z\sigma_{\tilde{b}}^2 \\ Z^*\sigma_{\tilde{b}}^2 & \sigma_{\tilde{b}}^2 + \sigma_\beta^2 \end{bmatrix}$$

where

$$\mathrm{E}[\tilde{B}\tilde{B}^*] = \sigma_b^2 + \sigma_\beta^2$$

$$\mathrm{E}[\tilde{B}\tilde{E}^*] = Z^*\sigma_b^2$$

$$\mathrm{E}[\tilde{E}\tilde{E}^*] = Z^2\sigma_b^2 + \sigma_\epsilon^2$$

and our least squares estimate is

$$\hat{Z} = \frac{\mathrm{cov}[\tilde{E}, \tilde{B}^*]}{\mathrm{var}[\tilde{B}]} = \frac{Z\sigma_b^2}{\sigma_b^2 + \sigma_\beta^2} = \frac{Z}{1 + \sigma_\beta^2/\sigma_b^2}$$

That is, our estimate of Z, $\hat{Z}$, is biased downwards by the unknown error in $\tilde{b}$. Another way to look at this is that our covariance matrix provides 4 constraints on 5 unknowns. We are doomed ...

In terms of cross spectra

$$\hat{Z} = \frac{E[\tilde{E}\tilde{B}^*]}{E[\tilde{B}\tilde{B}^*]}$$

there is correlated noise in the power spectrum of B. The remote reference technique was introduced (Gamble *et al.*, 1979) to reduce bias associated with noise in B by making a second measurement of the magnetic field, R, sufficiently far away (remote) that the noise sources are independent, relying on the fact that the magnetic source field is coherent over very large distances:

$$\tilde{R} = \tilde{b} + \eta$$

Now our random variable is

$$\mathbf{X} = \begin{bmatrix} \tilde{E} \\ \tilde{B} \\ \tilde{R} \end{bmatrix}$$

and our covariance matrix is

$$C = \begin{bmatrix} Z^2\sigma_b^2 + \sigma_\epsilon^2 & Z\sigma_b^2 & Z\sigma_b^2 \\ Z^*\sigma_b^2 & \sigma_b^2 + \sigma_\beta^2 & \sigma_b^2 \\ Z^*\sigma_b^2 & \sigma_b^2 & \sigma_b^2 + \sigma_\eta^2 \end{bmatrix}$$

which provides 9 constraints (three complex covariances and three variances) on 7 unknowns, more than soluble. Indeed, one could introduce a second impedance for the remote reference

$$\tilde{R} = Y\tilde{b} + \eta$$

which has a covariance matrix

$$C = \begin{bmatrix} Z^2\sigma_b^2 + \sigma_\epsilon^2 & Z\sigma_b^2 & ZY\sigma_b^2 \\ Z^*\sigma_b^2 & \sigma_b^2 + \sigma_\beta^2 & Y\sigma_b^2 \\ Z^*Y^*\sigma_b^2 & Y^*\sigma_b^2 & Y^2\sigma_b^2 + \sigma_\eta^2 \end{bmatrix}$$

and still solve the problem, because have increased the number of unknowns to 8, but the number of constraints remains at 9.

In terms of cross spectra

$$\hat{Z} = \frac{E[\tilde{E}\tilde{R}^*]}{E[\tilde{B}\tilde{R}^*]}$$

where with any luck $\tilde{B}\tilde{R}^*$ is now unbiased. Non-Gaussian noise can be handled by an iterative robust weighting W designed to reduce the effect of outliers

$$\hat{Z} = \frac{E[\tilde{E}W\tilde{R}^*]}{E[\tilde{B}W\tilde{R}^*]}$$

Both the effects of non-Gaussian noise and non-stationary source-field spectra can be handled by various robust estimation techniques (e.g. Egbert and Booker, 1986; Chave *et al.*, 1987).

Indeed, as you add uncorrelated measurements the number of constraints grows faster than the number of unknowns – every nth channel we add increases the covariance matrix to n^2 in size, but the number of unknowns just increases linearly. This was taken to extreme by the multivariate errors-in-variables approach of Egbert (1997), which uses a principle component analysis approach to multiple MT sites. Principle component analysis does an eigenvalue decomposition of a covariance matrix.

We make a data vector $\mathbf{X}_i$ of the Fourier components of the ith time segment of M segments by combining lots of channels (K of them) from of lots of sites J. For example, if we have two horizontal magnetic field channels and two horizontal electric field channels then $K = 4J$. Our model becomes

$$\mathbf{X}_i = \begin{bmatrix} \tilde{B}_{1i} \\ \tilde{E}_{2i} \\ \tilde{B}_{3i} \\ \tilde{E}_{4i} \\ \tilde{B}_{5i} \\ \vdots \\ \tilde{E}_{Ki} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ e_{21} & e_{22} \\ b_{31} & b_{32} \\ e_{41} & e_{42} \\ b_{51} & b_{52} \\ \vdots & \\ e_{K1} & e_{K2} \end{bmatrix} \begin{bmatrix} c_{1i} \\ c_{2i} \end{bmatrix} + \begin{bmatrix} \epsilon_{1i} \\ \epsilon_{2i} \\ \epsilon_{3i} \\ \epsilon_{4i} \\ \epsilon_{5i} \\ \vdots \\ \epsilon_{Ki} \end{bmatrix} = \mathbf{U}\mathbf{c}_i + \epsilon_i$$

Here the two columns of the $K \times 2$ complex matrix $\mathbf{U}$ are the ideal fields for linearly polarized NS and EW fields. These are what we want to find. The components of $\mathbf{c}_i$ define the polarizations of the source field for the ith data segment (that is, the linear combination of the ideal NS and EW fields that describes the field polarization during the ith data collection interval, and that can change for every interval), and ϵ_i is a vector of noise for the ith data vector. While we want to find $\mathbf{U}$, it turns out that can only do this up to a 2×2 multiplicative constant, because we can't separate the external source field polarization from the measured (internal + external) field. However, since we are only interested in ratios of electric to magnetic field, or possibly inter-station transfer functions, the undetermined constant doesn't matter. It turns out, (assuming we have a way to find $\mathbf{U}$), that the MT impedance for the jth site can be obtained by extracting the ideal electric and magnetic fields from the matrix $\mathbf{U}$ for that site:

$$\mathbf{Z}_j = \begin{bmatrix} ex_{j1} & ex_{j2} \\ ey_{j1} & ey_{j2} \end{bmatrix} \begin{bmatrix} bx_{j1} & bx_{j2} \\ by_{j1} & by_{j2} \end{bmatrix}^{-1} = \mathbf{U}_1 \mathbf{U}_2^{-1}$$

(here we have indexed the elements of $\mathbf{U}$ not by channel number, but to reflect the site number and component).

Our friend the covariance matrix is

$$\mathbf{S} = E[\mathbf{X}\mathbf{X}^*]$$

a $K \times K$ matrix which is also called the spectral density matrix since the elements of $\mathbf{X}$ are all Fourier estimates. We increase the reliability of $\mathbf{S}$ by averaging over the M estimates of the data sample.

If Σ_N is the covariance matrix of the noise in all channels, then an unbiased, and maximum-likelihood for Gaussian noise, estimate of $\mathbf{U}$ is obtained by solving the eigenvalue problem

$$\mathbf{S}\mathbf{u} = \lambda \Sigma_N \mathbf{u}$$

and taking the two largest eigenvectors as the columns of $\mathbf{U}$. But, we need to know Σ_N, which is rarely the case. However, we can approximate Σ_N by

$$\Sigma_N = \text{diag}(\epsilon_1^2 \, \epsilon_2^2 \, ... \, \epsilon_K^2)$$

which assumes the noise is uncorrelated between all channels. Egbert says that in this case $\mathbf{U}$ and Σ_N can both be solved in principle for $K \geq 5$, which is like a site with one remote reference channel, but he notes that this is optimistic. However, as we have already noted, our ability to solve the problem improves as K gets larger. Indeed, he shows that even if noise is correlated within the channels of one site, we can proceed as long as the noise is incoherent between sites and we process up a number of sites (in practice we find that 6 sites produces good results). He folds coherent noise into the model part of the vector equations, which means that there may be more than two eigenvalues:

$$\mathbf{X}_i = \mathbf{U}\mathbf{c}_i + \mathbf{V}\mathbf{b}_i + \epsilon_i$$

where $\mathbf{V}$ is a $K \times L$ matrix describing L sources of coherent noise with polarizations described by $\mathbf{b}$. The first step in the algorithm is an iterative method to estimate Σ_N. The second step is to compute an eigenvalue decomposition of the spectral density matrix scaled by the noise model:

$$\mathbf{S}' = \Sigma_N^{-\frac{1}{2}} \mathbf{S} \Sigma_N^{-\frac{1}{2}} = \mathbf{W} \Lambda \mathbf{W}^*$$

which gives a diagonal matrix of eigenvalues $\Lambda = \text{diag}(\lambda_1 ... \lambda_K)$ and a matrix $\mathbf{W}$ whose columns are the corresponding eigenvectors. The whole process is quite complicated, but the results work very well. In particular, the examination of the eigenvalue spectrum of $\mathbf{S}'$ provides very useful insight into the nature of signal, noise, and correlated noise. Where the MT plane wave solution works well, there will be two dominant eigenvalues. In the presence of coherent noise, or little signal to noise ratio, then $L > 0$ and the number of significant eigenvalues will be greater than 2.

Dimensionality and the impedance matrix

The nature of Z depends on the apparent dimensionality of the geology:

For 1D,

$$Z = \begin{bmatrix} 0 & Z_{xy} \\ -Z_{xy} & 0 \end{bmatrix}$$

that is, the diagonals are zero and the off-diagonals are equal and opposite. This captures the phase shift we saw when we derived the half-space equations in lecture 3.

For 2D,

$$Z = \begin{bmatrix} 0 & Z_{xy} \\ Z_{yx} & 0 \end{bmatrix}$$

that is the diagonals are still zero but the off-diagonals are different.

For 3D,

$$Z = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix}$$

That is, all elements are different and non-zero.

For 2D structure, $Z_{xx} = Z_{yy} = 0$ only when the coordinate system of the measurement is aligned to the 2D geometry of the geology. With a prior understanding of structural trends or geological strike, the experimenter will normally arrange for this to be approximately so. Otherwise, the tensor can be rotated mathematically to minimize Z_{xx} and Z_{yy}. Conventionally, the impedance with electric field along strike is called the transverse electric, or TE mode, and the impedance with across-strike electric fields is called the transverse magnetic, or TM mode.

Galvanic versus Inductive Effects

Until now we have only considered effects associated with electromagnetic induction. Even in the DC resistivity case we considered only geometric spreading in a homogeneous half-space. However, when we have an electric field component perpendicular to a contrast in conductivity, we have to consider conservation of charge, which basically says that the normal component of current density is continuous. From Ohm's Law, we observe that the normal component of the electric field must jump:

$$J_1 = J_2 \rightarrow \sigma_1 E_1 = \sigma_2 E_2 \quad .$$

Such a change in electric field is often called a "galvanic" effect. Note that there is no change in the phase of E across the conductivity boundary.

For a layered earth, magnetotelluric fields have no vertical component and so the behavior of the fields is purely inductive. However, for 2D and 3D conductivity structure, there will be lateral conductivity contrasts

which will generate galvanic effects. If these structures are spatially large, they can be included in the modeling and hopefully resolved in some sense. If they are small, it may be both a problem to include them in the model, and it will certainly be a problem resolving them. Lateral contrasts in conductivity can occur near the surface and have length scales smaller than the dimensions of the electric dipoles. These result in electric field measurements that are larger or smaller than would be observed over a (say) layered structure, and this bias then propagates into the magnitude of the apparent resistivity. Note that it does not necessarily affect the phase. In MT such a phenomenon is often called a "static shift", and in DC resistivity they are called "electrode effects". Various heuristic methods are used to correct the data, relying on the fact that such distortions tend to be proportional in electric field amplitude.

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