## SIO 223B-Problem Set 2

2.1 A series is sampled at a rate $f$; that is, the sample interval is $\Delta=1 / f$ seconds. A DFT is taken of $N$ points of the series. What is the frequency (in Hz ) corresponding to the $k$-th Fourier coefficient $X_{k}$ ? Note that $k$ runs from 0 to $N-1$; how do you reconcile this with the Nyquist frequency limit?
2.2 A common approach to getting finer sampling in frequency is to pad a finite sequence of length $N$ out to length $M$ by appending $N-M$ zeroes. Give the equation for each of the $M$ DFT coefficients obtained thereby (called, say, $X_{l}^{M}$ ) in terms of the $N$ coefficients $X_{k}$, gotten by taking the DFT of the original sequence. (Two hints: the answer is not $X_{k N / M}$, since $k N / M$ is not an integer; and you should use an expression for the original sequence $x_{n}$ ). Try to reduce the result to a single sum over the $X_{k}$ 's.
2.3 Show that if $x_{n}$ is an infinite periodic sequence (not just sinusoidal), $x_{n} * h_{n}$ is also, and has the same period (no exponentials are needed).
2.4 Show that if $x_{n}=-x_{N-1-n}$ then the DFT coefficient $X_{0}$ is 0 , for any $N$. Also show that if $x_{n}=x_{N-1-n}$ then $X_{N / 2}$ is 0 , for $N$ even; what happens for $N$ odd?
2.5 The Topex/Poseidon altimetry satellite has provided our best information on ocean tides. This system provided estimates of ocean height at a large number of locations with a repeat time of 9.9156 days. Calculate the apparent periods in days, in the satellite data, of the following tidal constituents (these are sinusoids, named by their 'Darwin symbols'; the frequencies are in cycles per day):

$$
\begin{array}{cc}
\mathrm{M}_{2}: 1.932274 & \mathrm{~N}_{2}: 1.895982 \\
\mathrm{~S}_{2}: 2.000000 & \mathrm{~K}_{2}: 2.005476 \\
\mathrm{~K}_{1}: 1.002738 & \mathrm{P}_{1}: 0.997262 \\
\mathrm{O}_{1}: 0.929536 & \mathrm{Q}_{1}: 0.893244
\end{array}
$$

How well will these frequencies be separated in the Topex/Poseidon data, assuming a mission length of 6 months; what if it is 5 years? Will any of these frequencies be confused with the annual variations in sea level, or even longerperiod changes?
2.6 A wheeled vehicle is filmed at a rate of $s$ frames per second as it accelerates from zero velocity with constant acceleration $a$. Define the apparent velocity of the wheels $v_{w}(t)$, by their apparent rotational velocity times their diameter $d$. Find a general expression for $v_{w(t)}$, and plot it for the case $a=1, s=20$, and $d=1$ (all SI units).
2.7. The attached plot shows readings of sea level at Avila Beach, California (about 200 miles north of here), made about 5 PM every day for about a year. Explain the variations you see in terms of undersampling of the tides, given the frequencies of the largest ones, as given in the table in problem 2.5 (left column semidiurnal tides, right diurnal, largest at the top):


