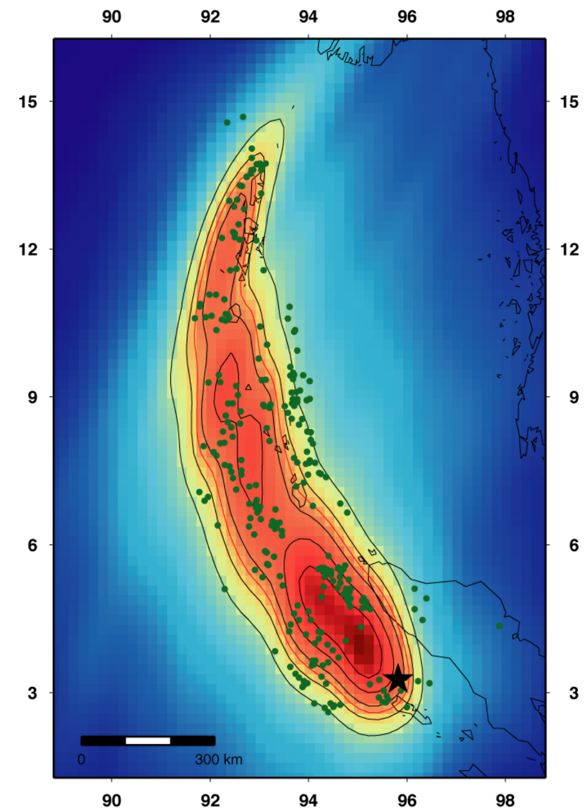
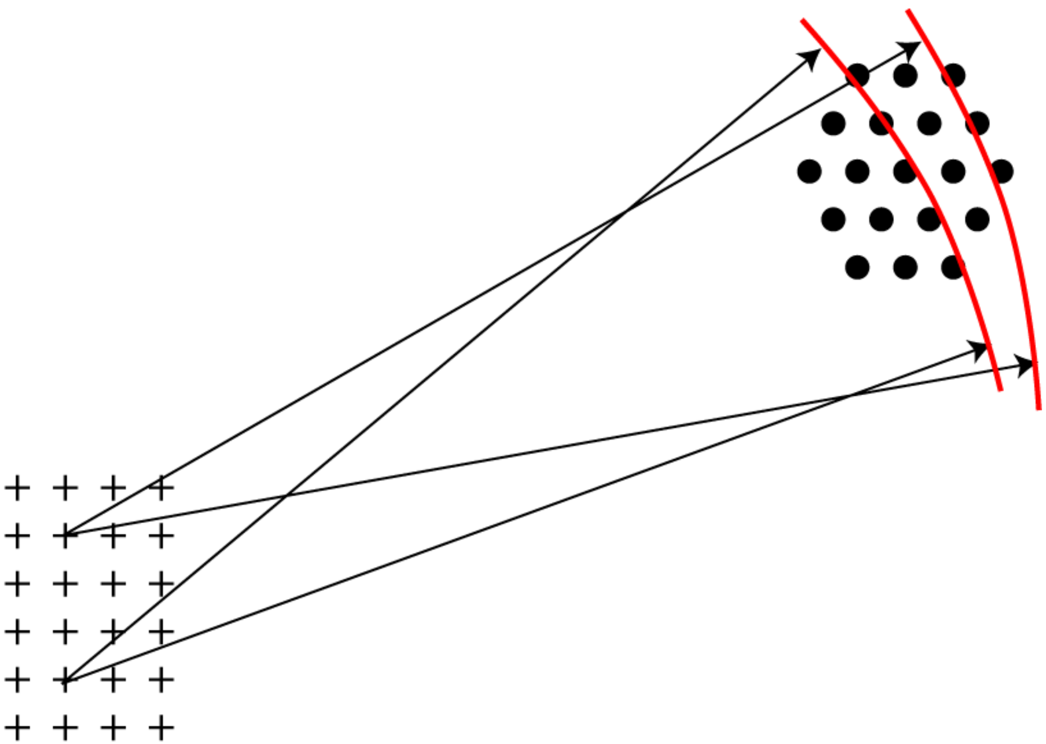
Back-projection methods

Peter M Shearer

SCEC-ERI Summer School, Oct. 1, 2014





http://igppweb.ucsd.edu/~shearer/SCECERI/

Peter Shearer's SCEC-ERI Back-projection Material

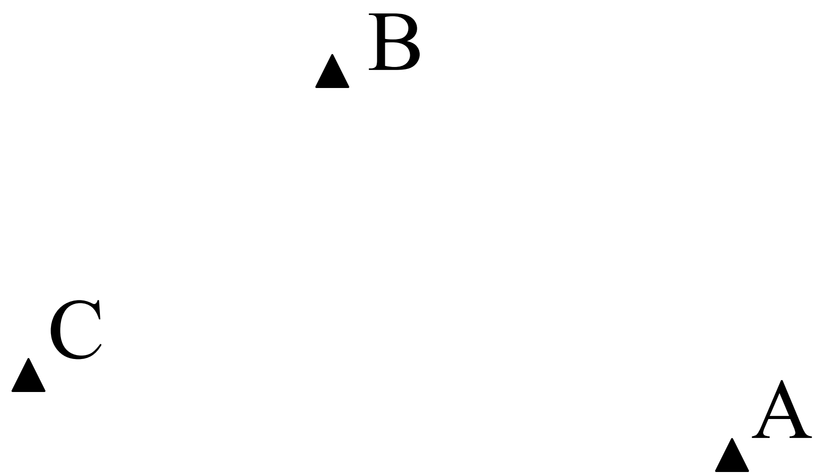
Lecture (PDF)
Notes (PDF) (includes computer exercise instructions)
Data for computer exercise (tremor_data.txt)
Station locations for computer exercise (stations.xy.txt)
Python code to plot results of computer exercise (plotimage.py)

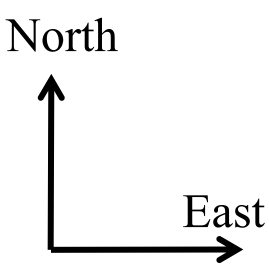
Back-projection is a very intuitive idea and can be related to:

- Earthquake location
- Time reversal
- Beam-forming and array processing
- Migration in reflection seismology
- Adjoint methods

Earthquake!

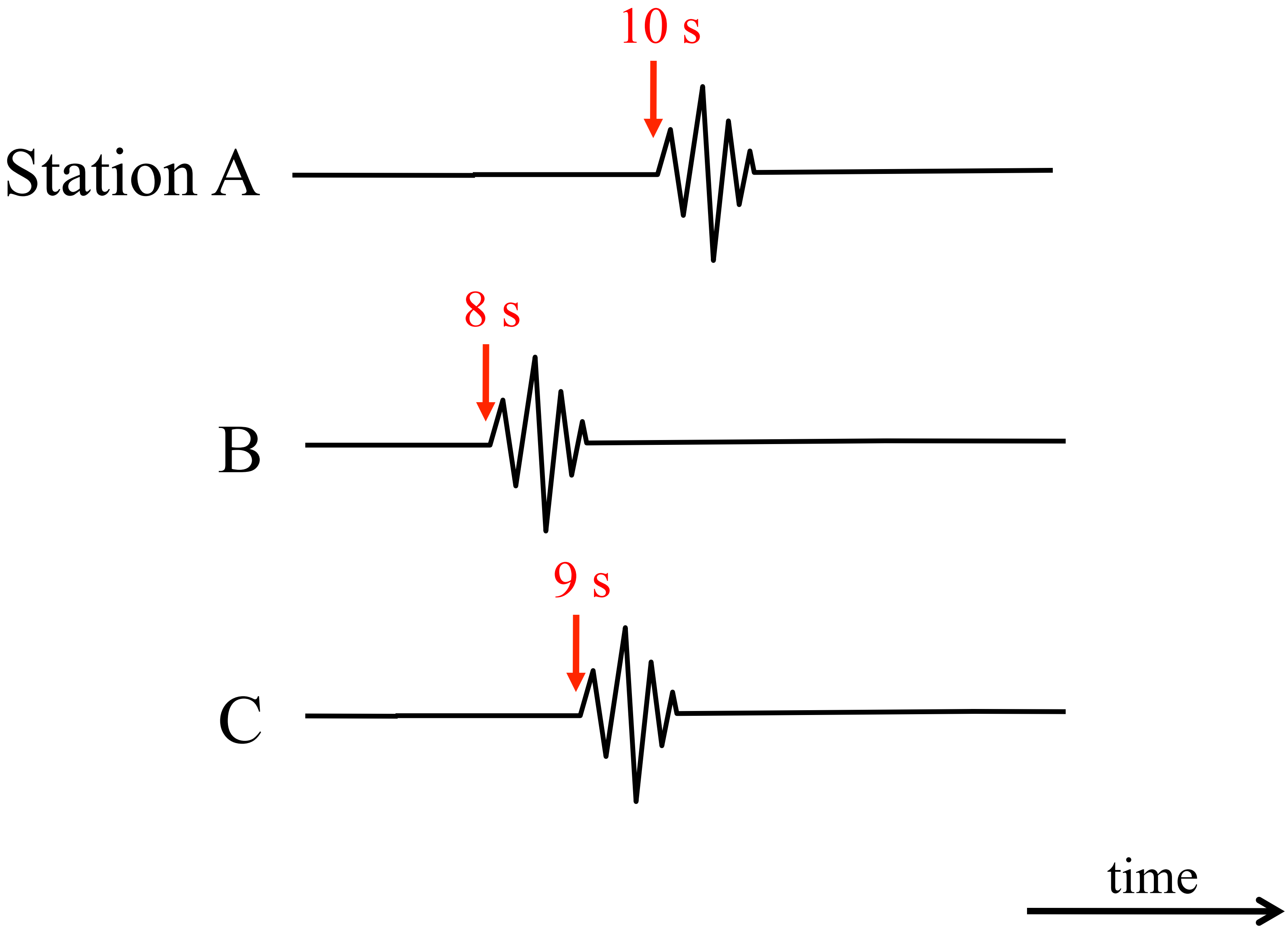
Seismic waves recorded at three stations:



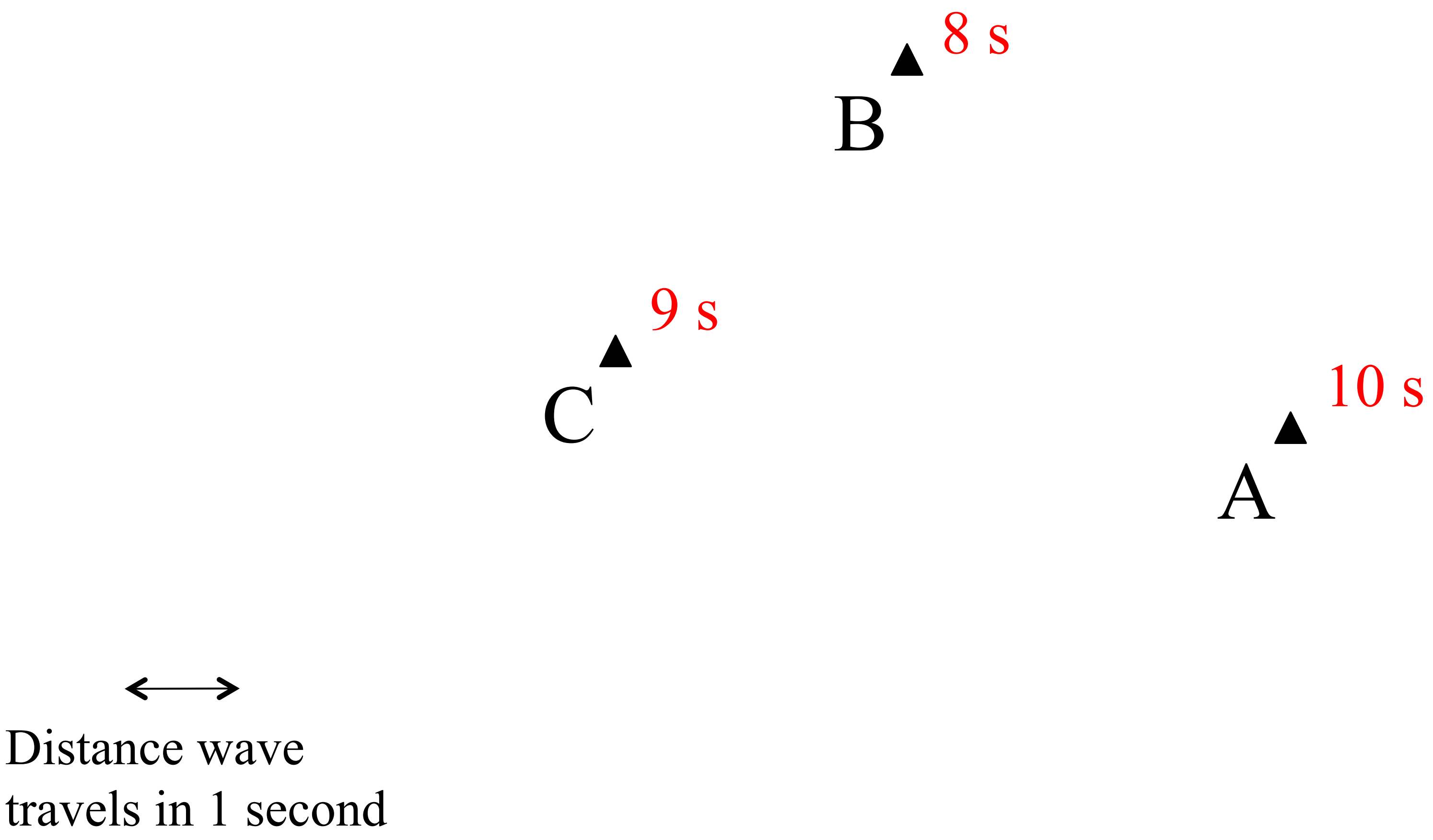


Where did it happen?

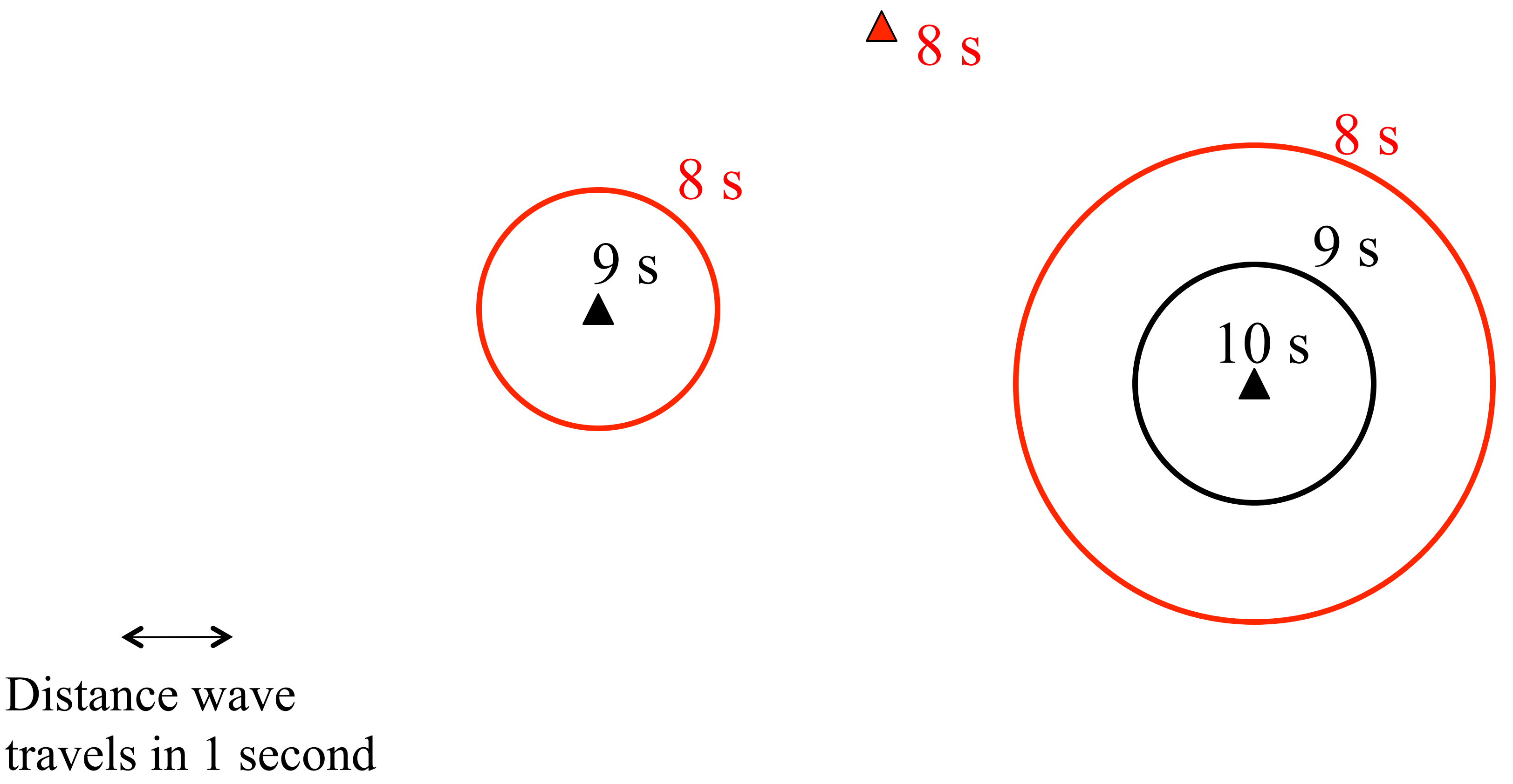
Measure seismic wave arrival times



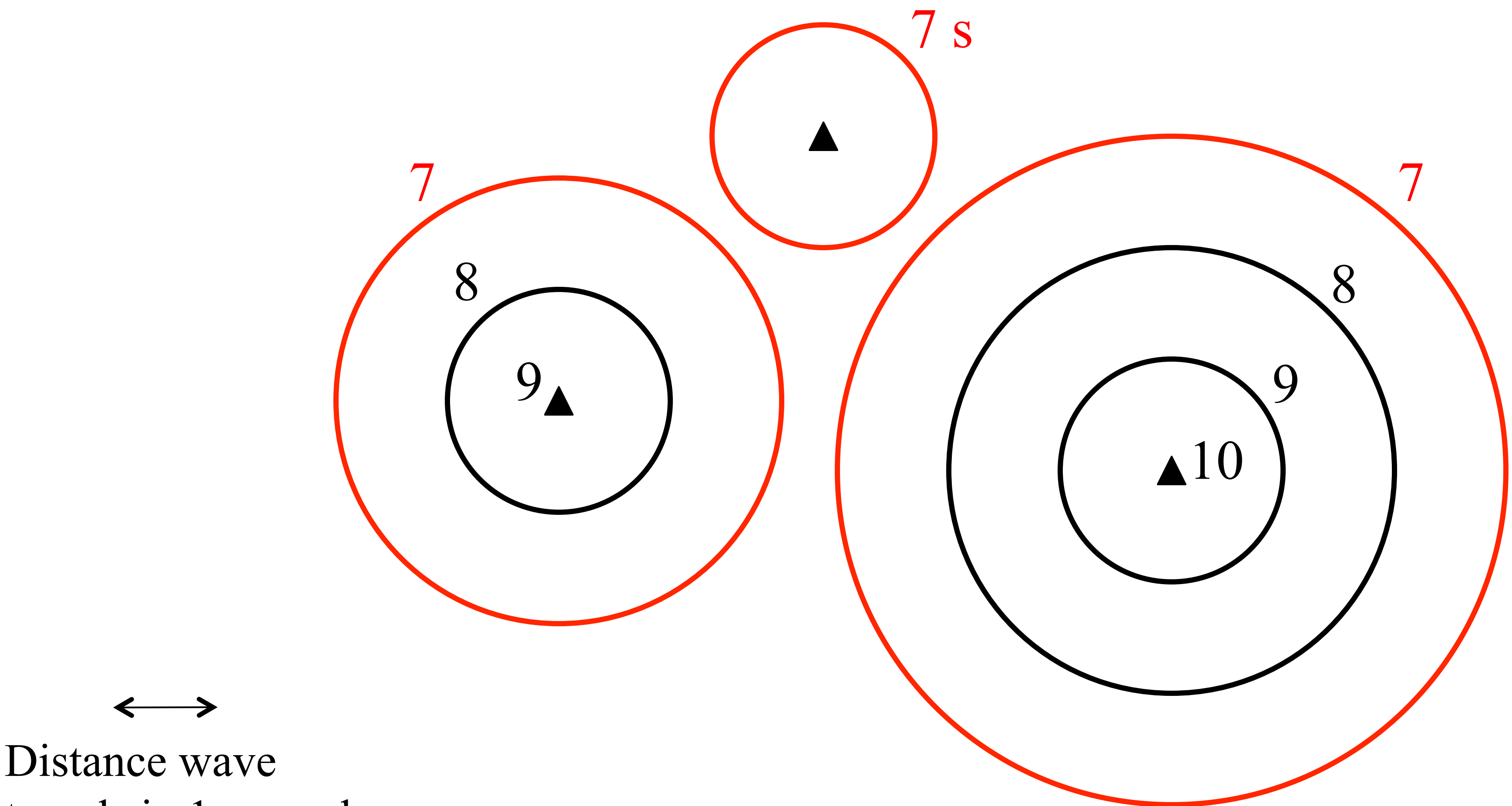
P-wave arrival times



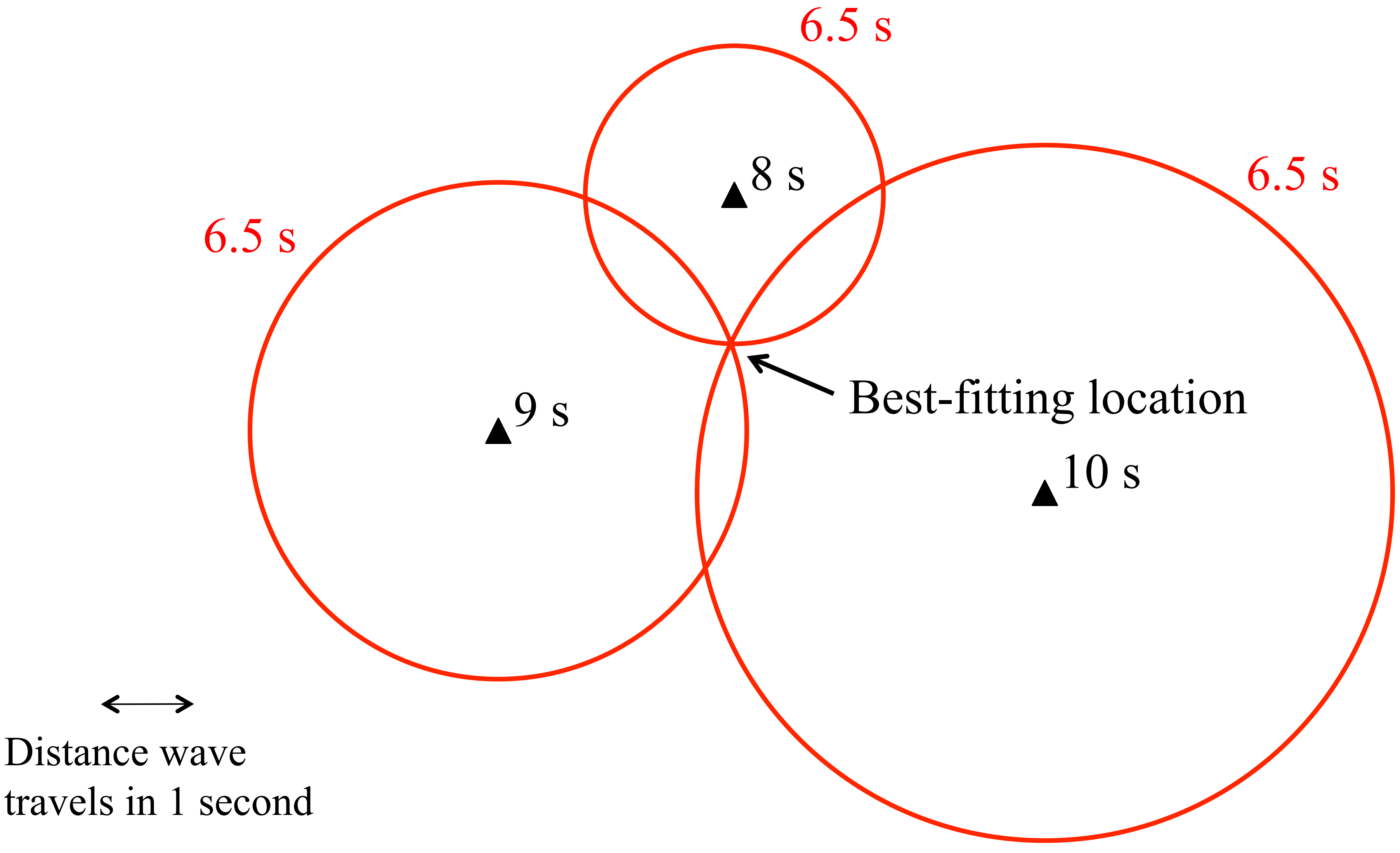
Possible event locations at 8 s (red circles)



Possible event locations at 7 s (red circles)

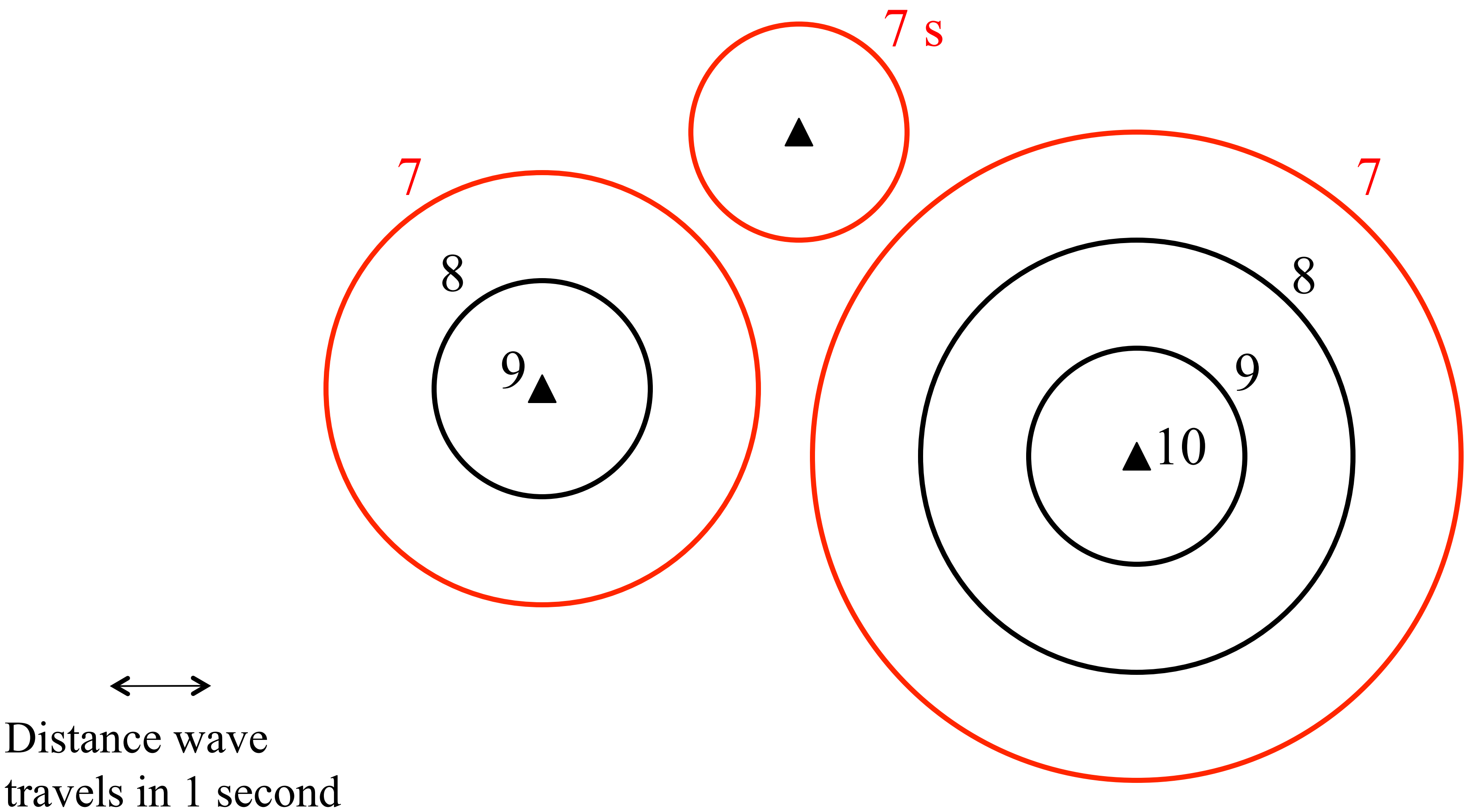


Possible event locations at 6.5 s (red circles)

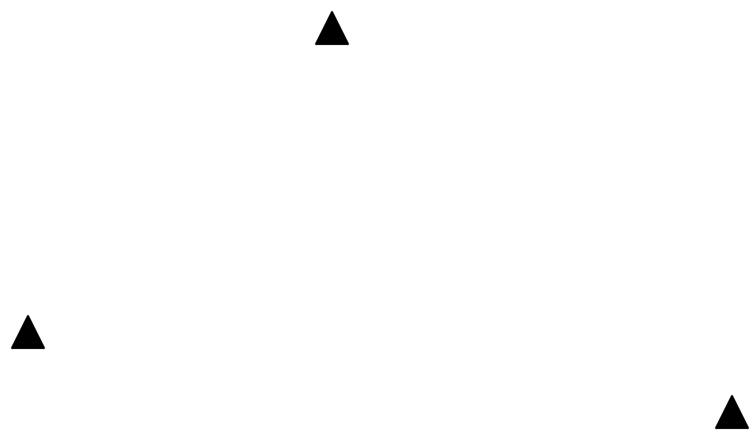


Time reversal and back-projection

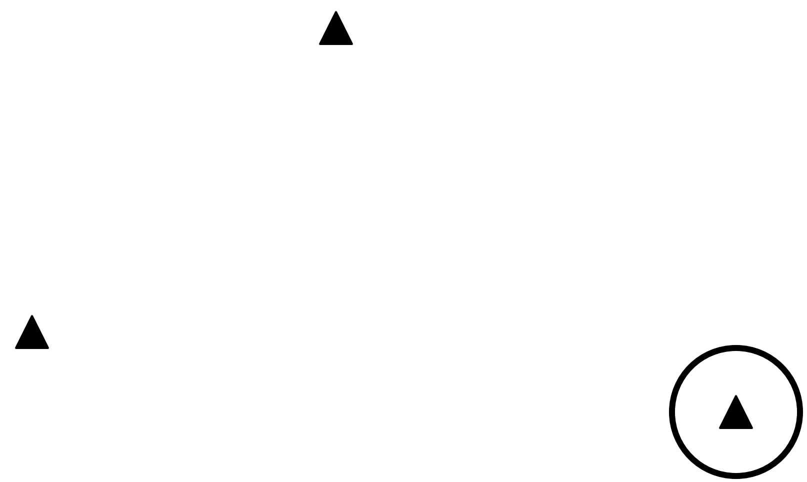
Possible event locations at 7 s (red circles)



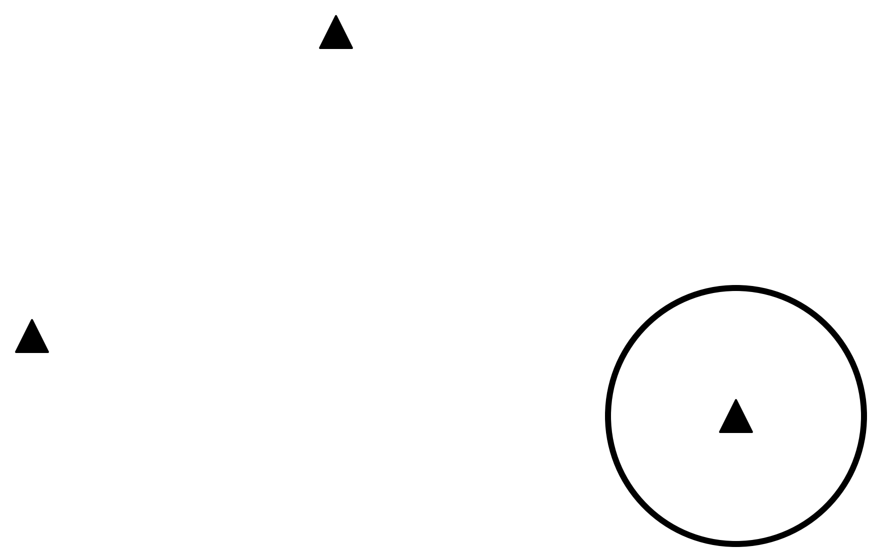
10.0 s



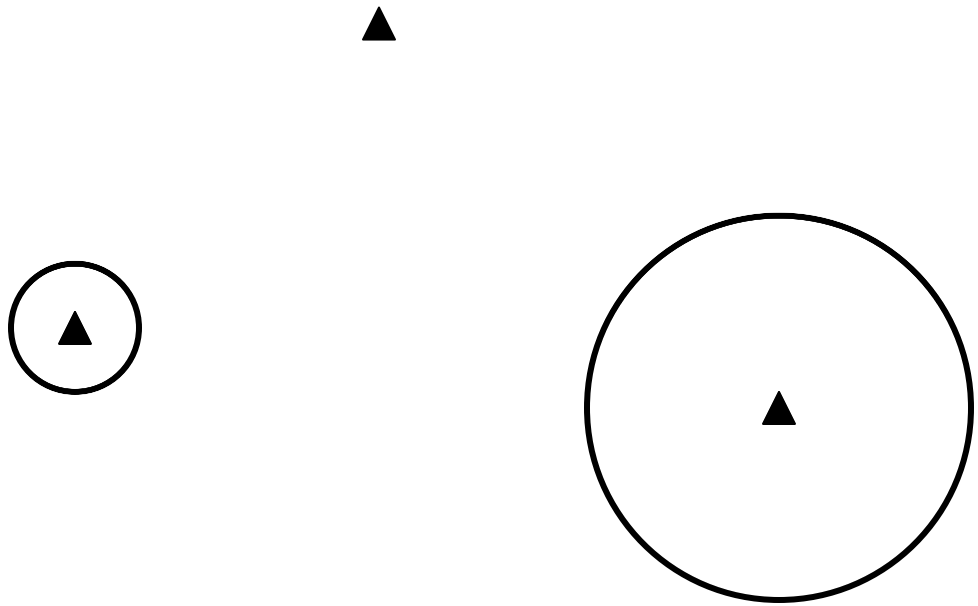
9.5 s



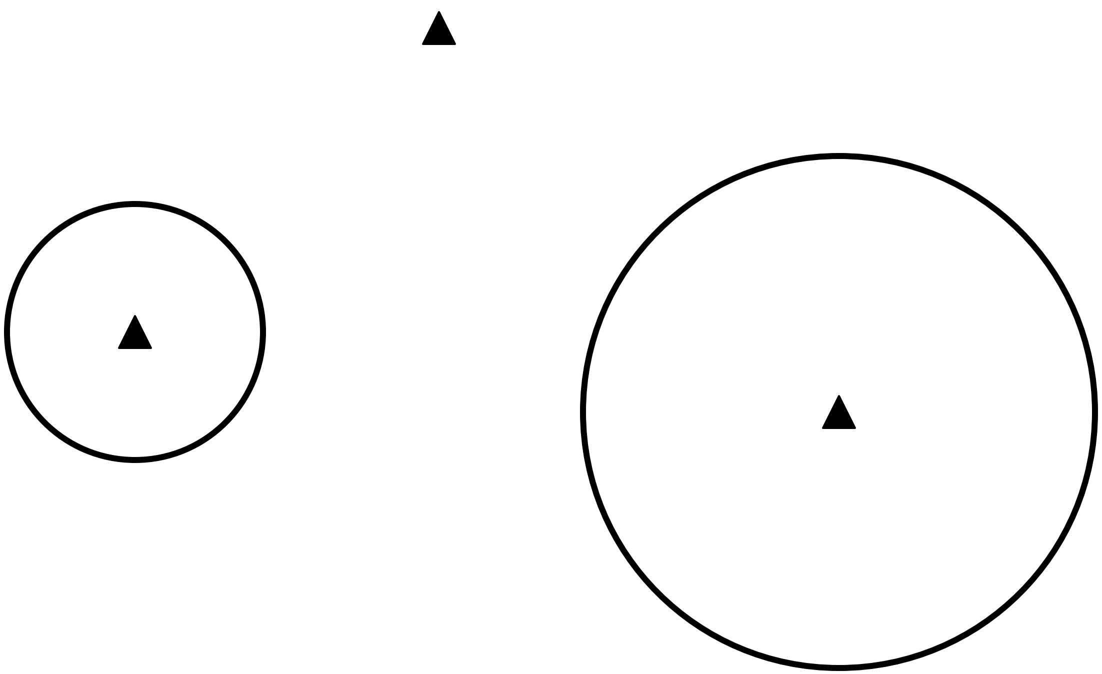
9.0 s



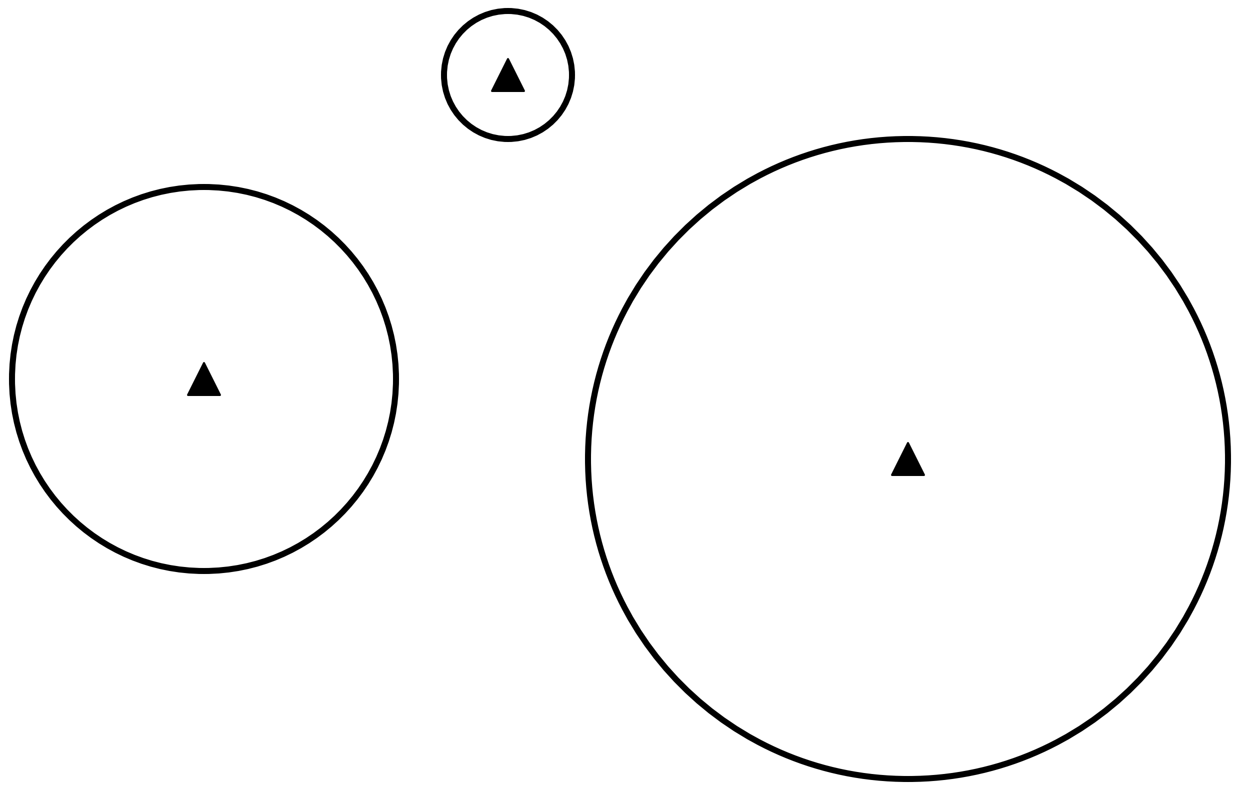
8.5 s



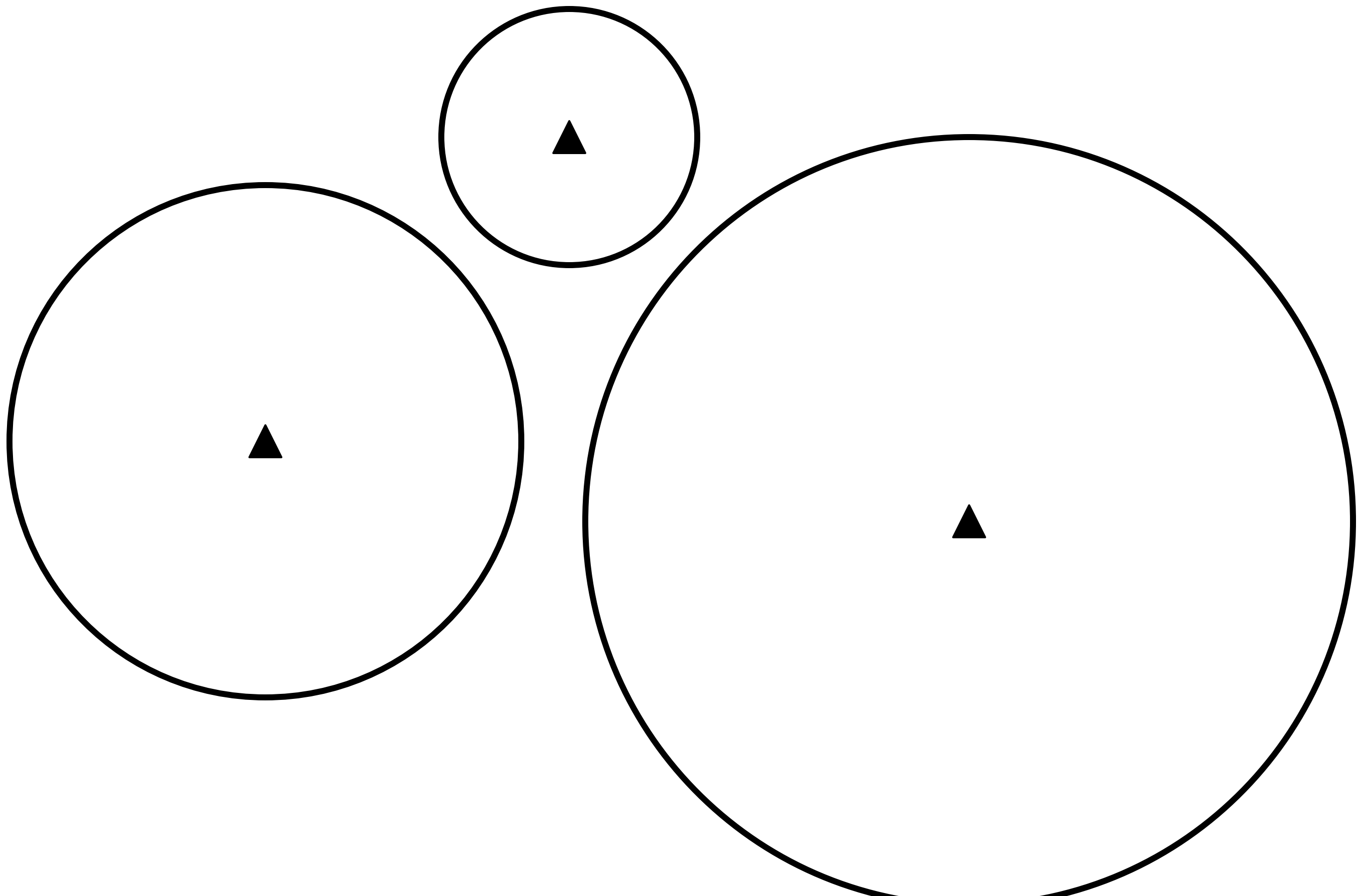
8.0 s



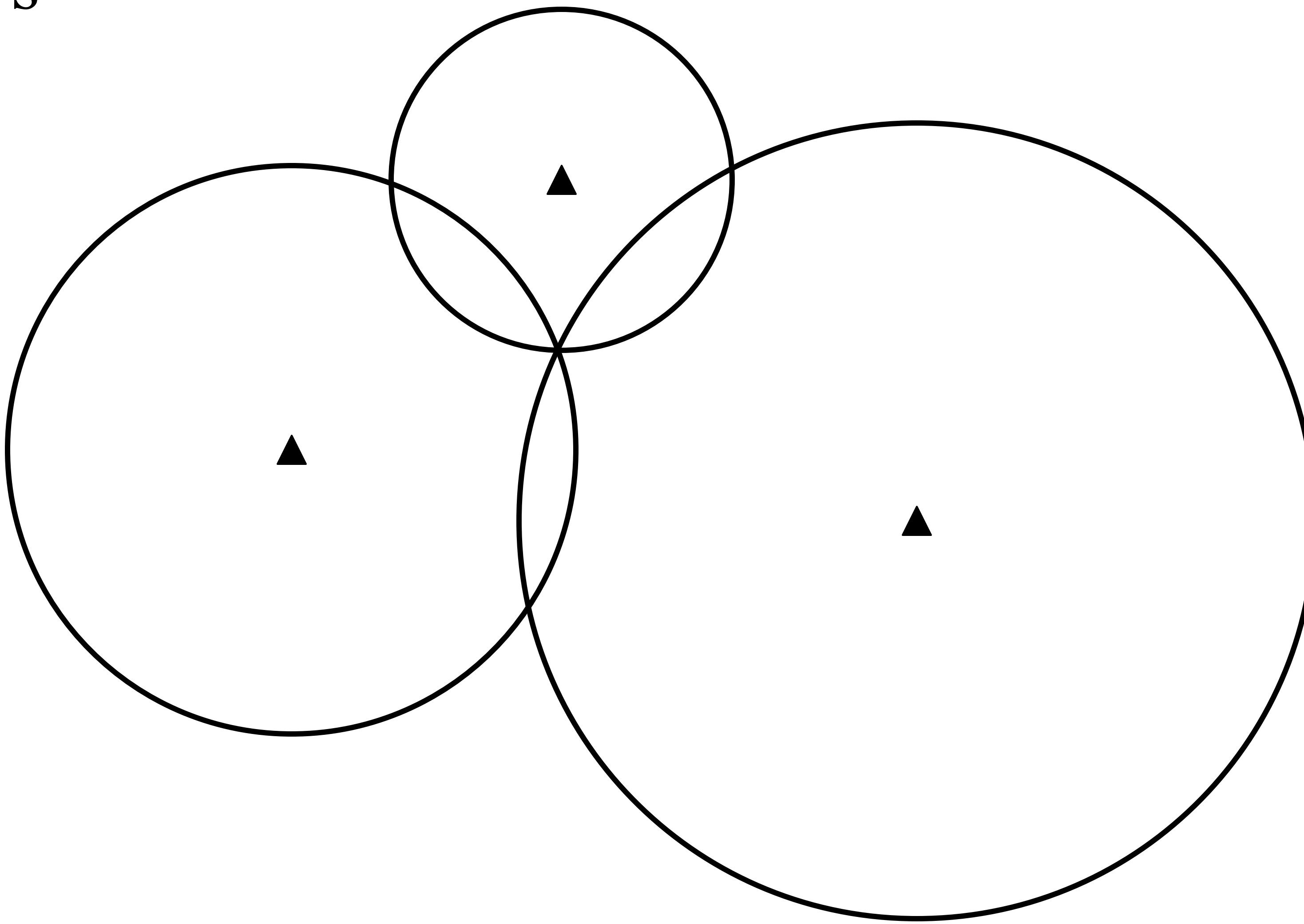
7.5 s

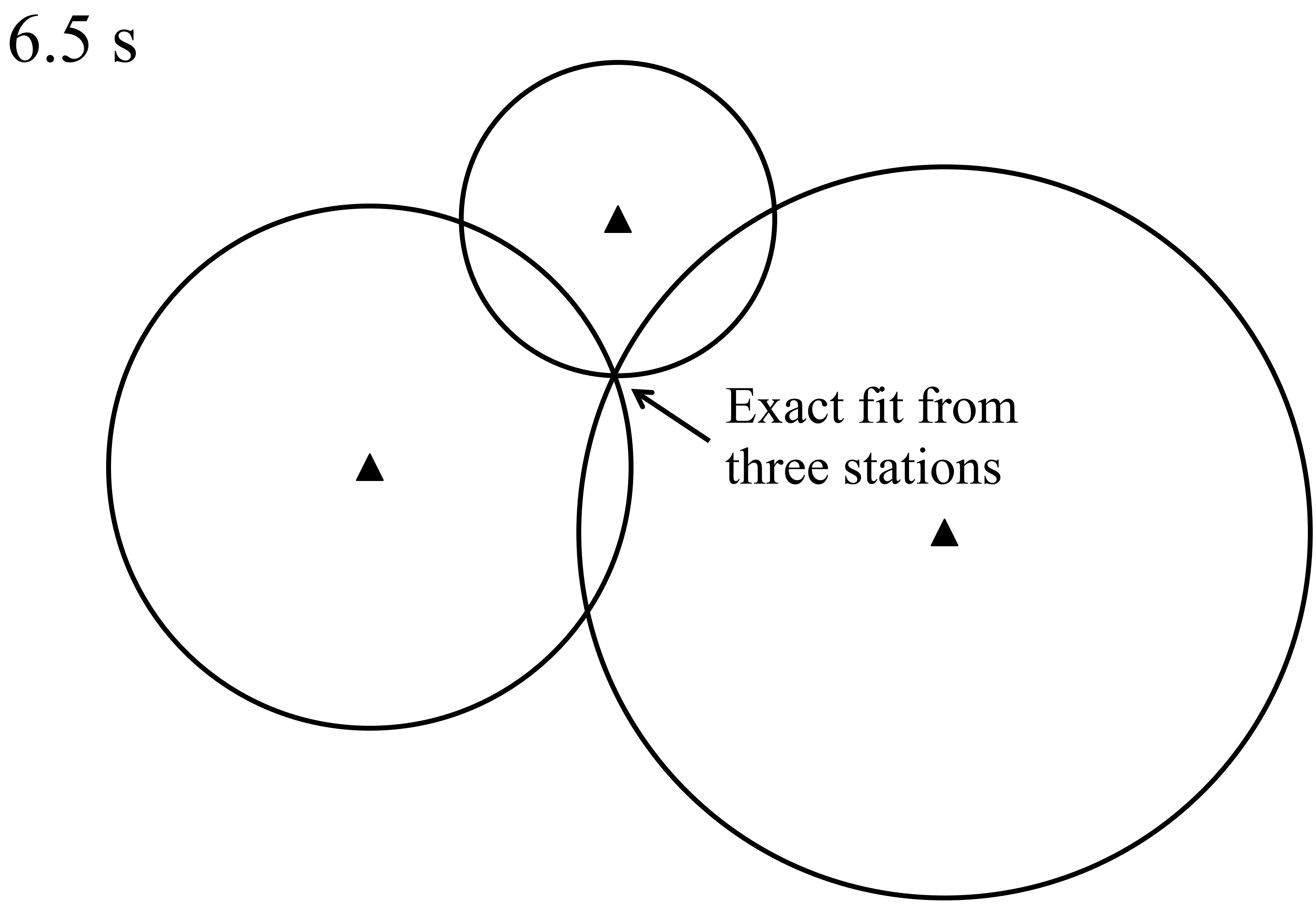


7.0 s

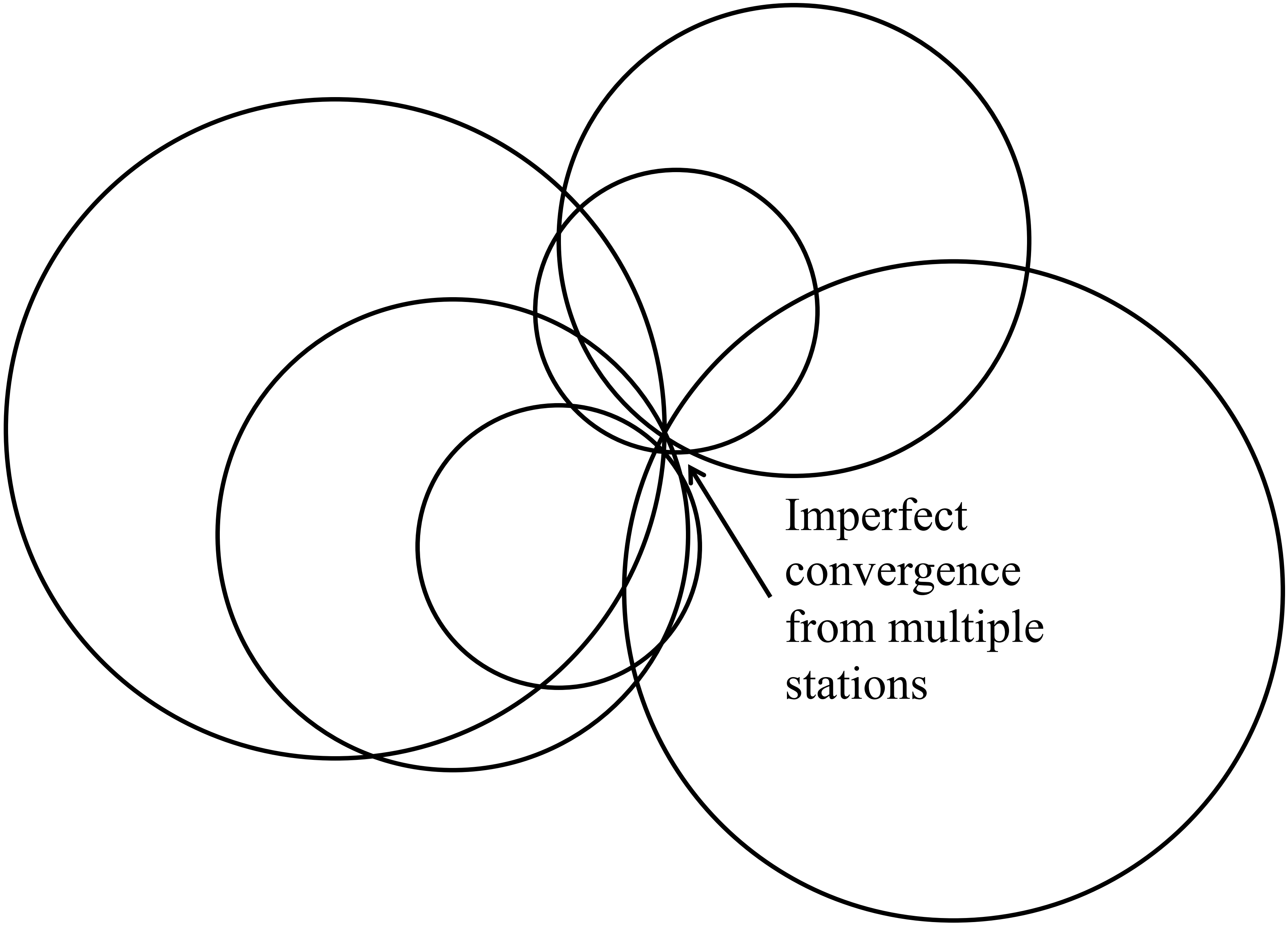


6.5 s





6.5 s
Exact fit from
three stations


Imperfect
convergence
from multiple
stations

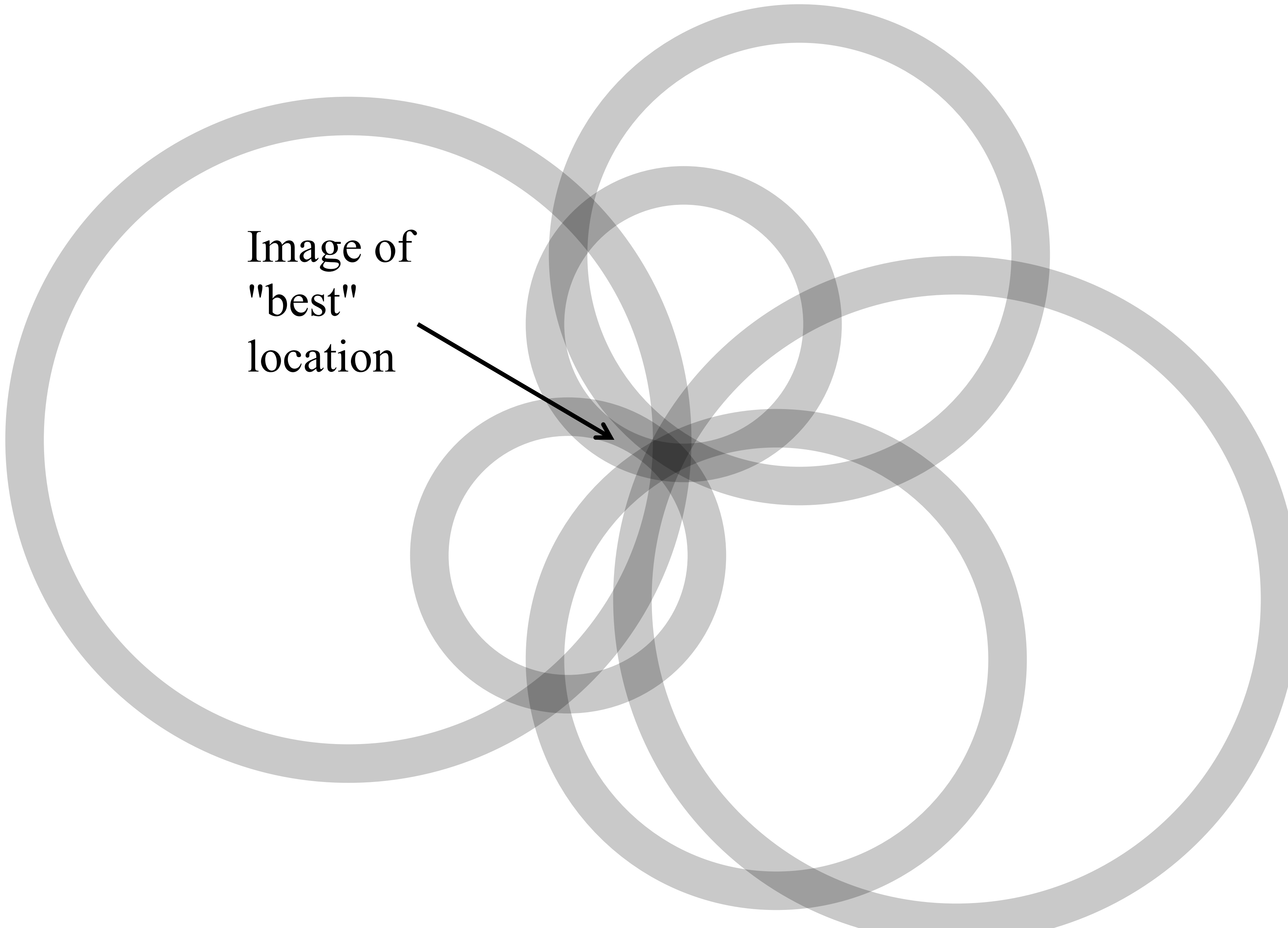
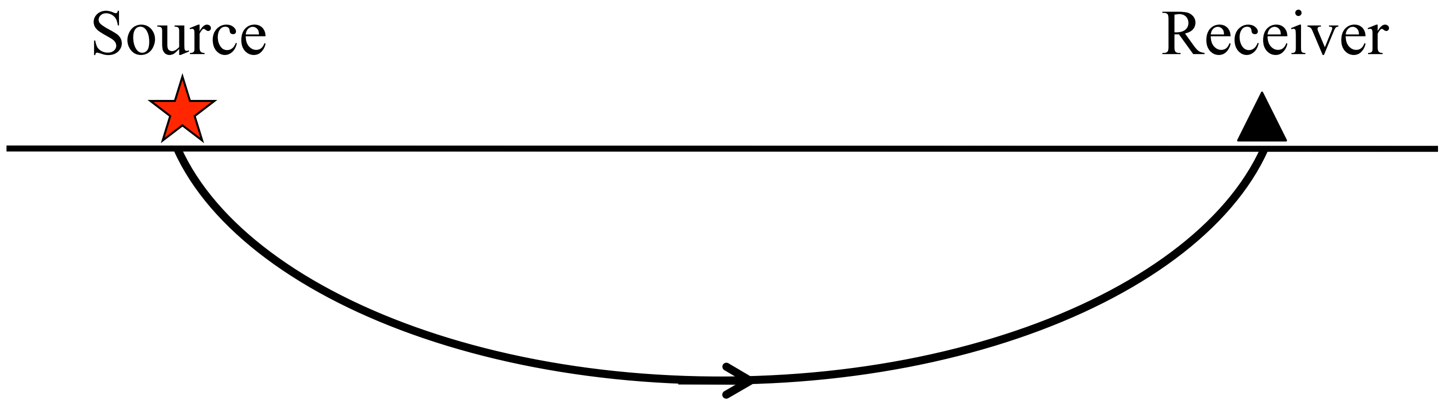
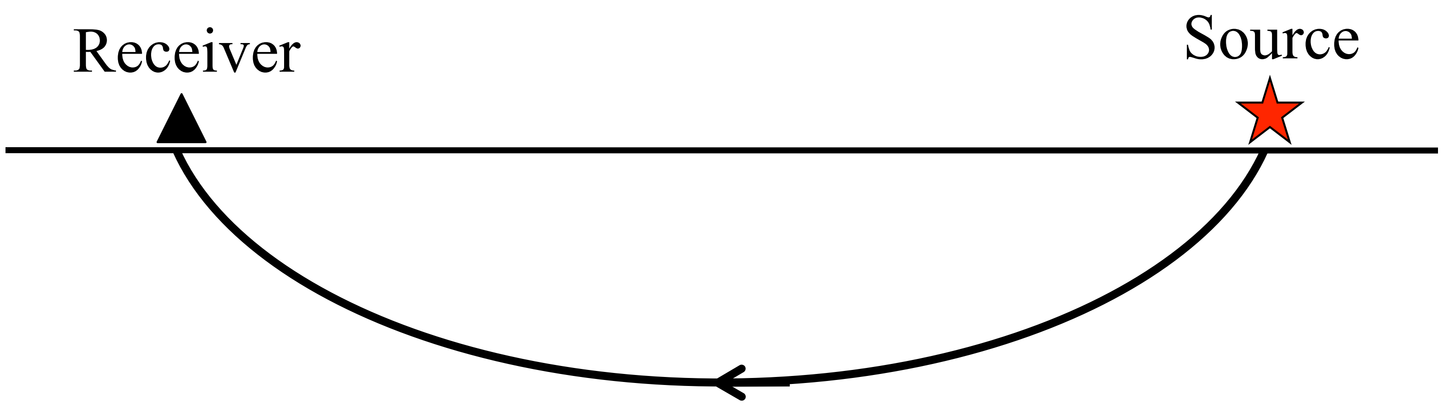


Image of
"best"
location

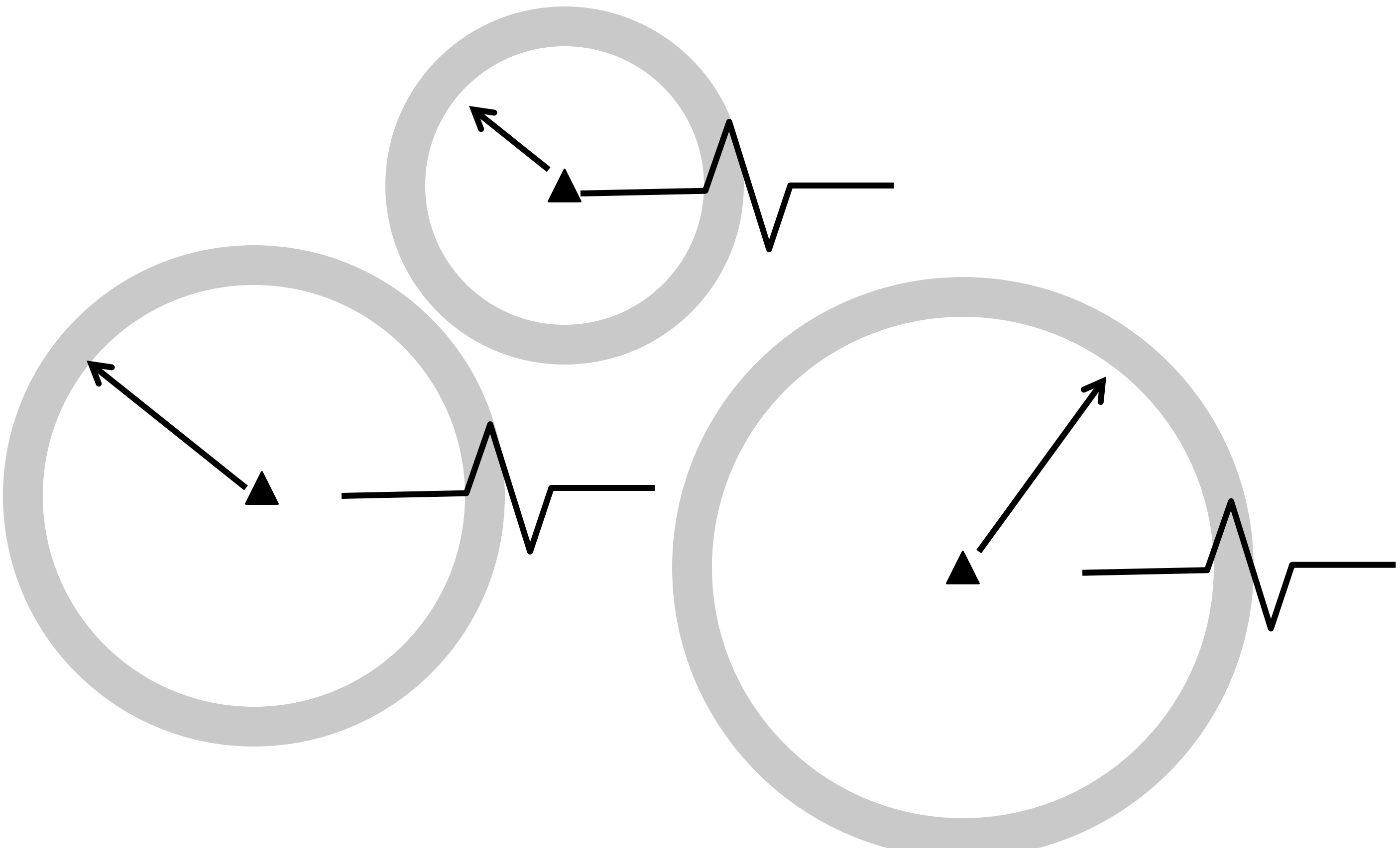
Source-receiver reciprocity



Both have same ray path and travel time

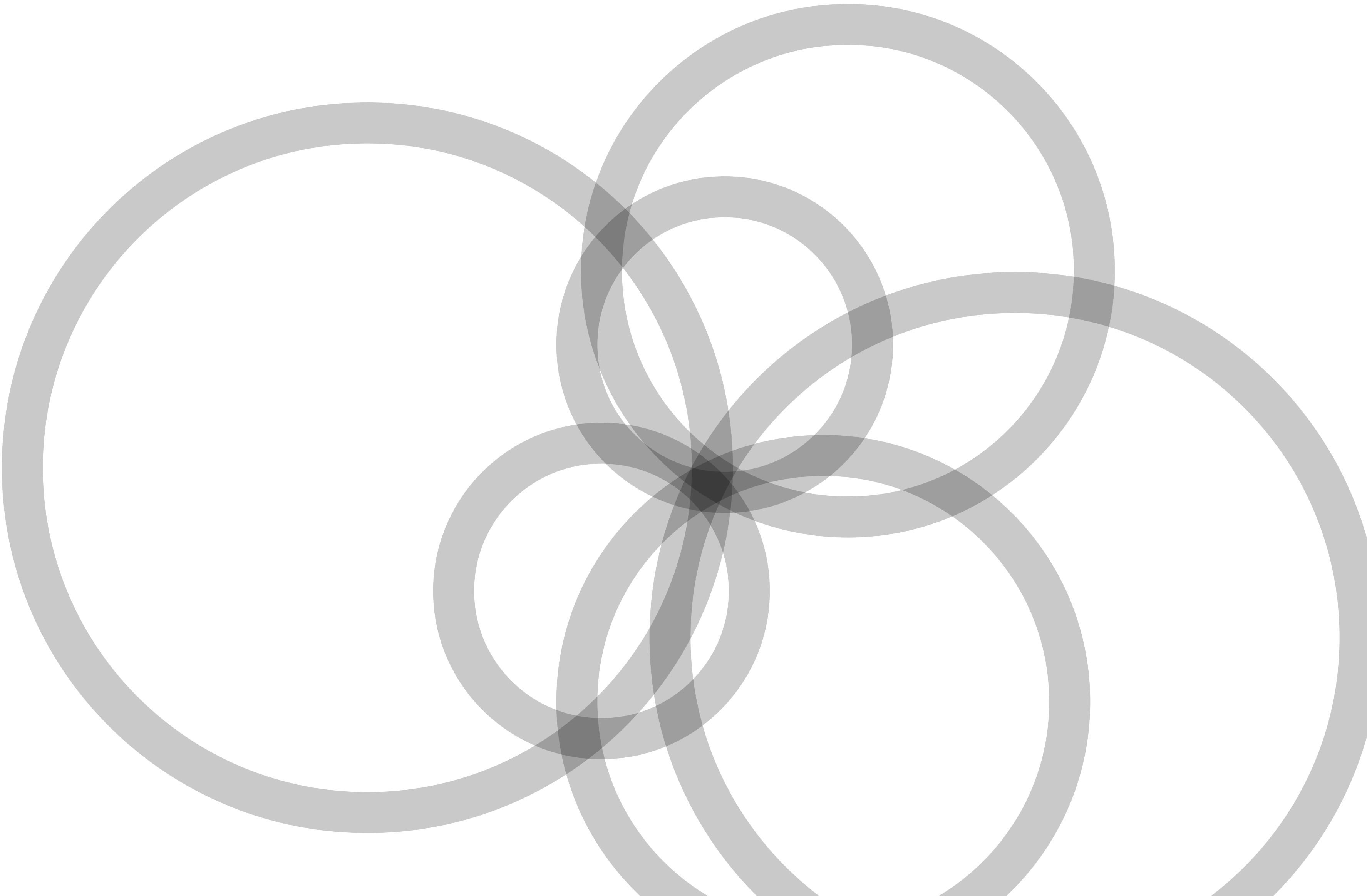


Location using time reversal (aka, back-projection)



Take recorded seismograms and project their waveforms *backwards* in time

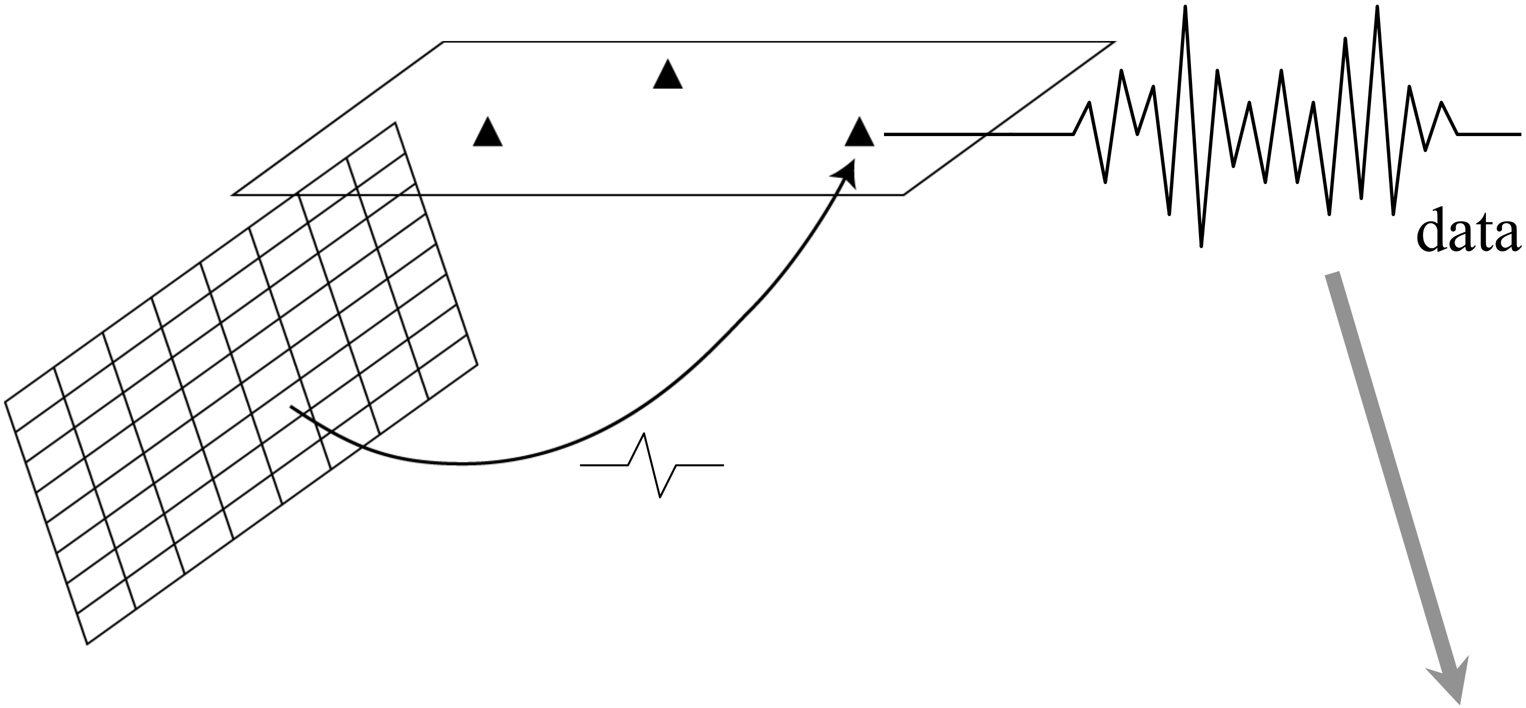
At the source origin time, their waves will constructively interfere at the source location:



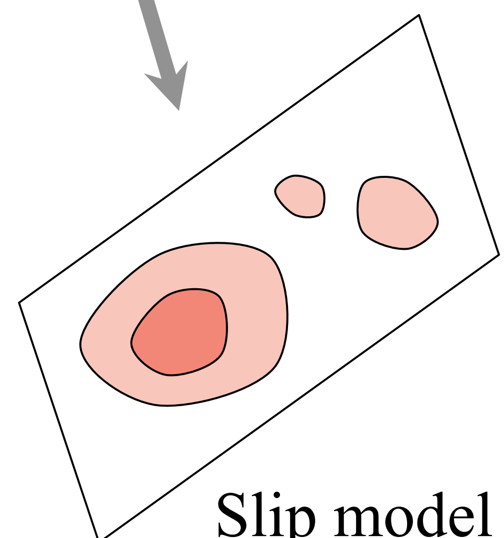
So what?

We already have better ways to locate earthquakes.

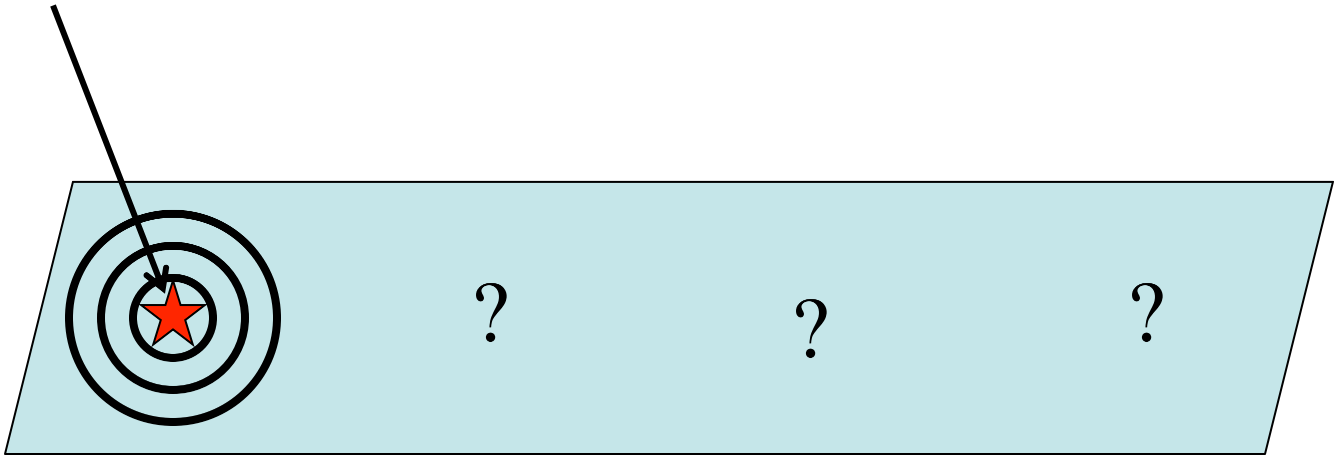
Finite source inversion



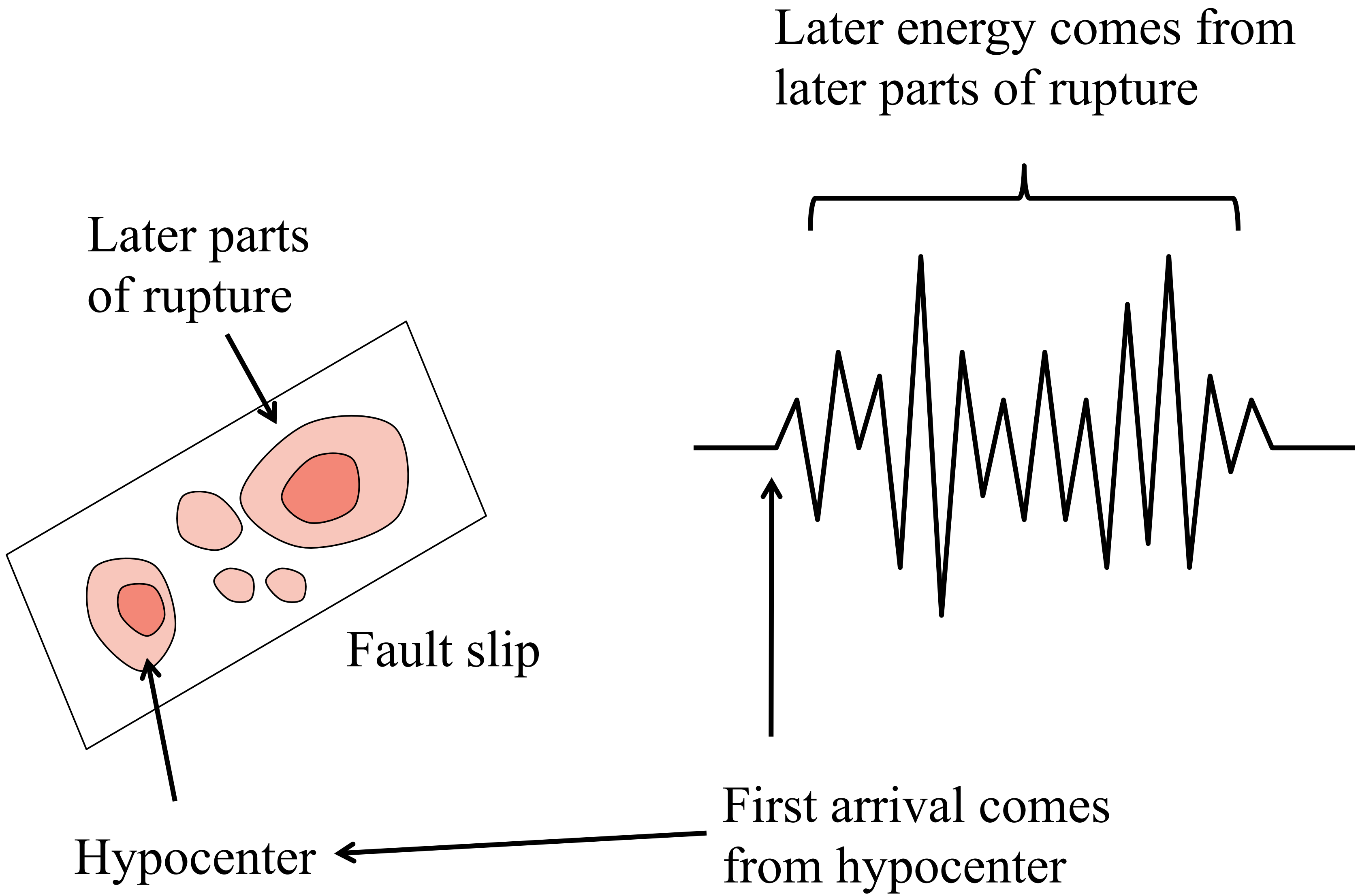
- Assume specific fault geometry & gridding
- Compute Green's function (synthetic seismogram) from each grid point to each station
- Set up and solve inverse problem for time-space slip model that predicts observed seismograms
- Only stable at relatively long periods



First-arrival locations define the earthquake *hypocenter*, where the rupture starts.

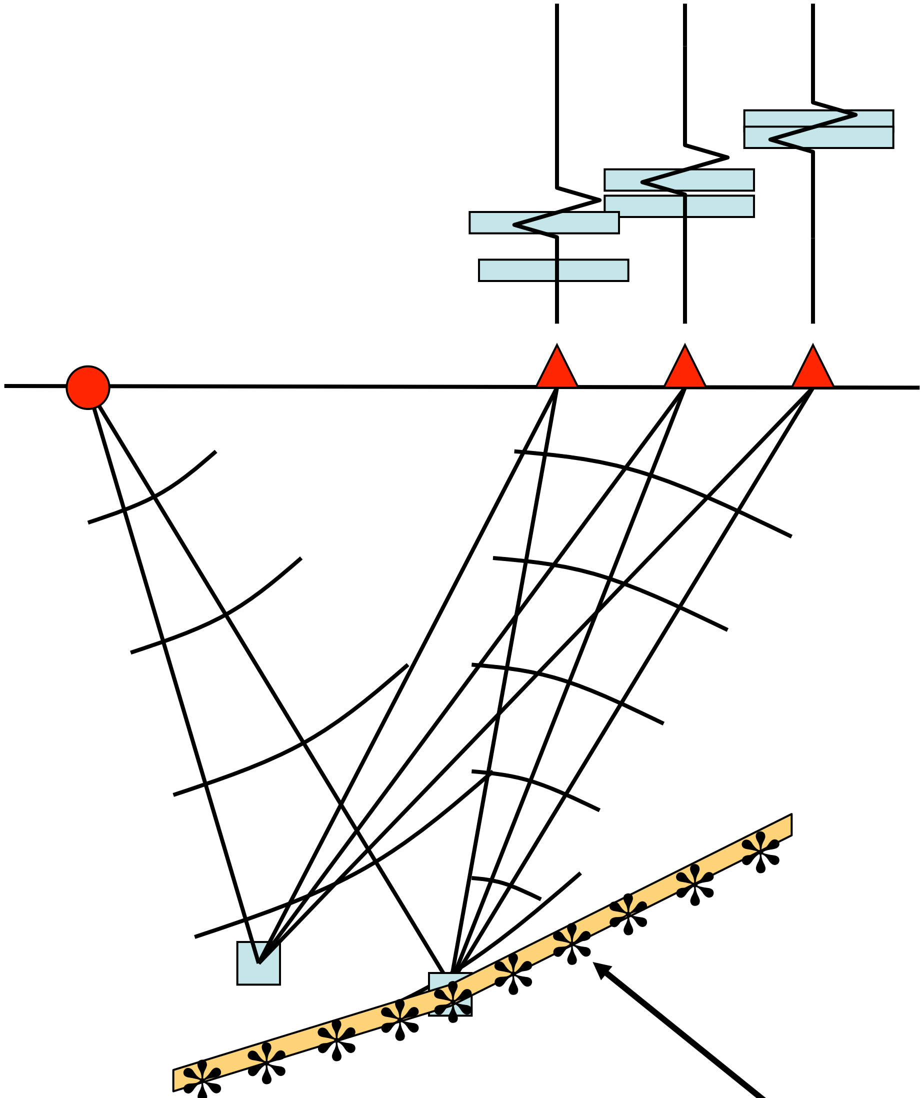


The seismic radiation from the rest of the rupture arrives later in the seismogram.



Back-projection potentially can locate sources of energy *throughout* the rupture

Migration in Reflection Seismology



Assume point scatterers

For each pixel in image, sum values from each trace at time of predicted source-to-scatterer-to-receiver travel time

Complete image is sum of individual point scatterers

“Exploding reflector” model

Many geophysical problems can be reduced to:

$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

Data (seismograms)

Green's functions (synthetic seismograms computed for assumed Earth model)

Model (slip function for each fault patch)

Least squares solution for model

$$\mathbf{m} = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{d}$$

Problem:

G^TG is invariably singular or ill-conditioned and may be far too large to easily invert

A practical inversion approach

$$\mathbf{m} = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{d}$$

Ignore the troublesome inverse term, i.e., set it to one → $(\mathbf{G}^T\mathbf{G})^{-1} \approx \mathbf{I}$

Then an estimate for the model is easily obtained → $\mathbf{m} \approx \mathbf{G}^T\mathbf{d}$

G^T is called the *adjoint* operator

Can we really get away with this?

With large real data sets, the answer is yes surprisingly often.

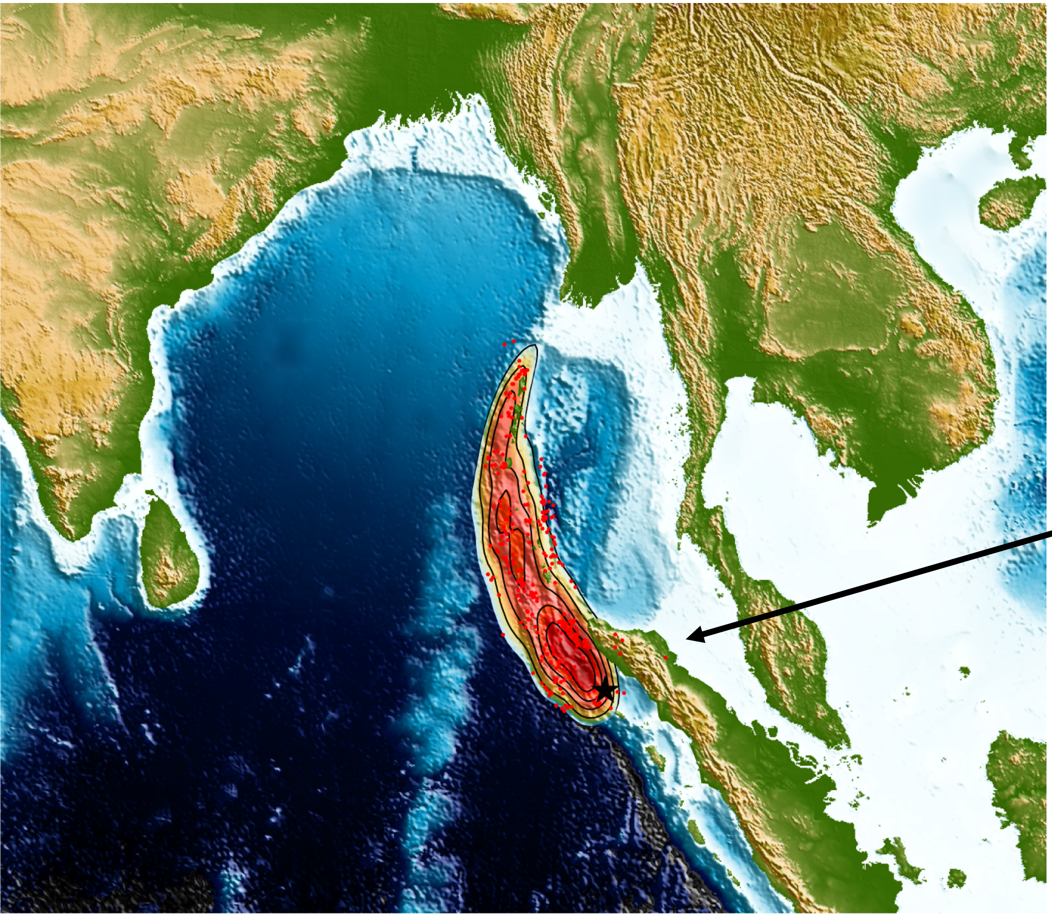
Jon Claerbout

Inverse theory is the fine art of dividing by zero (inverting a singular matrix).

…. in practice the adjoint sometimes does a better job than the inverse! This is because the adjoint operator tolerates imperfections in the data and does not demand that the data provide full information.

Back-projection to image earthquake rupture

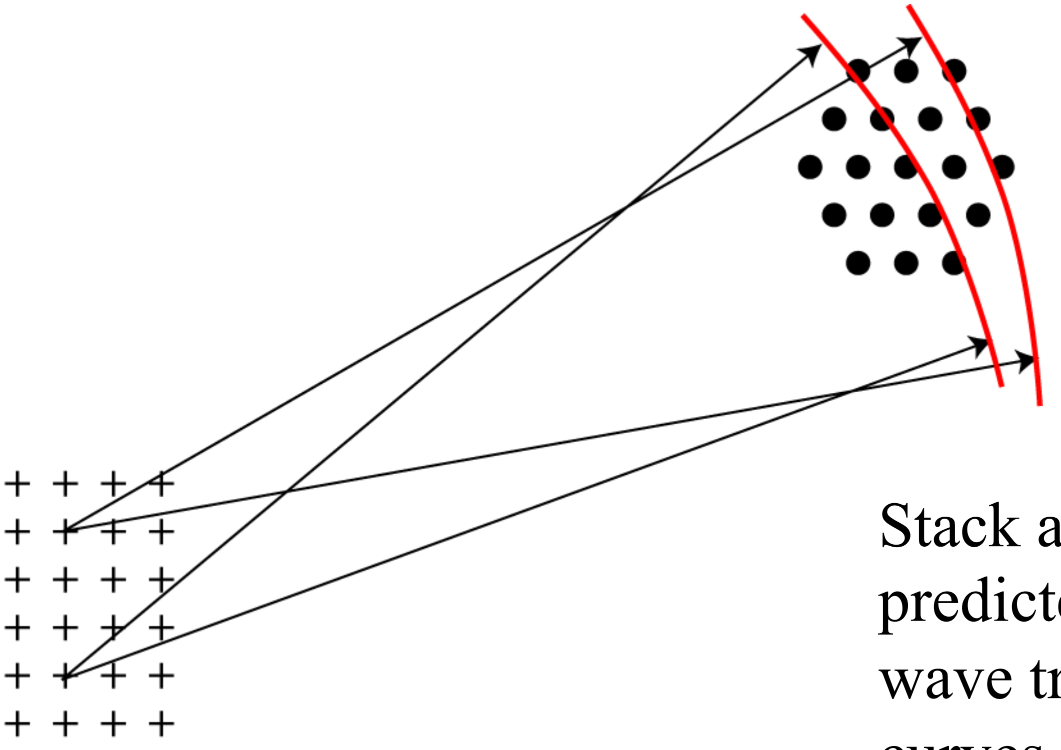
Japanese Hi-Net array of 700 stations



2004 Sumatra-Andaman earthquake

Miaki Ishii

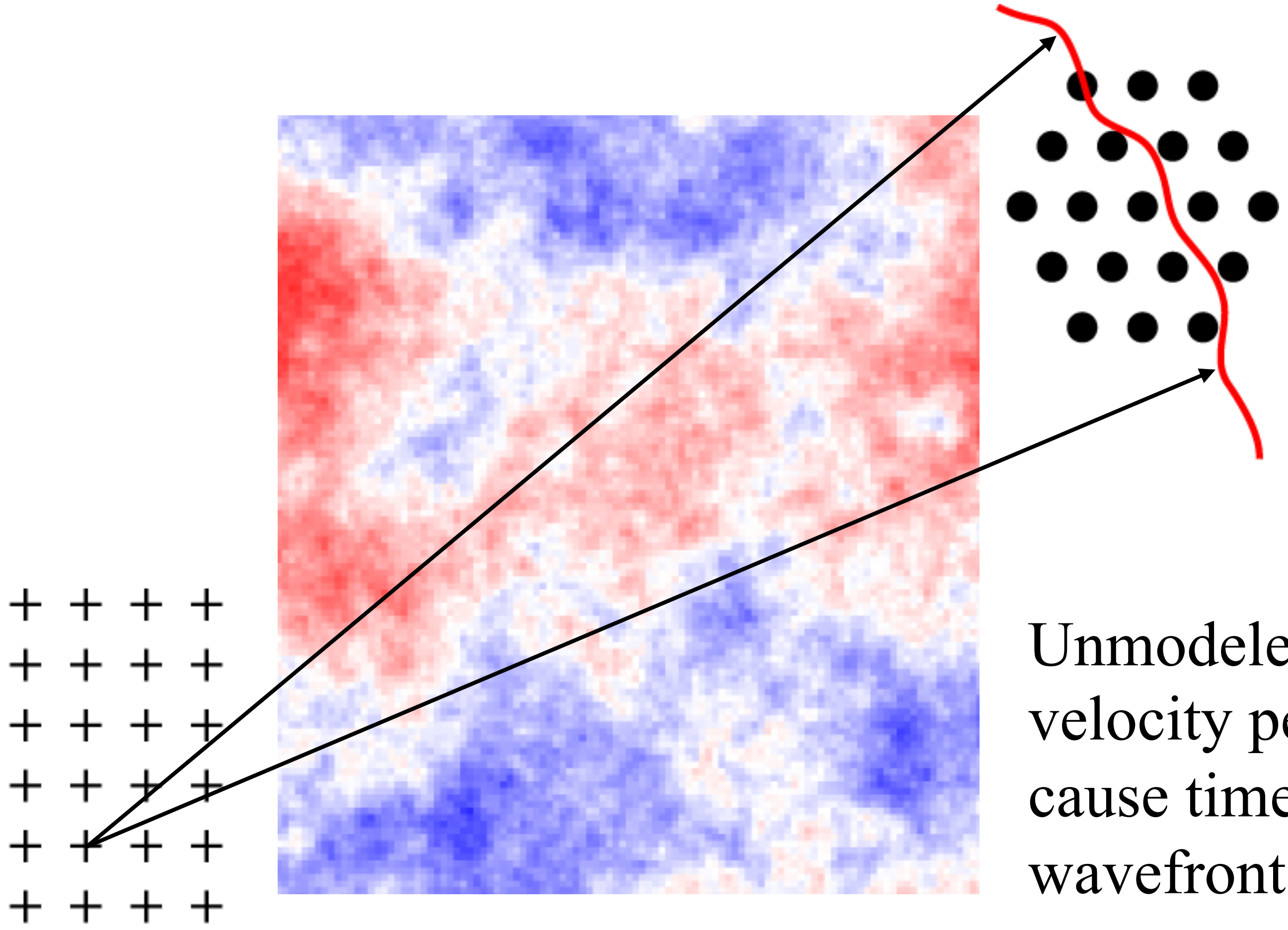
Source Imaging Using Back-projection



Stack along predicted P-wave travel time curves

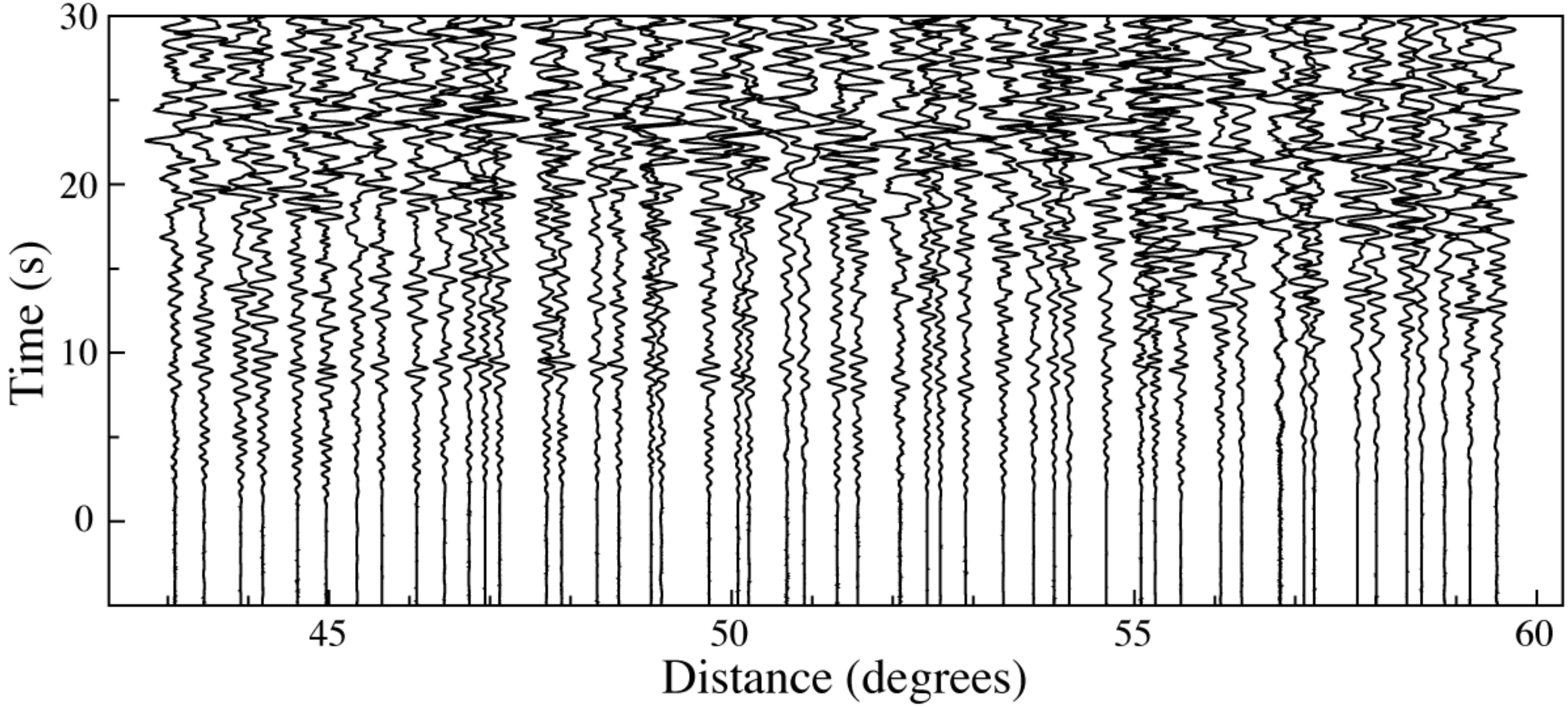
Assume grid of possible source locations

Problem: Incoherent stacking from time shifts from 3-D structure



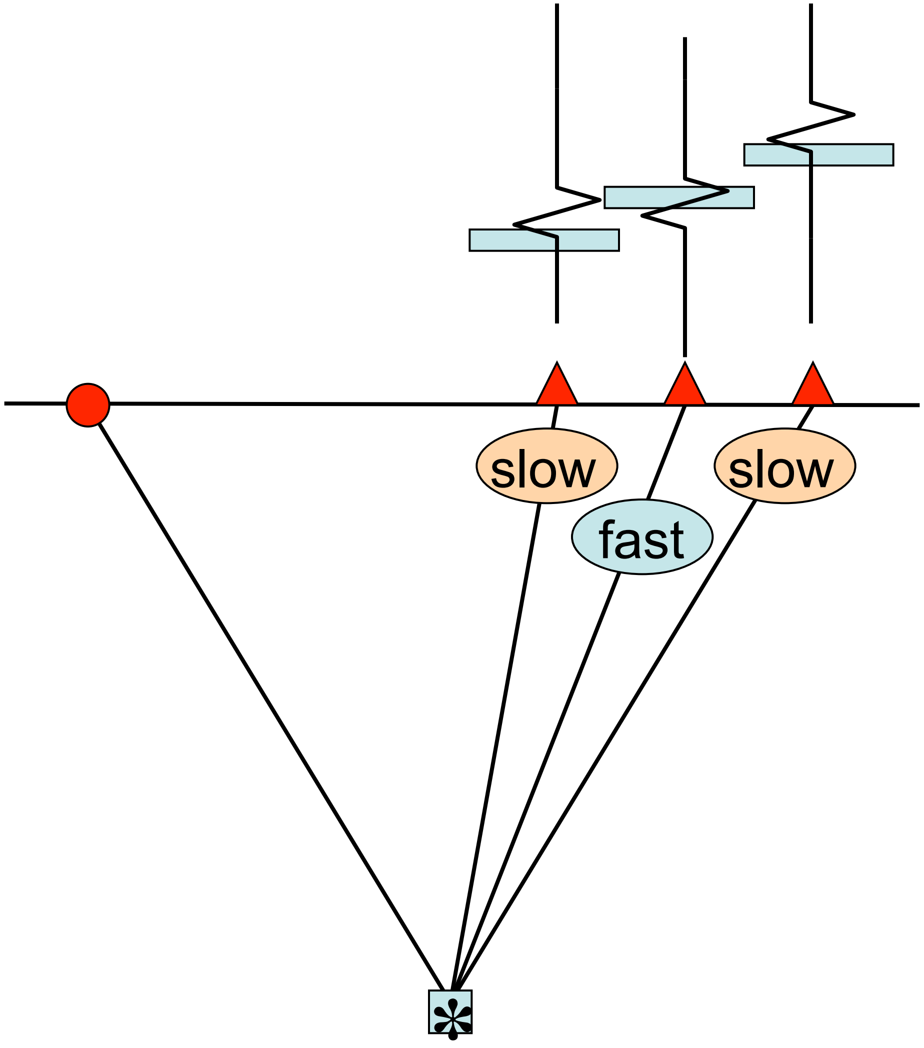
Unmodeled 3-D velocity perturbations cause time shifts in wavefront

Sumatra earthquake *P*-waves



Aligned on theoretical (iasp91) *P*-wave travel times

Migration in Reflection Seismology



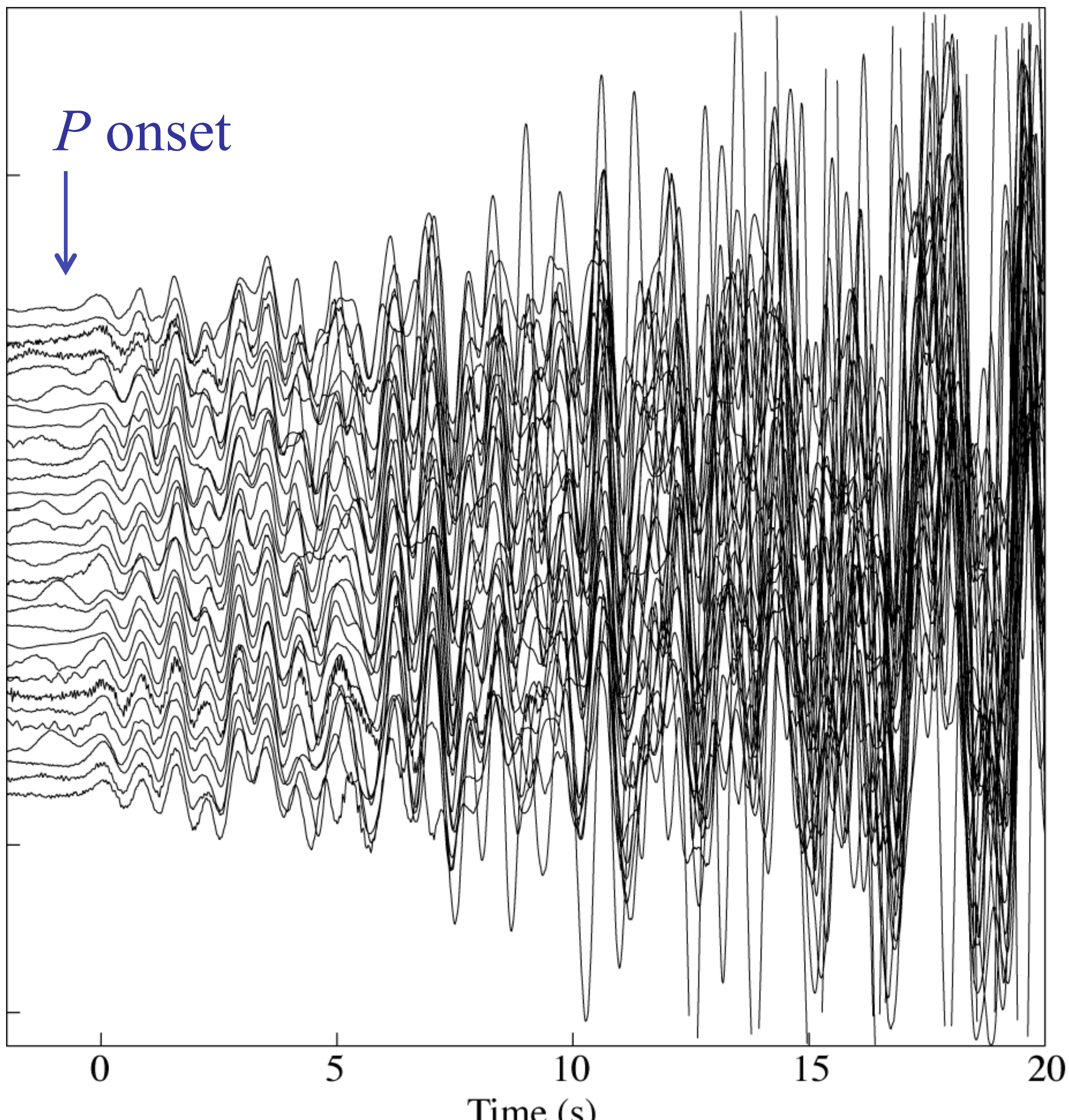
Problem:

Time shifts from 3-D structure can destroy stack coherence

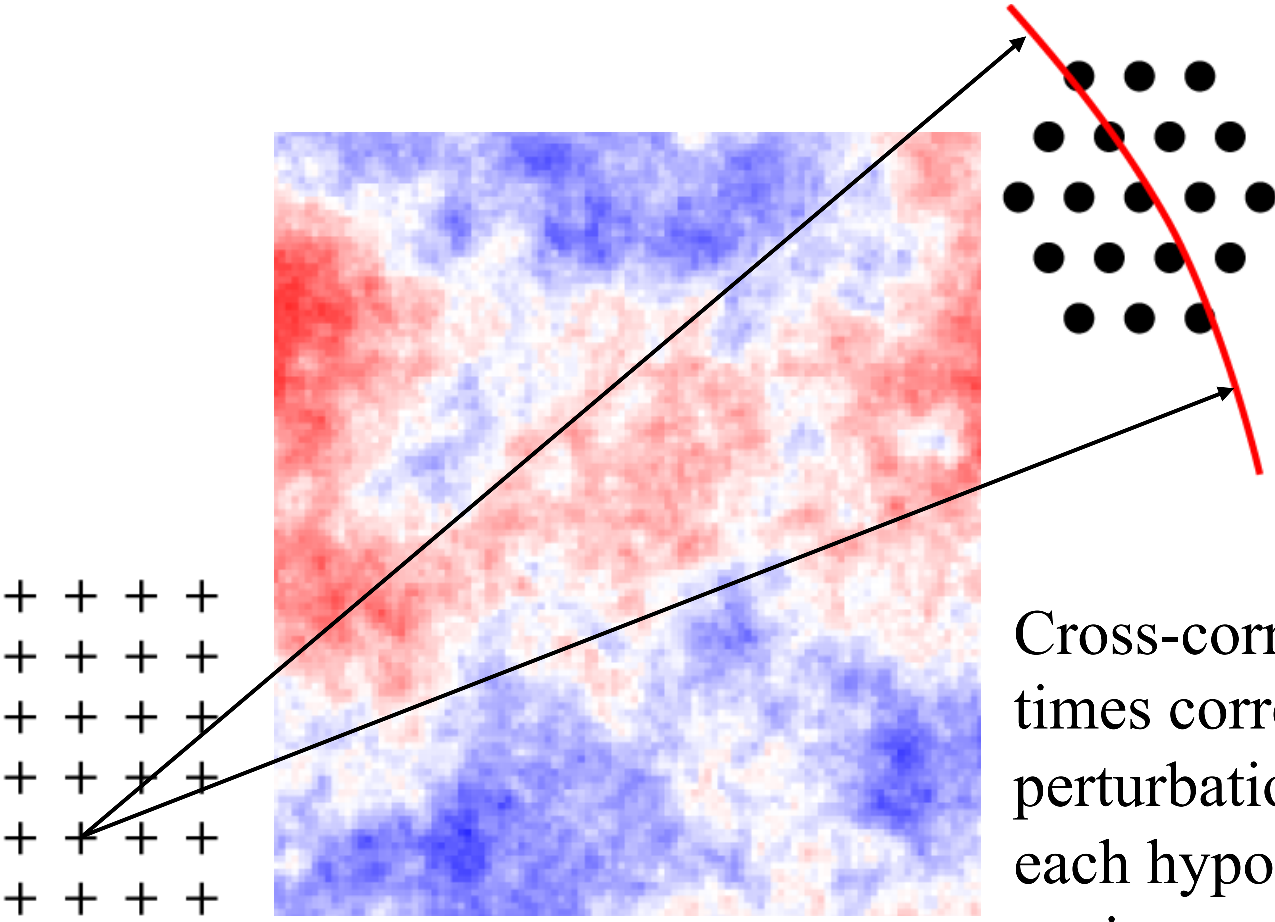
Solution:

Statics corrections (station terms)

Align *P*-waves with cross-correlation

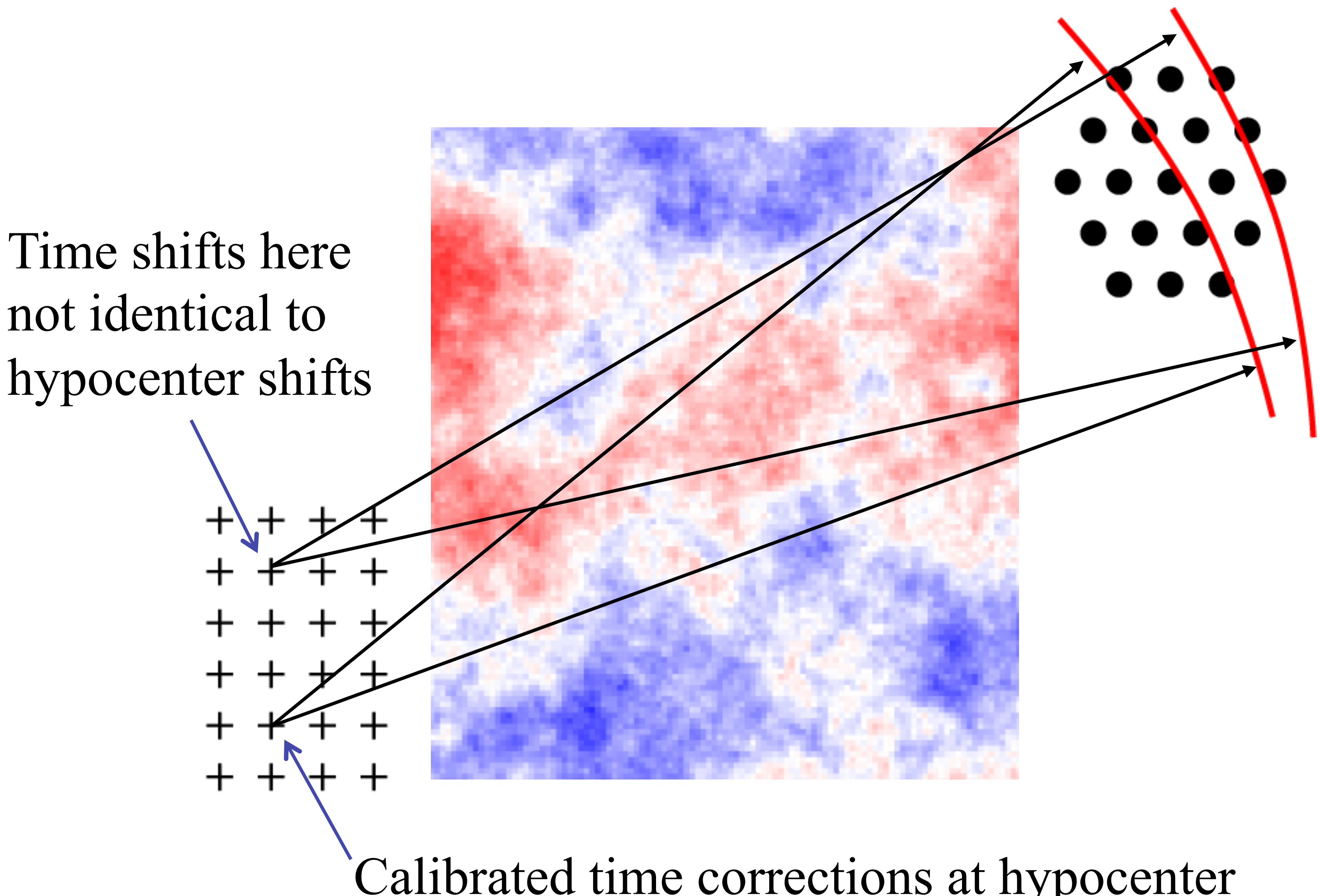


Method forces coherent stack at hypocenter

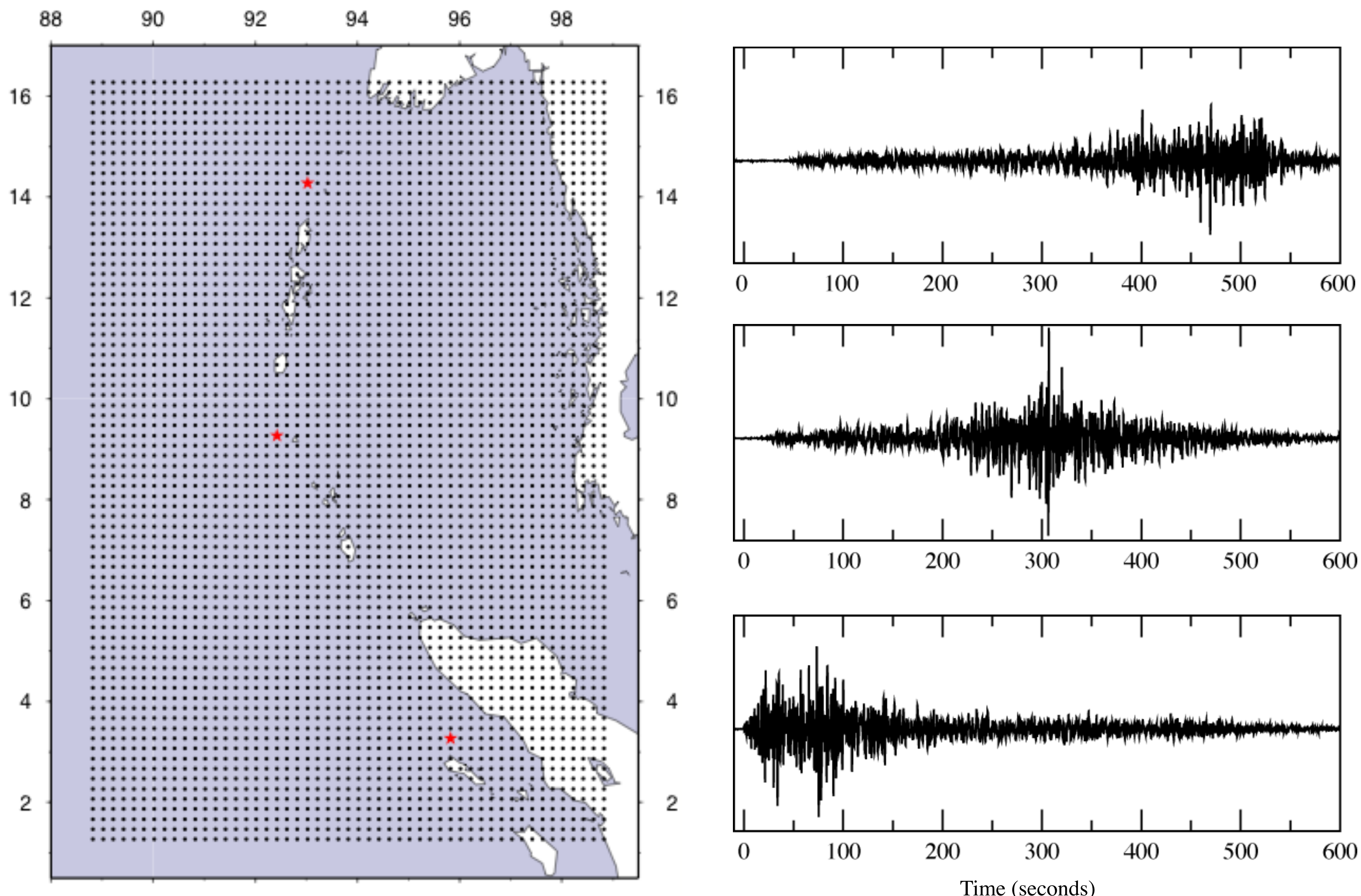


Cross-correlation times correct for perturbations along each hypocenter-station ray path

But coherence not guaranteed for sources offset from hypocenter

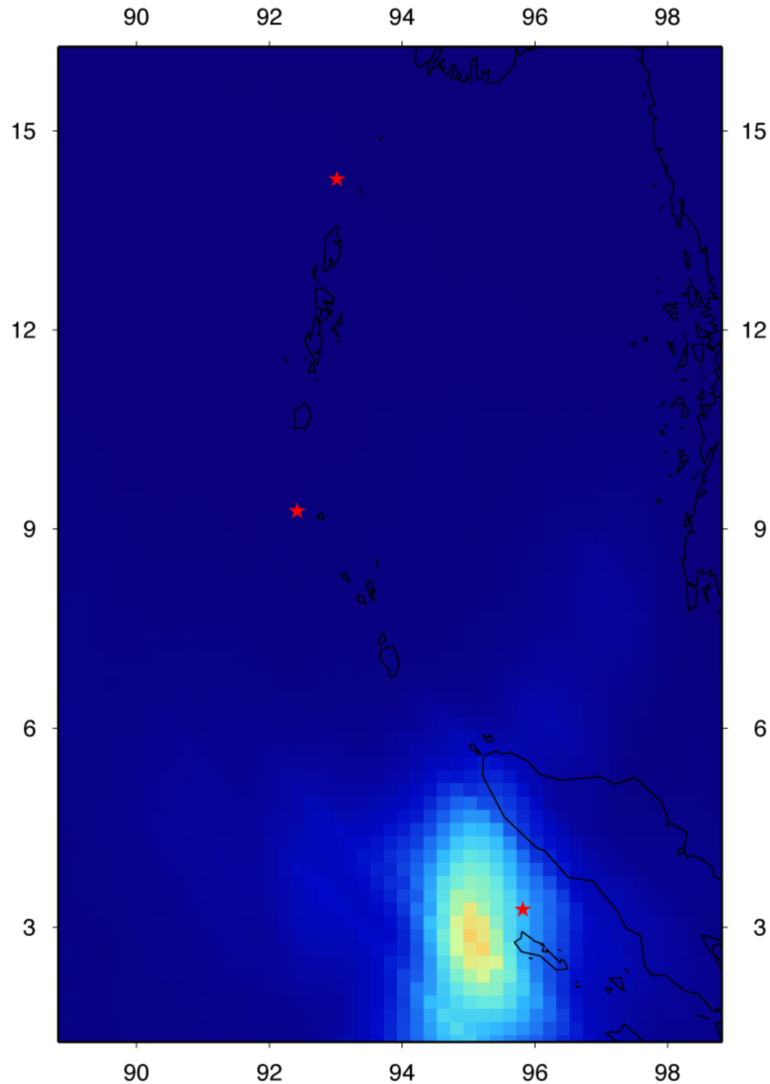


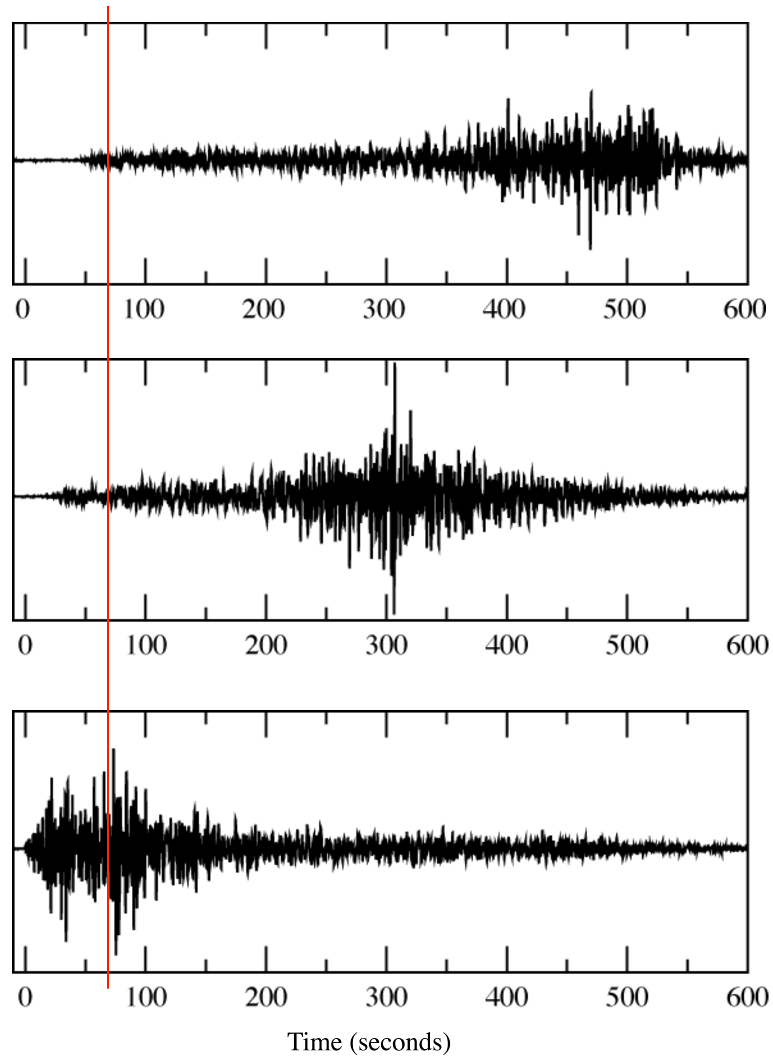
Stacks at different source points



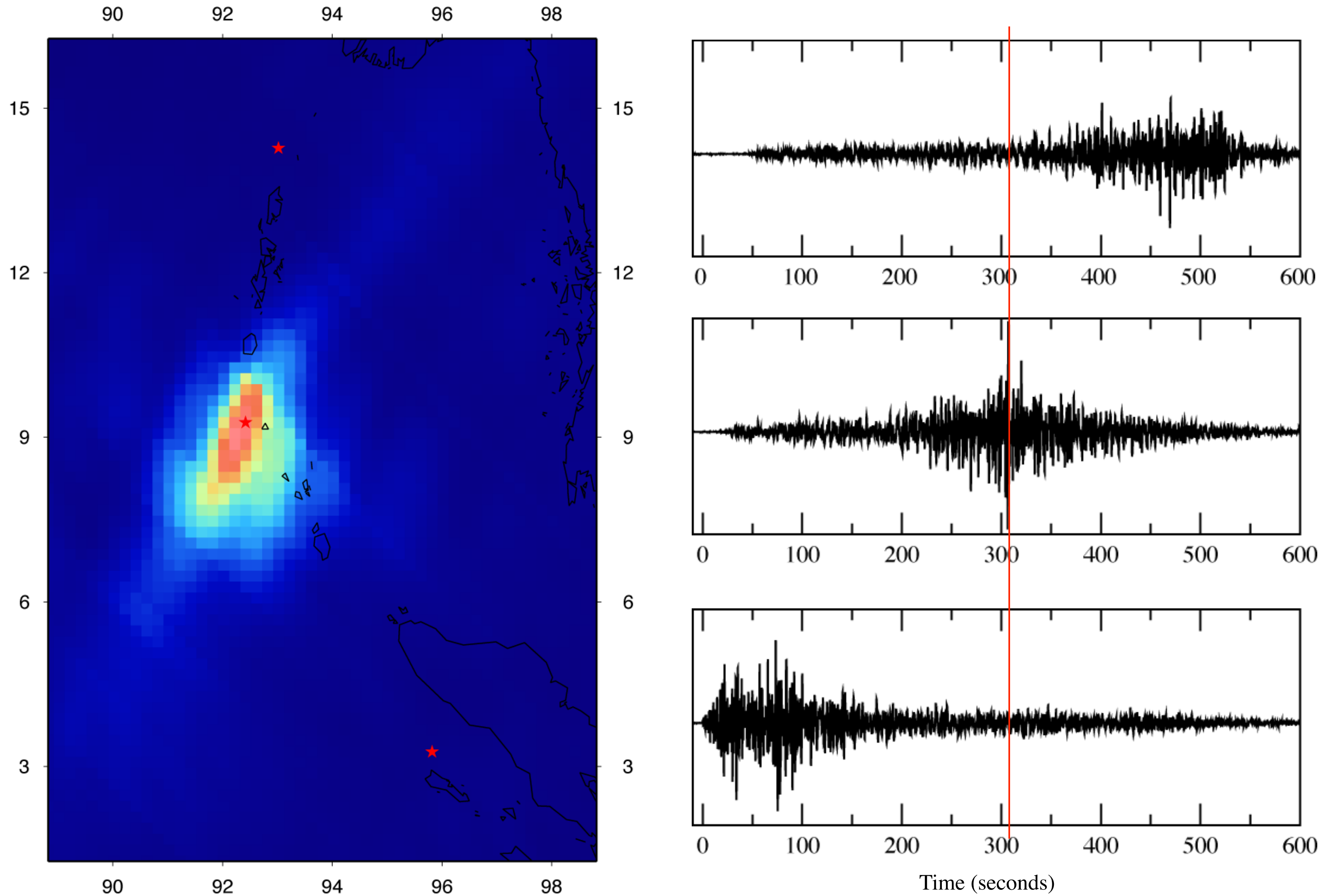
from *Ishii et al*. (2005)

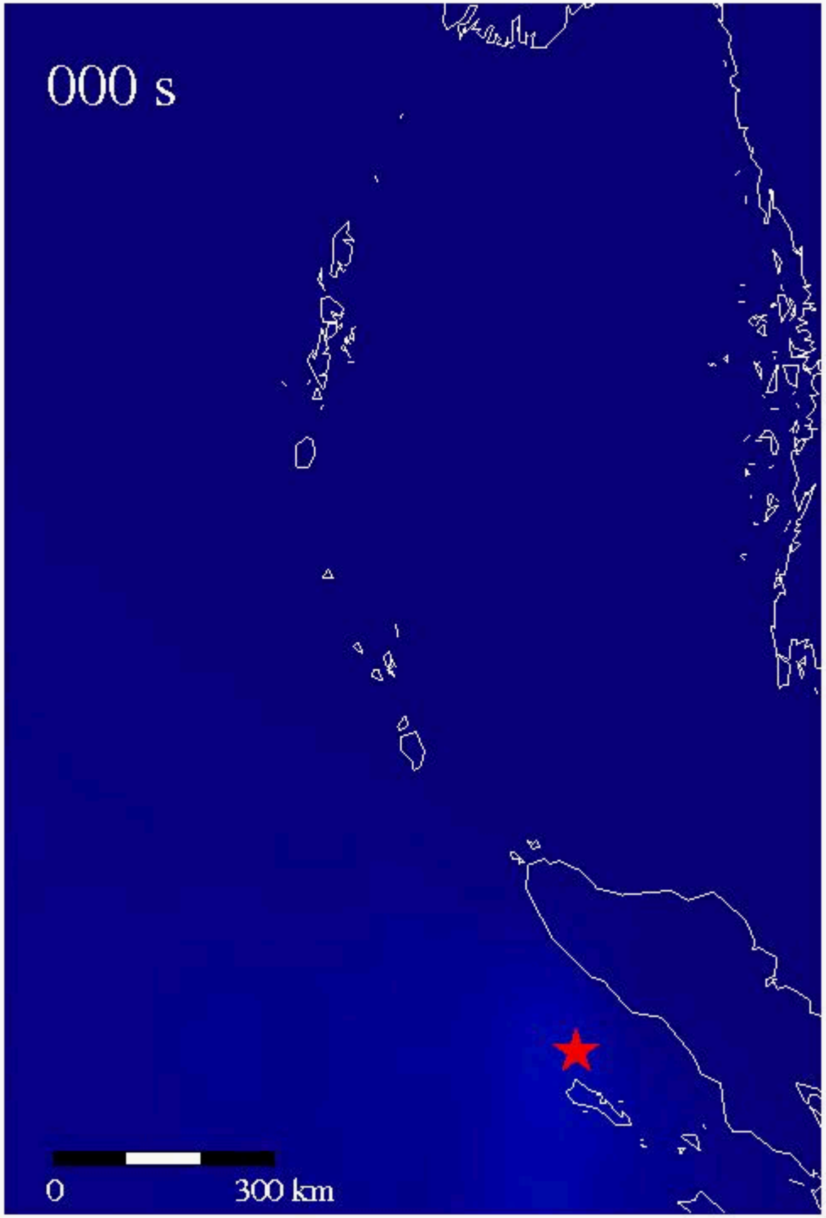
Stacks and Time Slice (60 seconds)



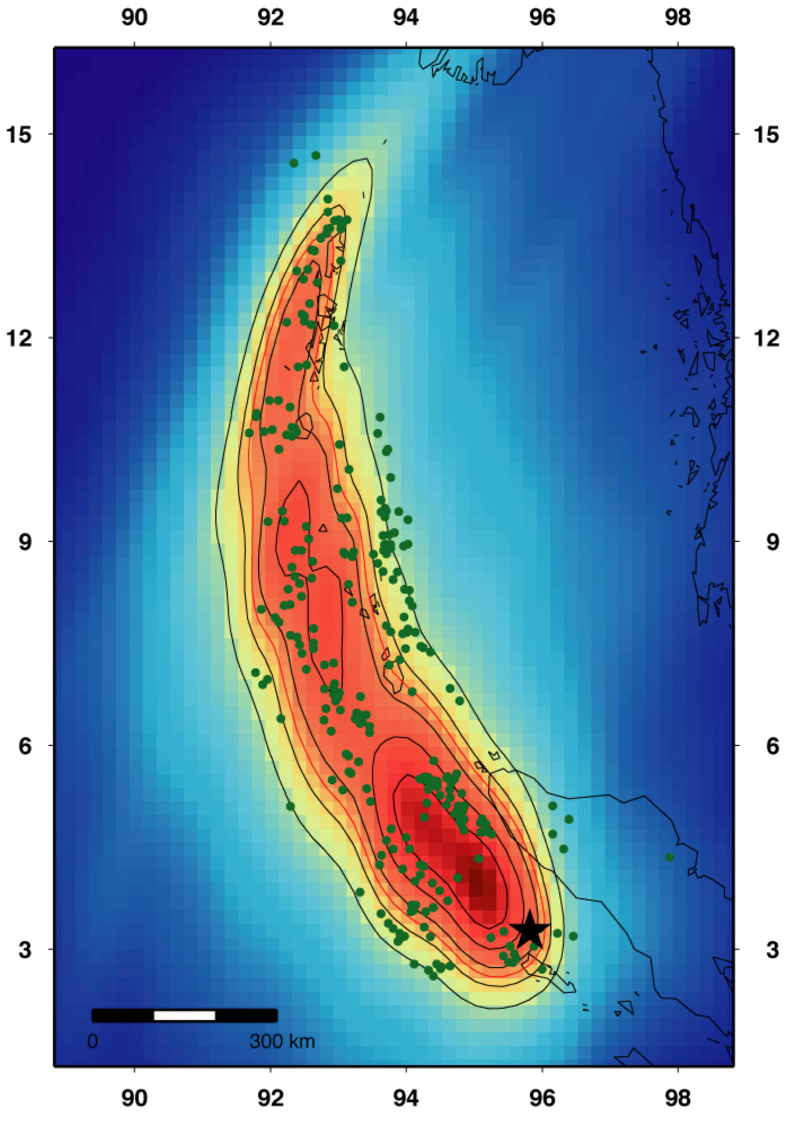


Stacks and Time Slice (300 seconds)

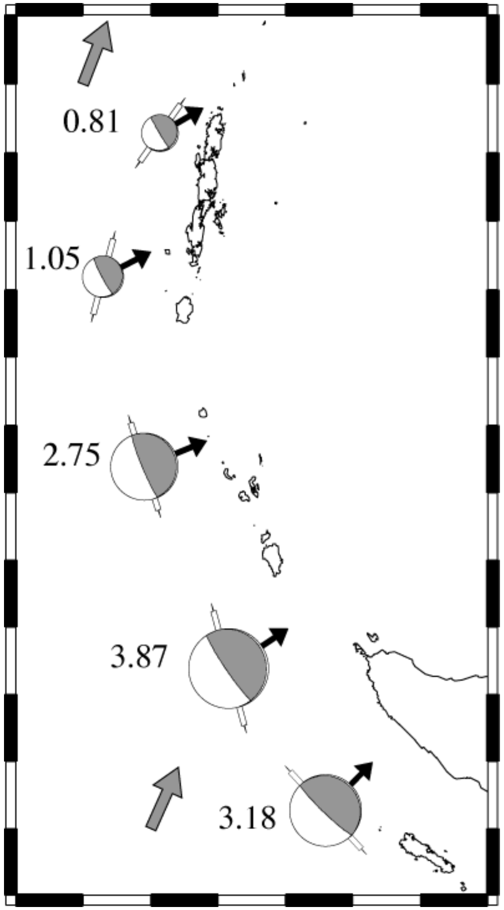


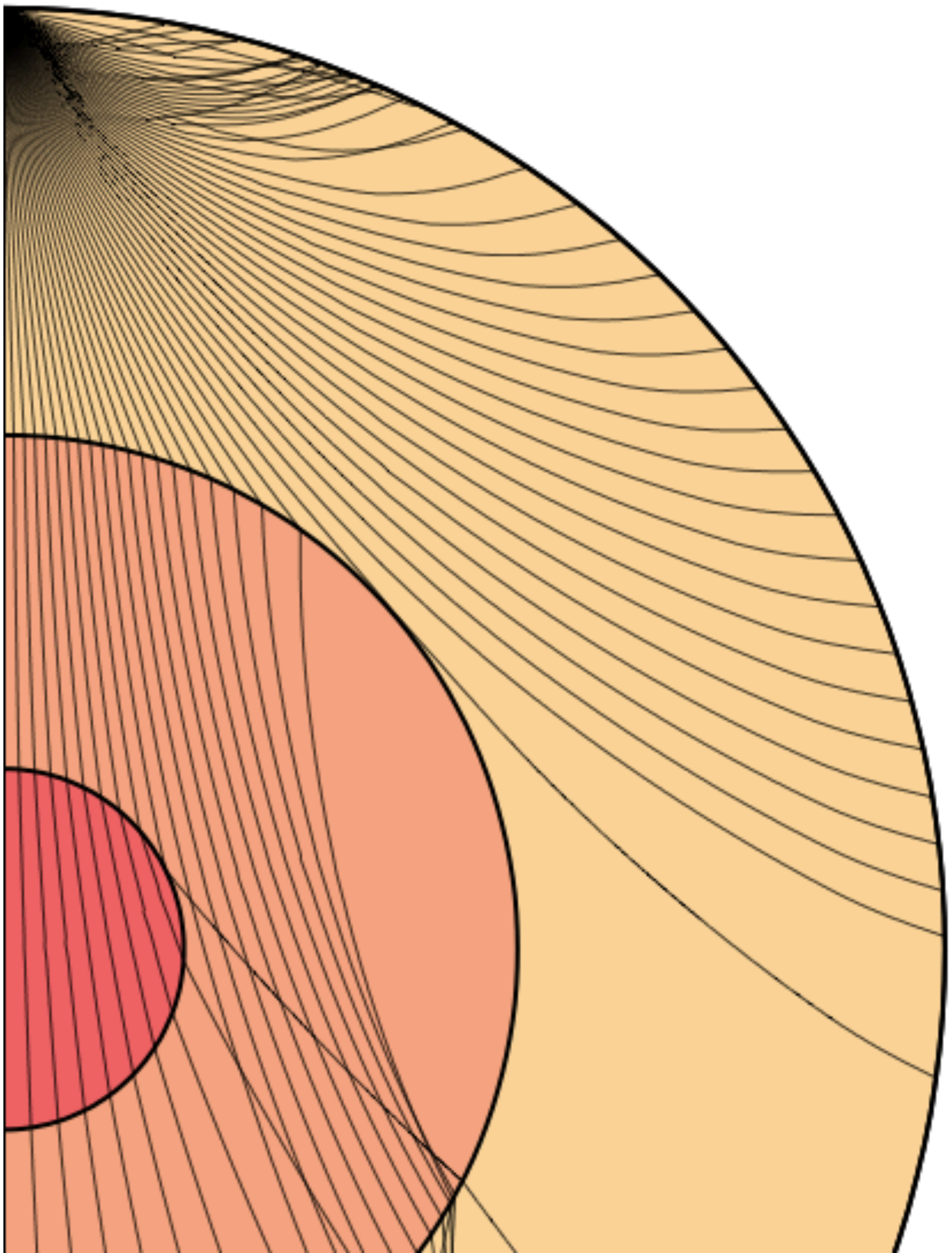

000 s
0
300 km

Short-period radiation from Hi-net backprojection (*Ishii et al.*, 2005)



Harvard multiple CMT solution (*Tsai et al.*, 2005)



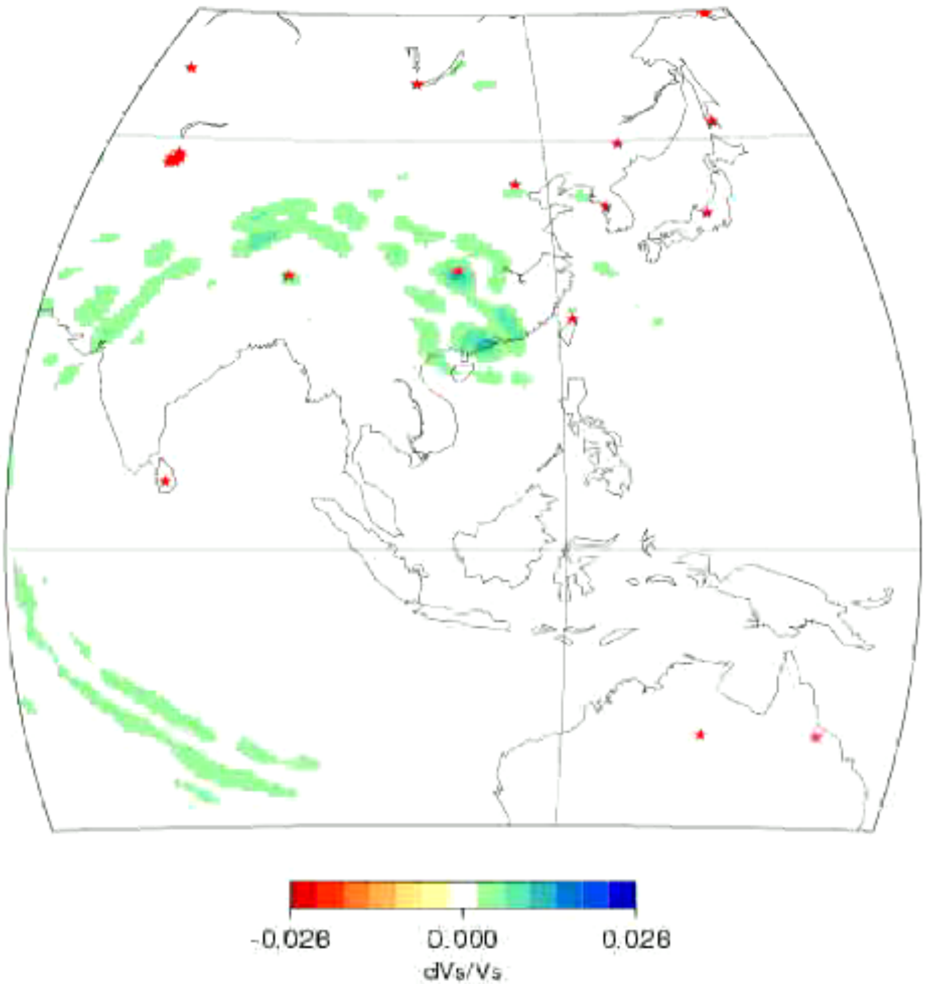


Technical Note:

Back-projection is a form of *time reversal* where we approximate the *P*-wave Green's function as a time-shifted delta function.

This works well for teleseismic arrivals between 30° and 90° where pulses are simple and amplitude variations are small.

Full waveform time reversal



Seismograms from 165 global stations sent back in time using normal mode Green's functions (> 150 s period). Image is mainly of surface waves.

from *Larmat et al*. (2006)

Back-projection advantages

- No need to assume a fault geometry, makes fewer assumptions than finite-slip inversions
- Easy to program, suited to near-real-time applications
- Many groups now produce back-projection images of major earthquakes

IRIS DMC now computes back-projection images for all large earthquakes

http://www.iris.edu/ds/products/backprojection/ has nice description of back-projection method

Back-projection cautionary note

Making images is easy!

The real problem is figuring out what parts of the images are reliable.

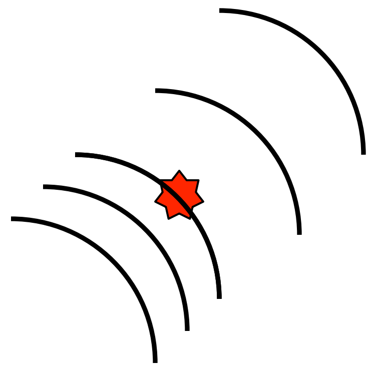
Details and Complications

- Maps high-frequency radiation, not slip (complementary to finite slip inversions)
- Subject to ‘sweeping’ artifacts
- Works best using regional arrays, not full global network, not clear why
- In principle, could be improved using aftershock calibration events, but *Ishii et al.* follow up study did not show much improvement

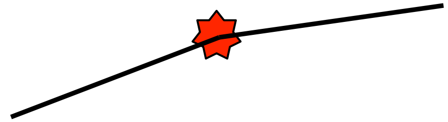
High-frequency radiation imaged by back-projection does not necessarily come from the fault patches with the largest slip

Enhanced high-frequency (HF) radiation

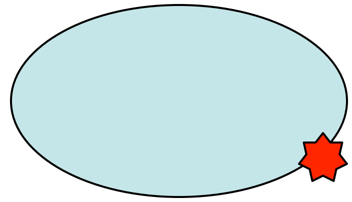
- HF radiation from areas of changes in slip and/or abrupt changes in rupture velocity (e.g., *Madariaga*, 1977; *Spudich and Frazer,* 1984)



- Near the initiation point of asperities or near changes in fault geometry (*Ide*, 2002)

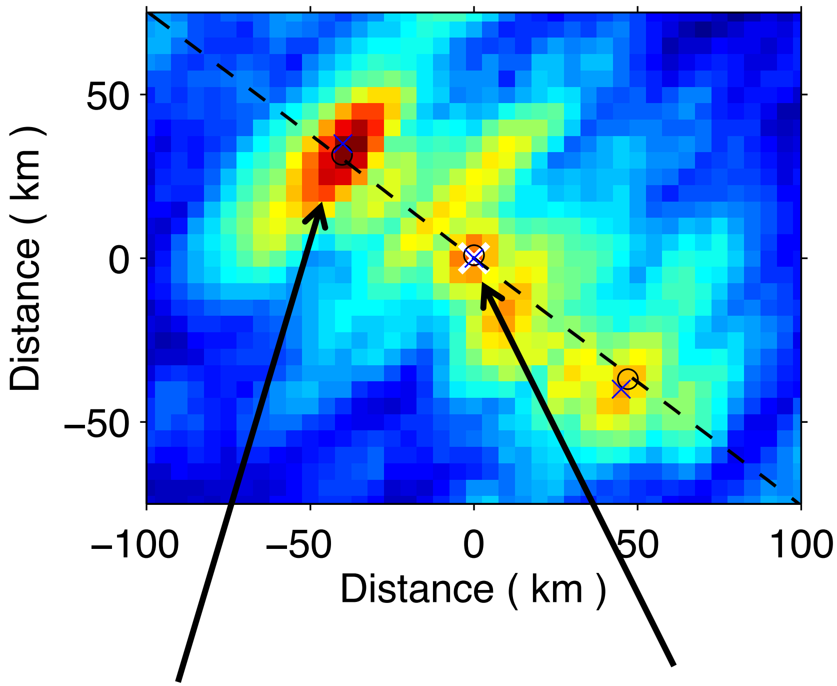


- Some observations indicate HF radiation is found at edges of major slip patches (*Nakayama and Takeo*, 1997; *Nakahara et al.*, 1998)



April 4, 2010, M 7.1 Baja earthquake

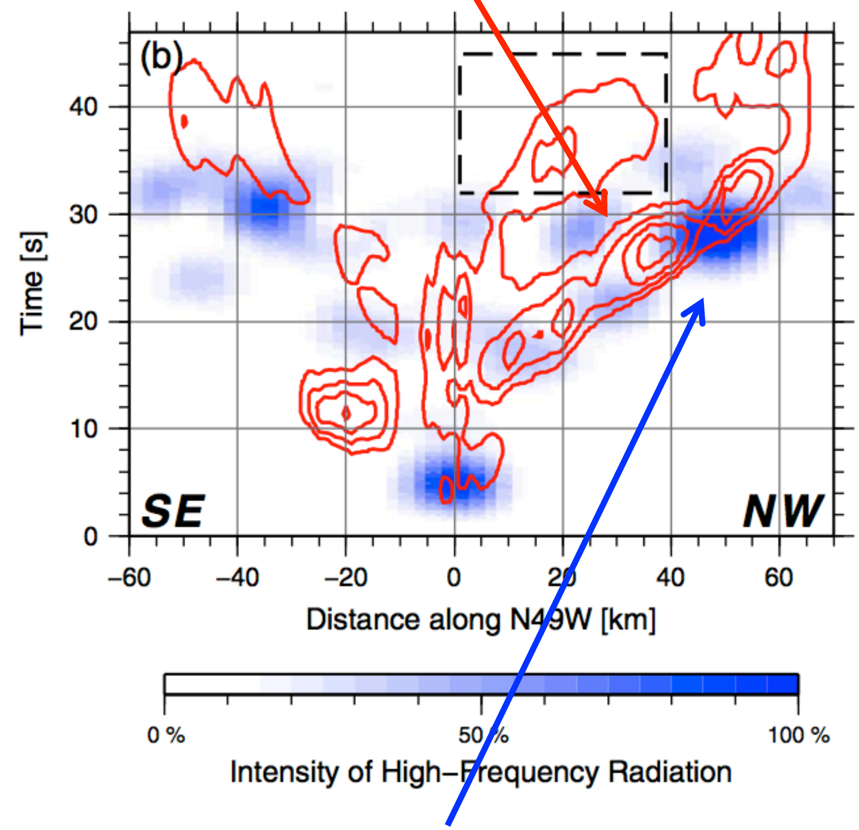
P-wave back-projection using GSN stations



Large energy burst 50 km NW at ~25 s

Hypocenter

Red contours show finite-slip inversion



Blue shows back-projection result

from *Uchide et al.* (2013)

“Sweeping” or “Swimming” artifacts often seen in back-projection animations. Do not confuse these with rupture propagation.

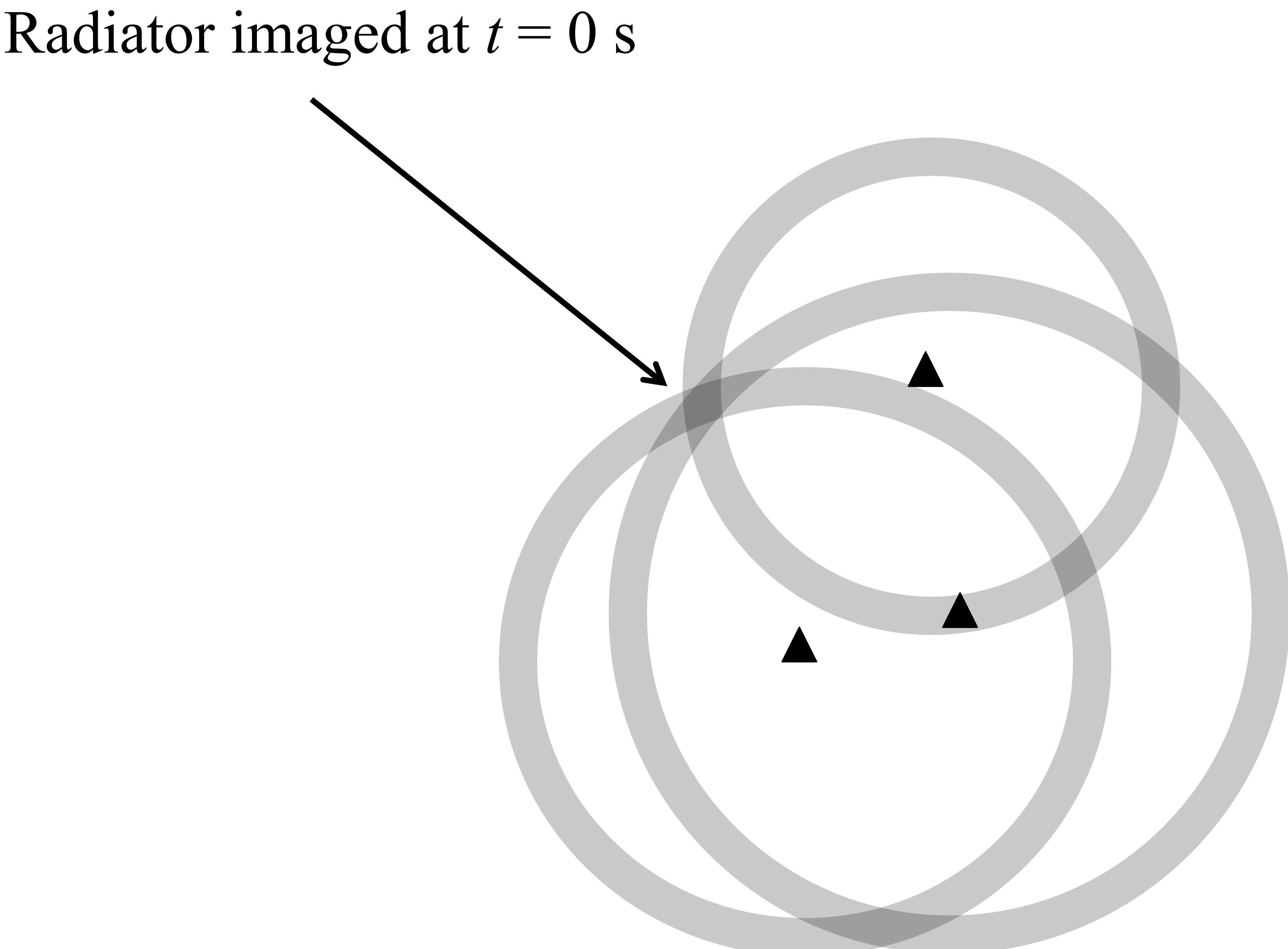

Radiator imaged at $t = 0$ s

Image at $t = 1$ s

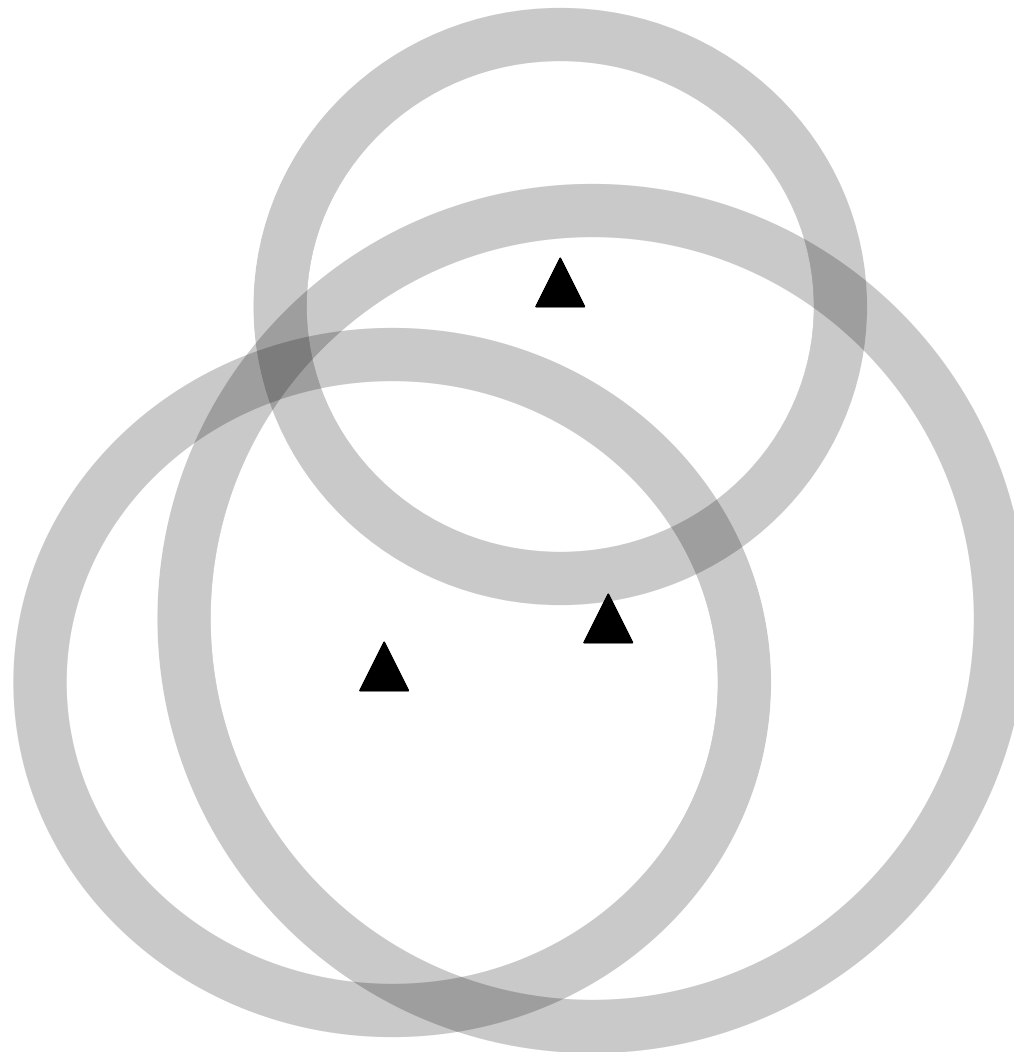


Image at $t = 2$ s

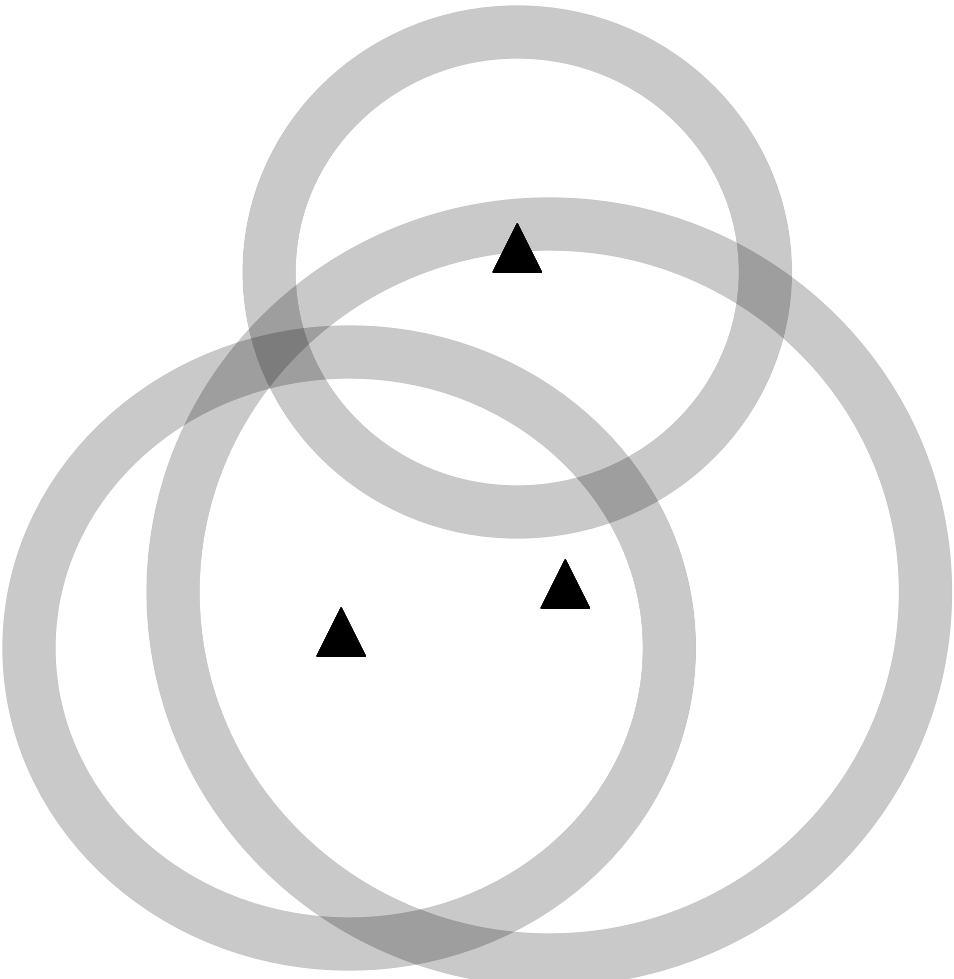
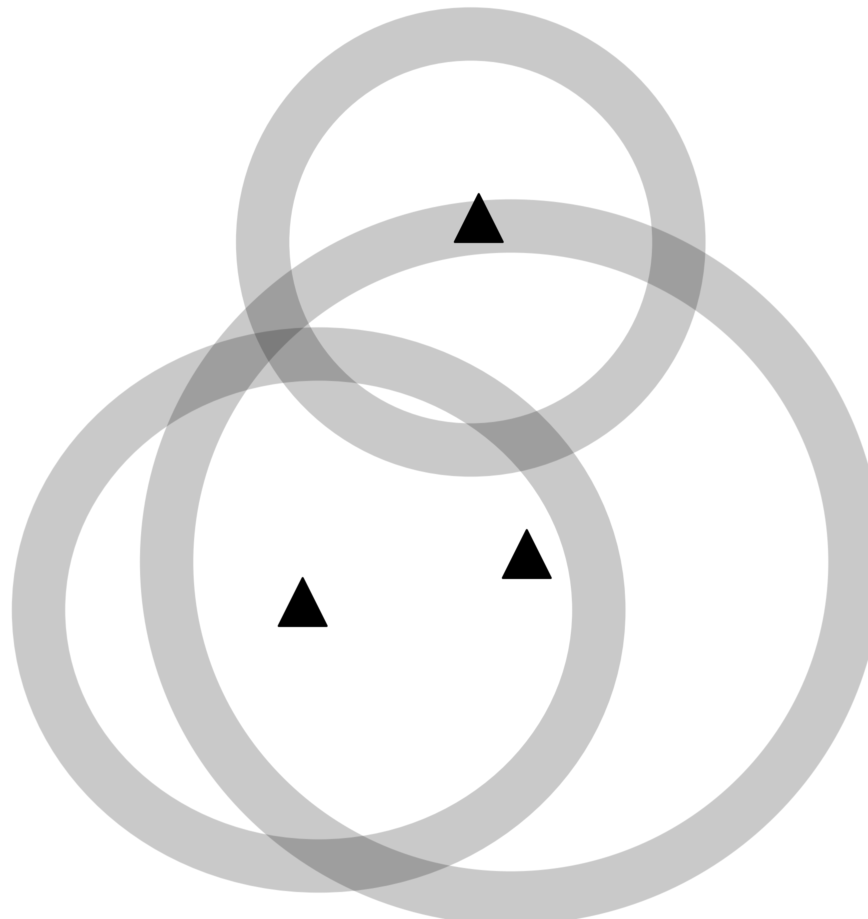
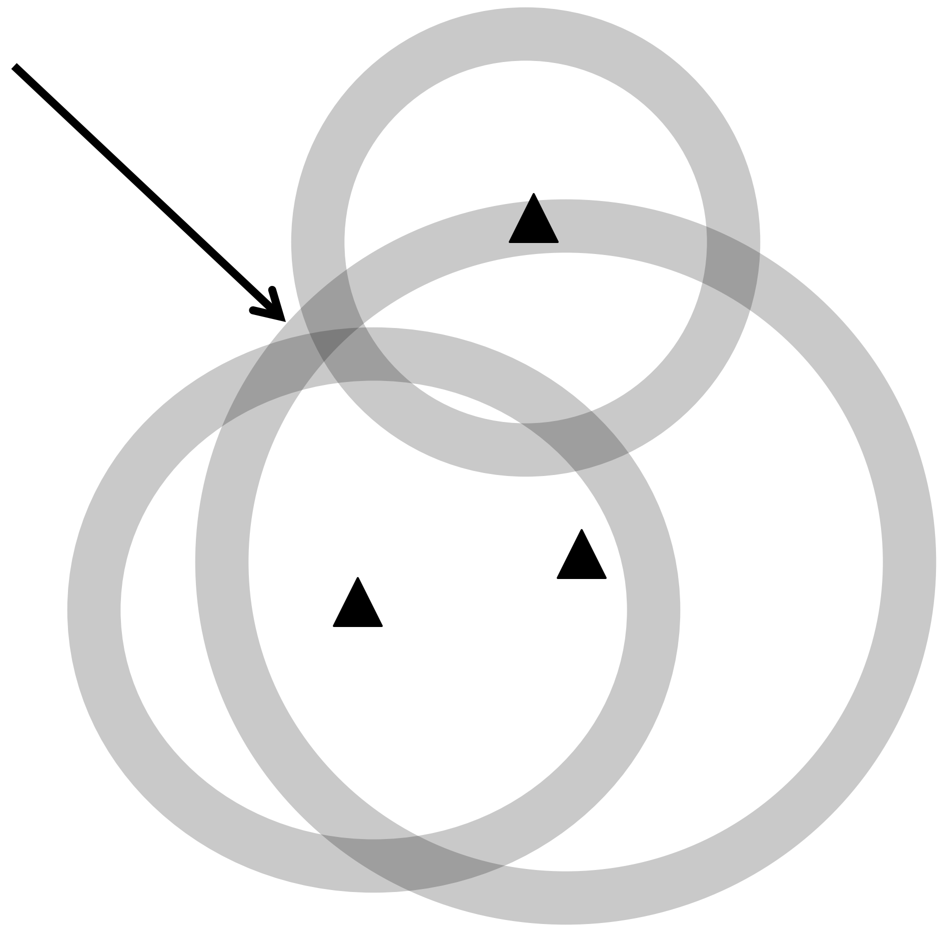


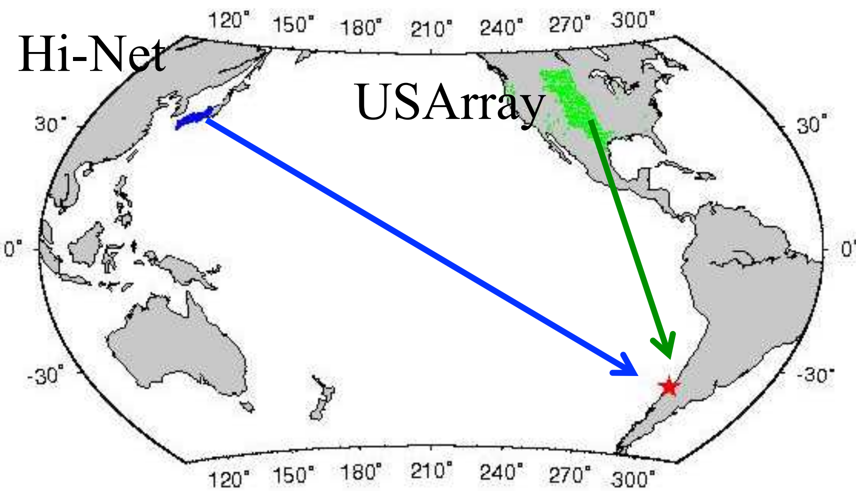
Image at $t = 3$ s

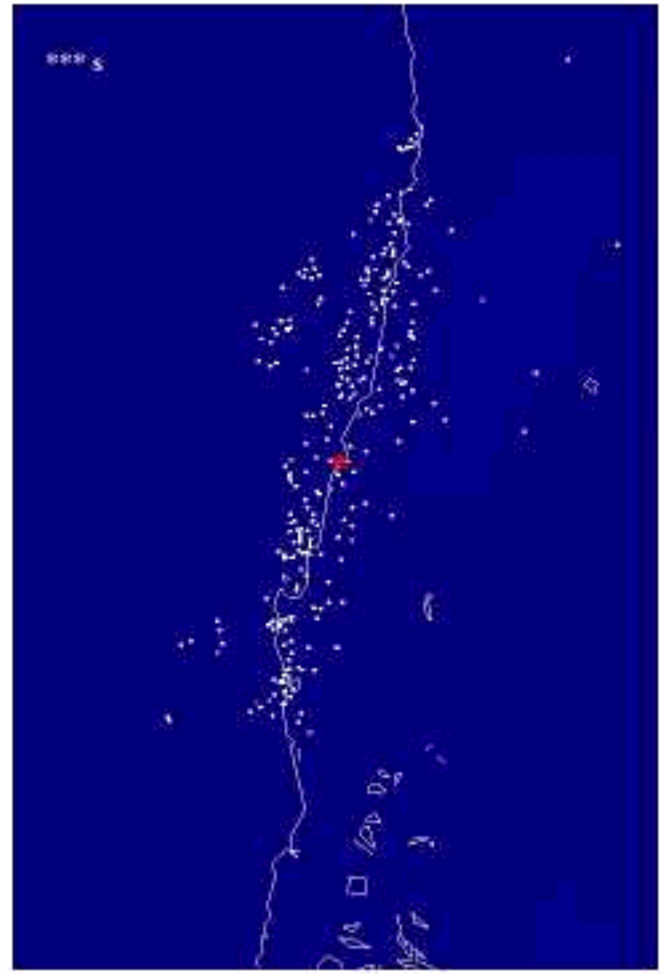


Back-projection features will sweep toward the stations with time



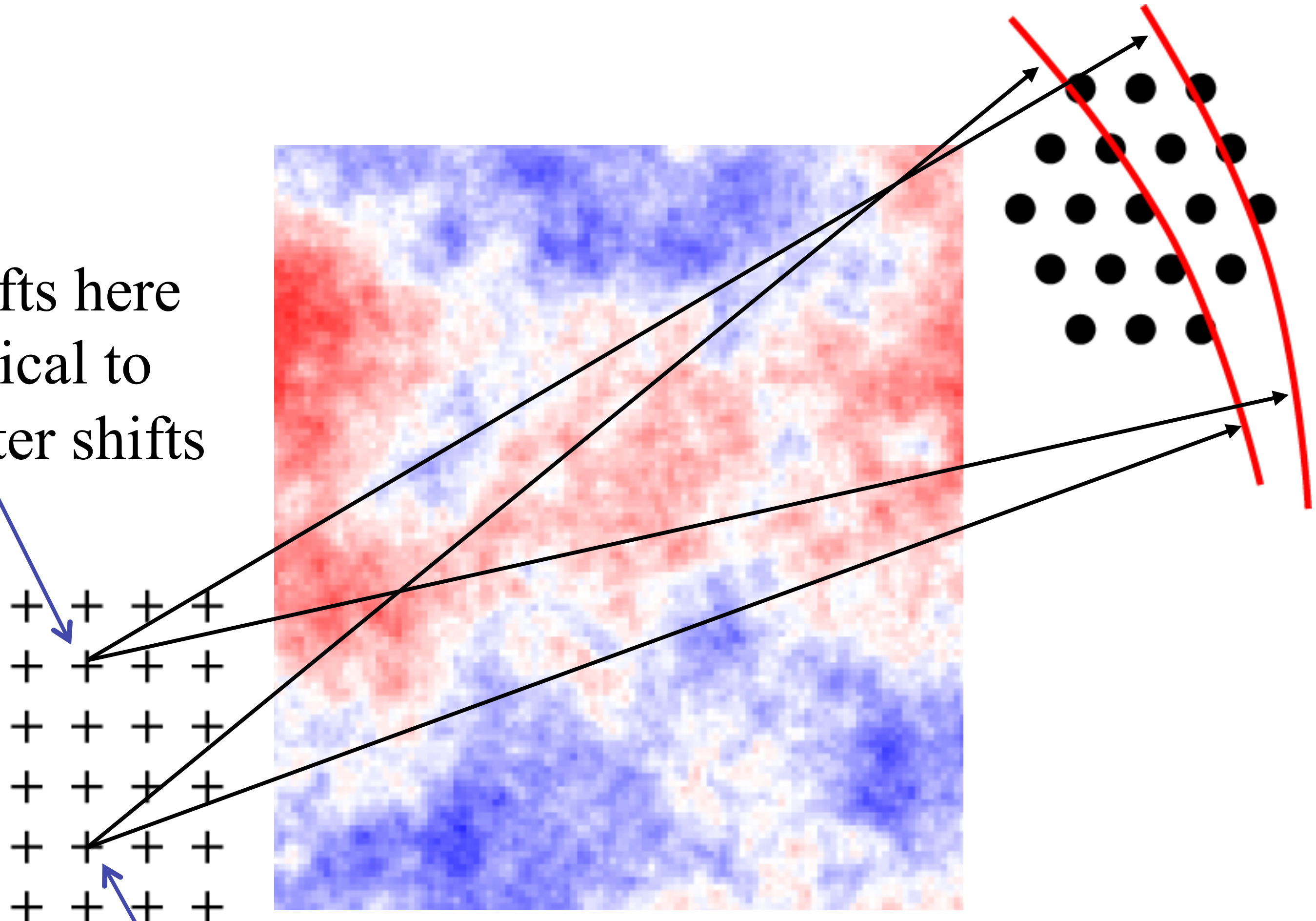
Sweeping artifacts

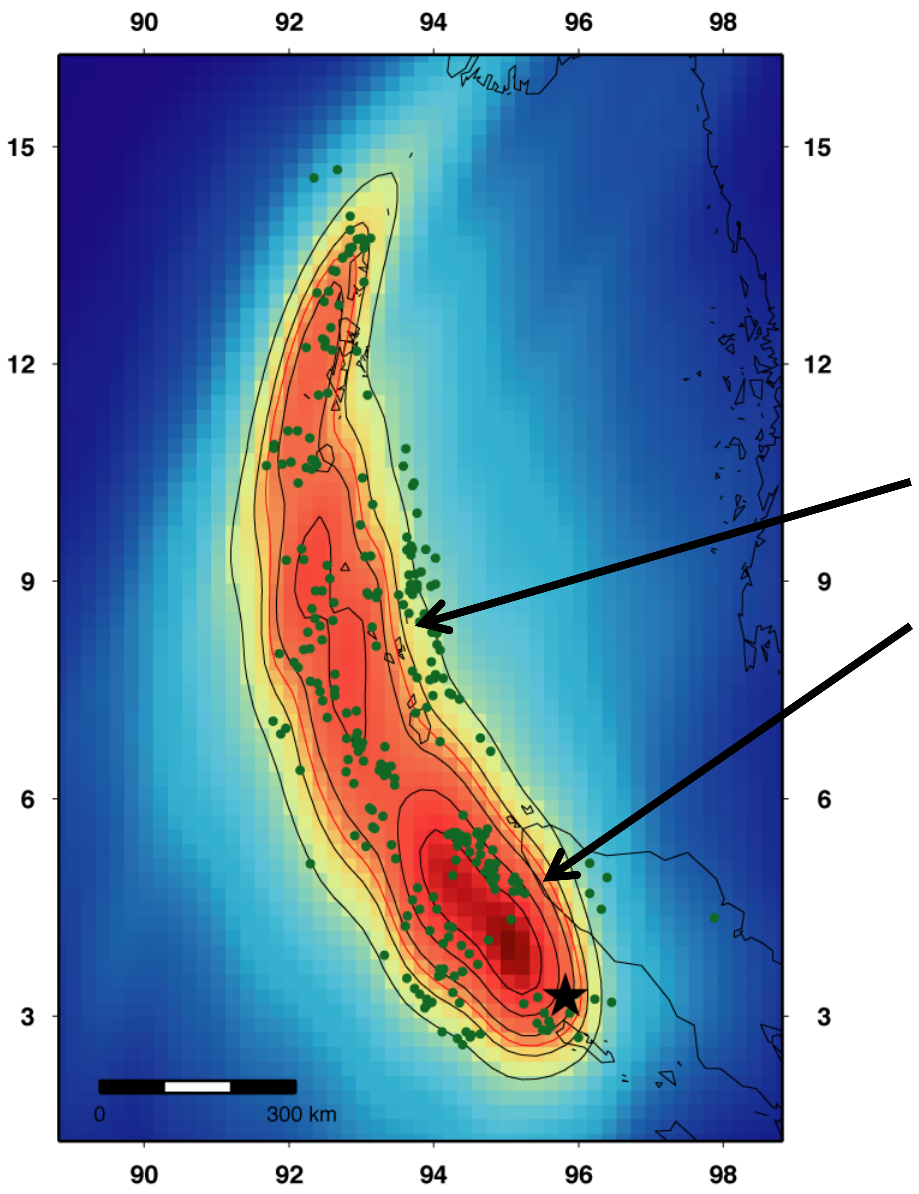




Back-projection of 2010 Chile earthquake by Kiser and Ishii

Loss of coherence in back-projection images occurs as one moves away from the hypocenter

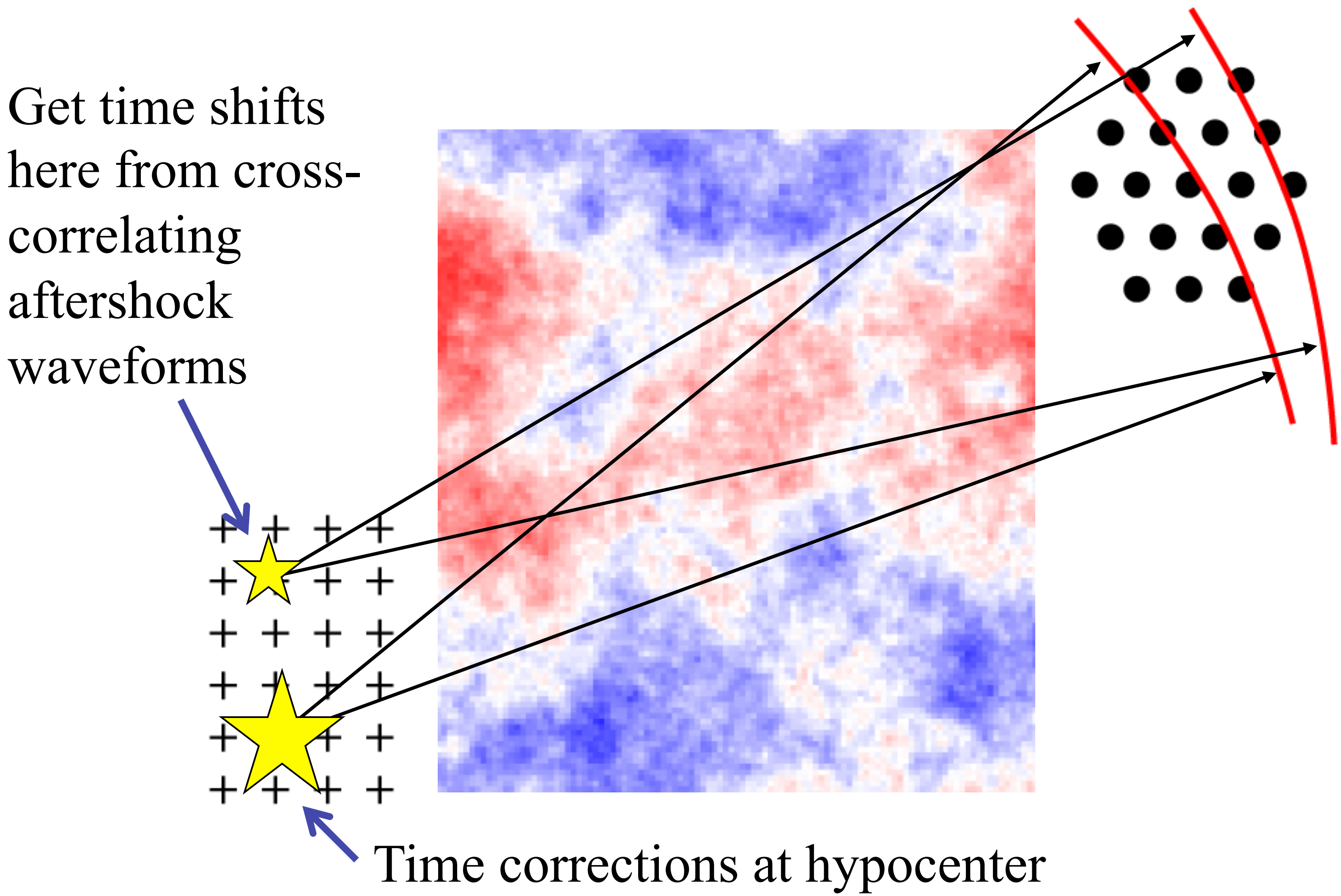




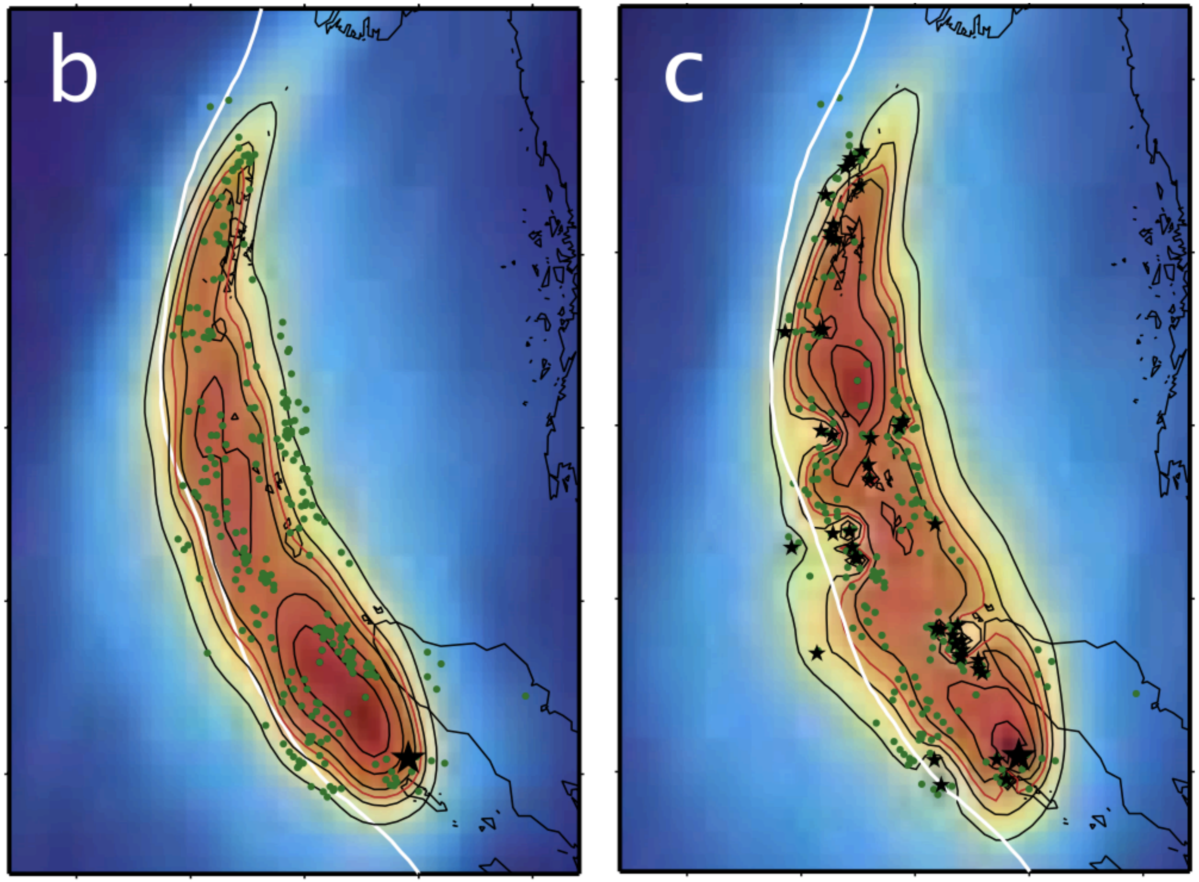
Are lower amplitudes imaged to north a real feature of the rupture?

Or an artifact of incoherent stacking?

Possible solution: Use aftershocks to calibrate timing corrections



Aftershock calibration for back-projection



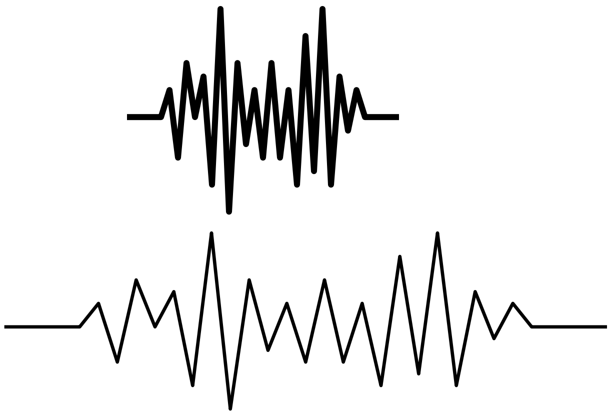
Time corrected using mainshock hypocenter

Time corrected using 46 aftershocks (black dots)

from *Ishii et al.* (2007)

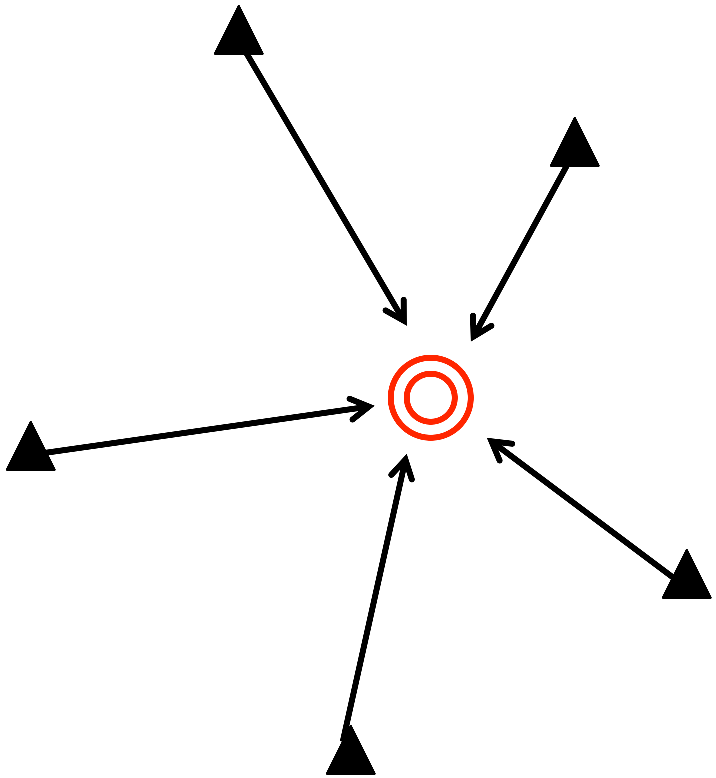
In theory, back-projection resolution kernels are smaller (i.e., better resolution) for:

Higher frequencies
(but incoherence limits
how high one can go)

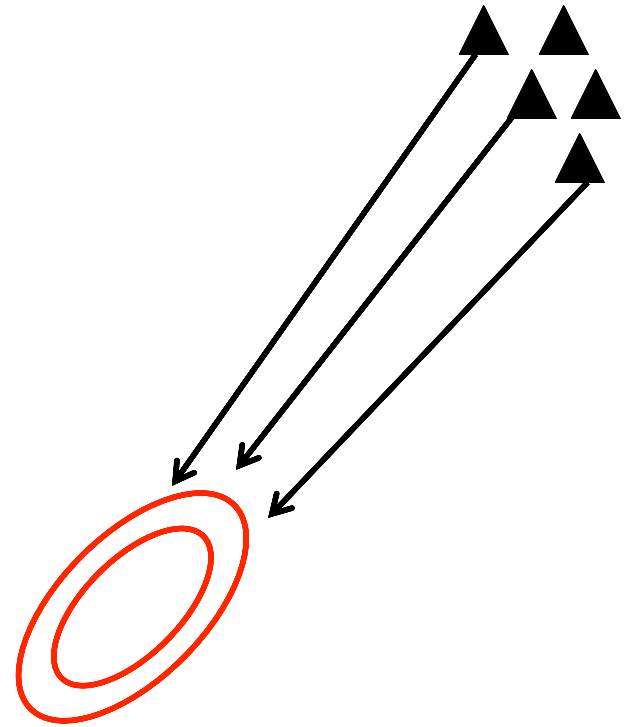


Better azimuthal station coverage
(but not always true in practice)

Theoretical Resolution Kernels



Global station distribution yields very tight kernel, should have much better resolution.



Regional array (Hi-Net, USArray) yields broader kernel, should have poorer resolution. But in practice usually works better! Why?

Back-Projection Research Challenge #1

- Develop a quantitative understanding of stacking coherence as a function of frequency, source size, and array geometry.
- One approach: Cross-correlate many small events to create empirical synthetics in different regions, conduct forward modeling tests.
- Use these results to develop methods to create higher resolution back-projection images.
- Useful for smaller earthquakes?

Back-Projection Research Challenge #2

- Develop methods to bridge the gap between finite-slip inversions at low frequencies and back-projection at high frequencies.
- Use results to constrain earthquake dynamics

Coherence between data and synthetics

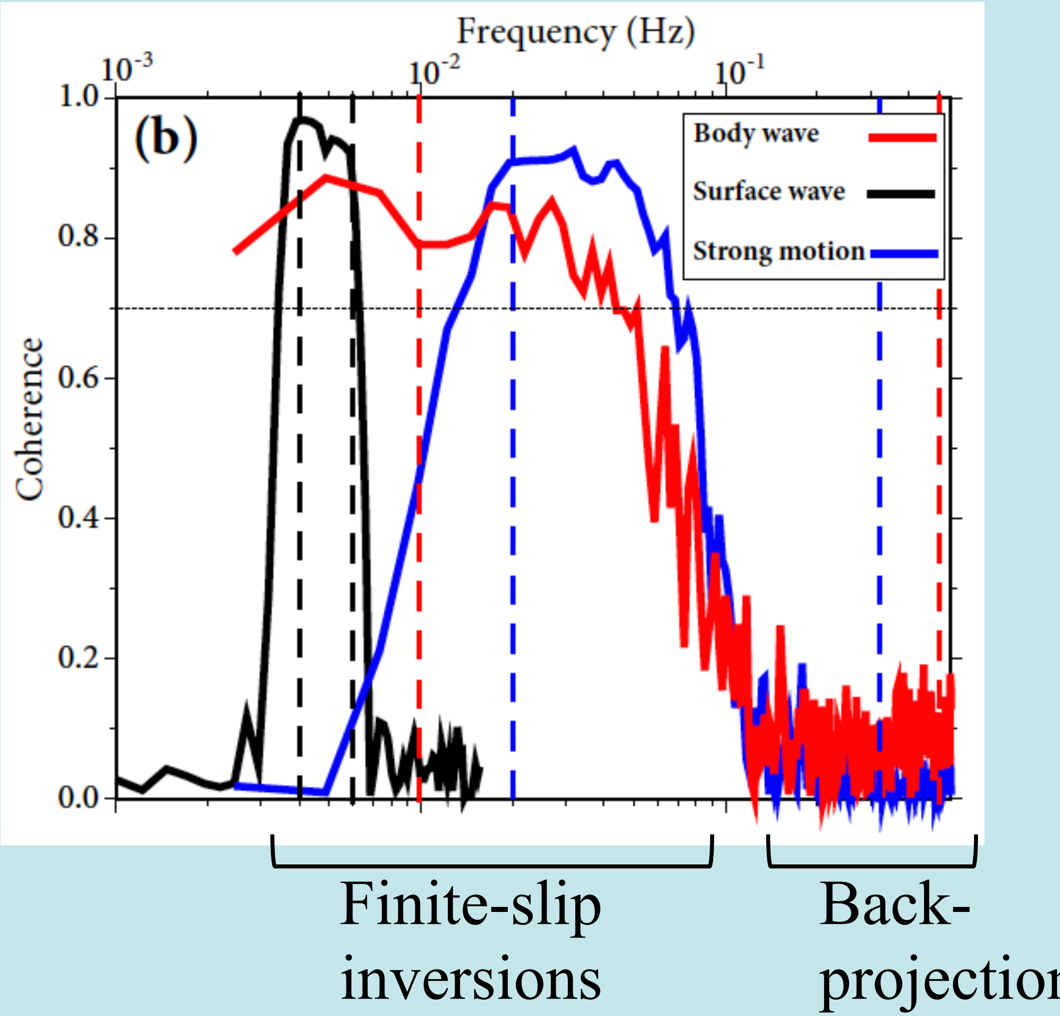


Figure from Chen Ji

Your Immediate Task: Computer Exercise 1

- Described at end of notes. Get needed files from: http://igppweb.ucsd.edu/~shearer/SCECERI/
- Data are provided. You must write your own program (e.g., F90, C, or Python) to back-project and image tremor sources.