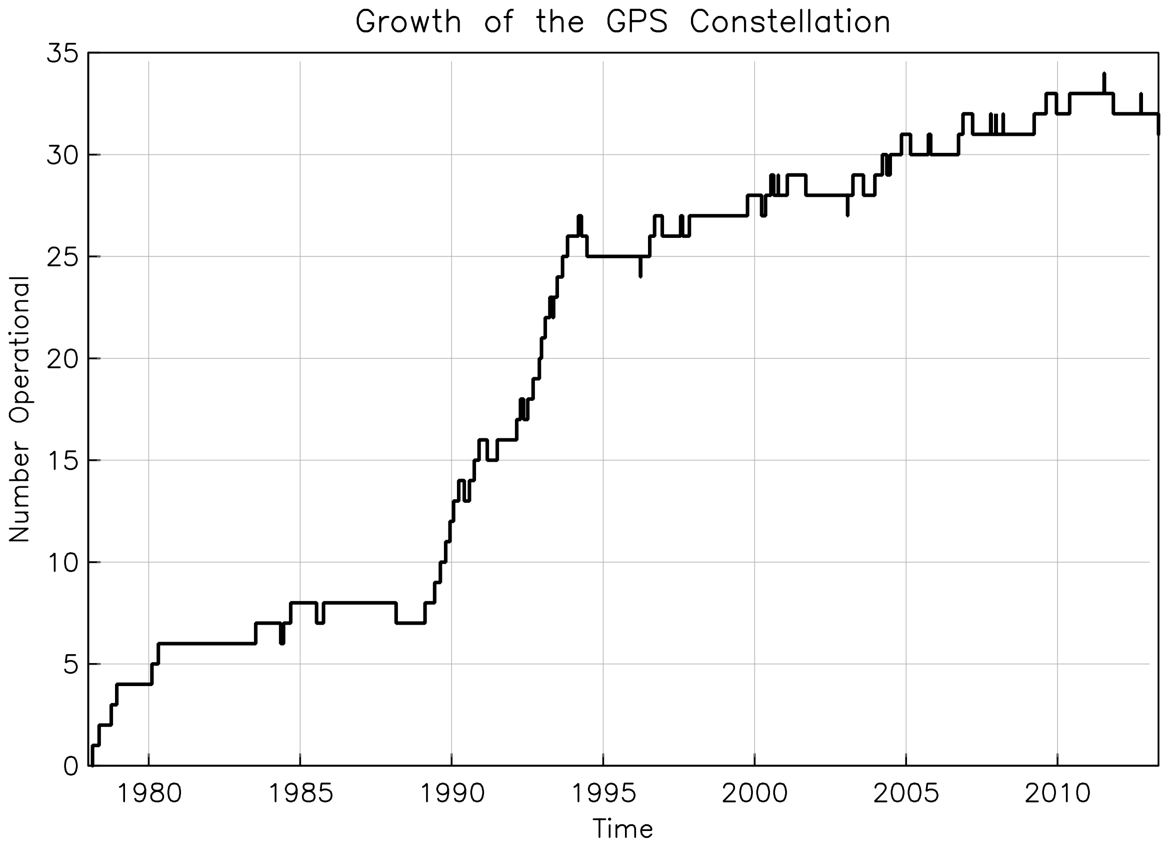
GPS: History, Operation, Processing

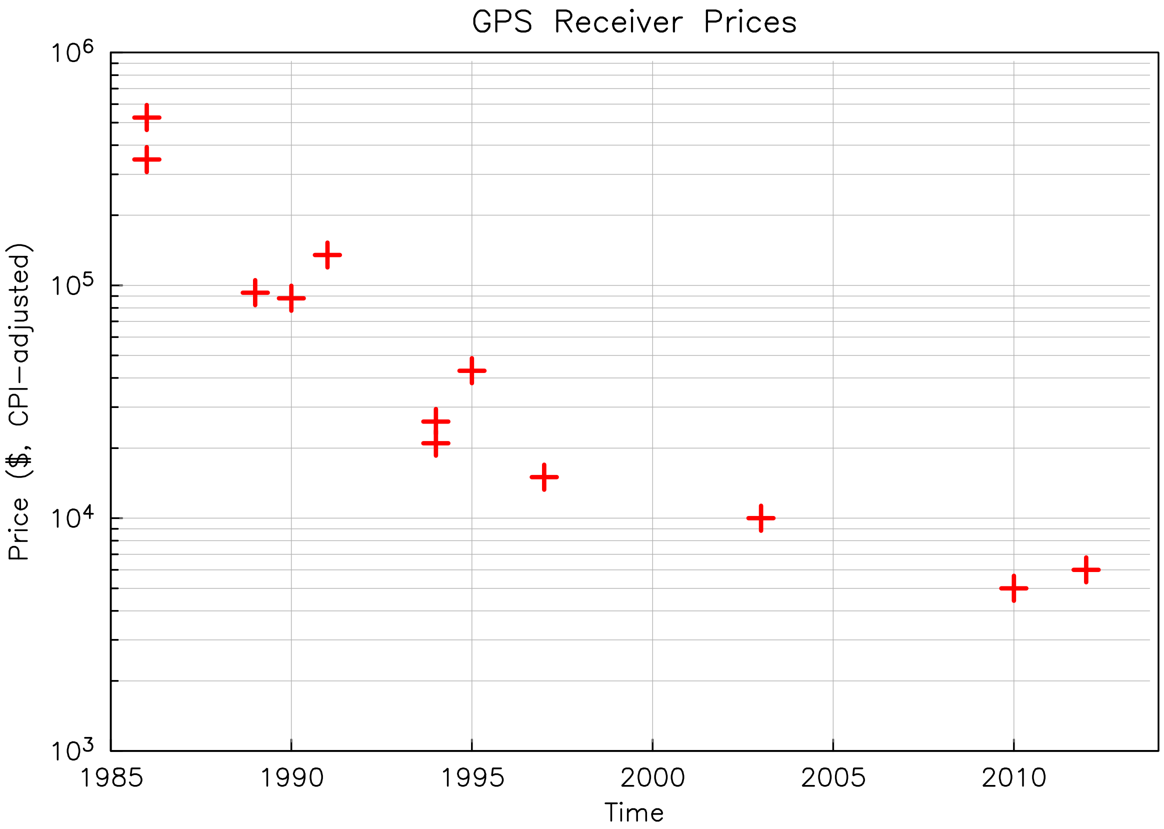
Important Dates

- **1970’s:** conceived as radionavigation system for the US military: real-time locations with few-meter accuracy.
- **1978:** first satellite launched
- **1983:** Declared to be a “dual-use” system, civilian and military.
- **Early 1980’s:** Academics realized it could be used for geodesy.
- **1986:** Initial measurements by academic institutions; start of UNAVCO.
- **Mid-1990’s:** Many more satellites; receiver costs drop substantially.
- **1994:** System becomes “operational”; International GPS Service produces high-quality orbits.
- **2000:** Selective Availability turned off – much better civil navigation.

GPS History: Celestial



GPS History: Mundane



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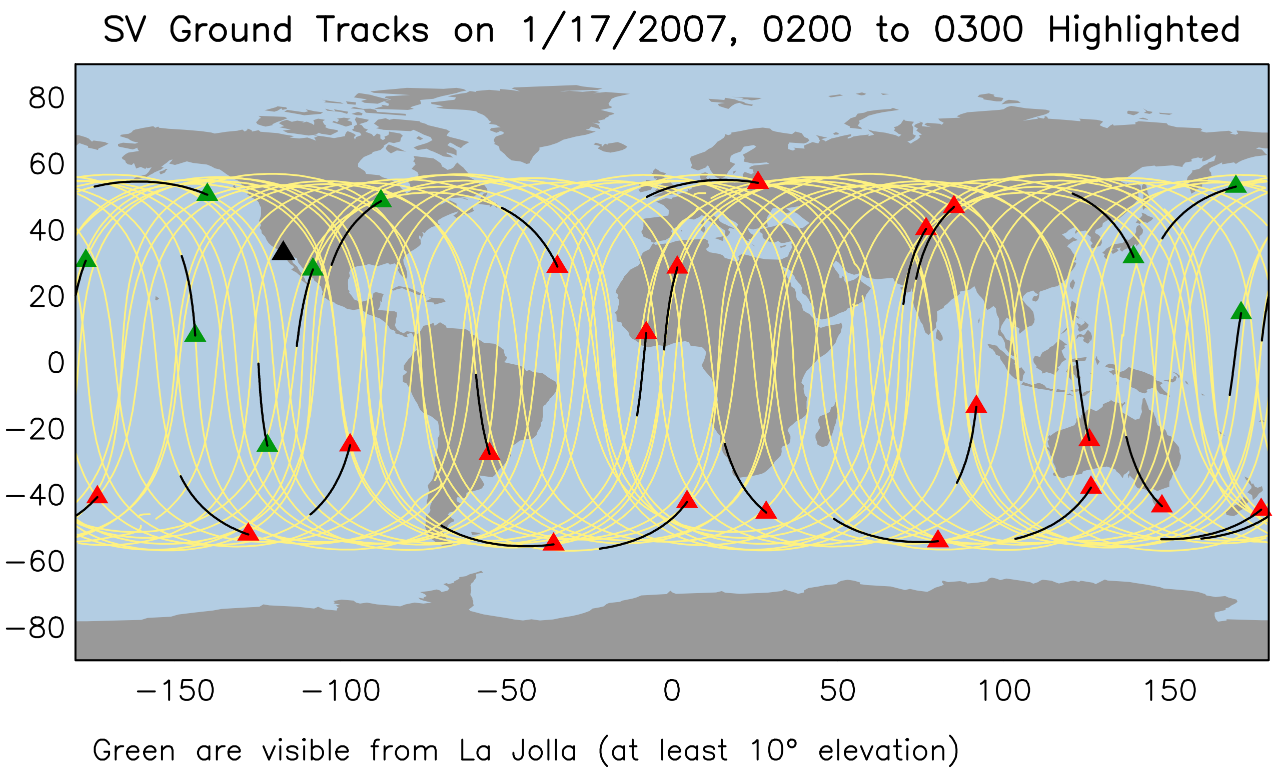
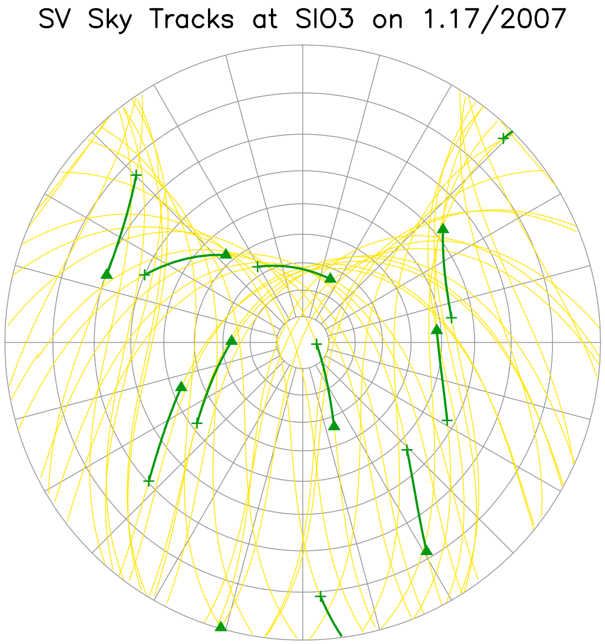
The processing packages all retain orbit determination, but we can usually assume that someone else (eg SOPAC) has found a precise orbit.

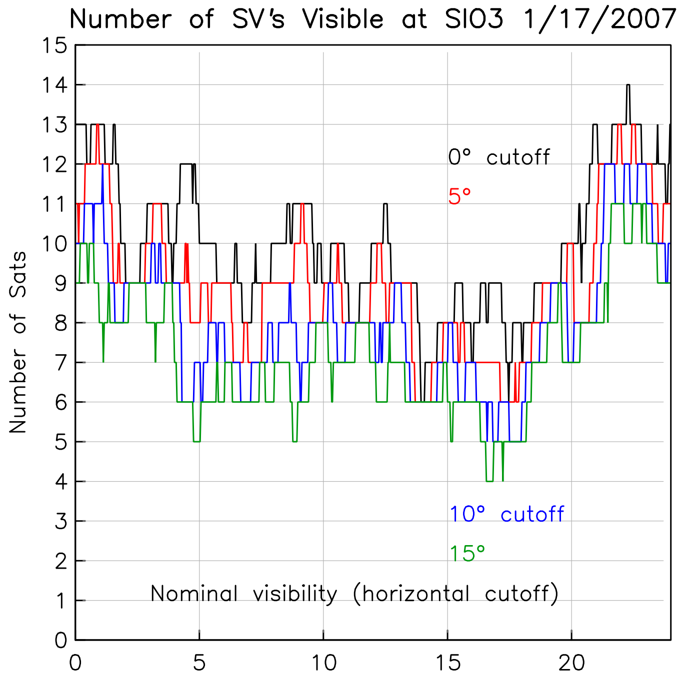
Where the Satellites Are (Looking Down)



Black lines show the movement over an hour.

Where the Satellites Are (Looking Up)





Viewed from near the Scripps Aquarium. Yellow is sky tracks over a day (notice the hole to the N), green shows the them over an hour. The number of satellites visible depends on the **elevation cutoff**. Fewer satellites means worse estimates.

What do the Satellites Transmit? (I)

All the radio signals are "L-band": frequencies about 1.5 GHz, wavelengths about 0.2 m. The two frequencies used for positioning are:

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Other frequencies are used for other purposes, and some have been added on the newest systems.

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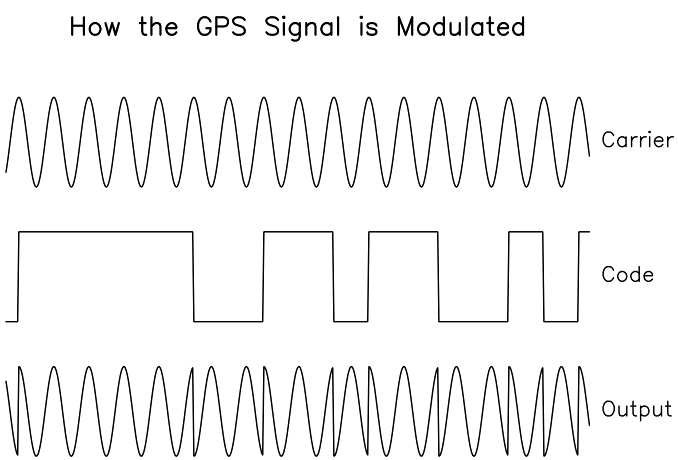
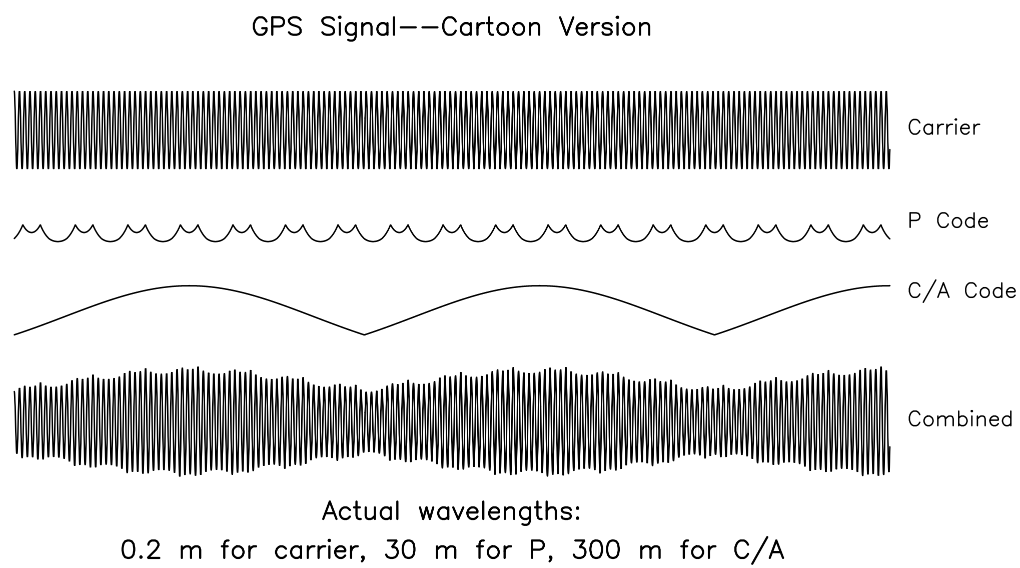
- Satellite position information, timing information, and lots more.

Carrier Modulation





Actual modulation is done (as on left) by changing the phase of the carrier.
Right-hand plot is a cartoon of how two codes amplitude modulate carrier.

Signal Wavelengths

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λ is 0.2 m for carrier, 30 m for P, 300 m for C/A; we can measure to about 0.01 of the wavelength, so 2 mm using the carrier, 3 m using C/A.

So for geodesy we use the carrier – ideally, after demodulating, which requires that we know the code.

Code is also useful for getting approximate positions, as a first step.

Basic Geodetic Observable: Carrier-Beat Phase

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We actually use the **carrier-beat phase**: the phase of $\exp(2\pi i[{}^{j}f - {}_{k}f]t)$ which is the difference between the carrier frequency from satellite j and the frequency of an oscillator in receiver k. This beat frequency is at much less than 1.5 GHz.

So this involves on two frequencies:

- ${}^{j}f$ depends on the satellite clock and the velocity of the satellite relative to the receiver (Doppler shift).
- ${}_{k}f$ depends on the receiver clock.

Note that we use superscripts for satellites, subscripts for receivers.

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- The radio waves not travel at c in the troposphere and ionosphere.

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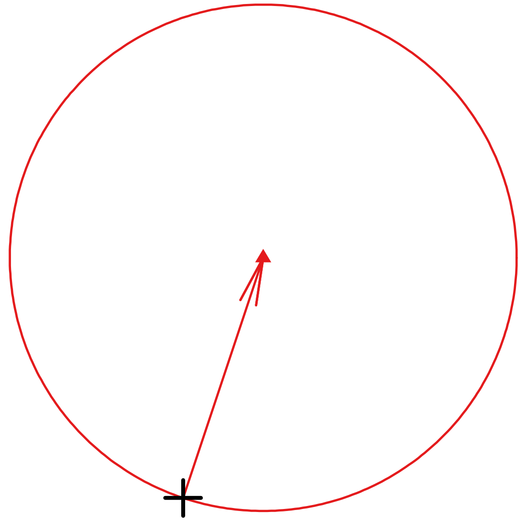

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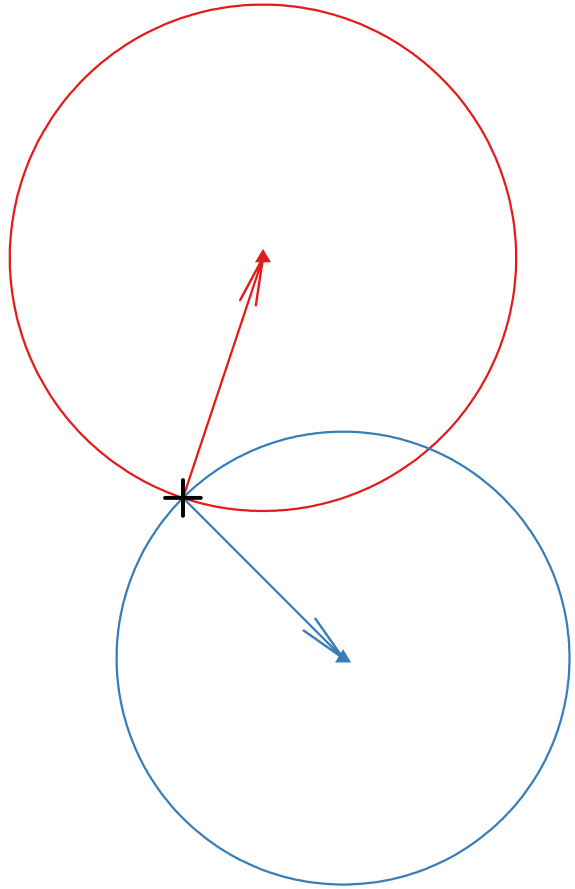

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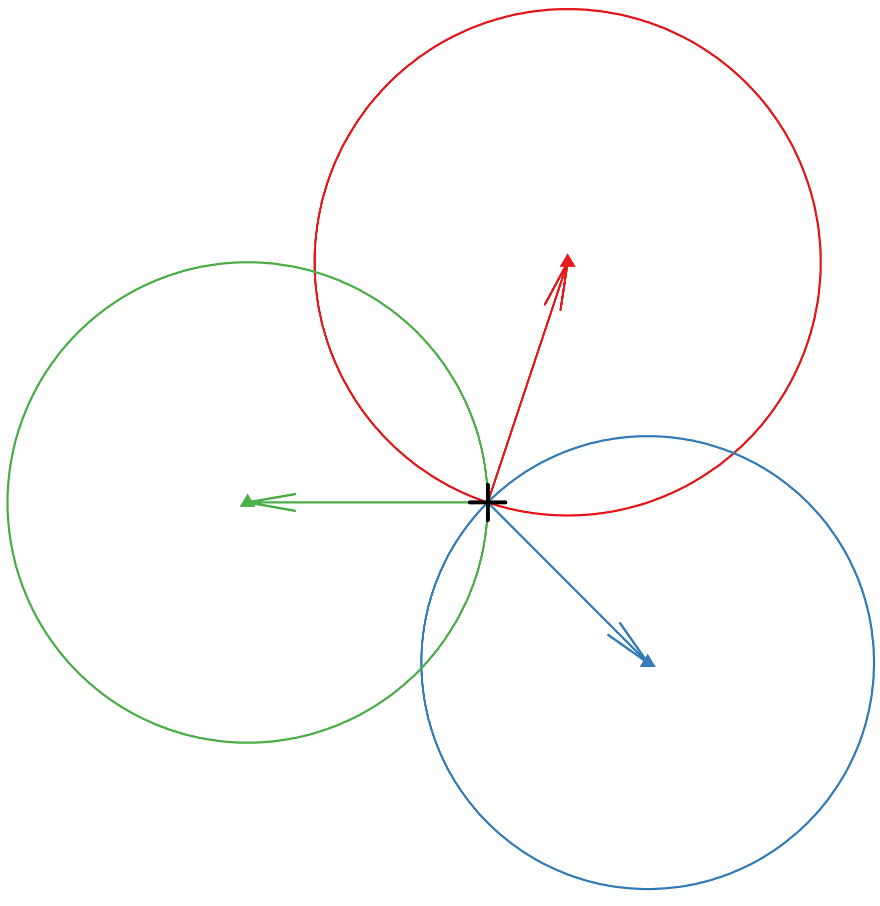

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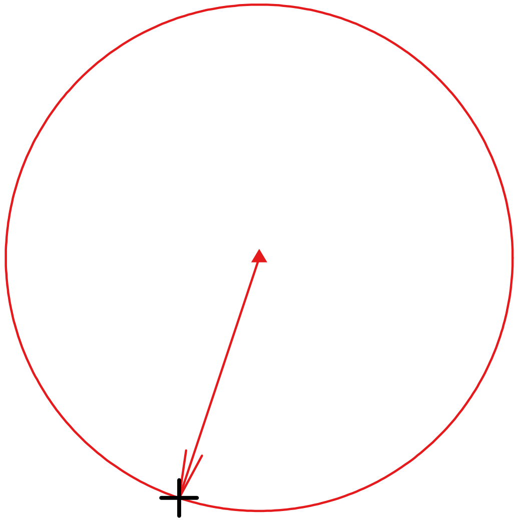

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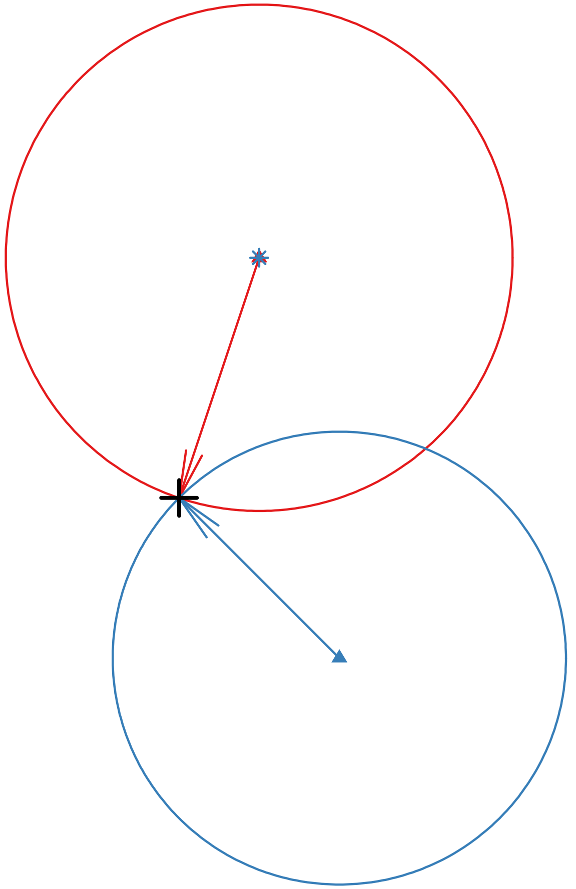

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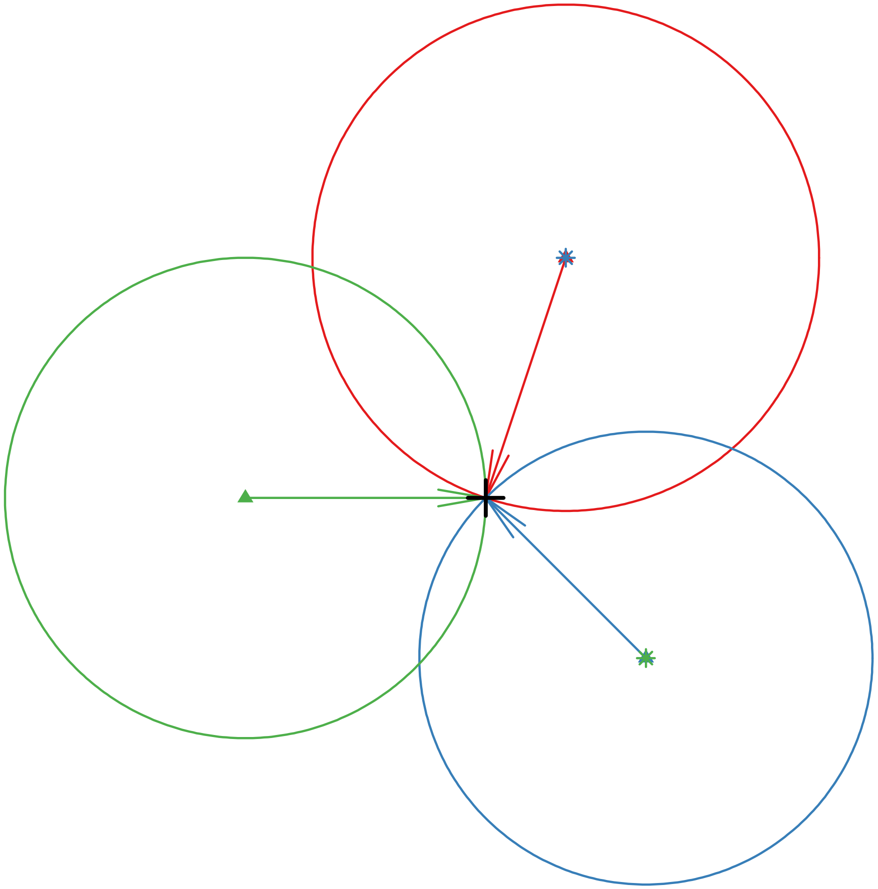

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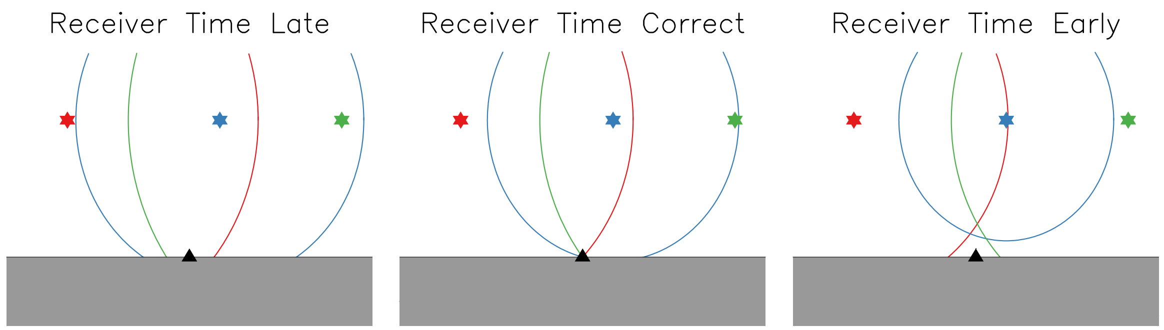

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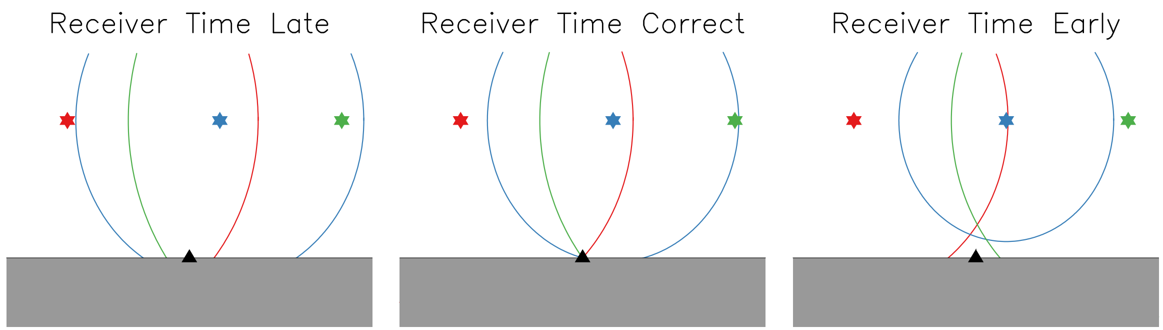

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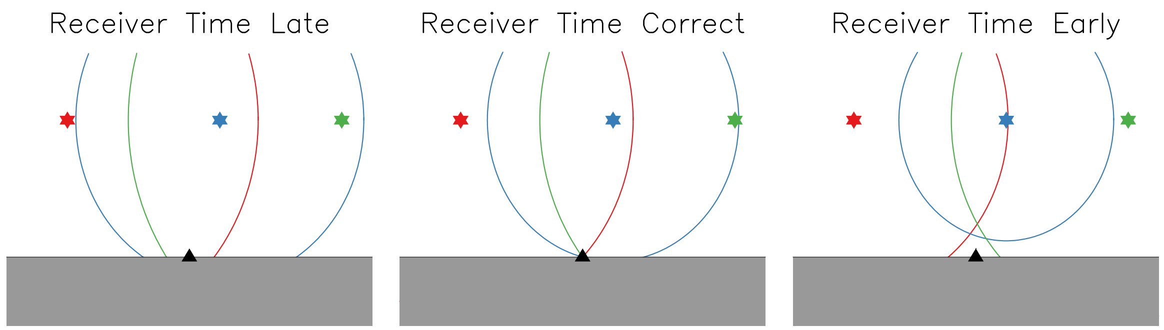

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But with *three* satellites, the wrong time will not fit any position – so we adjust the receiver time until it does. We then know both where we are, and what time it is (from the satellite clocks).

Differencing (I)

Algebraically, we create **combinations of observables** that remove the effects of clock errors. Consider:

$${}^{i}_{k}d - {}^{j}_{k}d = c({}^{i}_{k}t - {}^{i}t) - c({}^{j}_{k}t - {}^{j}t)$$

If the signals are received at the same time, the combination

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Likewise, differencing between receivers for the same satellite gives ${}^{j}_{k}d - {}^{j}_{l}d = c({}^{j}_{k}t - {}^{j}_{l}t)$ and the satellite clock drops out.

Differencing (II)

The next step is to form the difference of the between-receivers single difference, which is called a **double difference**:

$${}^{i}_{k}d - {}^{i}_{l}d - ({}^{j}_{k}d - {}^{j}_{l}d) = c({}^{i}_{k}t - {}^{i}_{l}t - {}^{j}_{k}t + {}^{j}_{l}t)$$

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However, this double-difference depends on the **relative position** of the receivers: the **baseline** between them.

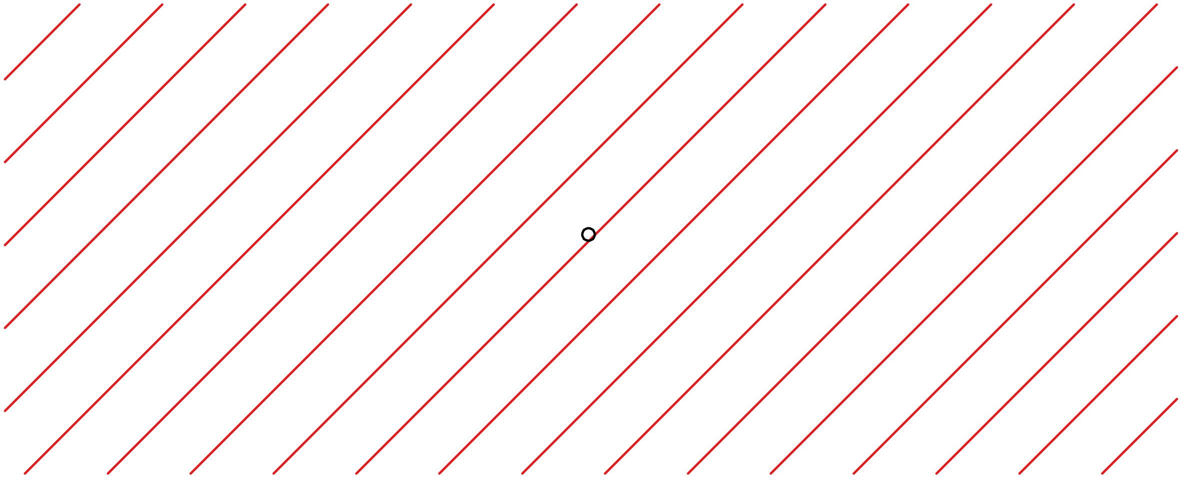
Ambiguity Resolution

Using the carrier-best-phase introduces **ambiguities**: since a sine wave repeats with wavelength λ, the distance to a satellite is (initially) uncertain to within a multiple of λ. Getting finding the correct multiple is **ambiguity resolution**.

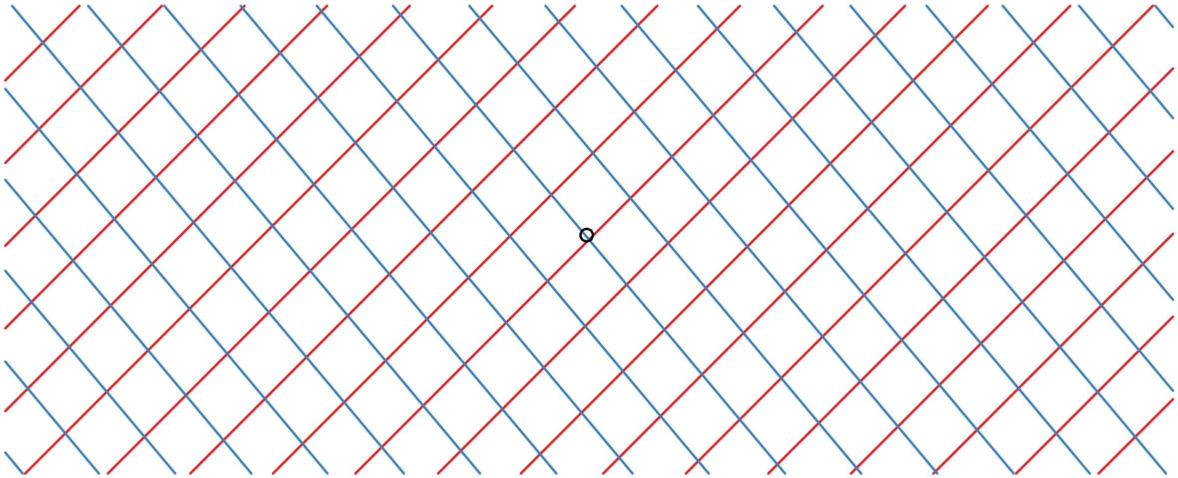
- Resolving the ambiguities removes unknowns from the solution and improves it.
- Because of error, our initial estimate of the ambiguity factor is never exactly an integer.
- Getting the wrong integer gives a very incorrect answer: unresolved is better than incorrect.
- Resolving ambiguities depends on having waves with different directions of arrival: multiple satellites, or the same satellite over a long time.
- **Single-epoch** estimates (needed for GPS seismology) involve ambiguity resolution from one time sample from multiple satellites.

Ambiguity Resolution: Point with Errors

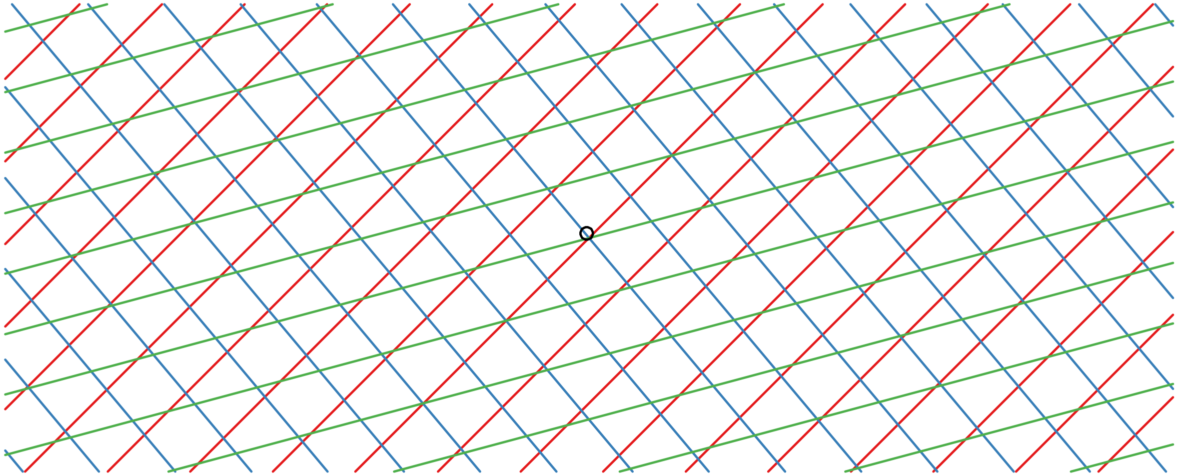
Ambiguity Resolution: First Satellite



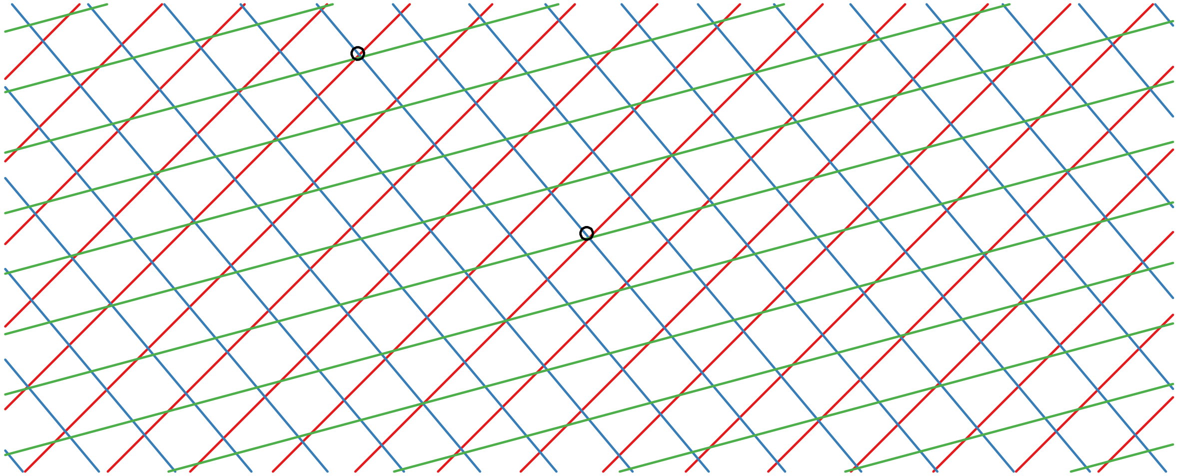
Ambiguity Resolution: First Two Satellites



Ambiguity Resolution: First Three Satellites



Ambiguity Resolution: Which Is It?



If we choose the wrong value for the ambiguity, we can be far from the correct value.

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Tropospheric delays can be up to 2.5 m, and change more slowly.

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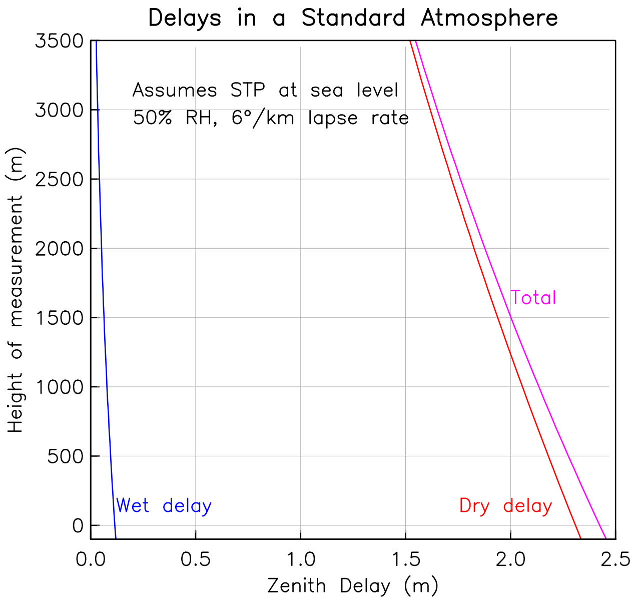
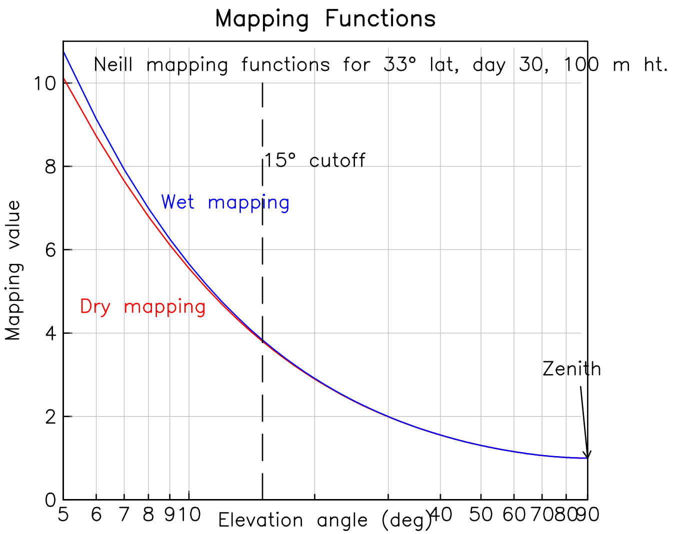
The **wet delay** cannot be modeled or be measured independently: it must be estimated using the GPS data.

We assume the delay looks like $Z(t)M(\theta)$ where

- $Z(t)$ is the **zenith delay**, varying with time.
- $M(\theta)$ is a **mapping function** of the elevation angle θ.

Propagation Delay: Models





Information Needed for Relative Positioning

- Data from the receiver at the point of interest: carrier-beat phases, and pseudoranges, to all visible satellites.
- The same for a reference receiver whose position is known.
- The positions of the satellites (“orbit information”), from the IGS or another procssing center.
- **Earth Orientation Parameters** used to connect a position on the Earth to inertial space, from the IERS,
- Satellite information (e.g. which ones are working) from the USNO.
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Information Needed for Point Positioning

- Data from the receiver at the point of interest: carrier-beat phases, and pseudoranges, to all visible satellites.
- The positions of the satellites (“orbit information”), from the IGS or another procssing center.
- A description of the satellite clock errors, from the IGS or another processing center.
- **Earth Orientation Parameters** used to connect a position on the Earth to inertial space, from the IERS,
- Satellite information (e.g. which ones are working) from the USNO.
- Antenna phase pattern (predetermined).

Steps in Relative Positioning

- Form double-difference combinations (not all, just a unique set).
- Get a preliminary position (or positions) using pseudorange information.
- Solve for:
  - Positions of unknown receivers (3 parameters)
  - Zenith delays at some time spacing, for each receiver.
  - Ambiguities at each receiver for all satellites observed there.
- Set ambiguity values to nearest integer, for all cases where this can safely be done.
- Repeat the solution for positions and zenith delays, with (we hope) many fewer parameters, since ambiguities have been resolved.

Local Effects I: Antenna Phase Delay

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First, we will have $U_0 e^{2\pi ift}[e^{i\phi_A(\theta_0,\beta_0)}]$ where ϕ_A is the phase shift introduced by the antenna itself, as a function of the elevation angle θ_0 and azimuth β_0 of the incoming signal; this shift includes any offset of the antenna "phase center" from the reference point on the antenna.

In general, this will be reduced if the same antenna types are used, or we have a model for ϕ_A: most antennas do.

Local Effects II: Multipath

In addition, we will have a term $U_0 e^{2\pi i f t}[\int_{\Omega^-} A(\theta, \beta) R(\theta, \beta) e^{i\phi_R(\theta,\beta)} d\theta\, d\beta]$ The integral term is meant to include all the "multipath" contributions, and so is an integral over Ω^-, which denotes the unit sphere excluding the direction of the direct wave. This includes

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None of these can be modeled well, so they are a source of noise that limits the precision of measurements made over short times; over long times this effect averages, somewhat.