SIOG 231: GEOMAGNETISM AND ELECTROMAGNETISM

Chapter 8: Earth's Geomagnetic Environment.

Introduction

Earth's electromagnetic environment arises from complex interactions between the internally-generated main magnetic field and the solar wind, the heating of the electrically conductive ionosphere, electric fields generated by lightning in the atmosphere, and by man-made sources of electromagnetic energy. We tried to summarize all these in a picture for a book chapter that we wrote some time ago:

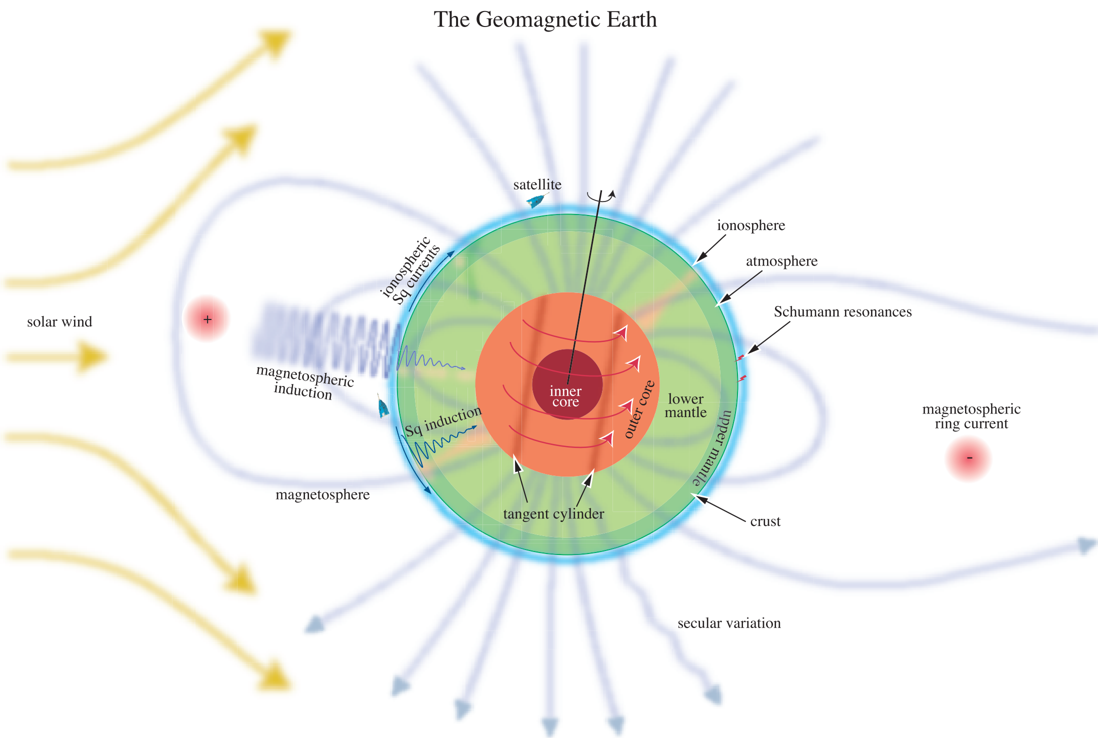


Figure 1. The electromagnetic environment of Earth (from Constable and Constable, 2004).

There is also a crustal magnetic field frozen into the rocks, which doesn't change with time (at least not on any time scale that we care about for the geophysical application of EM methods), but is spatially complex enough that for magnetic satellites orbiting Earth there is a time-varying magnetic field from the satellite's perspective.

Figure 2 below shows a (somewhat stylized) spectrum of the geomagnetic field from reversal periods to low radio frequencies. It approximates a $1/f$ spectrum over 20 decades of frequency (in this case $1/f^2$, since this is an amplitude spectrum). The upturn at the highest frequency is probably a result of radio noise leaking across the spectrum. At periods longer than one year, the energy is dominated by secular variation in Earth's internally-generated magnetic field. At periods shorter than several days the energy is from external variations in the magnetic field driven by the solar wind, rotation of Earth, lightning, and radio. All of these external variations can be used to probe Earth conductivity. Between about one month and one year, field variations are mixed and sometimes difficult to characterize. We are currently working on teasing out the EM response of the 11-year sunspot cycle, and hope to update this figure soon.

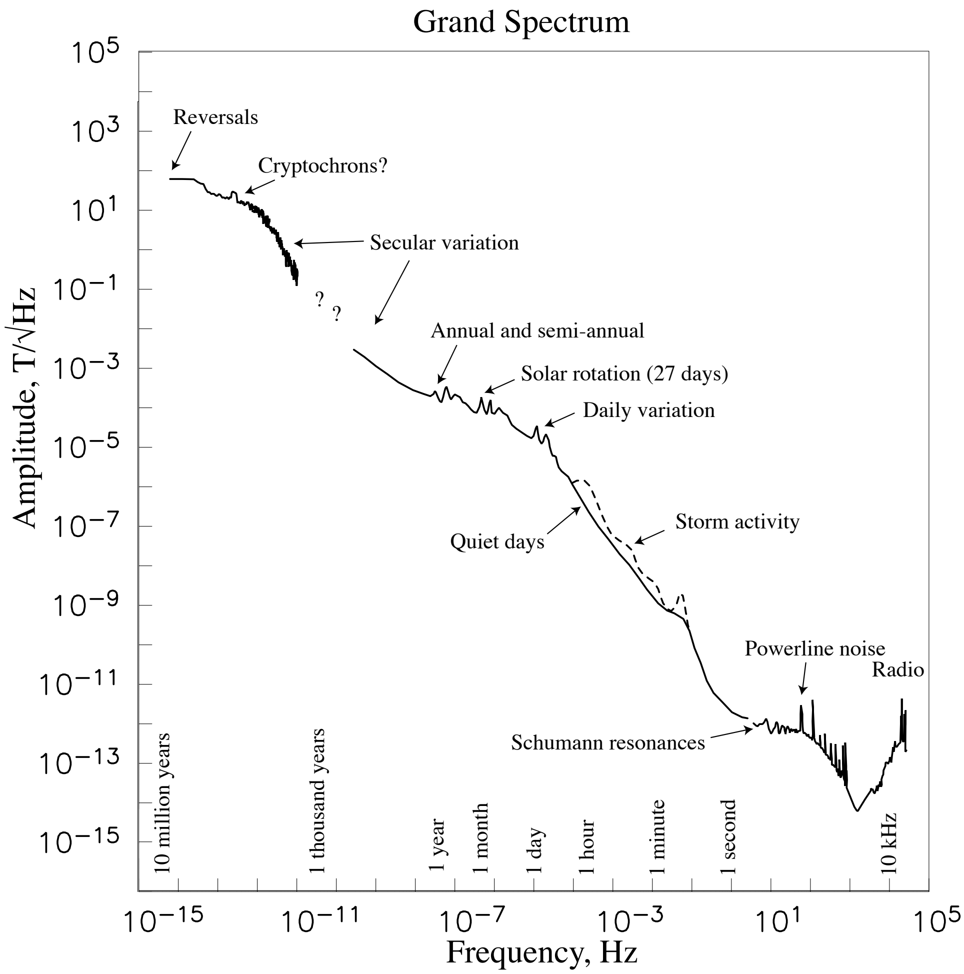


Figure 2. An approximate amplitude spectrum for Earth's magnetic field (to get a power spectrum, double the exponents and change the units to T^2/Hz) (from Constable and Constable, 2004).

The Magnetosphere

The Earth's magnetosphere is a consequence of the interaction between Earth's magnetic field, formed by the geodynamo in the core, and the solar wind, a stream of plasma (neutral ionized gas) emitted constantly, but with significant variability, by the sun. The solar wind is primarily electrons and protons, traveling at a speed of 300–1000 km/s, with energies of 1–10 keV. The total number of particles in the solar wind is around 10^{36} per second, or 1.3 – 1.9×10^9 kg/s. There is an interplanetary magnetic field embedded in the solar wind, amounting to about 5 nT at Earth, reversing along with the Sun's field every 11 years or so during the solar cycle. At Earth distance, the solar wind has a particle density of about 6×10^6 m^{-3}, moving at a speed of about 450 km/s on average. Like all plasmas, the solar wind is electrically conductive (about 10^4 S/m according to Campbell in his 1997 book). When the ionized particles hit Earth's magnetic field they will be subject to a $\mathbf{v} \times \mathbf{B}$ force, which deflects and excludes them. On the sun-side, the interaction forms a shock front called the bow shock (the solar wind is supersonic). On the night-side, the interaction is streamed out into the magnetotail. Thus, the magnetosphere can be thought of as a bubble in the solar plasma.

We can make a first-order estimate for where the boundary of the solar wind and Earth's magnetic field lies by balancing the pressure of the two systems. The pressure in the solar wind is akin to that of a gas, which in kinematic gas theory is the kinetic energy density (which has the right units, and is intuitively sensible if you think about gas molecules exerting pressure on a surface). The kinetic energy is mainly carried by the heavier protons of mass m_p moving with velocity v.

$$P_{KE} = \frac{1}{2}\rho v^2 = \frac{1}{2} N m_p v^2 \approx 1 \times 10^{-9} \text{ Pa}$$

where ρ is just the mass density and N is the particle density. Using the numbers quoted above, this is about 10^{-9} Pa at Earth.

The pressure of the magnetic field is also its energy density. How to get this? The energy density is closely related to inductance, and we can derive it from Faraday's Law for the magnitude of the EMF ε of a coil with N turns:

$$\varepsilon = N\frac{d\Phi_B}{dt} \quad .$$

The total number of flux linkages, $N\Phi_B$, depends on the current in the coil i:

$$N\Phi_B = Li$$

where the constant of proportionality L is the inductance (analogous to the capacitance). Thus the EMF is given by

$$\varepsilon = L\frac{di}{dt} \quad .$$

This is the definition of the inductance of a coil. Now if we multiply both sides by the current i we get

$$\varepsilon i = Li\frac{di}{dt} \quad .$$

We have power on the left hand side (energy per unit time) and so the right hand side is the rate of change of energy in the magnetic field U_B:

$$\frac{dU_B}{dt} = Li\frac{di}{dt} \quad .$$

We can integrate this to get

$$U_B = \frac{1}{2}Li^2$$

which is the stored magnetic energy in an inductance L carrying a current i. To go from this to energy density, we consider a solenoid with n turns per unit length. The inductance of a solenoid is given by

$$L = \frac{N\Phi_B}{i} = \mu_o n^2 l A$$

where A is the area and l is the length of the solenoid, and the magnetic field inside the solenoid given by

$$B = \mu_o i n \quad .$$

Because the magnetic field is uniform inside a solenoid, the energy density over a length l of the solenoid is

$$u_B = \frac{U_B}{Al} = \frac{\frac{1}{2}Li^2}{Al}$$

into which we can substitute for L and replace $(\mu_o ni)^2$ with B^2 to finally get

$$u_B = \frac{1}{2}\frac{B^2}{\mu_o}$$

(this is analogous to the energy density of an electric field in a vacuum: $u_E = 1/2\epsilon_o E^2$).

Equating this with the solar wind pressure obtained above we have

$$B^2 = \mu_o N m_p v^2$$

which is about 50 nT. However, this includes the effect of an image current on the solar wind side of the bow shock which serves to exclude Earth's magnetic field, and creates a field equal to Earth's field just inside the bow shock. So the component of this 50 nT due to Earth is only half, or 25 nT. Since Earth's field is dipolar and falls off with R^3, and has a value of about 30,000 nT at the equator, we have

$$(a/R)^3 = 30,000/25 = 1200$$

which reduces to about 11 Earth radii a, which is in good agreement with sun-side observations.

The Magnetospheric Ring Current.

The largest component of external field variations comes from a ring of current circulating westward at a distance of 3-9 Earth radii. The Van Allen radiation belts occupy this zone, named after James Van Allen who discovered them using data from Geiger counters flown on the first satellites. There is an inner belt, at 1–2 Earth radii, populated by low energy positive ions probably generated by cosmic rays interacting with the upper atmosphere, but this belt is not very dynamic. It does, however provide the outer belt with O^+ ions during magnetic storms. The outer belt, at 3–9 radii, is populated mostly by protons with energies of 10–200 keV that enter the magnetosphere from the solar wind, at the poles where the field geometry does not produce a lot of deflection, and the night-time magnetotail.

The solar wind is very dynamic, and variations in the wind produce variations in Earth's magnetic field and the density of particles in the radiation belts. The ultimate solar wind event is a coronal mass ejection (CME), in which around 10^{12} kg of particles are ejected with velocities up to 3,000 km/s. Coronal mass ejections are somewhat focussed in space, and if they intersect Earth's orbit they increase the number of particles in the radiation belts dramatically and produce what we call magnetic storms.

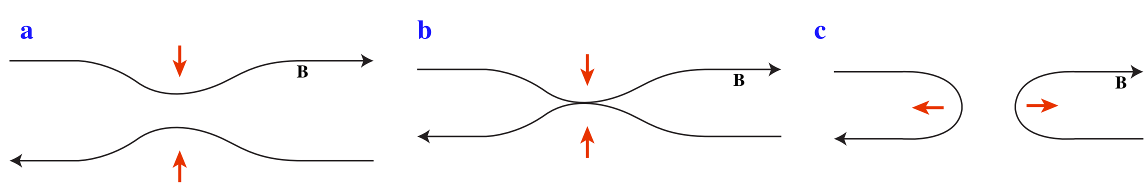


Figure 3. Cartoon of magnetic reconnection in the magnetotail. a: Coronal mass ejection from the solar wind puts pressure on the magnetopause and pushes the magnetic field lines together. b: The magnetic field lines reconnect. c: Kinetic energy is released as the lines of magnetic force spring away from the reconnection, injecting particles into the magnetosphere.

Both a CME and the resulting injection of particles into the magnetosphere depend on magnetic reconnection (Figure 3), in which magnetic energy is transformed into kinetic energy. In a solar prominence, lines of magnetic field extend from the sun in a loop, trapping the plasma along the field lines. If the magnetic field becomes unstable, and the sides of the loop come together and reconnect, the outer part of the loop springs away, carrying the plasma with it. If this CME reaches Earth, it puts additional pressure on the magnetosphere, and can push the sides of the magnetotail together. If the magnetic field lines in the magnetotail reconnect, the Earthwards loop of field that is formed springs towards Earth, carrying solar wind plasma with it and into the ring current.

Once inside the magnetosphere, the charged particles of the radiation belts are trapped to some extent by Earth's magnetic field, and move westwards through a mechanism called magnetic drift. To understand this we need to consider the nature of charged particles moving in a plasma. A charged particle moving with velocity **v** in a magnetic field **B** will be acted on by the Lorentz force

$$\mathbf{f} = q(\mathbf{v} \times \mathbf{B})$$

where we assume the particle has a charge of q. Since force is mass times acceleration ($f = ma$) the acceleration of the particle is

$$\frac{d}{dt}\mathbf{v} = \frac{q}{m}\mathbf{v} \times \mathbf{B}$$

If **v** is perpendicular to **B** then the particle moves in a circular path with frequency ω, and we can replace **v** with angular velocity and acceleration with angular acceleration:

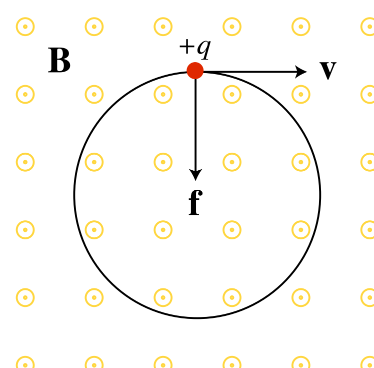
$$|\mathbf{v}| = r\omega \qquad \text{and} \qquad \left|\frac{d\mathbf{v}}{dt}\right| = r\omega^2$$

so

$$r\omega^2 = \frac{q}{m} r\omega|\mathbf{B}|$$

from which we can derive the cyclotron frequency (or gyrofrequency) as

$$\omega = \frac{q}{m}|\mathbf{B}|$$



and the radius of the circular motion as

$$r = \frac{m}{q}\frac{|\mathbf{v}|}{|\mathbf{B}|} \quad .$$

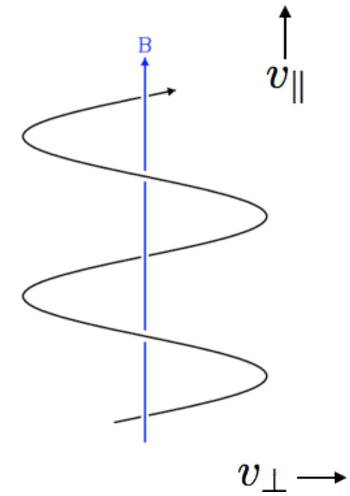
Another way of obtaining the radius is by balancing the centrifugal force with the Lorentz force:

$$\frac{mv^2}{r} = qvB$$

$$r = \frac{mv}{qB} \quad .$$

If there is only a component of velocity $v_\perp$ perpendicular to the direction of **B**, then the charged particle simply orbits in a circle. However, if there is a component of velocity $v_\|$ along the lines of the magnetic field, then there is no force on this component and the particle just keeps traveling along the field line with constant velocity. The result is a spiral motion around the field line.

And so it is for constant field strength. But, as the particles in the radiation belts spiral along the magnetic field lines towards the poles, the field gets stronger both because a dipole field is twice the size at the poles than the equator, and also the field lines dip into Earth. This results in *magnetic mirroring,* and the particles get reflected back along the field lines, bouncing back and forth between the polar mirror points and trapped in the magnetic field.



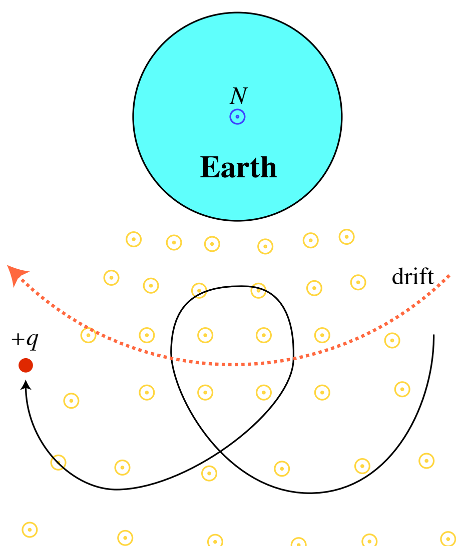
How does this work? The magnetic moment u of the particle is an adiabatic invariant and is given by $v_{\perp}$:

$$u = \frac{p_{\perp}^2}{2mB} = \frac{mv_{\perp}^2}{2B}$$

where $p_{\perp}^2$ is momentum perpendicular to the magnetic field. The total kinetic energy is given by the sum of both velocities

$$\varepsilon = \frac{1}{2}mv_{\parallel}^2 + \frac{1}{2}mv_{\perp}^2 \quad .$$

Both must be conserved. As B increases as the particles travel down the field lines towards the poles, $v_{\perp}^2$ must increase to conserve magnetic moment, or, equivalently, r gets smaller and so $v_{\perp}$ must increase to conserve angular momentum. As $v_{\perp}^2$ increases, $v_{\parallel}^2$ must decrease to conserve energy. At some point $v_{\parallel}^2$ must go negative, which is impossible because velocity cannot become imaginary, so the particle is excluded and $v_{\parallel}$ reverses.



Finally we get to the *geomagnetic ring current.* Consider a particle thus spiraling around a magnetic field line near the equator. The main magnetic field, originating inside the core, has a radial gradient and is larger closer to Earth. The radius of curvature of a particle will thus be smaller closer to Earth. Looking down from the North pole, a positive particle will circle clockwise and drift westward; an electron will do the opposite, and together they make a westward directed current that circles Earth between 3 and 9 Earth radii. This is magnetic drift.

The ring current is also sustained by a Hall current. If we consider also electric fields:

$$e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = \mathbf{f} = ma$$

when $\mathbf{E}$ is perpendicular to $\mathbf{B}$ we get a component of acceleration perpendicular to both fields, which for a radial electric field has a westward component also given by $v = -E/B$.

The ring current operates all the time, and if you think carefully, you will see that a westward current opposes Earth's main field. Sudden increases in the solar wind results in more charged particles being injected into the ring current, along with elevation of positively charged oxygen ions into the current from the lower radiation belts. This causes magnetic storms. A magnetic storm is characterized by a sudden commencement, a small but sharp increase in the magnetic field, associated with the sudden increased pressure of the solar wind on

Earth's magnetosphere. Following the commencement is a period of fluctuating magnetic field called the initial phase. The main phase of the storm is associated with a large decrease in the magnitude of the fields as the ring current is energized – the effect of the ring current is to cancel Earth's main dipole field slightly. Finally, there is a recovery phase in which the field returns quasi-exponentially back to normal. All this can happen in a couple of hours, or may last days for a large storm.

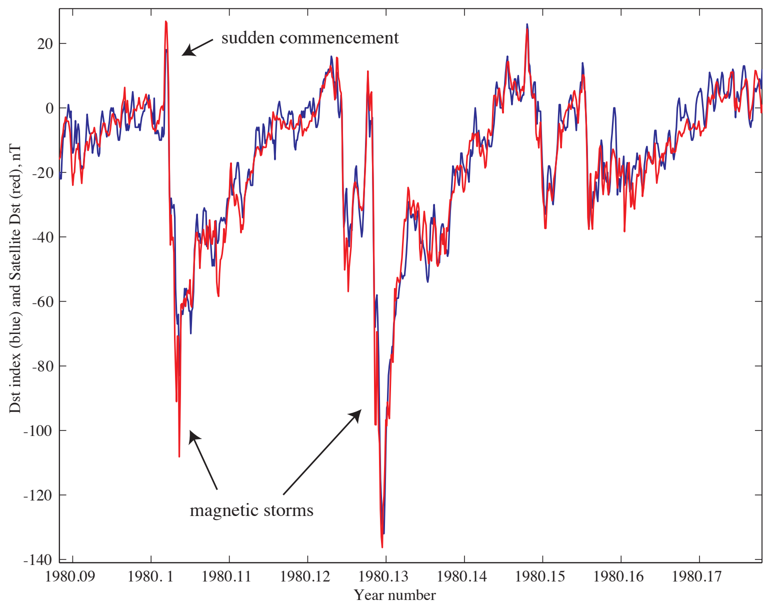


Figure 4. Dst index in red, and ring current magnetic field derived from Magsat data (blue) for one month in 1980. Two large storms can be seen, and the sudden commencement is very clear on the first one.

Acting like a huge single turn of wire around Earth, the ring current creates fields at Earth's surface of predominantly simple P_1^0 geometry. The frequency content is huge – from minute-by-minute fluctuations during a storm, the hours-long duration of a storm, a large peak at the 27-day rotation period of the sun (and harmonics), a semi-annual line associated with the geometry of the ecliptic and the sun's equatorial plane, and finally the 11-year solar cycle.

The strength of the ring current is characterized by the 'Dst' (disturbance storm time) index, an index of magnetic activity derived from a network of low to mid-latitude geomagnetic observatories that measures the intensity of the globally symmetrical part of the equatorial ring current. The current method for computing Dst is described in IAGA Bulletin No. 40, a report by Masahisa Sugiura which presents the values of the equatorial Dst index for 1957-1986. This can be found on the web at http://wdc.kugi.kyoto-u.ac.jp/dstdir/dst2/onDstindex.html. Honolulu, Hermanus, San Juan and Kakioka are the current contributing observatories. The actual morphology of the ring current fields is asymmetric about the day/night hemispheres, as injection of ions occurs preferentially as a function of solar time. Figure 4 above shows a month of the Dst index (red) along with an estimate of the ring current magnetic field (blue) from Magsat, a magnetic satellite that was operating at this time. Two large storms can be seen, and the sudden commencement is very clear on the first one.

Figure 5 shows a record of the Dst index (blue) for nearly l6 decades. The large negative spikes are individual magnetic storms. To illustrate the relationship between storm activity and the 11-year sunspot cycle, the

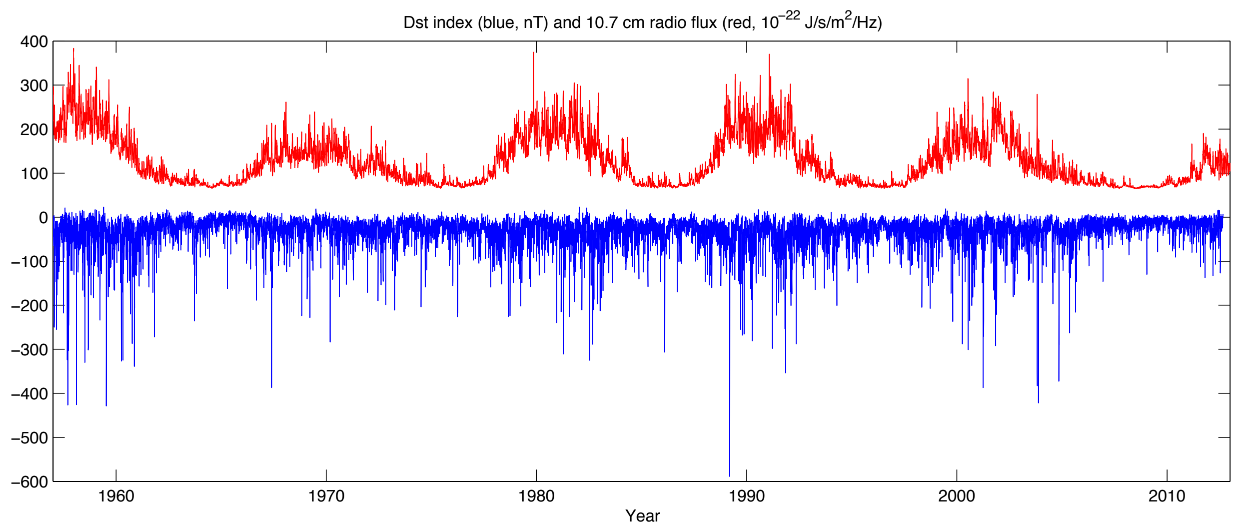


Figure 5. Dst index (blue) and solar activity measured as 10.7 cm radio flux (red).

10.7 cm solar radio flux is plotted in red (from Constable, 2007). The large magnetic storm in 1989 produced a Dst of -600 nT, and the induced currents shut down the Quebec power grid. A large coronal mass ejection, known as the Carrington event, hit Earth in 1859, and field strengths were estimated to be between 800 and 1700 nT. Such a storm today would do widespread damage to the global power grid.

Earth/ionosphere cavity.

The conductive ionosphere and conductive earth are separated by the resistive atmosphere to form a cavity. The electrical conductivity of the ground is of order 10^{-3} S/m, while the atmosphere near Earth's surface is about 10^{-14} S/m. What little conductivity there is results from ionization of oxygen and nitrogen by cosmic rays, whose flux increases with altitude. The mean free path of these particles also increases with decreasing atmospheric density, and by an altitude of 100 km, the start of the ionosphere, conductivity is the about same as that of the solid earth. Figure 6 below shows a profile of atmospheric conductivity.

The atmospheric cavity is excited by lightning strikes, and resonates with a characteristic frequency of around 8 Hz and harmonics (the circumference of Earth is 40,000 km, which if divided into the speed of light gives 7.5 Hz). This resonance is called the Schumann resonance, and this and individual lightning can be observed using induction coil magnetometers. The cavity is also excited by power-line noise (60 Hz in the USA and a few Asian countries, and 50 Hz elsewhere) which becomes ubiquitous in field measurements. Figure 7 shows a detail of the high frequency end of the grand spectrum, collected by Tom Nielson near Clark Dry Lake in the Anza-Borrego desert. Up to 7 harmonics of the 8 Hz Schumann resonance can be seen, as well as power line noise at 60 Hz and harmonics. The highest frequency lines are thought to be ULF mine-site communication signals. Also shown is an individual lightning strike recorded simultaneously in Germany, California, and Australia (from Fullekrug and Constable, 2000).

Daily variation and S_q.

The dayside ionosphere is heated as it passes beneath the sun, generating thermal tides which create strong winds of ionospheric particles. This produces a $\mathbf{v} \times \mathbf{B}$ EMF and generates electric currents flowing in the

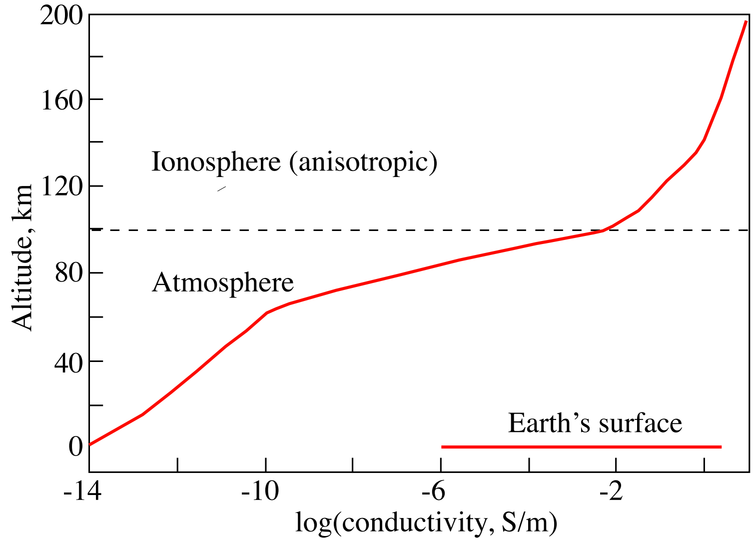


Figure 6. Electrical conductivity of the atmosphere and ionosphere. Redrawn from Bering, Few, and Benbook, 1998, October, Physics Today.

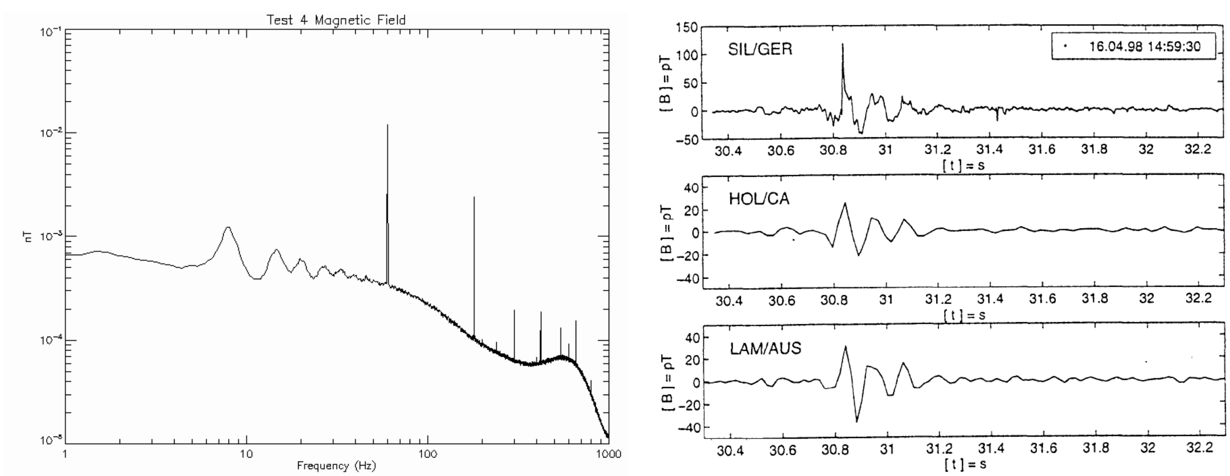


Figure 7. A high frequency spectrum of the magnetic field (left) and time series showing lightning strikes (right).

dayside ionosphere which have a pattern of two circulating current systems (one in each hemisphere, 10^4 amps or more) that are stationary in solar time and quasi-symmetric accross the magnetic equator, where the vertical component of **B** changes sign. Since Earth rotates beneath these current systems, a daily variation in magnetic field is seen at the surface of Earth, amounting to a few 10's of nT at mid-latitudes. This daily variation in the magnetic field is called S_q, for "solar quiet", because it is best seen in the absence of magnetic storms and other solar disturbances of the external magnetic field.

The currents a far weaker on the night side because the conductivity is lower and there is no thermal wind. The dayside ionosphere, ionized by radiation from the sun, is more conductive than the nightside.

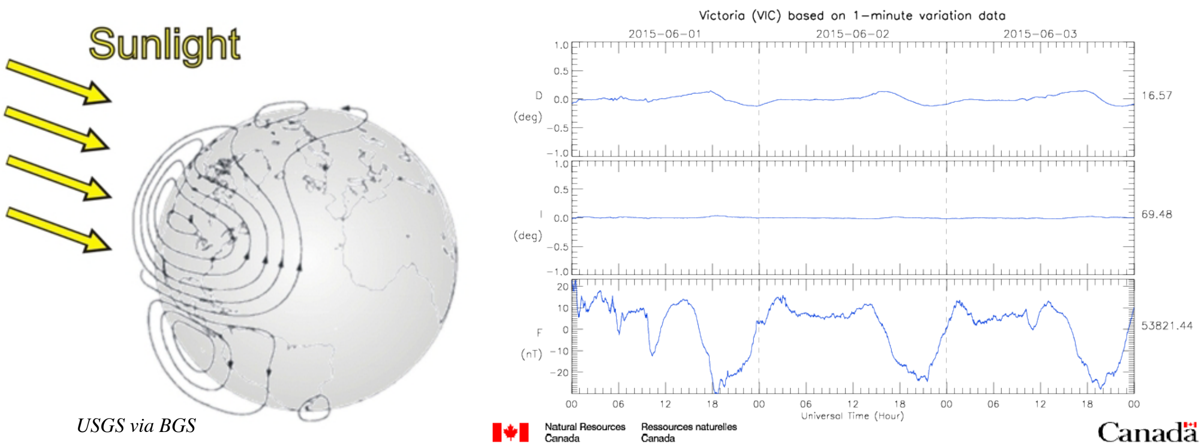


Figure 8. Left: the ionospheric S_q current system. Right: Three days of data from the VIC observatory.

The electrical conductivity of the ionosphere is anisotropic because there are different forces on charged particles depending on the strengths and directions of the electric and magnetic fields. Ionospheric conductivity is described by an anisotropic version of Ohm's Law:

$$\mathbf{J} = \begin{bmatrix} \sigma_P & \sigma_H & 0 \\ -\sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{||} \end{bmatrix} (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Here we have assumed that the magnetic field is in the z direction. The three conductivities are the parallel conductivity $\sigma_{||}$, the Hall conductivity σ_H, and the Peterson conductivity σ_P.

If the electric field is parallel to the magnetic field, charged particles moving in the direction of the electric field will experience no force from the magnetic field. The conductivity will be determined by the particle density N, the charge on the particle q, which we take to be the electronic charge, the mass of the particle m and the collision frequency ν. Both electrons e and protons p contribute to the current, and we have a parallel conductivity given by

$$\sigma_{||} = Nq^2 \left(\frac{1}{\nu_p m_p} + \frac{1}{\nu_e m_e} \right) \quad .$$

The drift velocities are opposite for electrons and protons, so both contribute to conventional current.

When the electric field is perpendicular to $\mathbf{B}$, charged particles traveling in the direction of the electric field will experience a Lorentz force and try to circulate around magnetic field lines, although collisions may thwart this. In the direction of the electric field, the conductivity is given by the Pedersen conductivity

$$\sigma_P = Nq^2 \left(\frac{\nu_p / m_p}{\nu_p^2 + \omega_p^2} + \frac{\nu_e / m_e}{\nu_e^2 + \omega_e^2} \right)$$

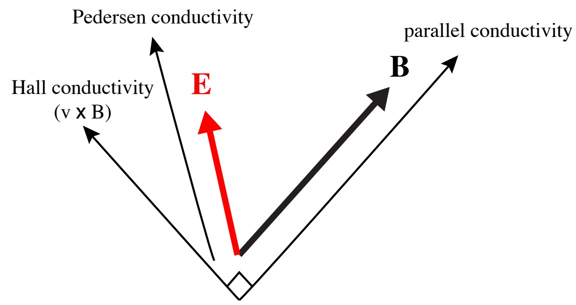
which depends both on collision frequency and cyclotron frequency ω. Again, the drift is opposite for protons and electrons.

To circle around the magnetic field, particles must also have a velocity in the direction perpendicular to both fields, and now the conductivity is called the Hall conductivity

$$\sigma_H = Nq^2\left(\frac{\omega_p/m_p}{\nu_p^2+\omega_p^2}+\frac{\omega_e/m_e}{\nu_e^2+\omega_e^2}\right)$$

which again depends on cyclotron frequency. However, here the sign of ω matters. Because both the charge and ω change sign, the particles all travel in the same direction, moving the plasma in a Hall drift.

So we see that conductivity depends on collision frequencies and cyclotron frequencies. Because the gyrofrequency depends on mass, conductivities are different for the proton and electron. If the collision frequency is much larger than the gyrofrequency ($\nu \gg \omega$) the Pedersen conductivity reduces to the parallel conductivity and the Hall conductivity decreases as ν^{-2}. In the magnetosphere, where particle collisions are rare, parallel and Pedersen conductivity is very high, but there is no net Hall current because both charges are drifting in the same direction.



As mentioned above, it is easier for charged particles to flow along the lines of **B**, but for most of Earth, there is a vertical component of **B** and so motion is inhibited by the boundary of the insulating atmosphere. At the magnetic equator the magnetic field is horizontal and northwards, with associated vertical and eastwards Hall currents. The vertical Hall current results in a vertical polarization of the ionosphere. The S_q current systems described above creates a large eastwards electric field with an associated Pedersen current, which is reinforced by an eastward Hall current. This is called the equatorial electrojet, with a peak current strength of around 100 kA, which creates an approximately 500 km wide belt of magnetic field variations which are roughly twice that of the normal daily variation.

The Global Electric Circuit.

Measurements of the vertical potential gradient at Earth's surface vary between slightly negative and several hundred volts positive upwards. The variations depend very much on local weather, atmospheric pollution, radon content, etc., but stable measurements can be made over the oceans during good weather. This was first observed in a systematic way from the research vessel Carnegie between 1909 and 1929 (at which time the vessel was destroyed by fire). Measurements at altitude can be made from balloons. These measurements show that the ionosphere is several hundred kV positive with respect to ground, and a leakage current of about 1000 A flows between the ionosphere and ground through an integrated transverse resistance of about 200 Ω. This leakage current supports the vertical electric field of 100 to 300 V/m at the surface. The Earth/atmosphere/ionosphere thus acts as one big capacitor with a value of about 1 farad and a time constant of a few thousand seconds.

Because of this leakage, there must be a return current sustaining the charge on the ionosphere. The major contributor to the return current is thunderstorm activity. During ice formation lighter particles become positively charged with respect to heavier water/slush particles, and upward convection currents carry the light, positively charged particles to the tops of thunderclouds. Cloud-to-ground negative charge flow in lightning completes the circuit, along with cloud to ionosphere discharges called "sprites" (only observed since 1989). Since thunderstorms preferentially form in the tropics, over land, in the afternoon, there is

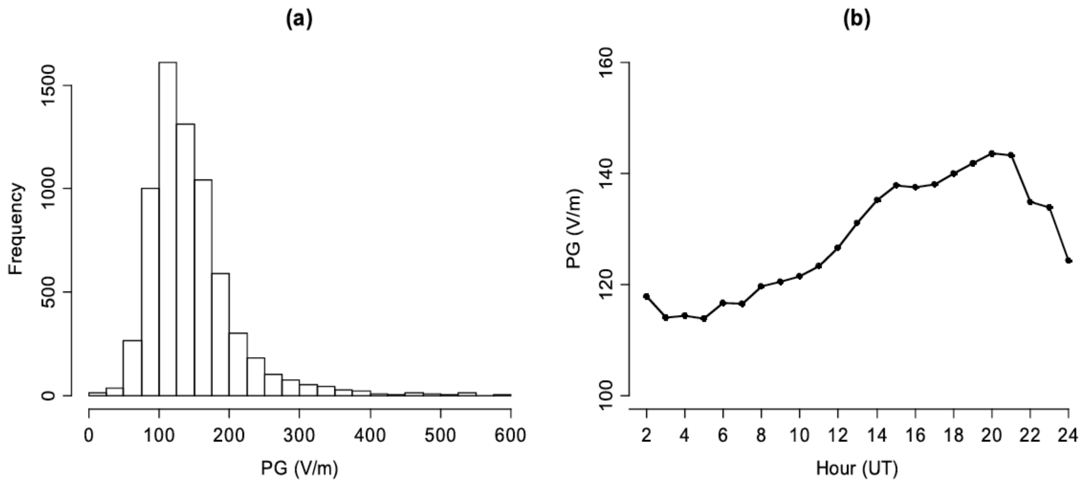


Figure 9. Left: Histogram of electric field data collected by the Carnegie between 1915 and 1929. Right: Means of the data collected during days of at least 12 hours fair weather, with data > 200 V/m excluded. From Rycroft *et al.* (2008).

an uneven distribution of lightning activity as Earth rotates the land mass of equatorial Africa though the afternoon local time, or about 16:00–18:00 UT. This recharging of the global capacitor was seen as an increased fair weather electric field at these times in the data collected by the Carnegie, and this is now called the Carnegie curve (Figure 9).

Figure 8 summarizes the global electrical circuit. C.T.R. Wilson, a Scottish physicist and meteorologist working at Cambridge, developed the first instruments for measuring atmospheric potential and developed the concept of the global electric circuit, and the electric currents that sustain charge on the global capacitor bear his name.

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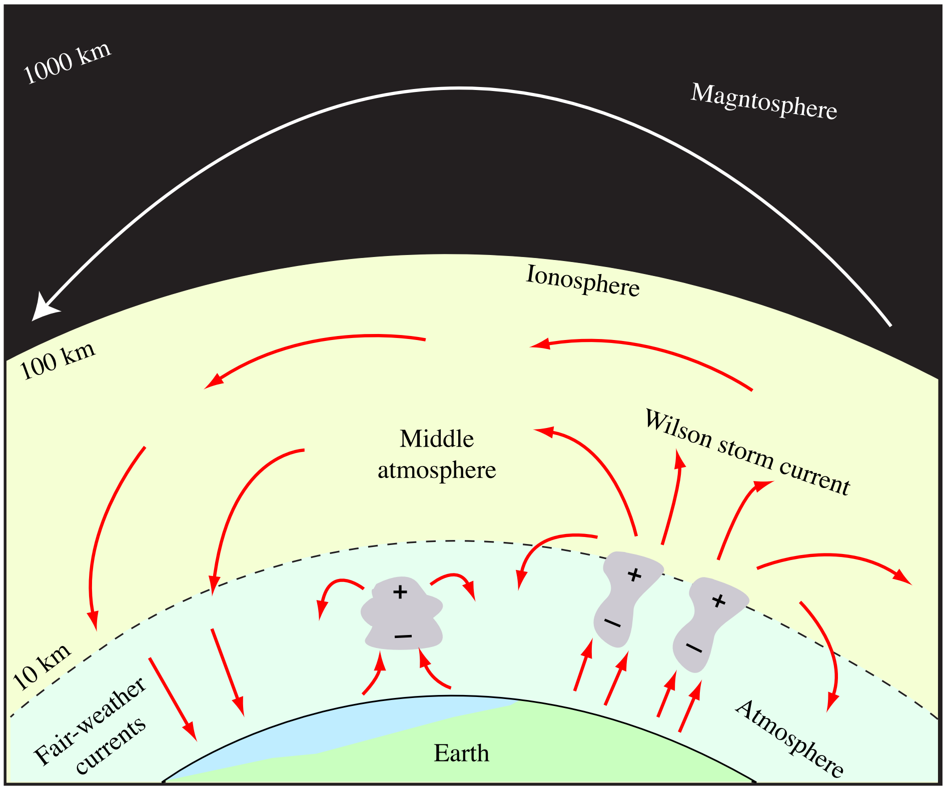

Figure 8. Current paths through the atmosphere and ionosphere. Redrawn from Bering, Few, and Benbook, 1998, October, Physics Today.