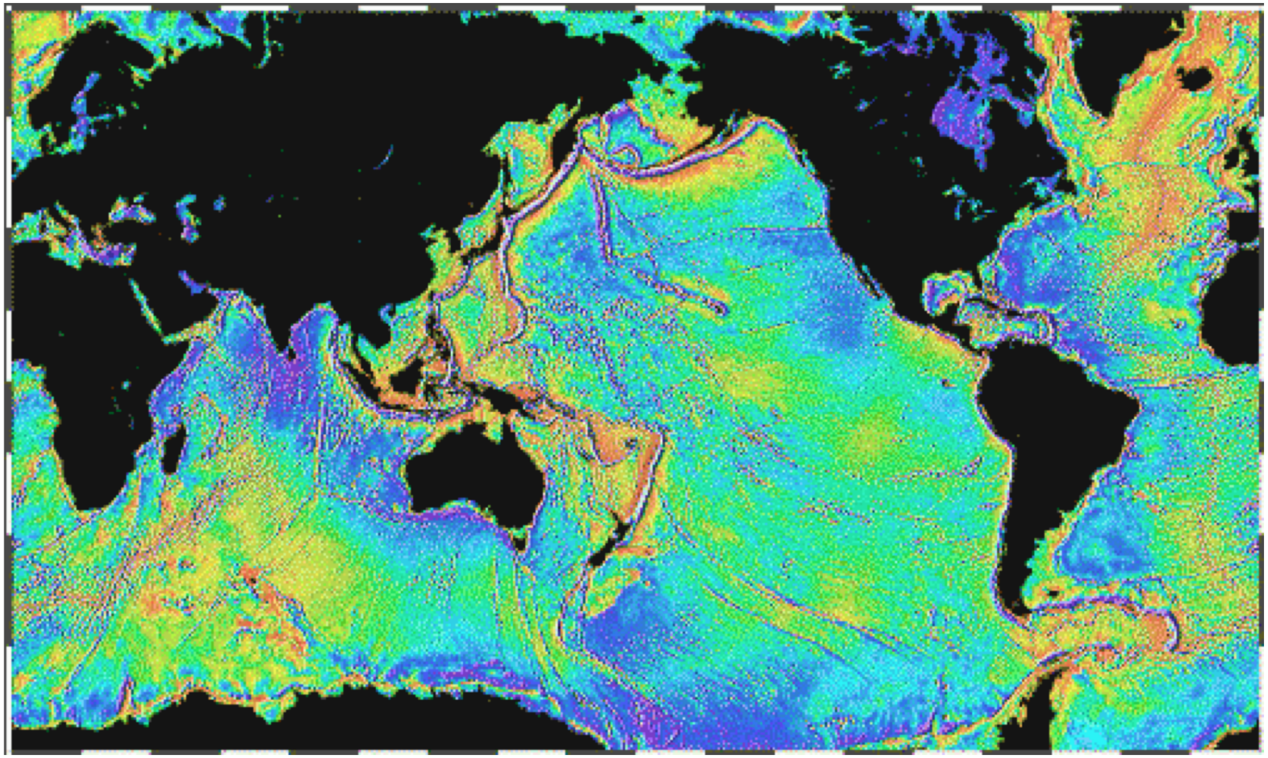
Lecture 16: Gravity, Isostasy and Flexure

Read pages 42-51 in KK&V

Homework 6



Gravity: theory and measurements

gravity - "weighing" the earth

Newton: $F = G\,(m_1\,m_2)/r^2$

and $F = ma$

$$g = \frac{GM_e}{R_e^2}$$

g = acceleration caused by attraction of one mass to Earth's mass

g = 980 cm/sec2 = 980 Gals = 980,000 mGals (milligal)

*This is for a stationary earth. For a rotating earth there are terms for centrifugal acceleration and nonspherical shape of earth amounting to <1%.

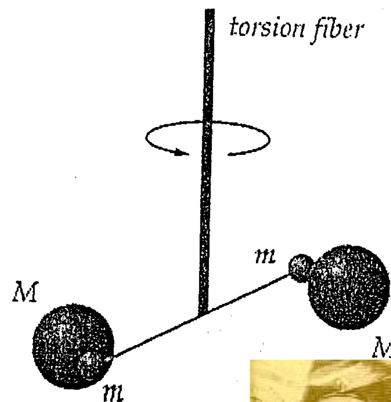
G = gravitational constant (6.67×10^{-11} Nm^2/kg^2)

Me = mass of Earth (5.973×10^{24} kg)

Re = radius of Earth

equatorial radius = 6.378139×10^6 m

polar radius = 6.35675×10^6 m



Cavendish experiment (1798)

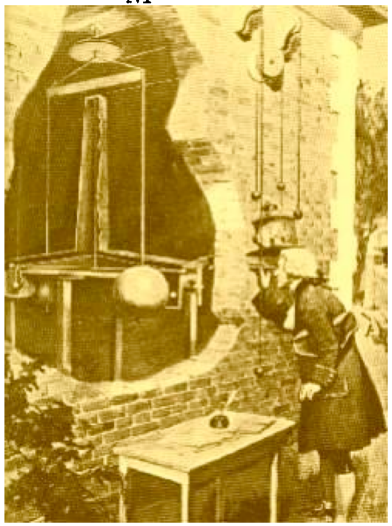
average density = 5.448 Mg/m^3

modern value = 5.517 Mg/m^3

At high pressures density of silicate minerals increases significantly

e.g. olivine at 1.7 Mbar, density = 5.5 Mg/m^3

pressure at center of earth: $P = 3.6\times10^{11}$ Pa $= 3.6\times10^6$ bar = 3.6 Mbar



So denser material (primarily Fe) needed to account for average density of earth

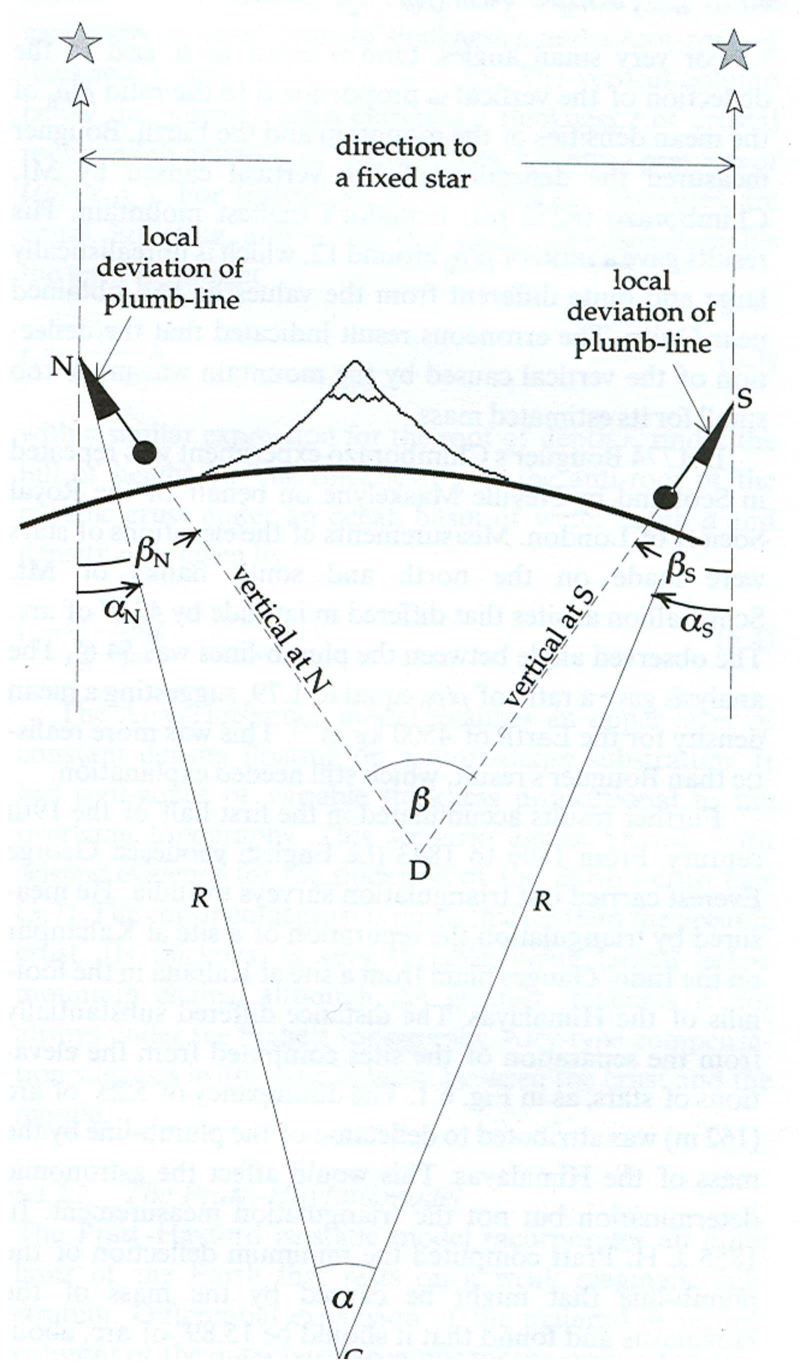
composition of crust known from direct sampling

meteorites, composition of solar system

core: 95+% Fe, some Ni and possibly light elements (O,S,K)

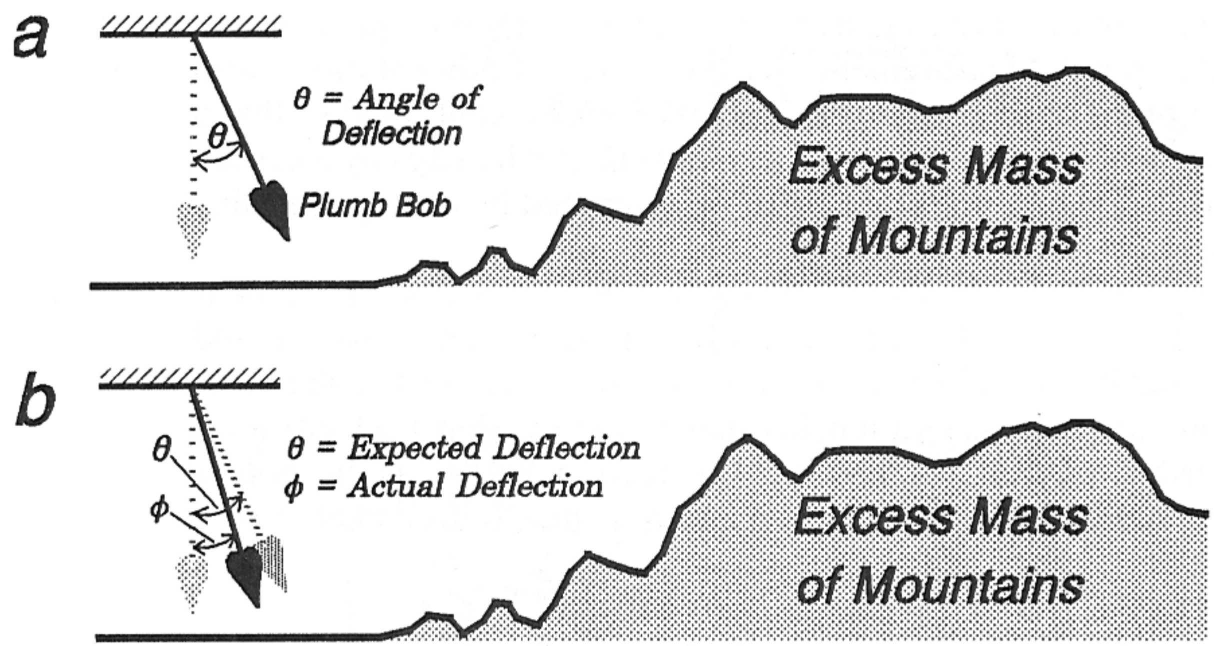
supported by presence of magnetic field

In XVIII and XIX centuries, surveys set out to measure the shape of the Earth

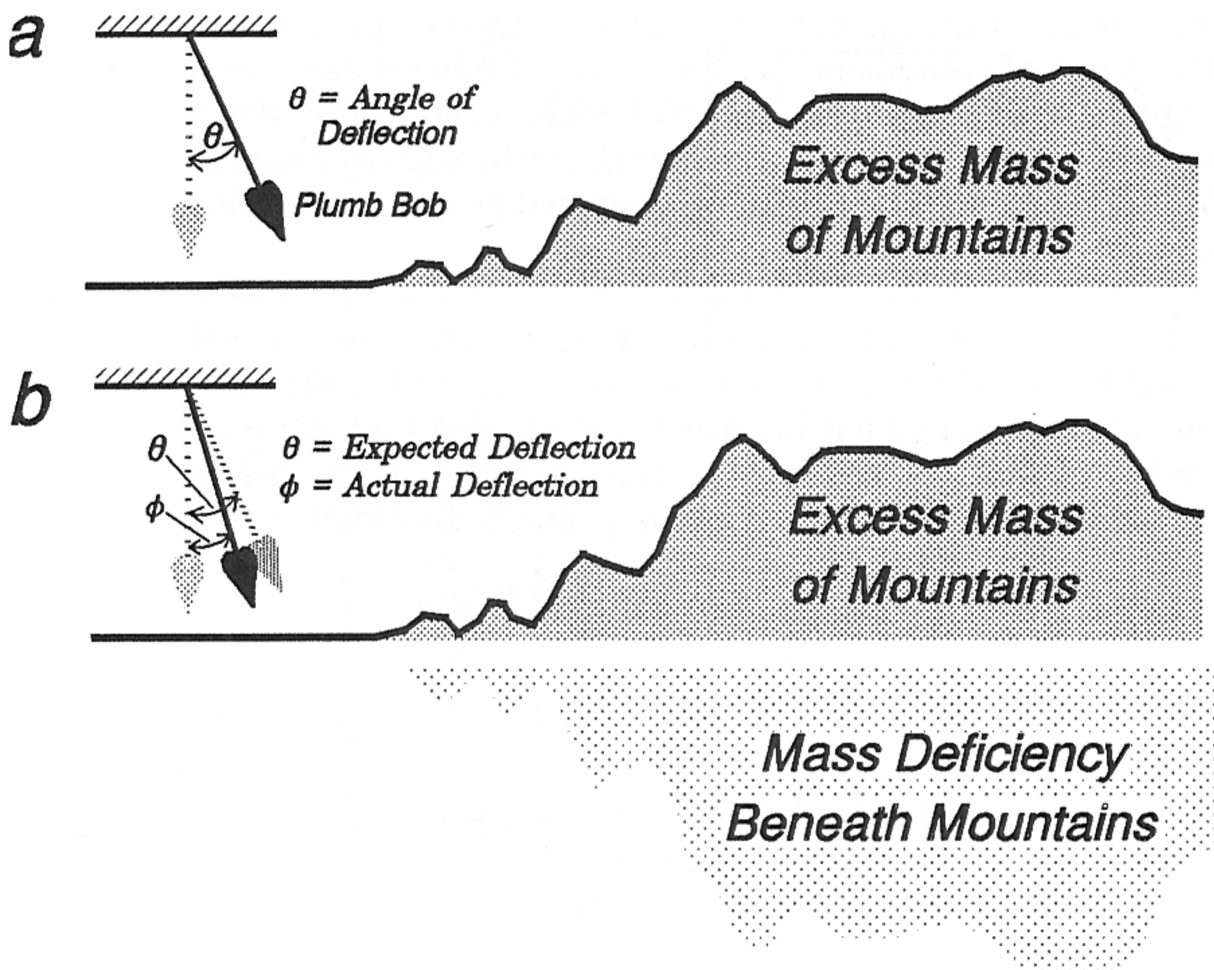


They used plumb bobs and expected them to be attracted toward adjacent mountains such as Andes and Himalayas

… but the plumb bobs did not deflect as much as expected

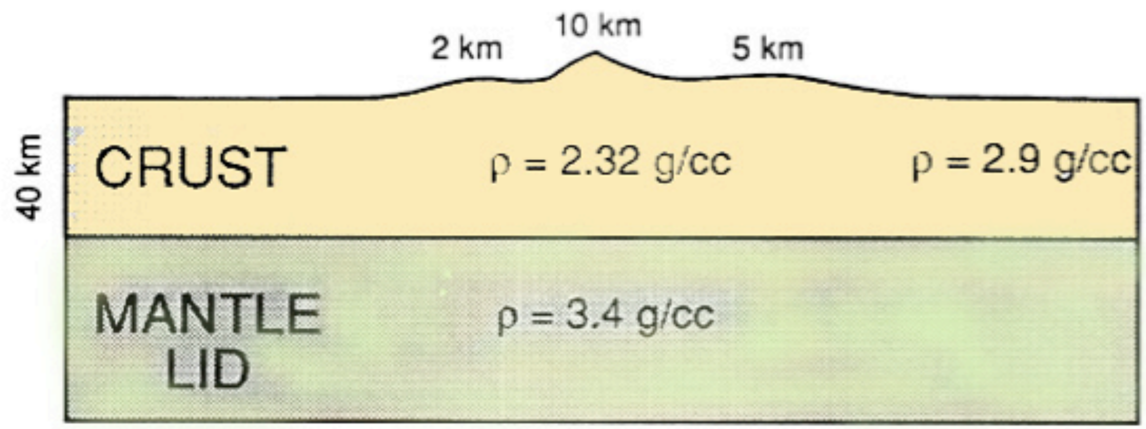


… smaller than predicted deflection could result from either overestimated mass (density) of the mountains, or some deficit of mass below the mountains



Isostasy – the state of gravitational equilibrium between the Earth's crust and mantle, as if the (lighter) crust were floating on a (heavier) mantle

Pratt Isostasy



Airy Isostasy

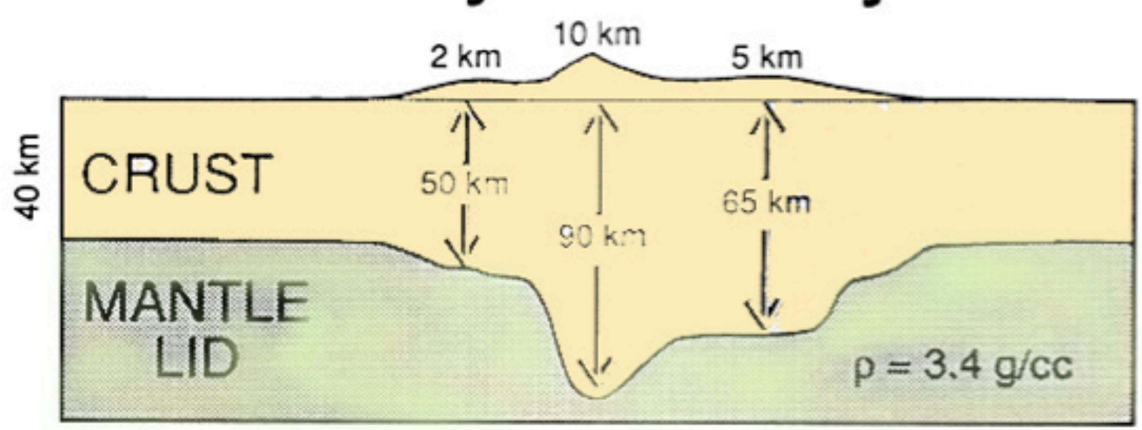


Fig. 1.10. John Henry Pratt 1 year after his appointment as Chaplain to the Bishop of Calcutta. Portrait courtesy of D. P. McKenzie of Cambridge University.

where G is the universal gravitational constant and d is the distance between the two masses. In the case that M and P are points on the surface of a spherical Earth, then it follows (Fig. 1.11) that the component of the attraction at a station P due to the elementary mass at M *in the direction of gravity* is given by

$$dg = \frac{G\ dm\ \sin(\xi)}{d^2}$$

where

$$\xi = \frac{\theta_s}{2} \qquad \text{and} \qquad d = 2r_e \sin\left(\frac{\theta_s}{2}\right)$$

$$\therefore dg = G\ dm \frac{\sin\left(\frac{\theta_s}{2}\right)}{4r_e^2 \sin^2\left(\frac{\theta_s}{2}\right)} \tag{1.2}$$

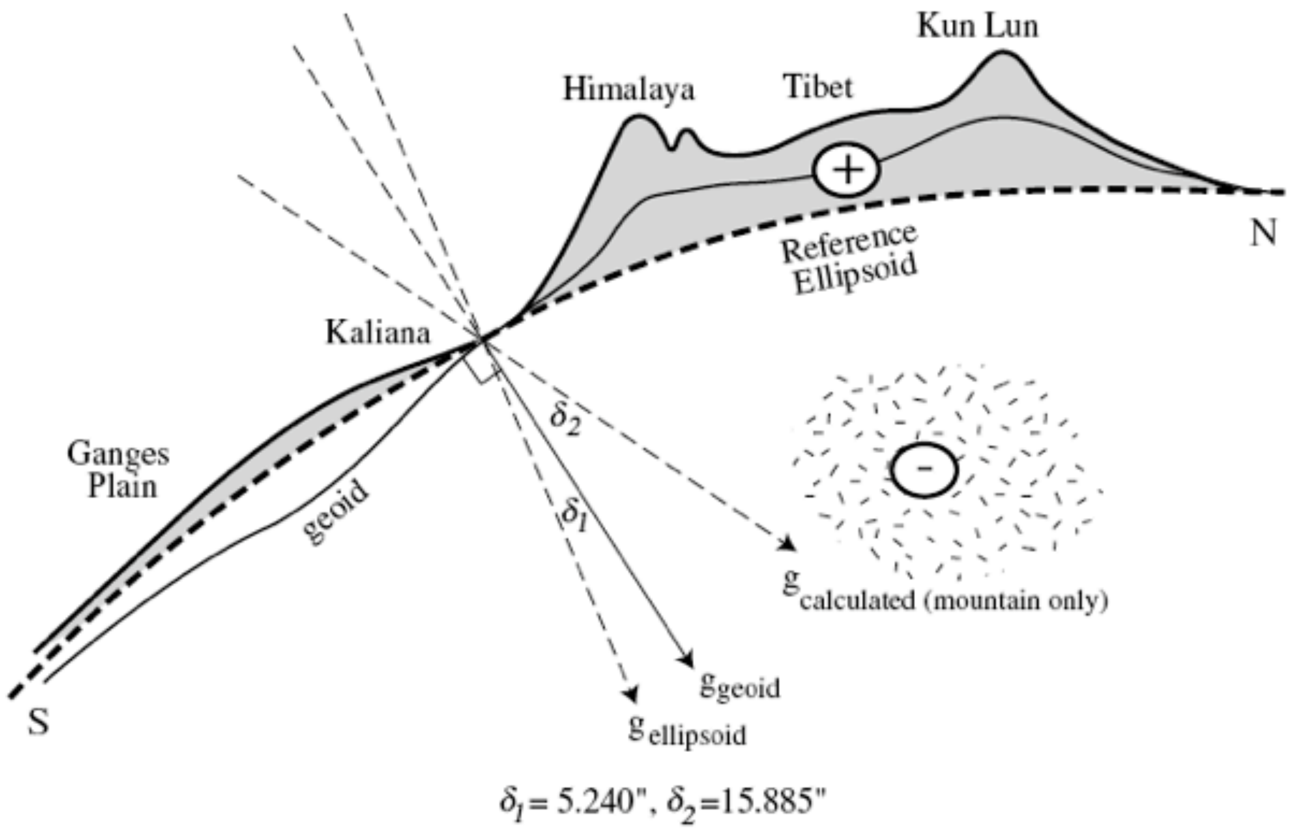
Pratt used Eq. (1.2) to calculate the gravitational effect of the Himalayas at Kaliana and Kalianpur, and published the results in a 75-page long paper (Pratt, 1855). He found that the gravitational effect of the mountains is large enough to deflect the plumb line by 15.885″, more than three times the observed value (Fig. 1.12). Pratt was satisfied, despite problems with not knowing the detailed topography of the Himalayas, that he had correctly computed the effect of the mountains at Kaliana and Kalianpur. He concluded his paper by saying that he did not understand the cause of the discrepancy and that the problem should be investigated further.

Fig. 1.13. George Biddell Airy, Professor of Astronomy at Cambridge University and Astronomer Royal. Portrait courtesy of D. P. McKenzie of Cambridge University.

1.4 Isostasy According to Airy

Shortly after Pratt's paper, G. B. Airy (1801–92; Fig. 1.13), the Astronomer Royal, presented a paper to the Royal Society in which he offered an explanation for the discrepancy. Unlike Pratt, Airy was not surprised by the discrepancy. Indeed, he thought that it should have been anticipated.

Airy's argument was based on his belief that the outer layers of the Earth consisted of a thin crust that overlay a fluid layer of greater density than the crust. He referred to the fluid layer as "*lava*". Airy compared the state of the crust lying on the lava to timber blocks floating on water. He wrote (Airy, 1855, p. 103):



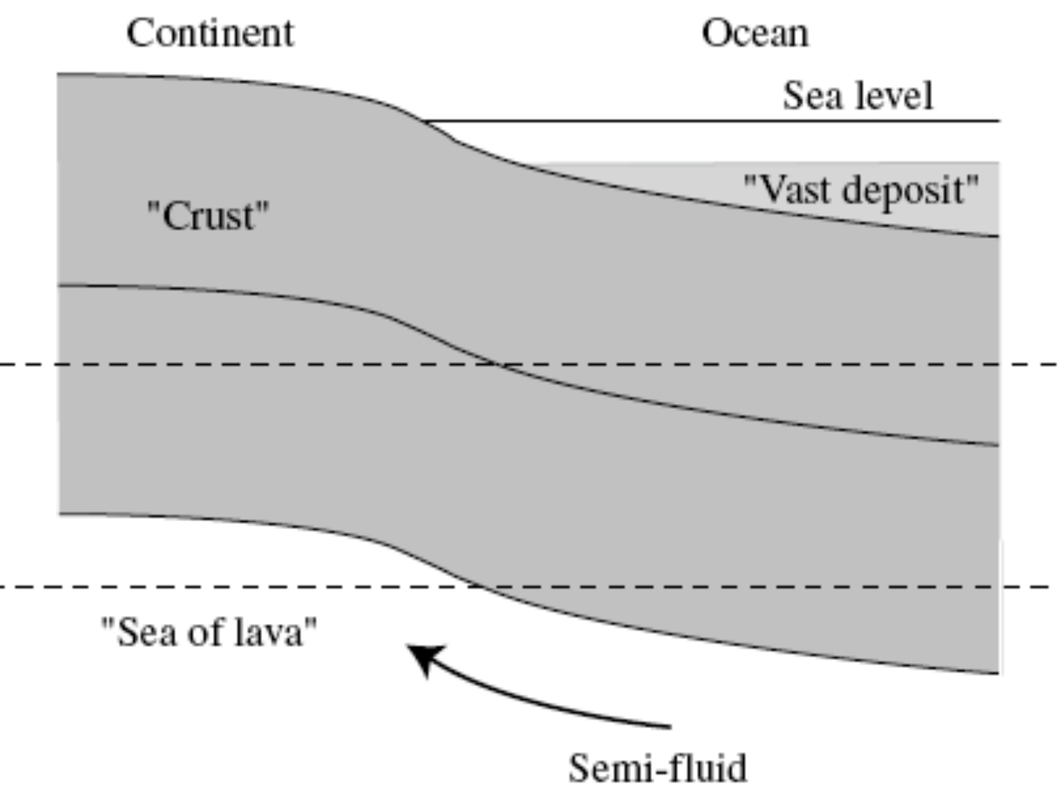


Fig. 1.6. The adjustment of the crust to a “vast deposit” by flow in the underlying “sea of lava”. Reproduced from a figure in Herschel (1836) with permission of the Royal Society.

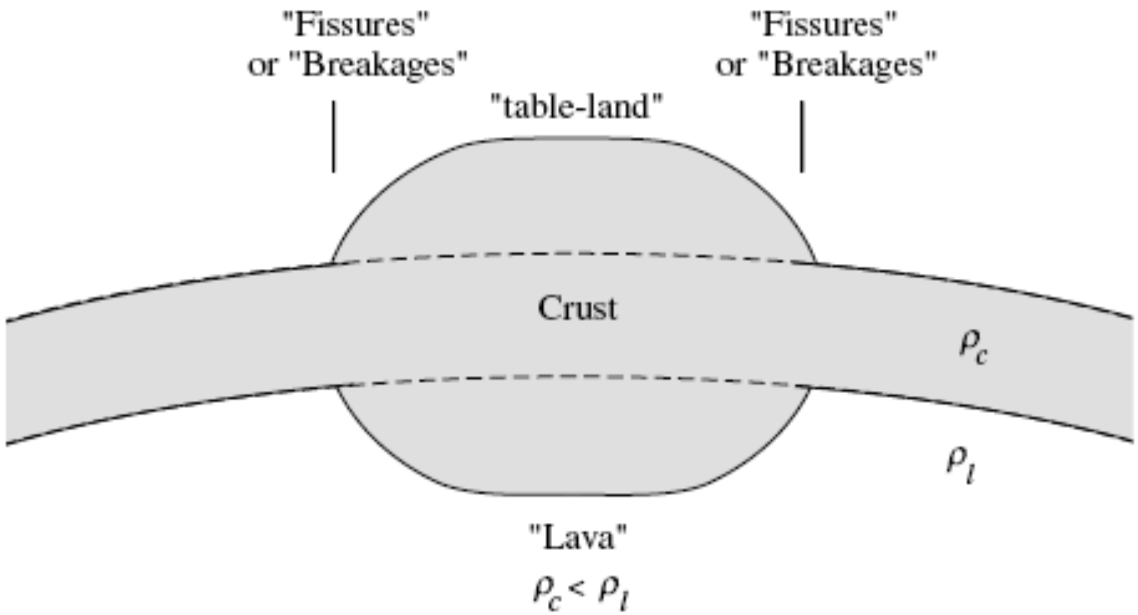
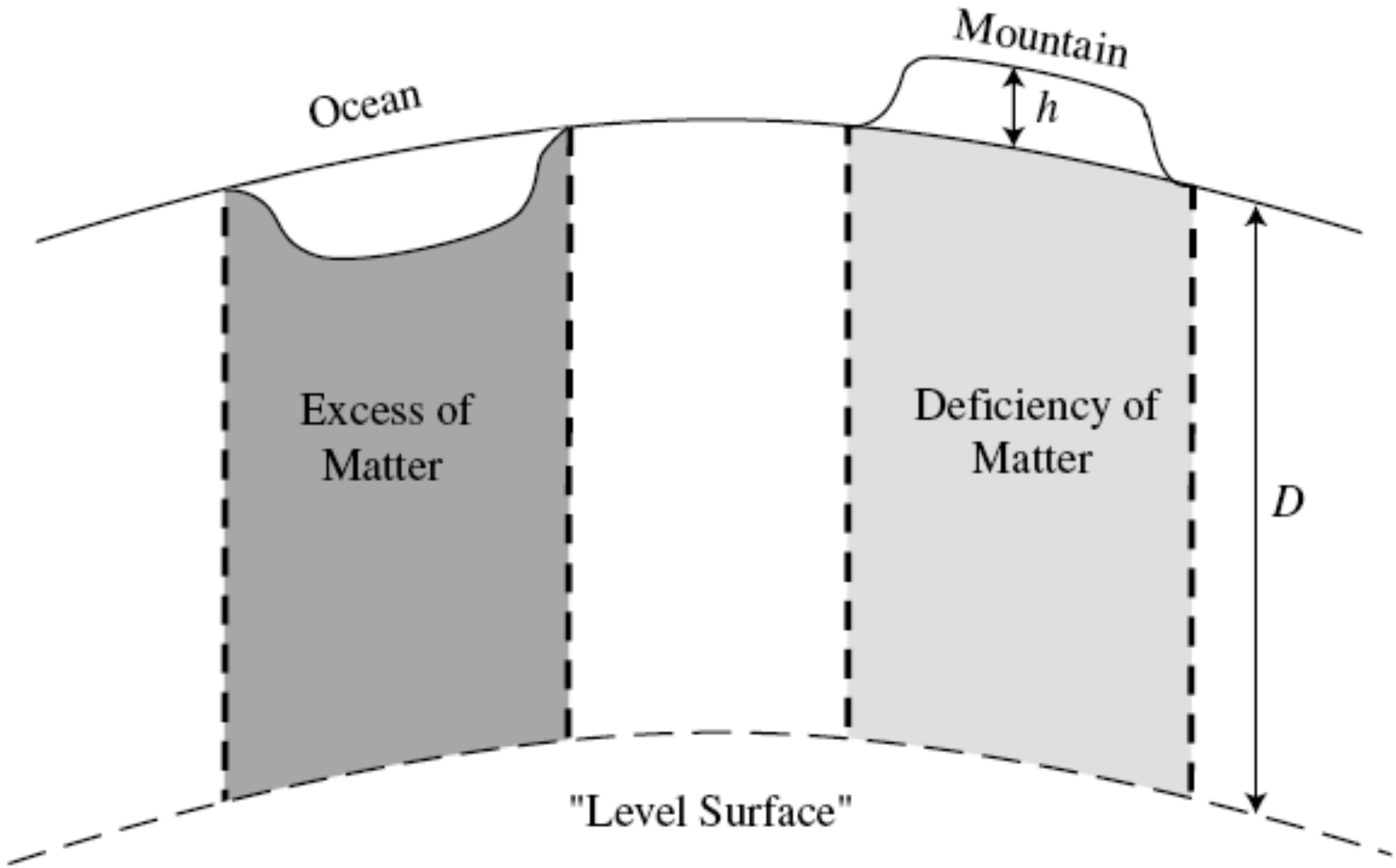
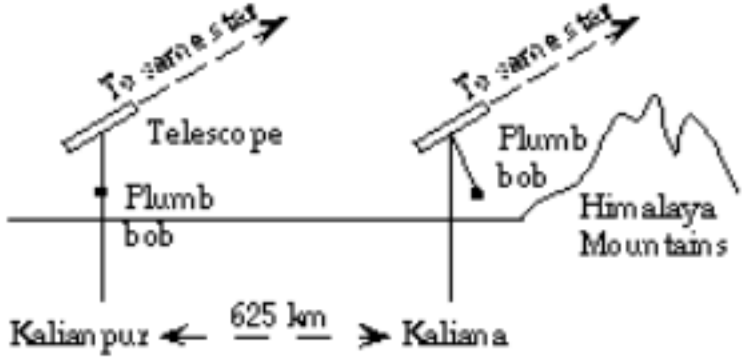
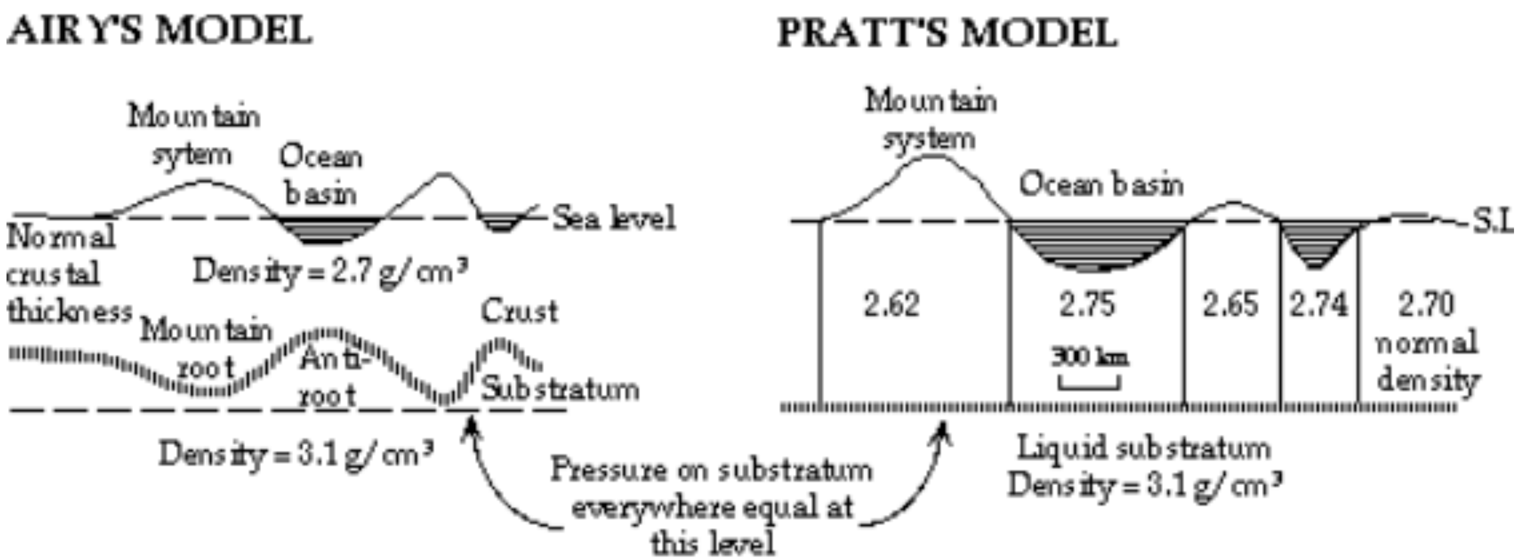


Fig. 1.14. Airy's hypothesis of a crust that "floats" upon "lava". Reproduced from Fig. 2 of Airy (1855) with permission of the Royal Society.


Ocean
Mountain
h
Excess of Matter
Deficiency of Matter
D
"Level Surface"


To same star
To same star
Telescope
Plumb
bob
Plumb
bob
Himalaya
Mountains
Kalianpur
625 km
Kaliana

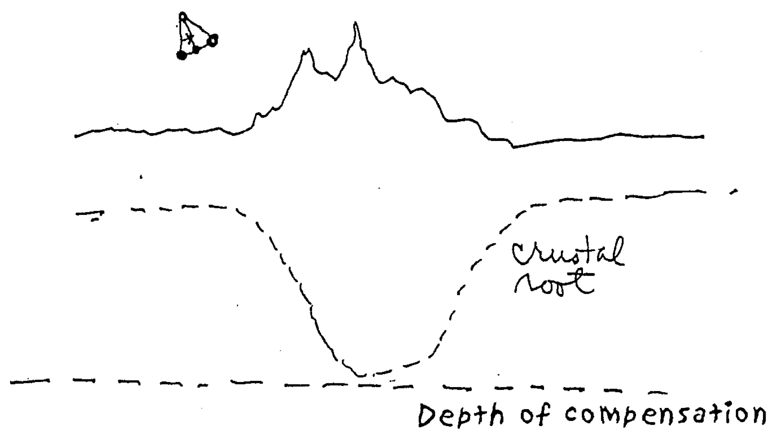

AIRY'S MODEL
Mountain
sytem
Ocean
basin
Sea level
Normal
crustal
thickness
Density = 2.7 g/cm³
Crust
Mountain
root
Anti-
root
Substratum
Density = 3.1 g/cm³
PRATT'S MODEL
Mountain
system
Ocean basin
S.L
2.62
2.75
2.65
2.74
2.70
normal
density
300 km
Liquid substratum
Density = 3.1 g/cm³
Pressure on substratum
everywhere equal at
this level

Isostasy

iso = equal
stasis = standing

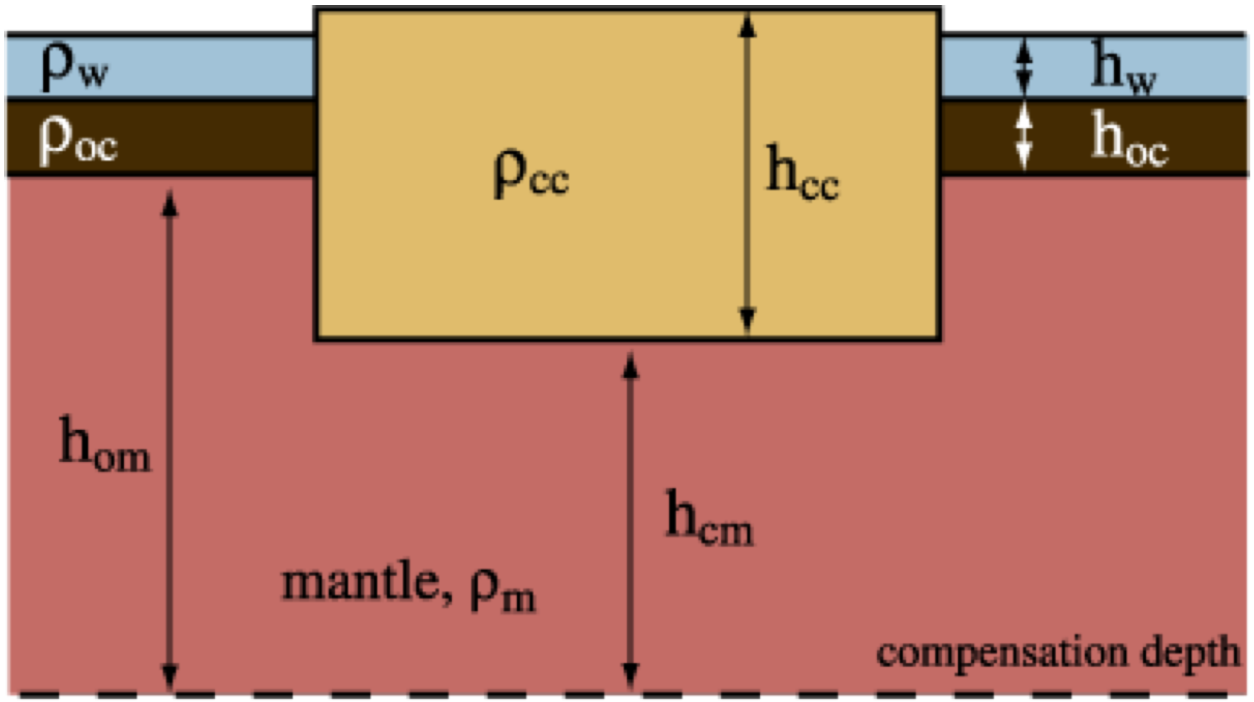
French surveyors in Peru in the 1740's noticed that the Andes deflected a plumb bob much less than anticipated.

Also surveys in India in the 1800s showed compensation of the Himalayas.



Principle: Beneath a certain depth, known as the depth of compensation, the pressures generated by all overlying materials are everywhere equal.

Think of an iceberg.



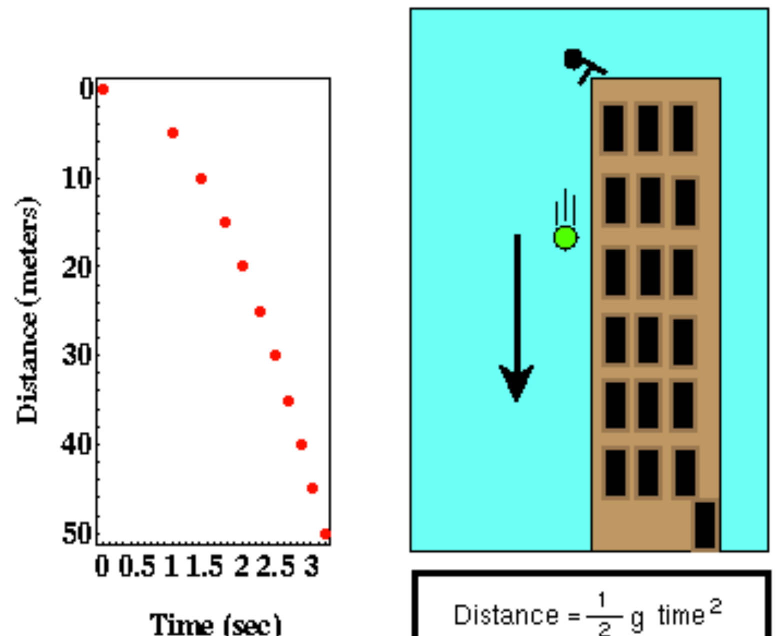
$$\rho_w h_w + \rho_{oc} h_{oc} + \rho_m h_{om} = \rho_{cc} h_{cc} + \rho_m h_{cm}$$

How do we measure gravity?

Falling Body Measurements

The gravitational acceleration can be measured directly by dropping an object and measuring its time rate of change of speed (acceleration) as it falls. By tradition, this is the method we have commonly ascribed to Galileo Galilei. In this experiment, Galileo is supposed to have dropped objects of varying mass from the leaning tower of Pisa and found that the gravitational acceleration an object undergoes is independent of its mass. He is also said to have estimated the value of the gravitational acceleration in this experiment. While it is true that Galileo did make these observations, he didn't use a falling body experiment to do them. Rather, he used measurements based on pendulums.

It is easy to show that the distance a body falls is proportional to the time it has fallen squared. The proportionality constant is the gravitational acceleration, *g*. Therefore, by measuring distances and times as a body falls, it is possible to estimate the gravitational acceleration. To measure changes in the gravitational acceleration down to 1 part in 40 million using an instrument of reasonable size (say one that allows the object to drop 1 meter), we need to be able to measure changes in distance down to 1 part in 10 million and changes in time down to 1 part in 100 million!! As you can imagine, it is difficult to make measurements with this level of accuracy.

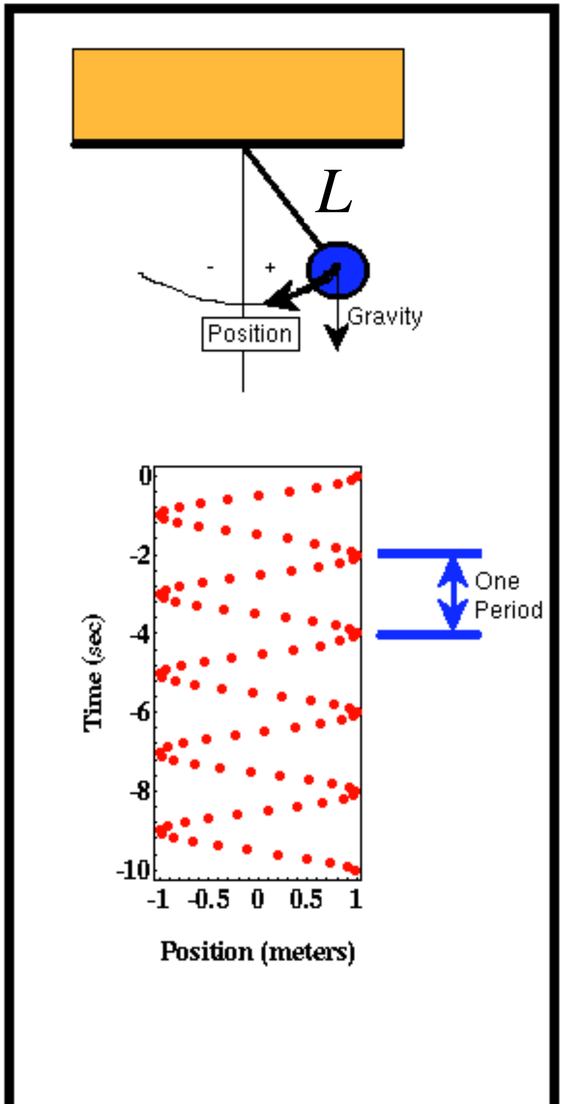


$$\text{Distance} = \frac{1}{2} g \text{ time}^2$$

$$g = \frac{2 \text{ Distance}}{\text{time}^2}$$

Pendulum Measurements

Another method by which we can measure the acceleration due to gravity is to observe the oscillation of a pendulum, such as that found on a grandfather clock. Contrary to popular belief, Galileo Galilei made his famous gravity observations using a pendulum, not by dropping objects from the Leaning Tower of Pisa.



If we were to construct a simple pendulum by hanging a mass from a rod and then displace the mass from vertical, the pendulum would begin to oscillate about the vertical in a regular fashion. The relevant parameter that describes this oscillation is known as the period* of oscillation.

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*The period of oscillation is the time required for the pendulum to complete one cycle in its motion. This can be determined by measuring the time required for the pendulum to reoccupy a given position. In the example shown to the left, the period of oscillation of the pendulum is approximately two seconds.

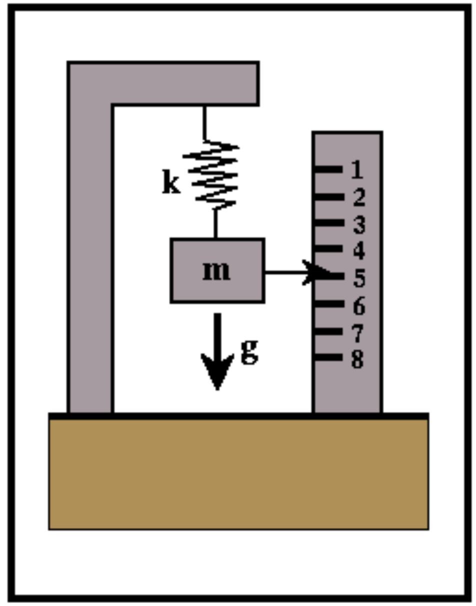
The reason that the pendulum oscillates about the vertical is that if the pendulum is displaced, the force of gravity pulls down on the pendulum. The pendulum begins to move downward. When the pendulum reaches vertical it can't stop instantaneously. The pendulum continues past the vertical and upward in the opposite direction. The force of gravity slows it down until it eventually stops and begins to fall again. If there is no friction where the pendulum is attached to the ceiling and there is no wind resistance to the motion of the pendulum, this would continue forever.

Because it is the force of gravity that produces the oscillation, one might expect the period of oscillation to differ for differing values of gravity. In particular, if the force of gravity is small, there is less force pulling the pendulum downward, the pendulum moves more slowly toward vertical, and the observed period of oscillation becomes longer. Thus, by measuring the period of oscillation of a pendulum, we can estimate the gravitational force or acceleration.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

It can be shown that the period of oscillation of the pendulum, *T*, is proportional to one over the square root of the gravitational acceleration, *g*. The constant of proportionality, *k*, depends on the physical characteristics of the pendulum such as its length and the distribution of mass about the pendulum's pivot point.

Mass and Spring Measurements

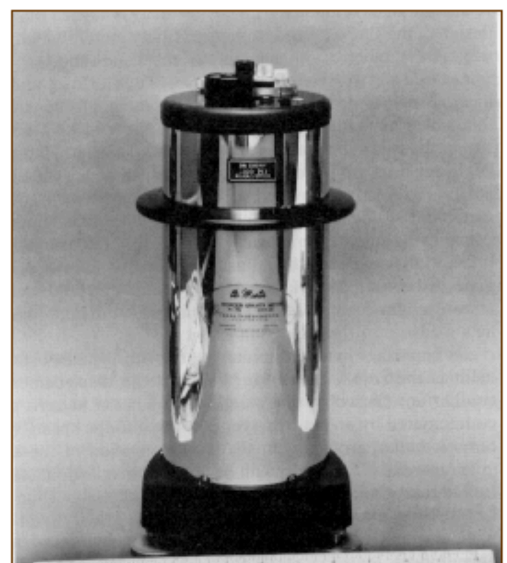


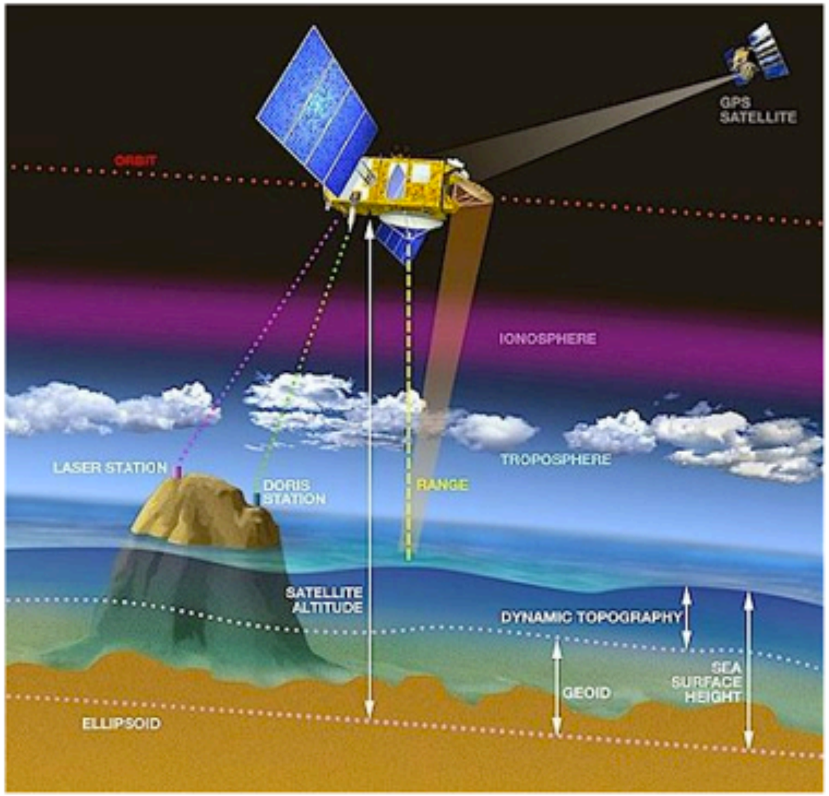
The most common type of gravimeter* used in exploration surveys is based on a simple mass-spring system. If we hang a mass on a spring, the force of gravity will stretch the spring by an amount that is proportional to the gravitational force. It can be shown that the proportionality between the stretch of the spring and the gravitational acceleration is the magnitude of the mass hung on the spring divided by a constant, *k*, which describes the stiffness of the spring. The larger *k* is, the stiffer the spring is, and the less the spring will stretch for a given value of gravitational acceleration.

$$x = \frac{m\,g}{k}$$

Like pendulum measurements, we can not determine *k* accurately enough to estimate the absolute value of the gravitational acceleration to 1 part in 40 million. We can, however, estimate variations in the gravitational acceleration from place to place to within this precision. To be able to do this, however, a sophisticated mass-spring system is used that places the mass on a beam and employs a special type of spring known as a *zero-length* spring.

Instruments of this type are produced by several manufacturers; *LaCoste and Romberg*, *Texas Instruments (Worden Gravity Meter)*, and *Scintrex*. Modern gravimeters are capable of measuring changes in the Earth's gravitational acceleration down to 1 part in 100 million. This translates to a precision of about 0.01 mgal. Such a precision can be obtained only under optimal conditions when the recommended field procedures are carefully followed.



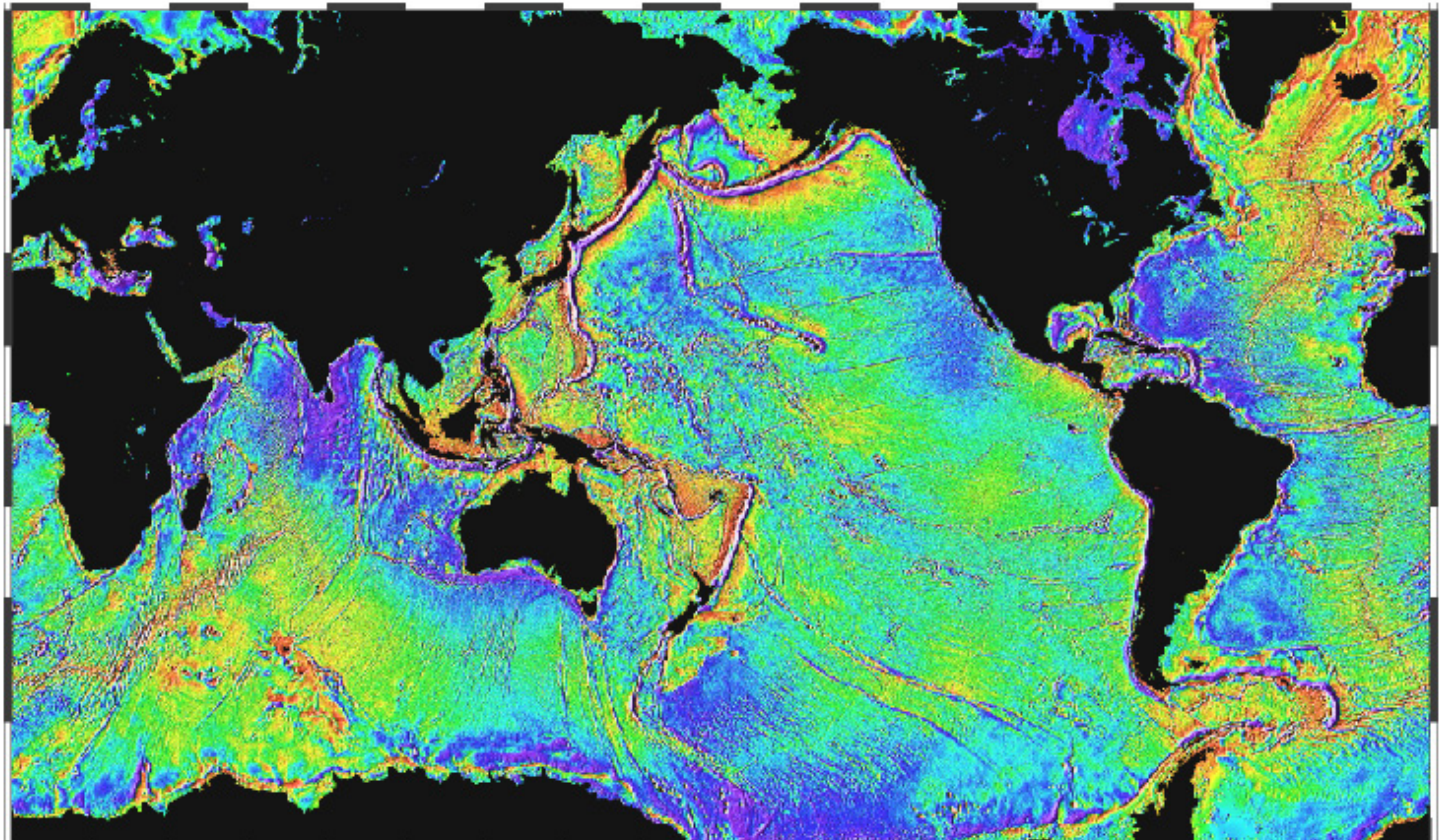


The principle of altimetry
(Credits CNES/D. Ducros)

Satellite-to-surface distance: Range

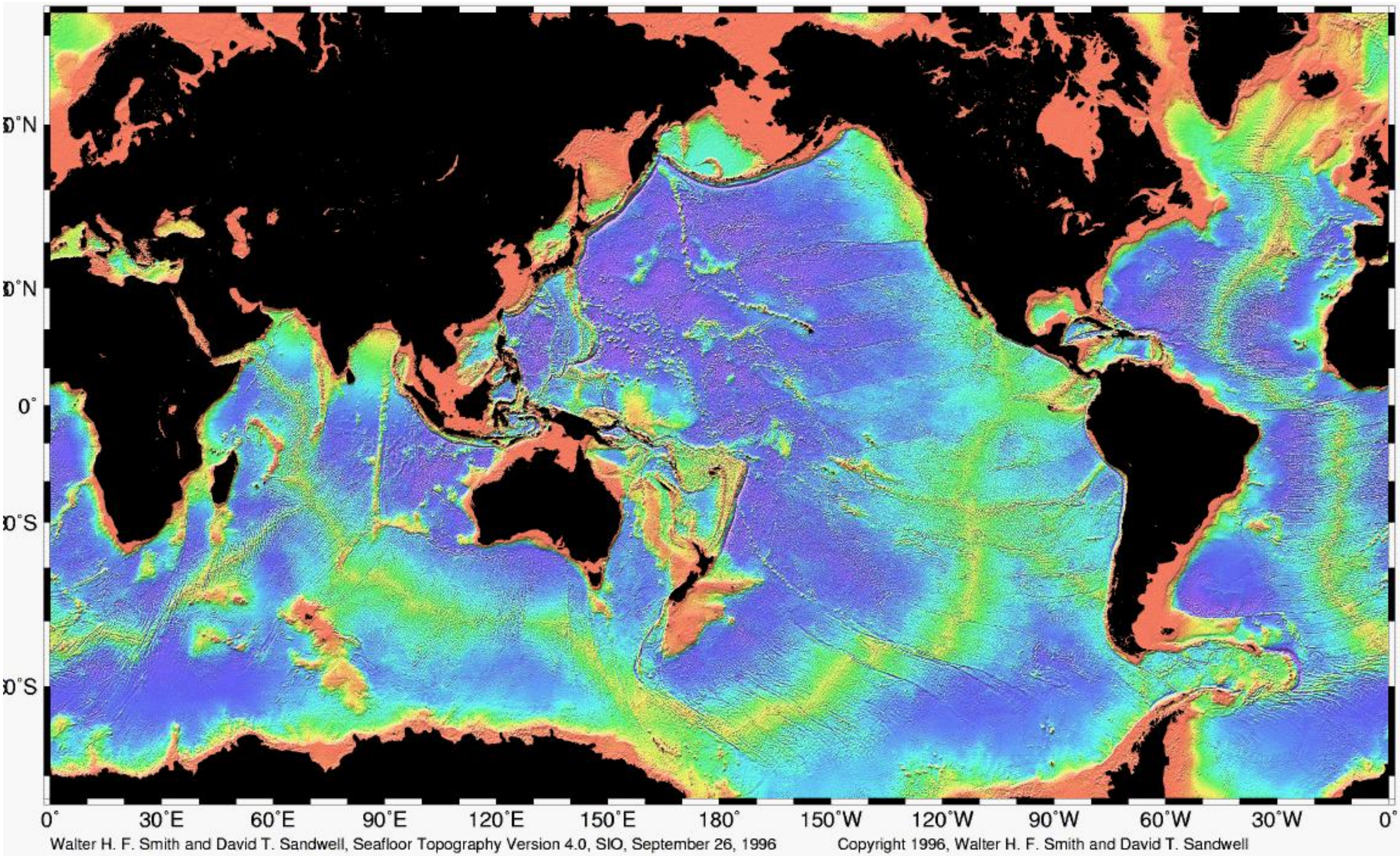
Radar altimeters on board the satellite transmit signals at high frequencies (over 1,700 pulses per second) to Earth, and receive the echo from the surface (the "waveform"). (the 'waveform'). This is analysed to derive a precise measurement of the time taken to make the round trip between the satellite and the surface. This time measurement, scaled to the speed of light (the speed at which electromagnetic waves travel), yields a **range R** measurement (see From radar pulse to altimetry measurements for Further details).
However, as electromagnetic waves travel through the atmosphere, they can be decelerated by water vapour or ionisation. Once these phenomena have been corrected for, the final range can be estimated with great accuracy (see data processing).
The ultimate aim is to measure surface height relative to a terrestrial reference frame. This requires independent measurements of the satellite's orbital trajectory, i.e. exact latitude, longitude and altitude coordinates.

A straightforward calculation converts sea surface height to gravity



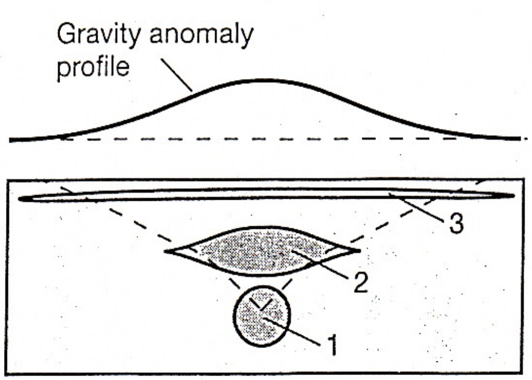
FREE-AIR GRAVITY, BASED ON GEOSAT AND ERS-1 SATELLITE RADAR ALTIMETRY (VERSION 7.2, D. SANDWELL & W. SMITH).

“Predicted” topography based on satellite radar altimetry measurements combined with shipboard bathymetry measurements



Use gravity to map hidden structures

The interpretation is ambiguous unless you have an independent way of mapping the shape of the source



Three shapes, all have same anomaly

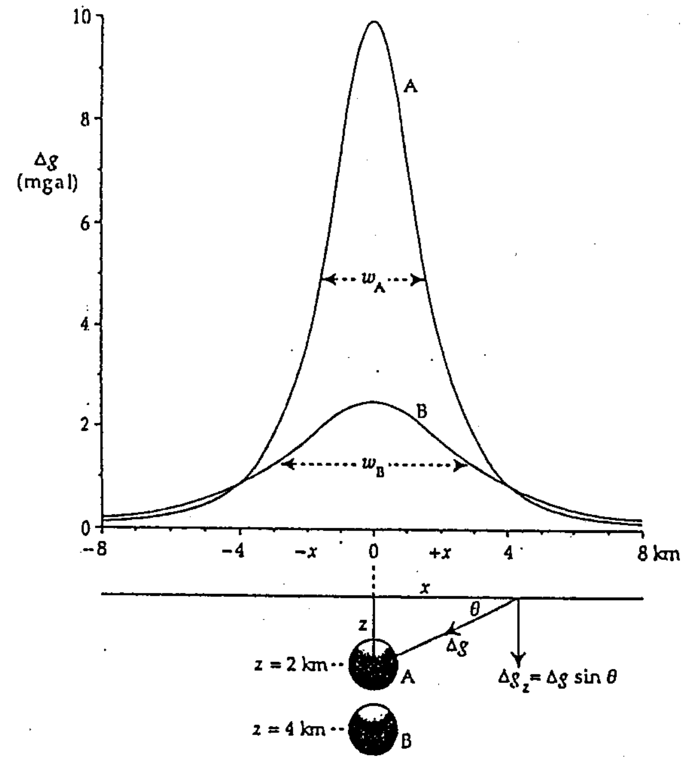


Fig. 2.47 Gravity anomalies for buried spheres with the same radius R and density contrast $\Delta\rho$ but with their centers at different depths z below the surface. The anomaly of the deeper sphere B is flatter and broader than the anomaly of the shallower sphere A.

Lowrie (1997)

gravity anomalies: compare observed (g_0) to "expected" values (γ_0)

γ_0 = gravity predicted on reference ellipsoid

We use gravity anomalies to study isostatic compensation

Free-air correction

Accounts for the $1/r^2$ decrease in gravity with distance from the center of the Earth. A given gravity measurement was made at an elevation h, not at sea level, recall:

$$g = \frac{GM_E}{R_E^{\,2}}$$

$$g = \frac{GM}{r^2} \qquad \frac{\partial g}{\partial r} = -\frac{2g}{r} = -.31\,\text{mgal/m}$$

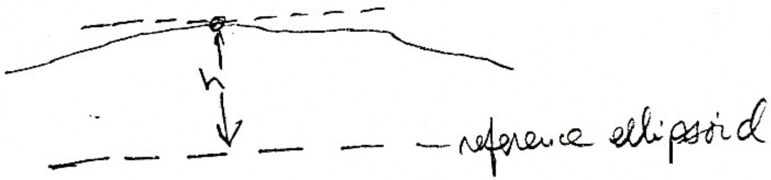
$$\Delta g_{FA} = g_0 + \underline{\underline{.31\,h}} - \gamma_0$$ free-air anomaly

Free-air gravity anomalies

Over oceans, no free-air correction is needed

Bouguer Anomaly

Correct for mass between observation point and reference level

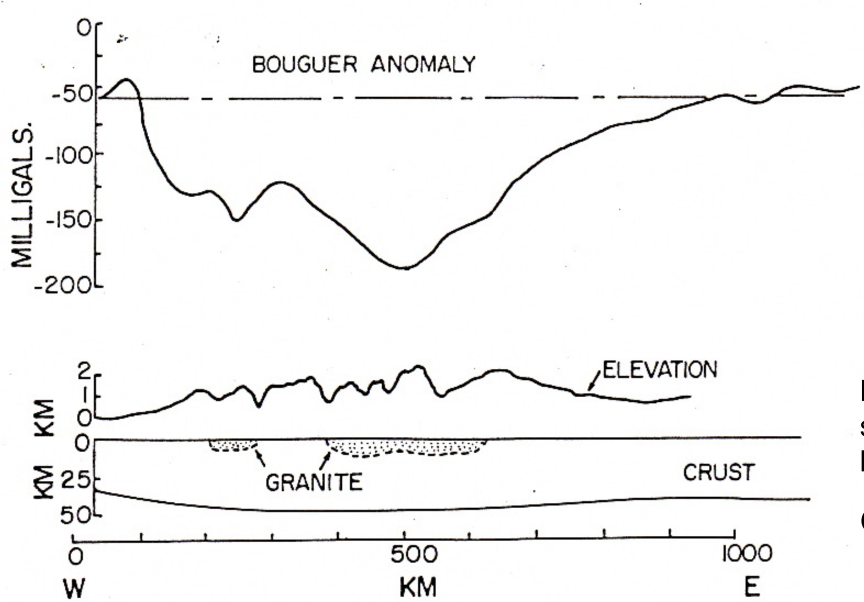


Approximate effect with a slab of thickness h:

Bouguer correction = $2\pi \Delta\rho G h$ $= .04 \Delta\rho$ mgal/m

(or ≈ .1 mgal/m over continent)

Bouguer Anomaly = $\Delta g_B := \Delta g_{FA} - 2\pi \Delta\rho G h$



Note: VE is not the same above and below sea level

Garland (1971)

Pierre Bouguer was one of the French surveyors in the Andes in the 1740's

The bouguer anomaly is negative over a mountain range and reflects the low-density crustal root associate with Airy isostatic compensation

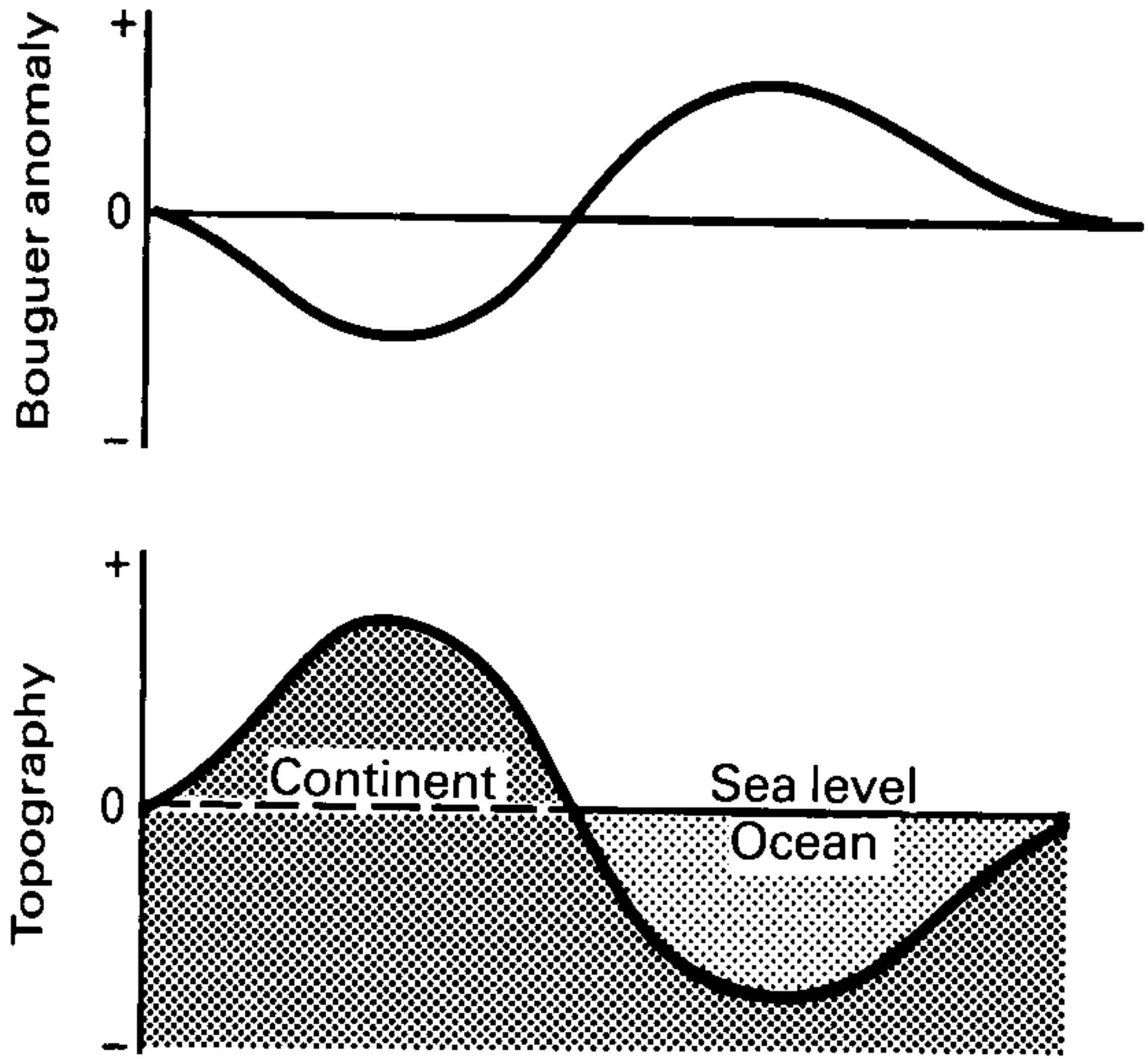


Fig. 2.28 Inverse correlation of Bouguer anomalies with topography indicating its isostatic compensation.

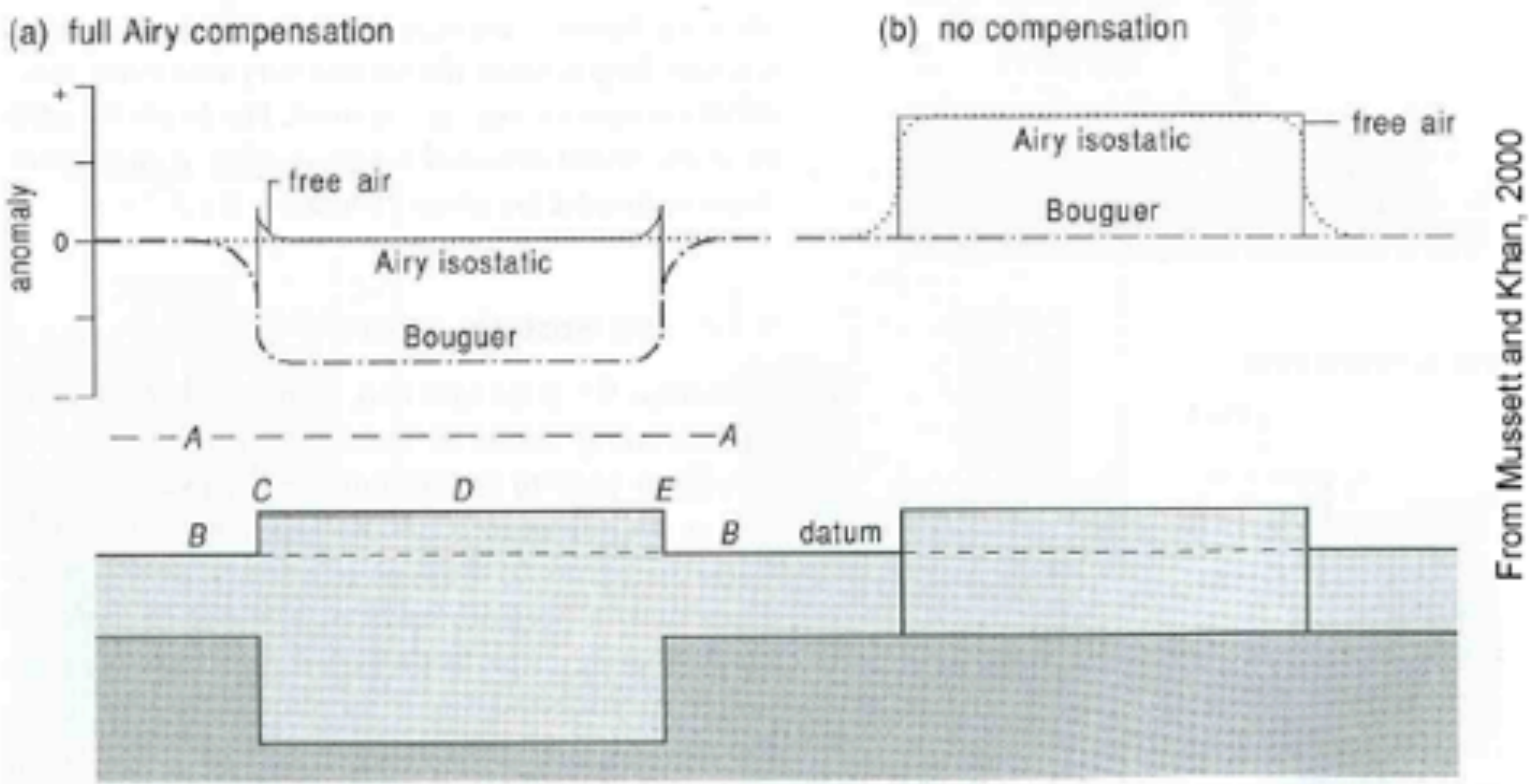
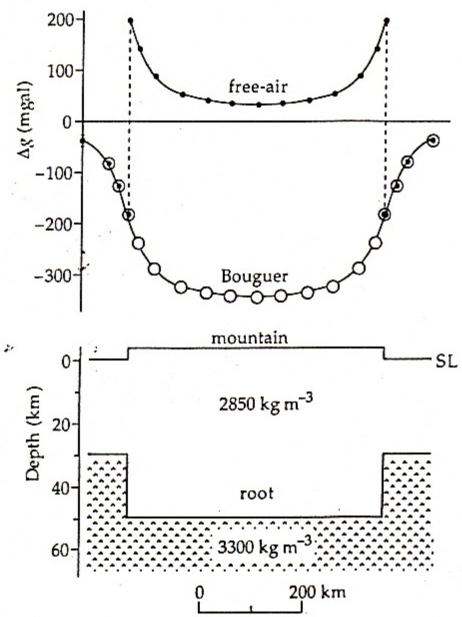
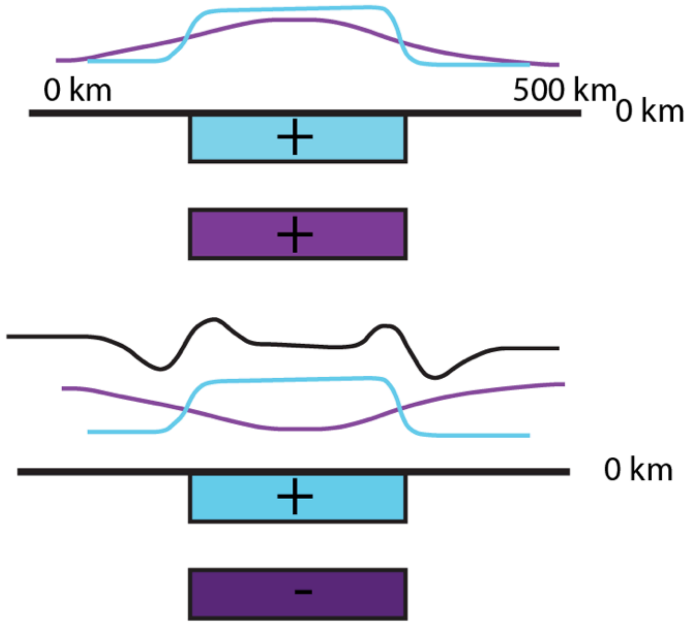


Figure 9.11 Anomalies with and without isostatic compensation.

Another way of thinking of it:
With the Bouguer anomaly you only see the effect of the root (think of example shown earlier)



KK&V (2009)



Simple model from earlier:

Let's move zero level to bottom of top body

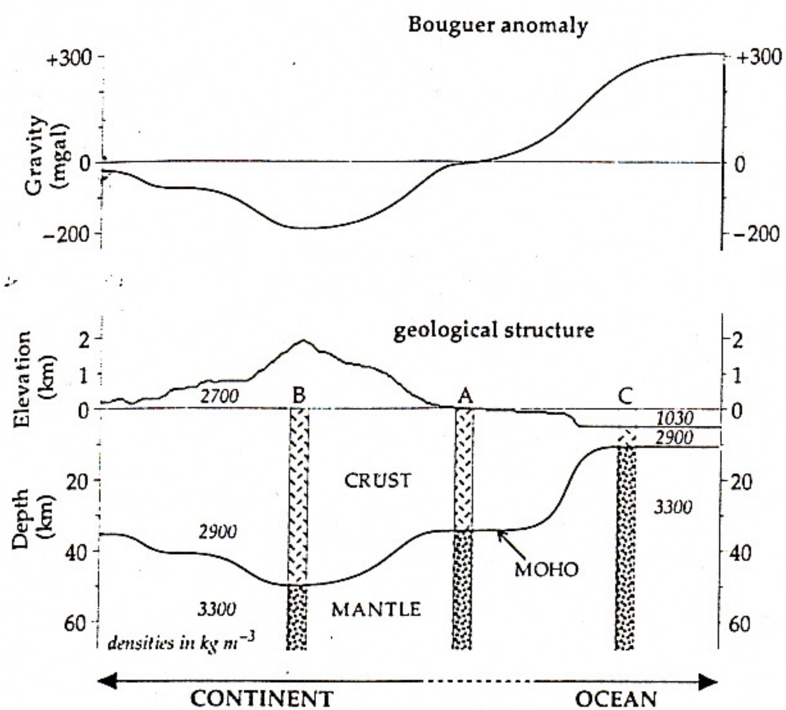
Composite anomaly = free-air anomaly
Bouguer correction = effect of top body
Bouguer anom = composite – Bouguer correction

Bouguer anomaly over the oceans

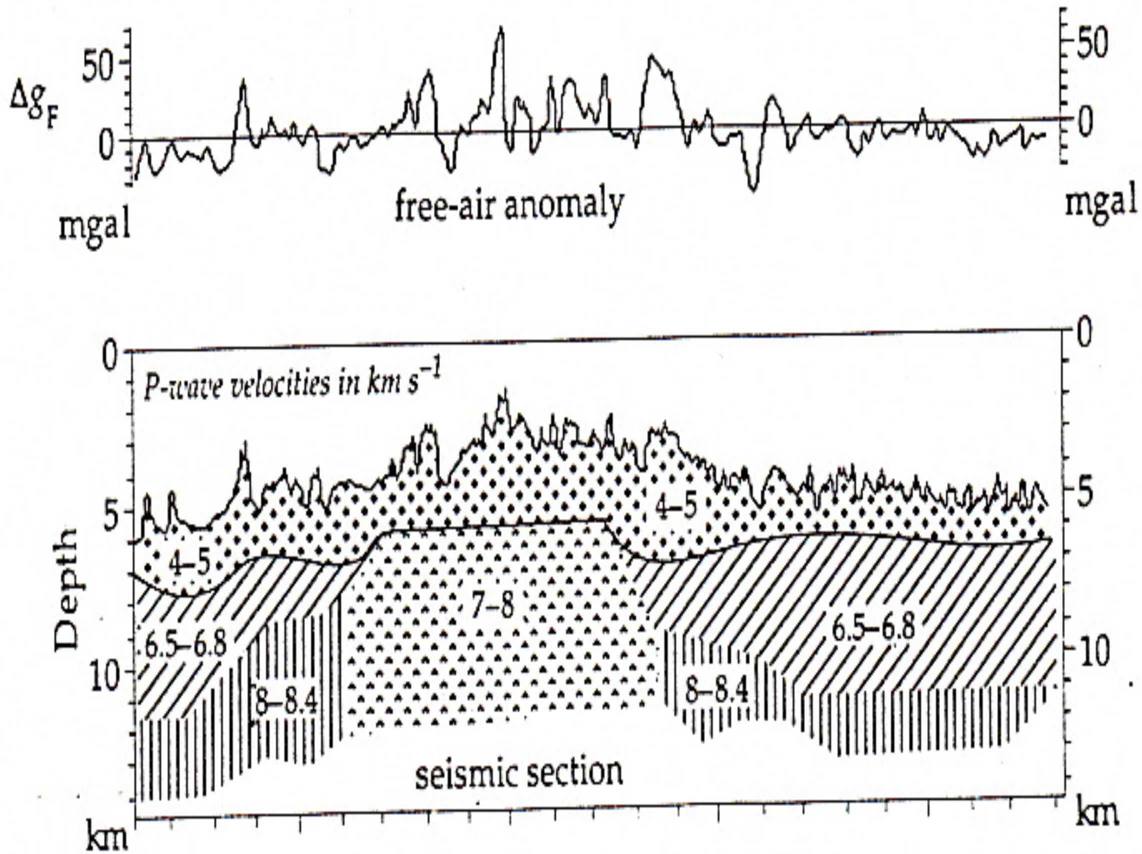
over water: $\Delta g_B = \Delta g_{FA} + 2\pi \Delta\rho G h$

Replace water with rock with same density as adjacent continent

Reveals “anti-root” (thin crust) beneath oceans



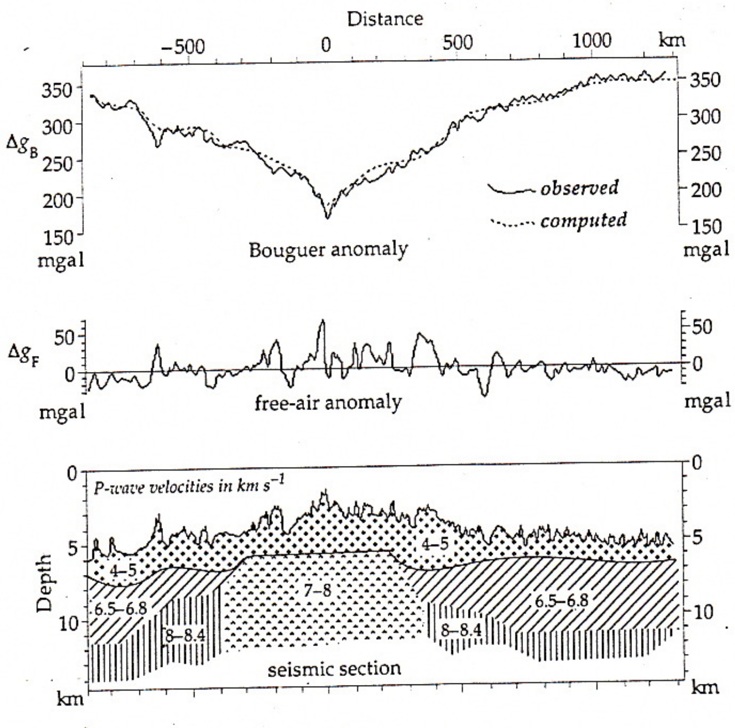
Lowrie (1997)

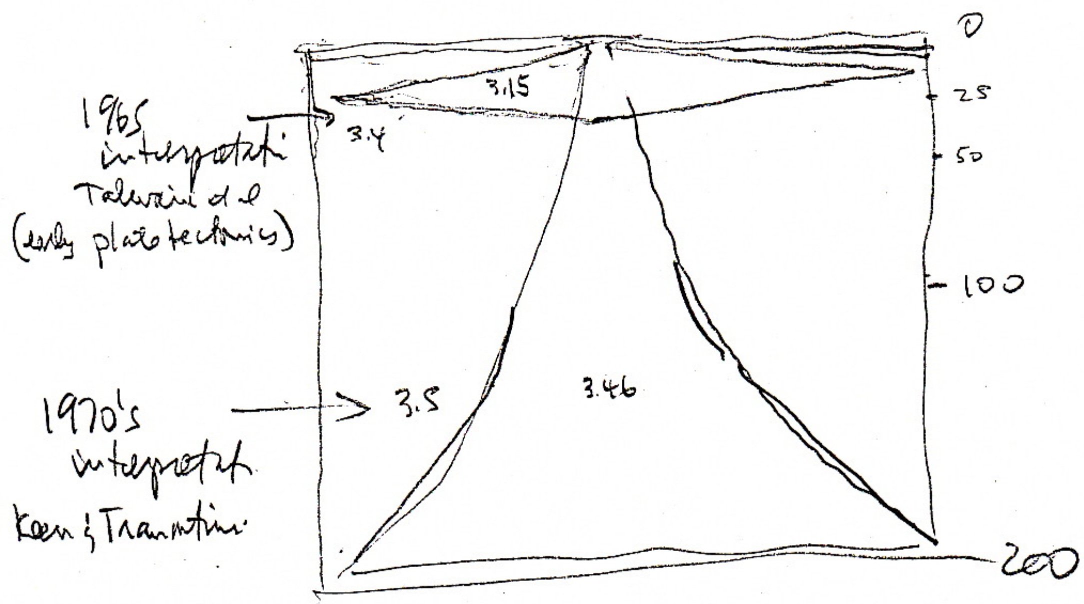


What does the Bouguer anomaly look like over a mid-ocean spreading center like the Mid-Atlantic ridge?

And what does that tell us?

Note: topography gets shallower but free-air anomaly stays near zero (on average)





Bouguer anomaly drops by 200 mgals over the mid-ocean ridge:

The mid-ocean ridges are generally isostatically compensated:
Free-air anomaly is roughly zero

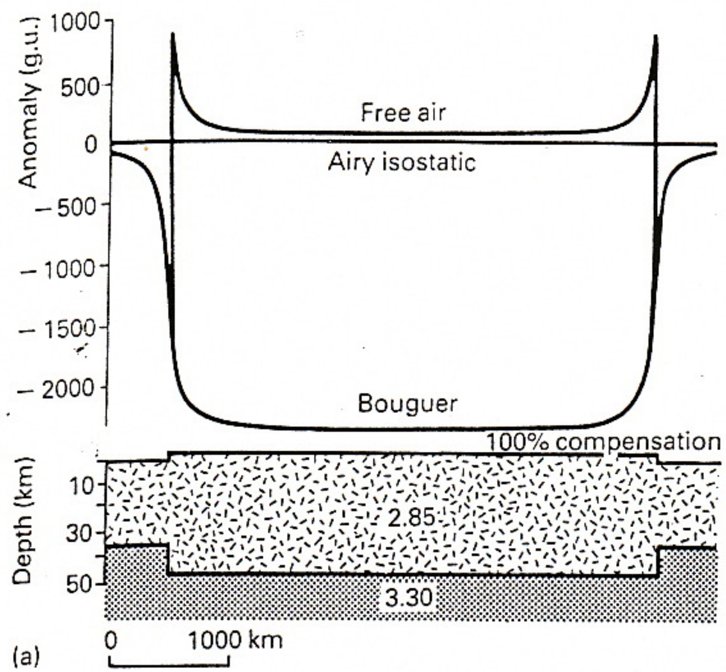
Since the ridge axis is shallower than flanks, the Bouguer correction is smaller, so get a drop in the Bouguer anomaly centered on ridge axis

Reveals body of warm (less dense) asthenospheric material

Note ambiguity in size of body

Isostatic anomaly = Bouguer anomaly minus the gravity anomaly of an ideal root

$$\Delta g_I = \Delta g_B - A_{root}$$



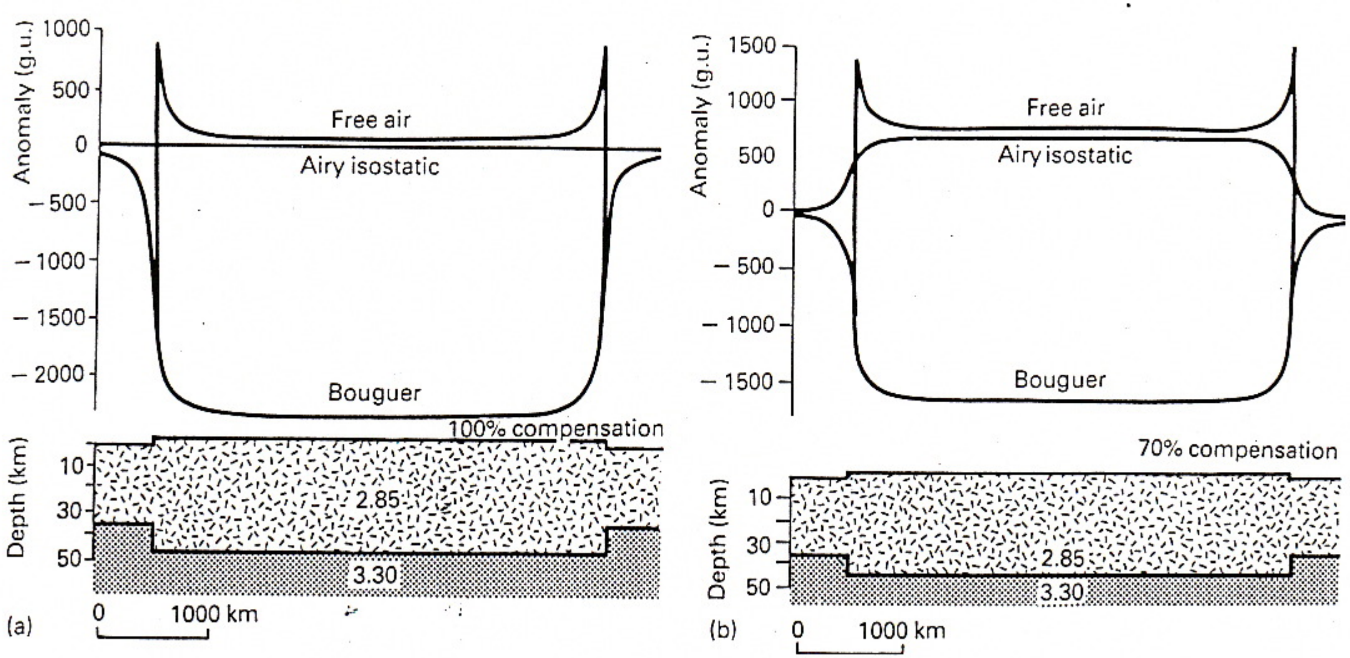
Isostatic anomaly (IA)

You calculate the shape of root by assuming Airy isostasy and typical crustal densities

Isostatic anomaly should be zero if your model is correct

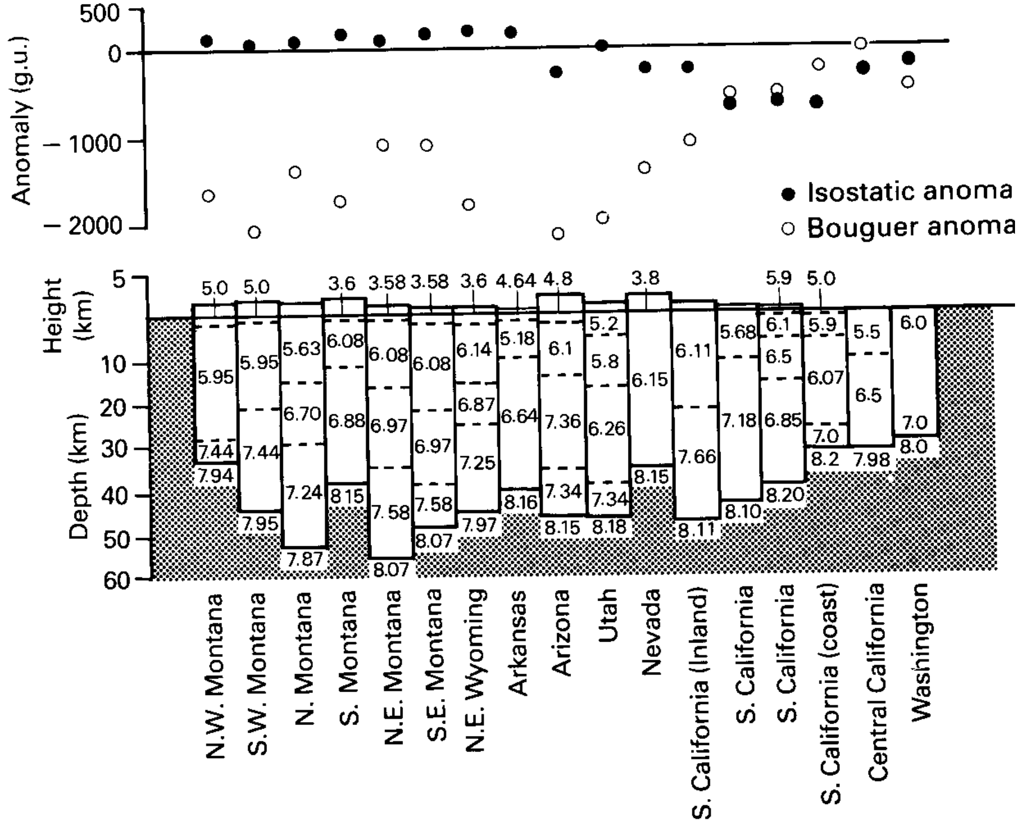
Of course, you don't really know the shape of the root …

If the compensation is not 100% then the isostatic anomaly will not be zero (which is essentially the reason for calculating isostatic anomalies)

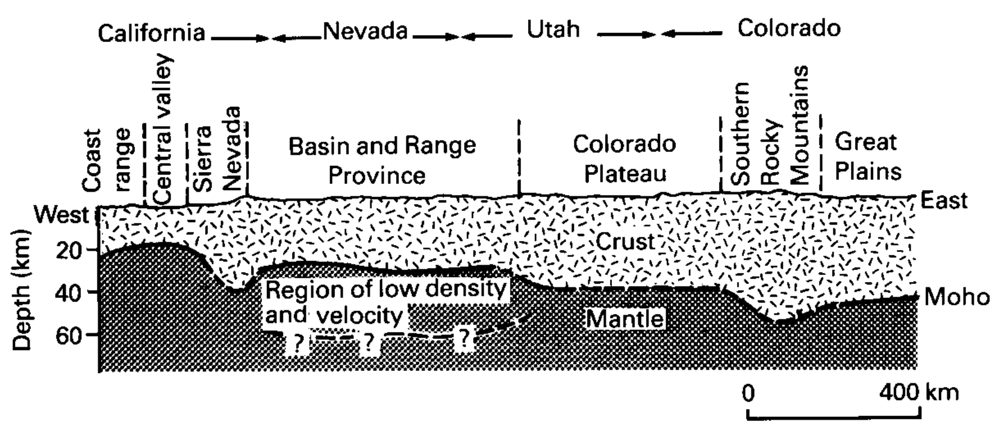


The root on the right is not as large as the one on the left but the “correction” was made for an “ideal” root so the isostatic anomaly is positive

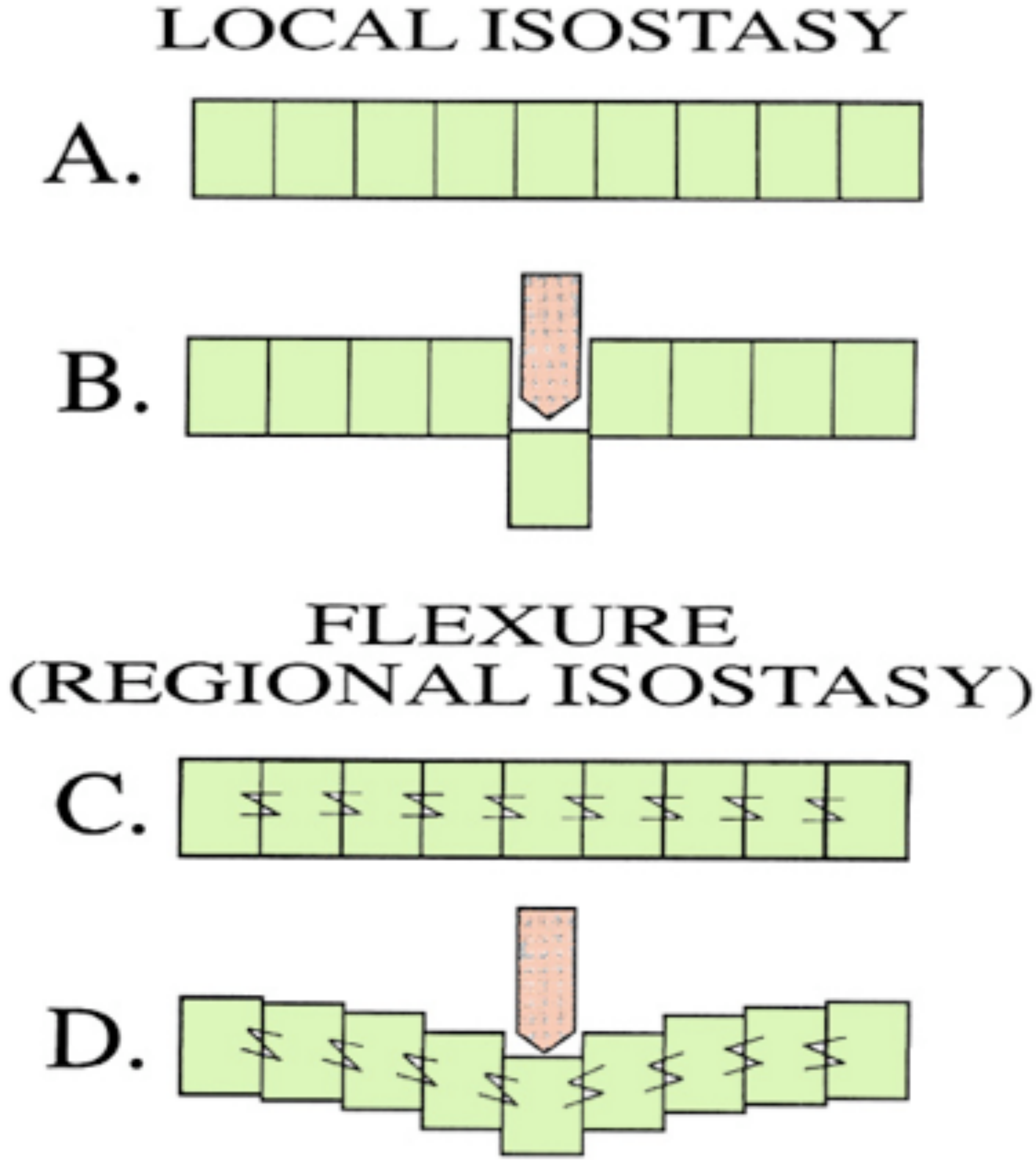
Why might the compensation not be 100% ?

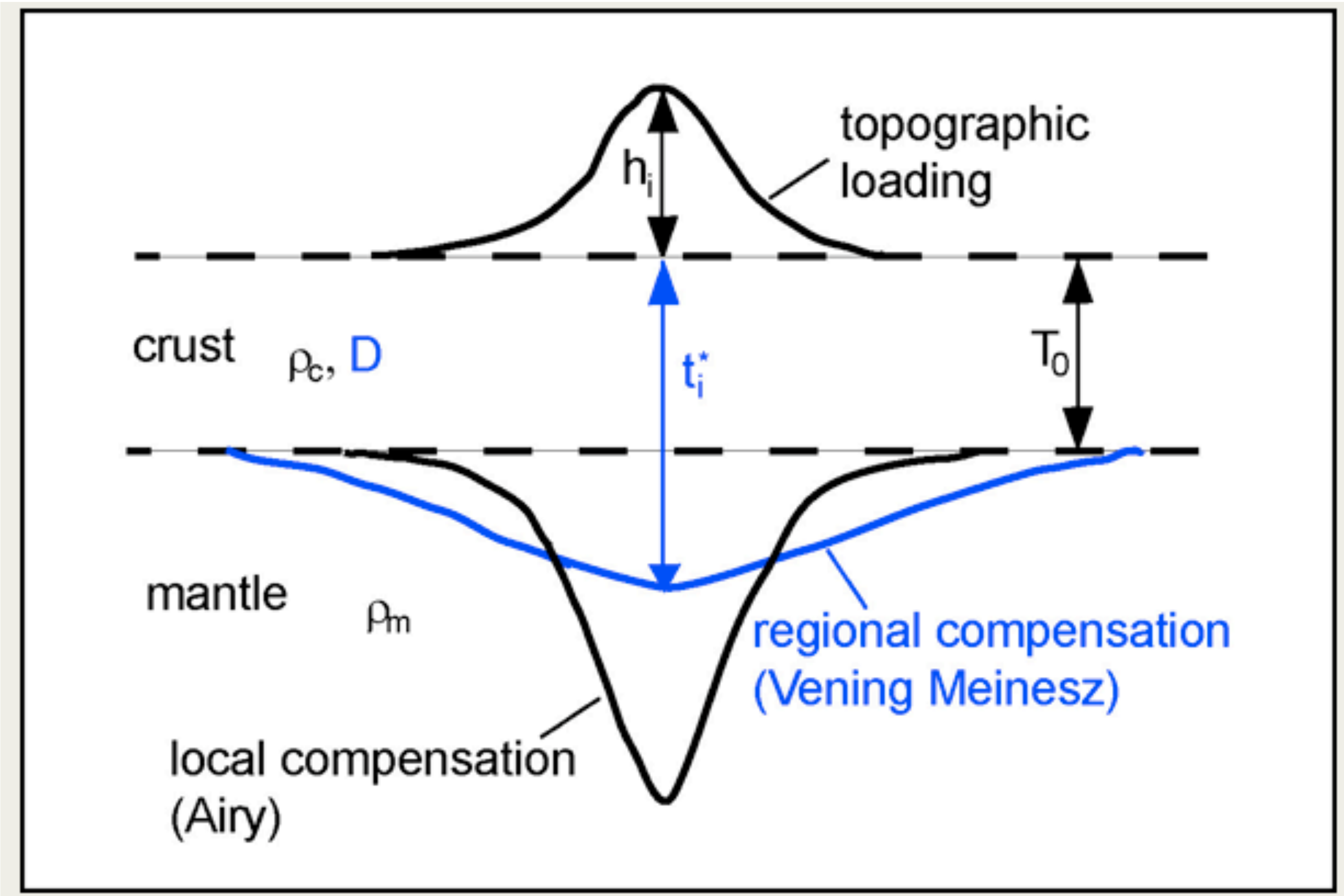


Anomaly (g.u.)
500
0
– 1000
– 2000
● Isostatic anoma
○ Bouguer anoma
Height (km)
Depth (km)
5
10
20
30
40
50
60
N.W. Montana
S.W. Montana
N. Montana
S. Montana
N.E. Montana
S.E. Montana
N.E. Wyoming
Arkansas
Arizona
Utah
Nevada
S. California (Inland)
S. California
S. California
S. California (coast)
Central California
Washington



California
Nevada
Utah
Colorado
Coast range
Central valley
Sierra Nevada
Basin and Range Province
Colorado Plateau
Southern Rocky Mountains
Great Plains
West
East
Crust
Moho
Region of low density and velocity
Mantle
Depth (km)
20
40
60
0
400 km


LOCAL ISOSTASY
A.
B.
FLEXURE
(REGIONAL ISOSTASY)
C.
D.



topographic
loading
h_i
crust
ρ_c, D
t_i^*
T_0
mantle
ρ_m
regional compensation
(Vening Meinesz)
local compensation
(Airy)

Flexure

Earths's crust acts as a rigid plate of finite thickness and deforms "elastically" in response to a load.

Before plate tectonics, these concepts were used to analyze the effects of loading and unloading of glaciers, mountains, lakes, deltas.

The "plate" of plate tectonics

Flexure generates large gravity anomalies because the load is not compensated locally.

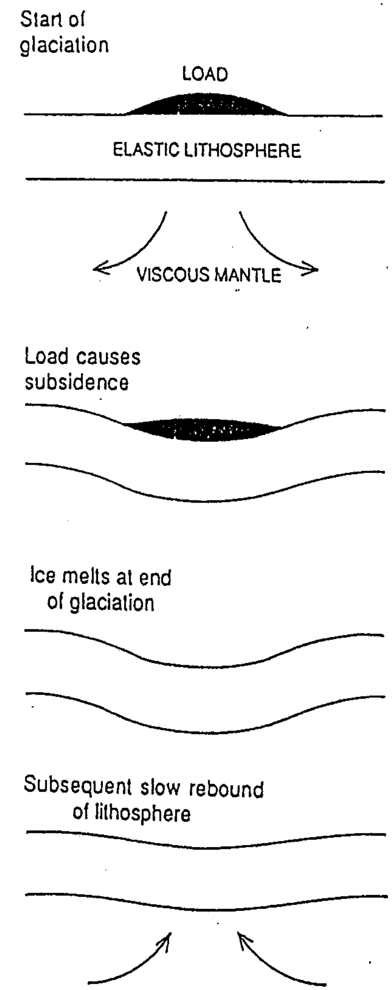


Figure 5.16. The deformation and uplift which occur as a result of loading and unloading of an elastic lithospheric plate overlying a viscous mantle.

Fowler (1990)

Regional Isostatic Compensation

- The effect of the load can therefore be distributed over a wide area, depending on the flexural rigidity of the supporting material.
- A common model of regional isostatic compensation is that of an elastic plate that is bent by topographic and subsurface loads.
- The *flexural rigidity* of the plate (D) determines the degree to which the plate supports the load.
    - A thin, weak diving board bends greatly, especially near the diver.
    - A thicker board of the same material behaves more rigidly – the diver causes a smaller deflection.
    - The *flexural rigidity* (resistance to bending) thus depends on the *elastic thickness* of each board

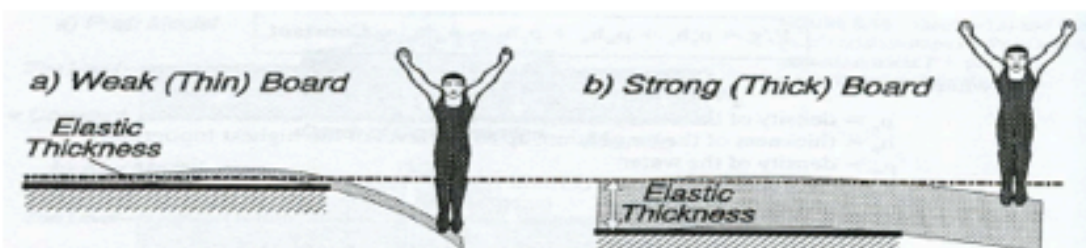


FIGURE 8.22 *Flexural rigidity.* a) A thin diving board (small elastic thickness) has low flexural rigidity. b) A thick board (large elastic thickness) has high flexural rigidity.

From Lillie, 1999

Regional Isostatic Compensation

The deflection of a 2-D plate due to a linear load depressing the plates surface (assuming a fluid material below) is given by:

$$D(d^4w/d^4x)+(\rho_b-\rho_a)gw=q(x)$$

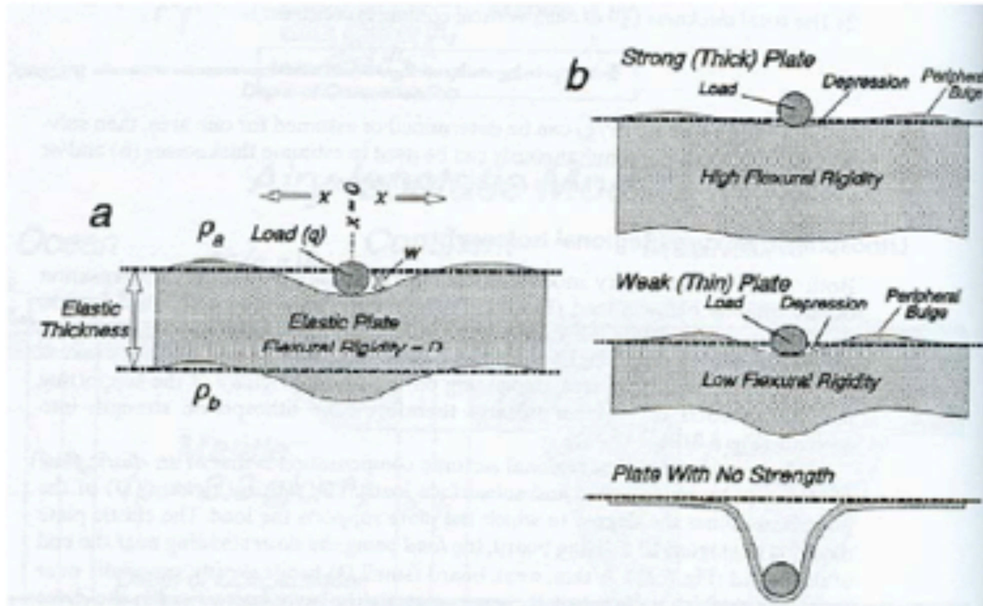


FIGURE 8.23 a) Parameters for two-dimensional model of a plate flexed by a linear load. Both the plate and load extend infinitely in and out of the page. See text for definition of variables. b) Positions of depressions and bulges formed on the surface of a flexed plate. A strong plate has shallow but wide depressions. The depressions and peripheral bulges have larger amplitudes on a weak plate, but are closer to the load. A very weak plate collapses into local isostatic equilibrium.

From Lillie 1999

Where:

D=Flexural rigidity

w = vertical deflection of plate at x

x = horizontal distance from load to a point on the plates surface

ρ_a = density of material above plate

ρ_b = density of material below plate

g = gravitational acceleration

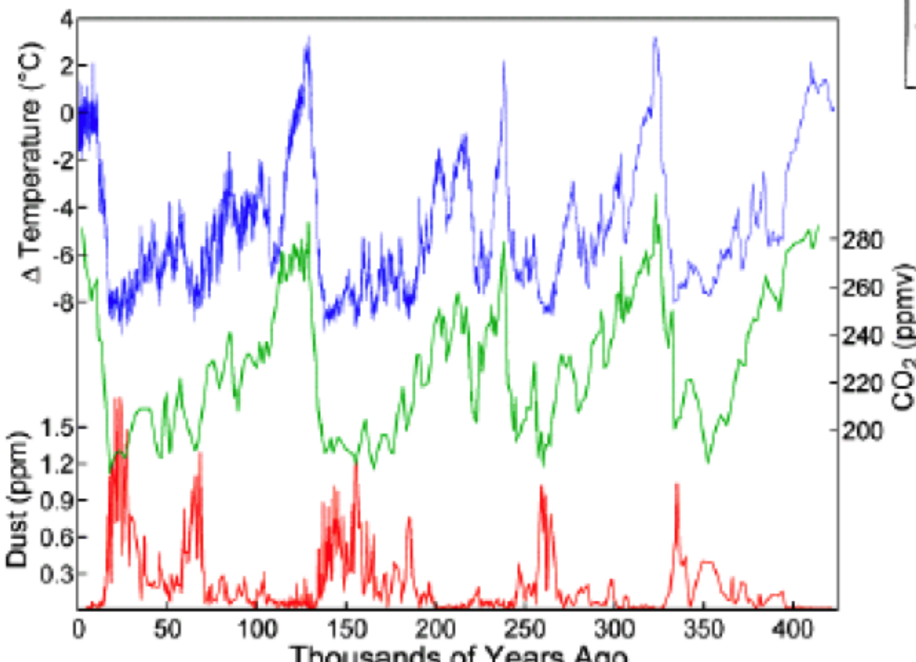
q(x) = load applied to the plate at x

Strong plate: Long wavelength small amplitude (w) deflection.

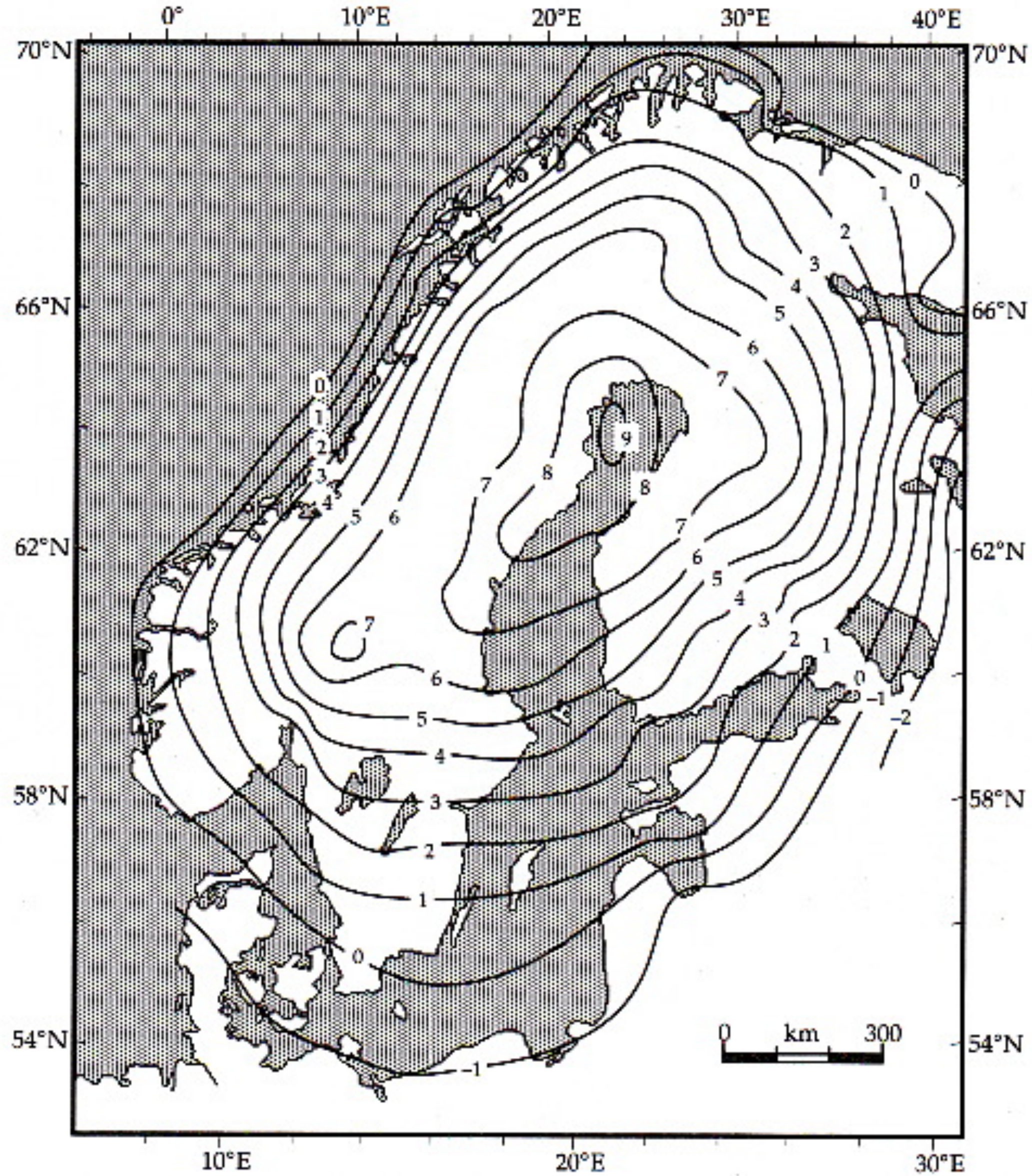
Weak plate: Short wavelength large amplitude deflection.

A special case of flexure and isostasy…

11,000 years ago, large parts of N. Europe and N. America were covered by ice sheets up to 3 km thick.



Ice sheets melted rapidly ~10,000 years ago as a result of global climate change.

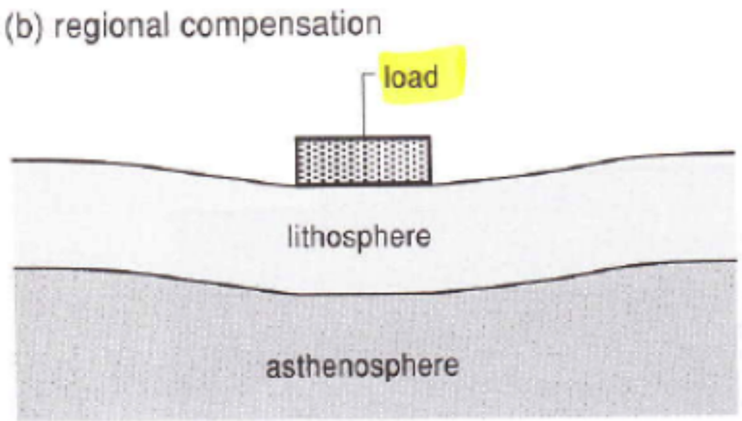


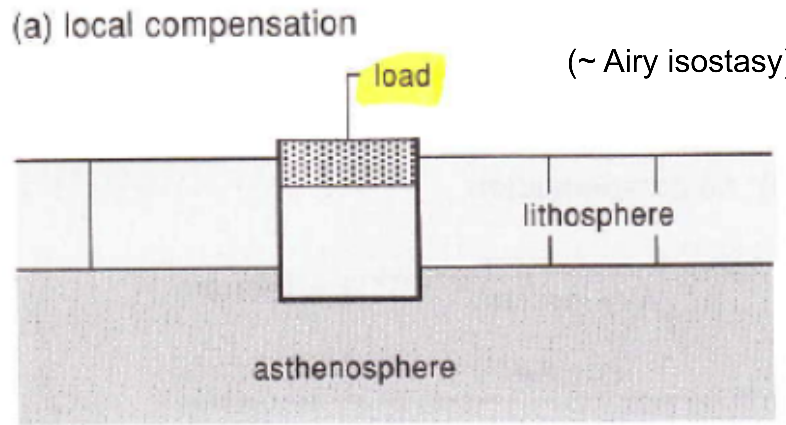
(Lowrie, 1997)

Glacial rebound: the best constraint on the viscosity of the upper mantle:

This figure shows the rate of vertical crustal movement relative to mean sea-level in mm/yr

Local versus regional compensation





(~ Airy isostasy)

The Lithosphere (crust) is strong enough to support the load (weight) of the mountain. But, the lithospheric strength is finite and the surface of the lithosphere is bowed down 'regionally' to support the load.

The Lithosphere (crust) is NOT strong enough to support the load (weight) of the mountain. In fact, in the limit, the lithosphere is broken on either side of the load and has a near zero strength. The load is supported by the hydrostatic pressure of the asthenosphere pushing on the bottom of the loaded block.

(Dueker, U. Wyoming)

Regional compensation of Hawaiian Island Chain

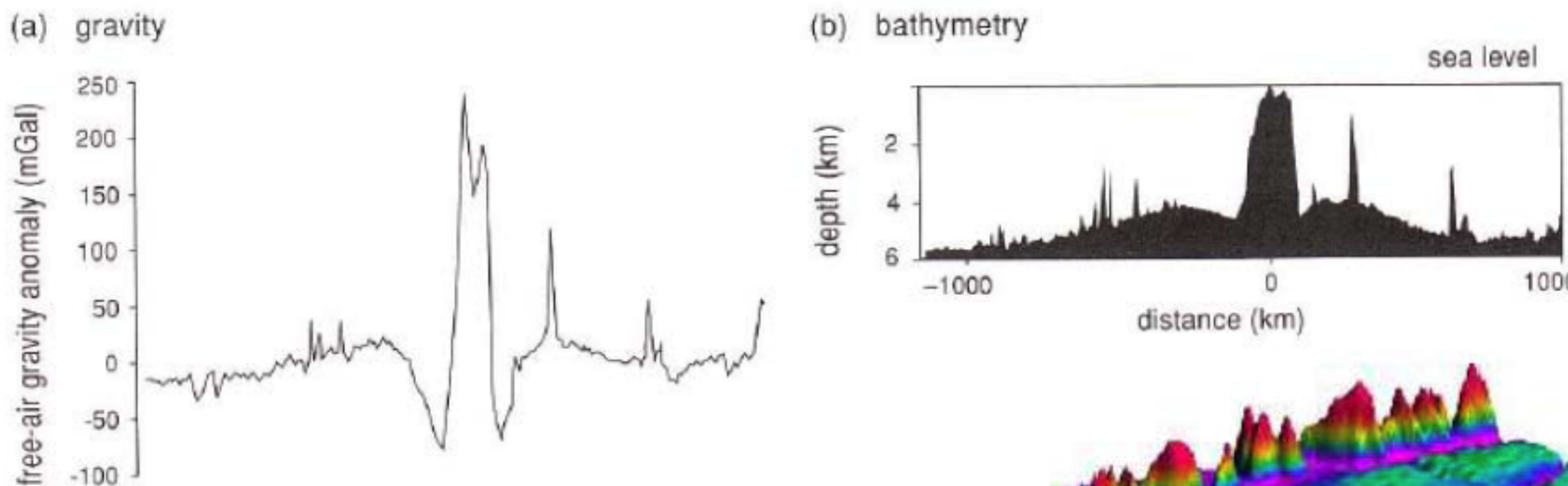
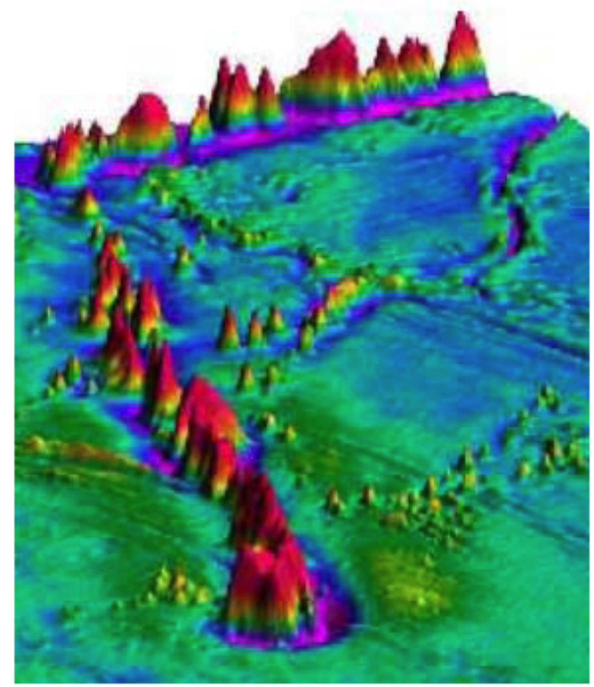
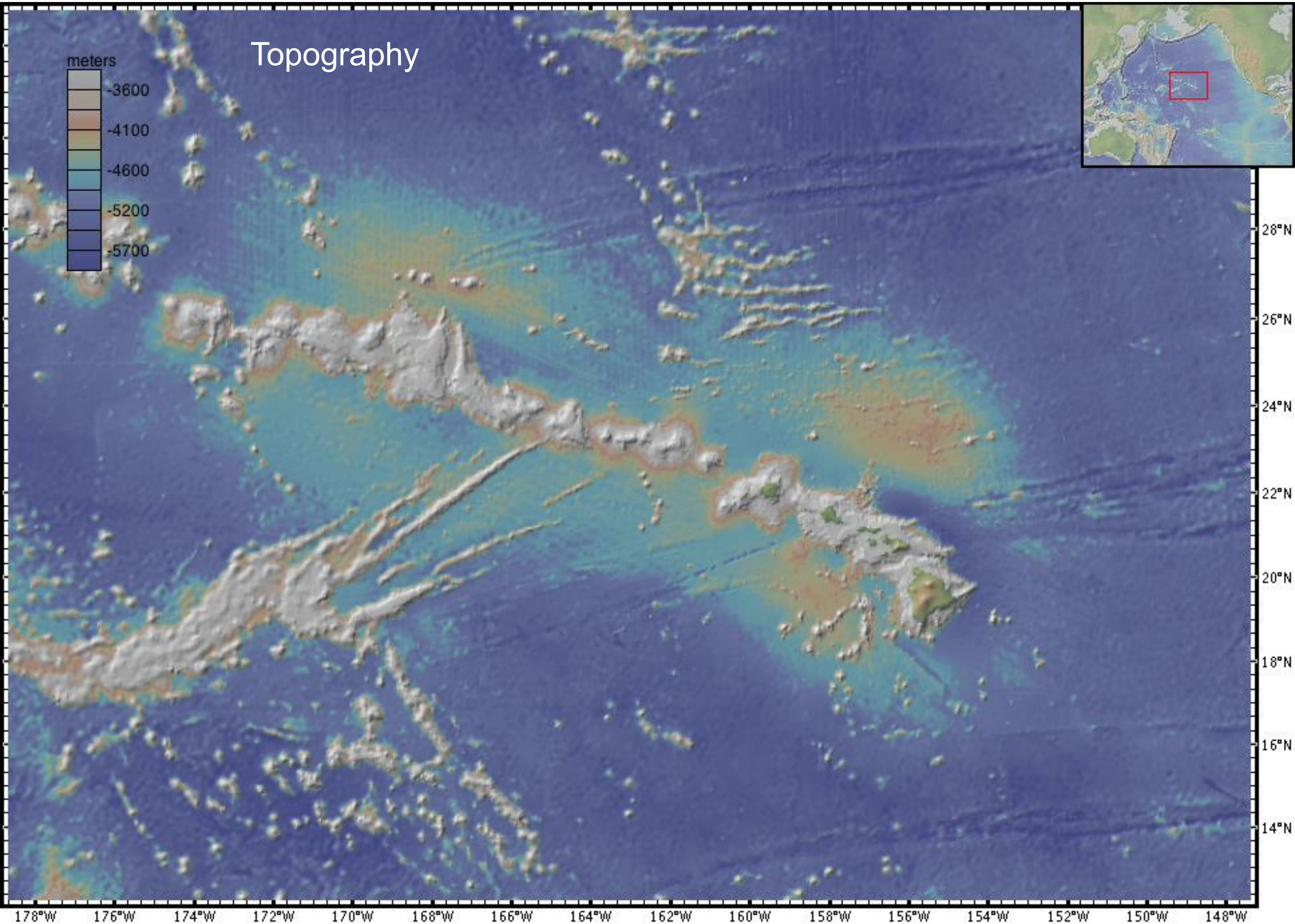


Figure 9.12 Bathymetry and gravity anomaly across Hawaiian Islands.

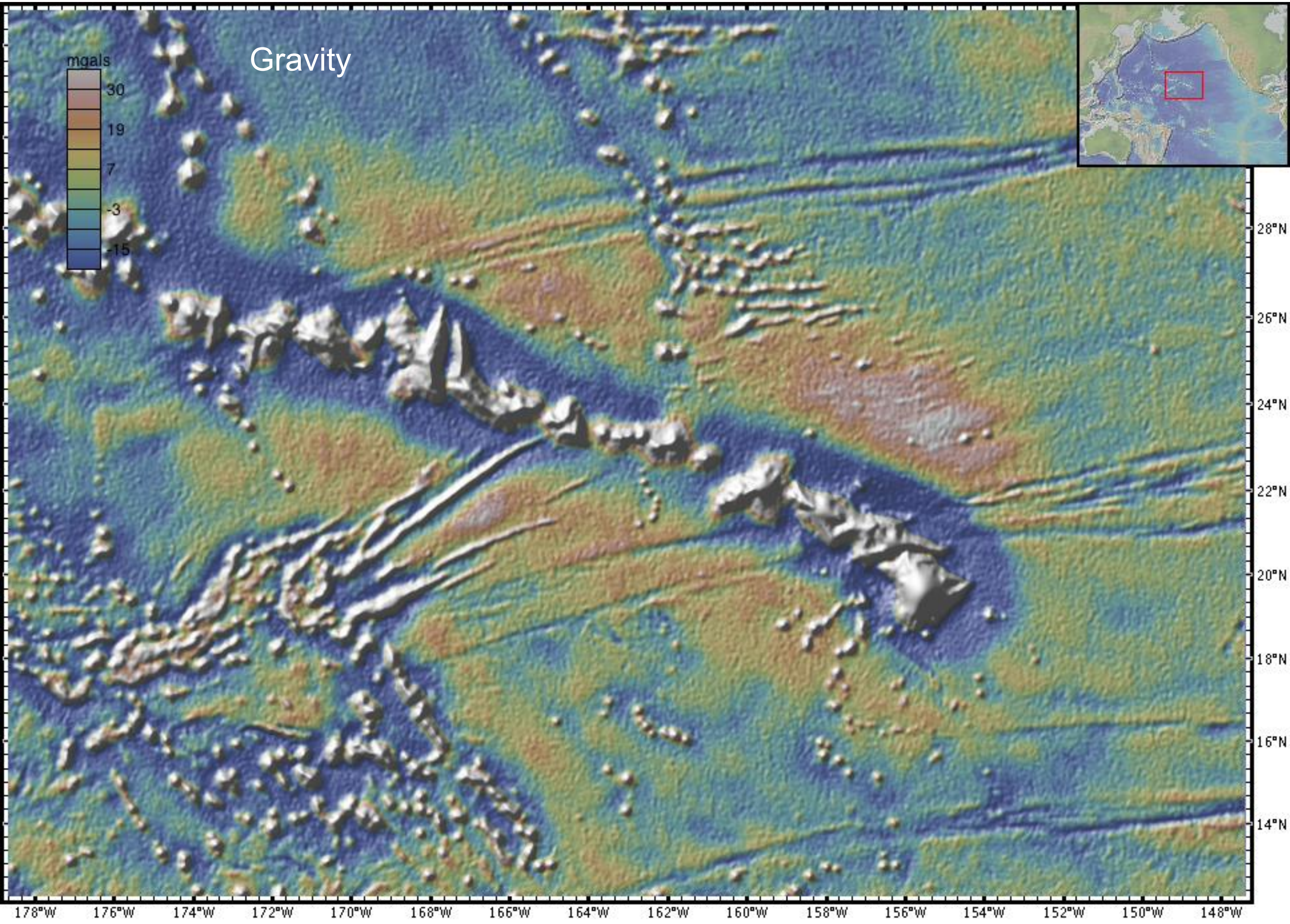


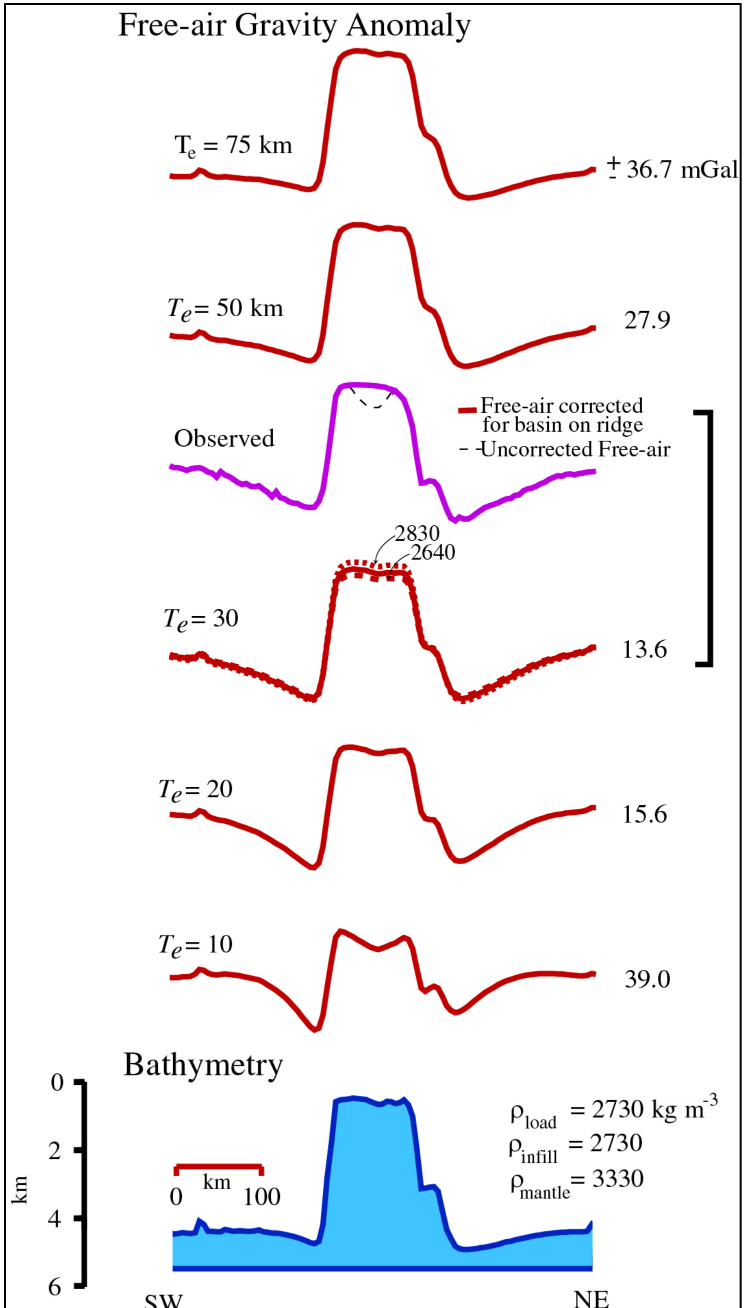
Note the strong correlation between the bathymetry and the free air gravity profile. This is because this topography is NOT isostatically compensated but is 'regional' compensated by the strength of the lithosphere. Also, note the downwarps to either side of the big island caused by the load that down-flexes the lithosphere

(Ken Dueker, U. Wyoming)

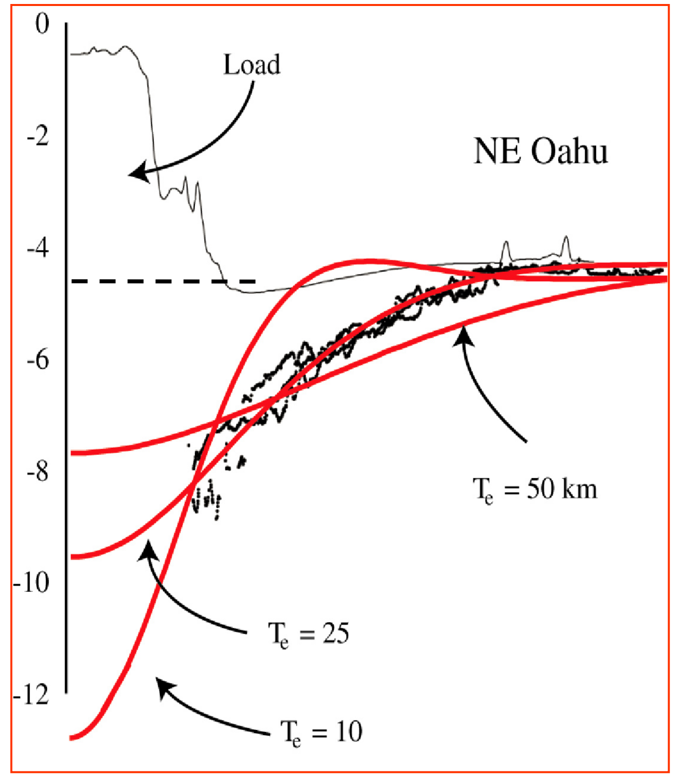


Topography
meters
-3600
-4100
-4600
-5200
-5700
28°N
26°N
24°N
22°N
20°N
18°N
16°N
14°N
178°W
176°W
174°W
172°W
170°W
168°W
166°W
164°W
162°W
160°W
158°W
156°W
154°W
152°W
150°W
148°W


Gravity
mgals
30
19
7
-3
-15
28°N
26°N
24°N
22°N
20°N
18°N
16°N
14°N
178°W
176°W
174°W
172°W
170°W
168°W
166°W
164°W
162°W
160°W
158°W
156°W
154°W
152°W
150°W
148°W



T_e can be estimated by comparing the amplitude and wavelength of the observed gravity anomaly to the predicted anomaly based on an elastic plate model.



Note that T_e is much smaller than the total thickness of the lithosphere