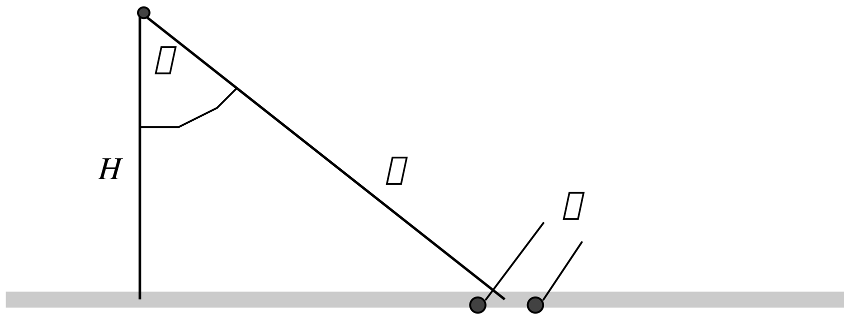
SYNTHETIC APERTURE RADAR (SAR) SUMMARY

Range Resolution (end view)



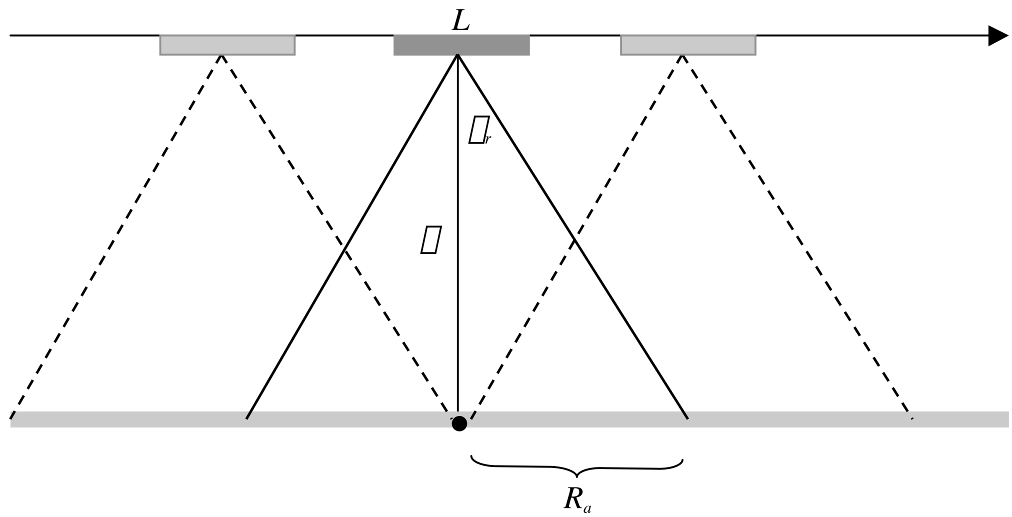
θ	-	look angle
H	-	spacecraft height
B	-	bandwidth of radar
τ	-	pulse length $\approx 1/B$
C	-	speed of light

$$\Delta r = \frac{C\tau}{2}$$ - slant range resolution

$$R_r = \frac{C\tau}{2}\frac{1}{\sin\theta}$$ - ground range resolution

Note the range resolution is infinite for vertical look angle and improves as look angle is increased. Also note that the range resolution is independent of the height of the spacecraft H. The range resolution can be improved by increasing the bandwidth of the radar. Shorter wavelength will enable higher bandwidth because the bandwidth is only a small fraction of the carrier frequency.

Azimuth Resolution (top view)



L	-	length of radar antenna
ρ	-	nominal slant range $H/\cos\theta$
λ	□	wavelength of radar

$$\sin\theta_r = \lambda/L \qquad \text{- diffration resolution}$$

$$R_a = \rho\sin\theta_r = \frac{\lambda\rho}{L} = \frac{\lambda H}{L\cos\theta} \qquad \text{- half - length of ground illumination}$$

Form a synthetic aperture that has a length of $2R_a$. The improved azimuth resolution is

$$R_a^{'} = \frac{\lambda\rho}{2R_a} = \frac{L}{2} \qquad \text{- theoretical resolution of strip - mode SAR}$$

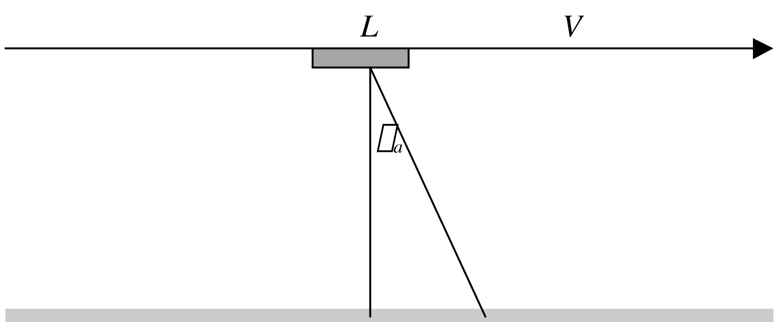
Note the azimuth resolution is independent of spacecraft height and improves as the antenna length is reduced. One could form a longer synthetic aperture by steering the transmitted radar beam so it follows the target as the spacecraft (aircraft) flies by. This is called spotlight-mode SAR.

Resolution of ERS SAR

The ERS SAR has a bandwidth of 15.6 MHz, an antenna length of 10 m and a look angle of 23˚. The ground range resolution is about 25 m and the maximum azimuth resolution is 5 m. In practice, one averages several "looks" together to improve the quality of the amplitude (backscatter) image. In the case of ERS, one could average 5 looks to for a resolution cell of 25 m by 25 m.

Required Azimuth Sampling

The discussion above suggests that we should make the antenna length *L* as short as possible to improve azimuth resolution. However, to form a complete aperture without aliasing longer wavelengths back into shorter wavelengths we must pulse the radar every at an along-track distance of L/2 or shorter. Consider the maximum Doppler shift from a point that is a illuminated at a maximum distance ahead of the radar.



f_o	-	carrier frequency = C/λ
V		velocity of spacecraft relative to the ground

The maximum doppler shift occurs at a maximum angle of

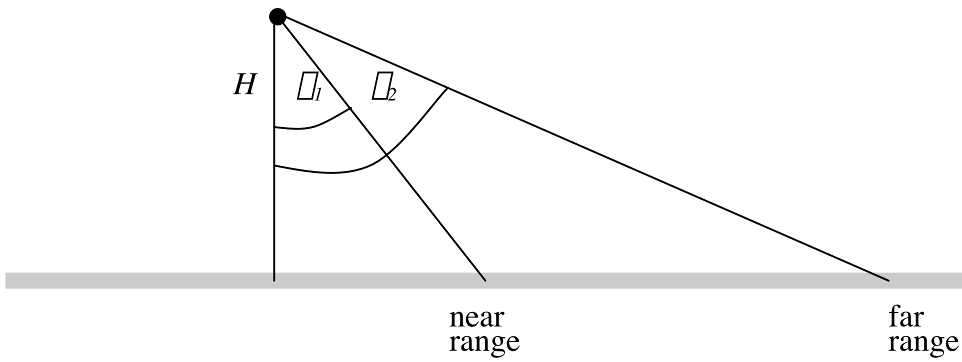
$$\sin\theta_a = \frac{\lambda}{L}$$

$$\Delta f = 2 f_o \frac{V}{C} \sin\theta_a = \frac{2C}{\lambda}\frac{V}{C}\frac{\lambda}{L} = \frac{2V}{L}$$

This corresponds to a maximum along-track distance between samples of *L*/2. For ERS this corresponds to a minimum pulse repetition frequency (PRF) of 1400 Hz. The actual PRF of ERS is 1680 Hz.

Other Constraints on the PRF

The PRF cannot be too large or the return pulses from the near range and far range will overlap in time.



$$PRF < \frac{1}{t_2 - t_1}$$

$$t_1 = \frac{2H}{C\cos\theta_1} \qquad t_2 = \frac{2H}{C\cos\theta_2}$$

$$\Delta t = \frac{2H}{C}\left(\sec\theta_2 - \sec\theta_1\right)$$

$$\frac{2V}{L} < PRF < \frac{C}{2H}\left(\sec\theta_2 - \sec\theta_1\right)^{-1}$$

For ERS the look angles to the rear range and far range are 18˚ and 24˚, respectively. Thus the maximum PRF is 4777 Hz. The actual PRF of 1680 is safely below this value and the real limitation is imposed by the speed of the data link from the spacecraft to the ground. A wider swath or a higher PRF would require a faster data link than is possible using a normal X-band communication link of 105 Mbps.

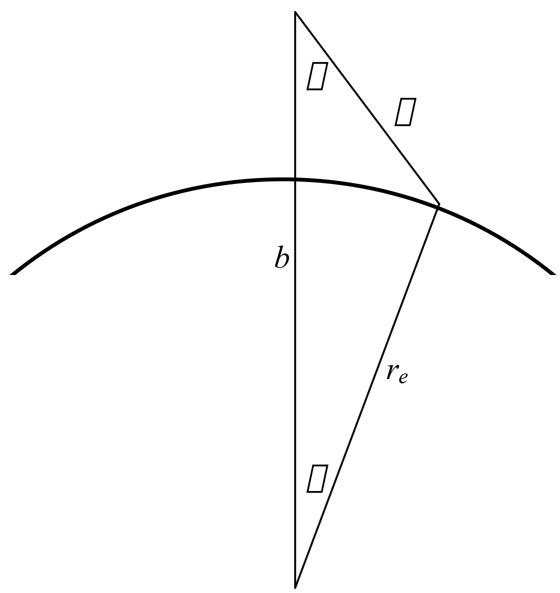
Look Angle and Incidence Angle for a Spherical Earth

We will make the approximation that the earth is locally spherical although we will adjust the local radius of the Earth using the WGS84 ellipsoid. While this approximation is good locally, there can be large differences between the elliptical earth model and the local radius for long SAR swaths. This difference will produces topographic fringes in long-strip interferograms depending on the interferometric baseline. We will deal with this issue later by including the ellipsoidal radius minus the local radius into a topographic correction term. The local earth radius is given by

$$r_e(\phi) = \left[\frac{\cos^2\phi}{a^2} + \frac{\sin^2\phi}{c^2}\right]^{-1/2}$$

where ϕ is latitude, and a and c are the equatorial (6378 km) and polar (6357 km) radii, respectively.

The look angle of the radar depends only on the local earth radius r_e, the range to the sphere ρ, and the height of the spacecraft above the center of the Earth b.



Using the Law of cosines one finds

$$\theta = \cos^{-1}\frac{\left(b^2 + \rho^2 - r_e^2\right)}{2\rho b}$$

On a sphere, the incidence angle is greater than the look angle by the angle α. As an exercise, develop a formula for the incidence angle.