GEOPHYSICAL RESEARCH LETTERS, VOL. 22, NO. 21, PAGES 2905-2908, NOVEMBER 1, 1995

Monument motion and measurements of crustal velocities

Hadley O. Johnson and Duncan Carr Agnew

Institute of Geophysics and Planetary Physics, Scripps Institution of Oceanography
University of California, San Diego

Abstract. It is usually assumed in geodetic studies that measurement errors are independent from one measurement to the next and that the rate of deformation (velocity) is constant over the duration of the experiment. Any temporal correlation between measurements can substantially affect the uncertainty in this velocity estimate when it is determined from the time series of measurements. One source of possible long-term correlation is motion of the geodetic monument with respect to the "deep" crust. Available measurements suggest that this motion introduces errors that have the form of a random walk process. We show how such errors affect the uncertainty of velocity estimates. For a geodetic experiment of set duration we calculate the velocity uncertainty as a function of the number of observations and of the relative amount of correlated and uncorrelated noise. We find that 1) neglecting long-term temporal correlations makes the uncertainty in the estimated velocities much too small, and that 2) when the correlated and independent noise sources are of similar magnitude, the expected improvement in uncertainty from having more measurements ($1/\sqrt{N}$) is not realized; there is almost no improvement in some cases. We have also examined the effect of outliers ("blunders") on the velocity uncertainty; for a frequency of outliers typical of geodetic field campaigns, the previous two conclusions remain unchanged. These results suggest that long-term correlations have a large effect on estimating deformation rates; unless these correlations are small, frequent observations give little advantage. If frequent observations are planned, the amount of correlated noise due to monument instability must be kept small if the full capabilities of the measurement technique are to be realized.

Introduction

It is common in actively deforming areas to make repeated geodetic measurements and to estimate rates of deformation by fitting a linear trend to these measurements assuming no significant episodic deformation has occurred (e.g., a seismic or volcanic event). To determine this trend (the velocity) and its uncertainty, some estimate of the data covariance matrix is needed. Usually the errors from one measurement to another are assumed to be independent, making the covariance matrix diagonal. However, for geodetic measurements of tectonic motion another potential source of error is the instability of the geodetic monument which can introduce large temporal correlations into the data. Ideally the position of a geodetic monument would reflect only deep-seated tectonic deformations. For this to be true requires the monument to be unaffected by non-tectonic processes such as desication weathering, water withdrawal, and landslides to name only a few of the many possibilities.

Wyatt [1982] and *Wyatt* [1989] show that continuous measurements of near surface monuments from a sensitive strainmeter at Piñon Flat Observatory in southern California have power spectra that rise at low frequencies approximately as f^{-2}. In addition, *Langbein et al.* [1993] show power spectra from frequent geodetic measurements which also suggest an f^{-2} dependence. This behavior is a characteristic of a random walk (Brownian motion) where the monuments move about as if they are particles under the influence of small random forces, which is a plausible model for the physics of the problem. We demonstrate here the effect such an error source has on estimates of secular velocities, and especially on the uncertainty of those estimates, through simulations with synthetic data. In an earlier paper [*Johnson and Wyatt*, 1994] we used a similar technique to understand the effect of noise from monument motion (and other noise sources) on estimates of fault-model parameters.

The effect of long-range correlations on estimates of uncertainty has long been noted by practicing statisticians (e.g., *Jeffreys* [1939]), but has not been much studied in the statistical literature until the last few years; *Beran* [1992] describes much of this recent work. *Arnadottir et al.* [1992] have shown the importance of acknowledging the existence of spatial correlations which are introduced into the geodetic data (in this case primarily leveling) by the processing strategy. We treat a different case here in which the correlations are temporal in nature; it should be noted that insofar as this sort of correlation is present, as it must be at some level due to monument motion, it is an issue for all geodetic data.

Technique

We assume that the measured position of a reference point (in one dimension) is given by:

$$P(t_i) = P_0 + r\,t_i + \varepsilon(t_i) \qquad (1)$$

where P_0 is an initial position and r is velocity. We further assume that the error, $\varepsilon(t_i)$, is the sum of two terms, one from the measurement system and the other from monument motion:

$$\varepsilon(t_i) = a\alpha(t_i) + b\beta(t_i) \qquad (2)$$

Paper number 95GL02661
0094-8534/95/95GL-02661$03.00

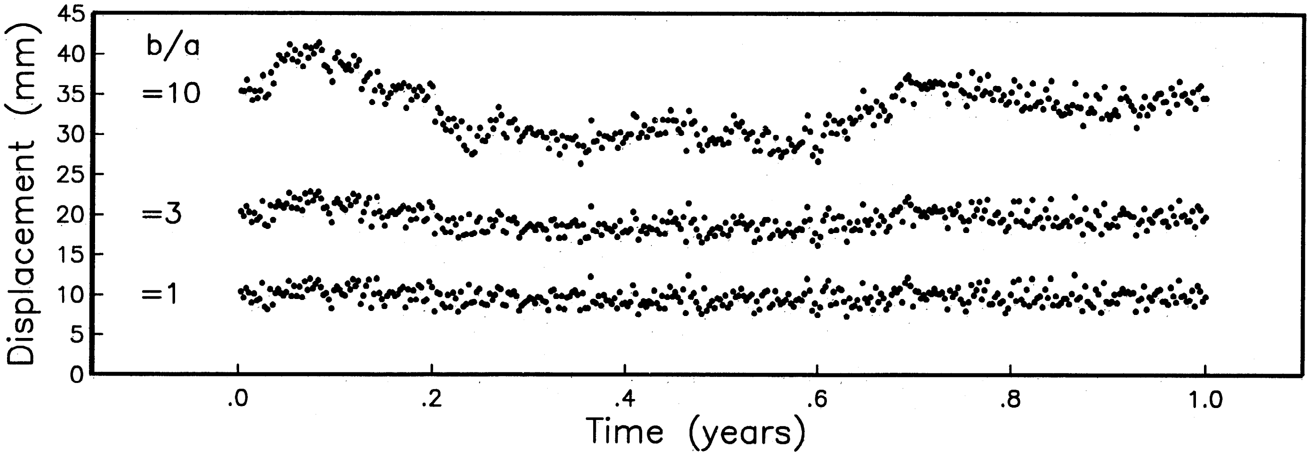

Figure 1. Three synthetic time series representing one year of daily position estimates as described by equations (1) and (2) with $a = 1$. The relative amount of the random walk error component (b) increases by a factor of 10 from bottom to top. Only in the top two traces is the correlated nature of the time series evident.

where $\alpha(t_i)$ is a sequence of independent, unit-variance Gaussian random variables, $\beta(t_i)$ is a random walk sequence, and a and b are scale factors that control the relative amount of each type of error (we use millimeters as our basic unit of position and years for time).

The random walk sequence is generated by the cumulative addition of Gaussian random variables, and is nonstationary. Our scaling of $\beta(t_i)$ in equation (2) is such that its standard deviation after time t is given by 1 mm $\times \sqrt{t}$. Thus, a large number of realizations of $\beta(t_i)$ would have a standard deviation of one millimeter after one year, two millimeters after four years, and so on. Since analyses of actual geodetic data in California (done by us and by J. Langbein [personal communication]) suggest that monument instabilities as large as a few mm/$\sqrt{\text{yr}}$ are typical, we can thus reasonably suppose b to be in the range 0.1 to 3. Likewise, errors of 0.5–3 mm for a seem representative for the measurement error of current regional GPS systems.

Given a time series of the form (1), we fit to it a model with

$$P(t) = \hat{P}_0 + \hat{r}\,t \tag{3}$$

where $\hat{P}_0$ and $\hat{r}$ are the estimated initial position and velocity. Since we are interested only in velocity estimates, we set $P_0 = 0$, and do not discuss it further. We may also, again without loss of generality, set $r = 0$. We use a 2-norm fitting criterion (least squares), in which case the standard theory for regression gives matrix equations to determine $\hat{r}$ and its standard error, $\sigma_{\hat{r}}$. To find these, we must also know the covariance matrix, Σ_{PP}; for the error model given by (3), this is

$$\Sigma_{PP} = a^2\mathbf{I} + b^2\mathbf{T} \tag{4}$$

where **I** is the identity matrix and **T** is

$$\mathbf{T} = f_s^{-1}\begin{bmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 1 & 1 & \cdots \\ 0 & 1 & 2 & 2 & \cdots \\ 0 & 1 & 2 & 3 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

where f_s^{-1} is the sampling frequency in years^{-1}. (A derivation of the **T** matrix is contained in the appendix of *Johnson and Wyatt* [1994] where it is called the Σ_{vv} matrix. Note, however, that there is an error in the **Z** matrix in that derivation: the right-most column of zeros should be removed.) For the actual computation we use a Cholesky factorization of the covariance matrix to transform the correlated time series into an uncorrelated one. In this new system the transformed data are independent and the parameter estimation proceeds easily.

Table 1. Slope and standard deviation estimates for the time series in Figure 1.

	Error sources included in covariance matrix			
	Measurement system (only)		Measurement system & Monument instability	
b/a	$\hat{r}$	$\sigma_{\hat{r}}$	$\hat{r}$	$\sigma_{\hat{r}}$
10.0	−0.993	0.181	−0.662	10.054
3.0	−0.386	0.181	−0.312	3.054
1.0	−0.213	0.181	−0.263	1.057

Units of $\hat{r}$ and $\sigma_{\hat{r}}$: mm/yr.

Results and Discussion

What does a correlated time series look like? Or, more importantly, how much correlation can be "hidden" in a time series before it can easily be seen by eye? Figure 1 attempts to demonstrate the answer to these questions by showing three synthetic time series of the form described by equations (1) and (2) but with different relative amounts of correlated noise. Each series represents one year of daily measurements. The same sequences of pseudo-random numbers were used to generate each series. The scaling of the independent errors is the same (a = 1, for a 1 mm standard deviation) in all three series; the scaling, b, of the random walk error term ranges from 1 at bottom to 10 in the top trace. Only in the top two series is the influence of the random walk noise term evident.

Table 1 shows the results of fitting a model in the form of equation (3) to each of these time series, using different covariance matrices: the full matrix described by (4), and one which ignores the correlations ($b = 0$ in (4)). For all

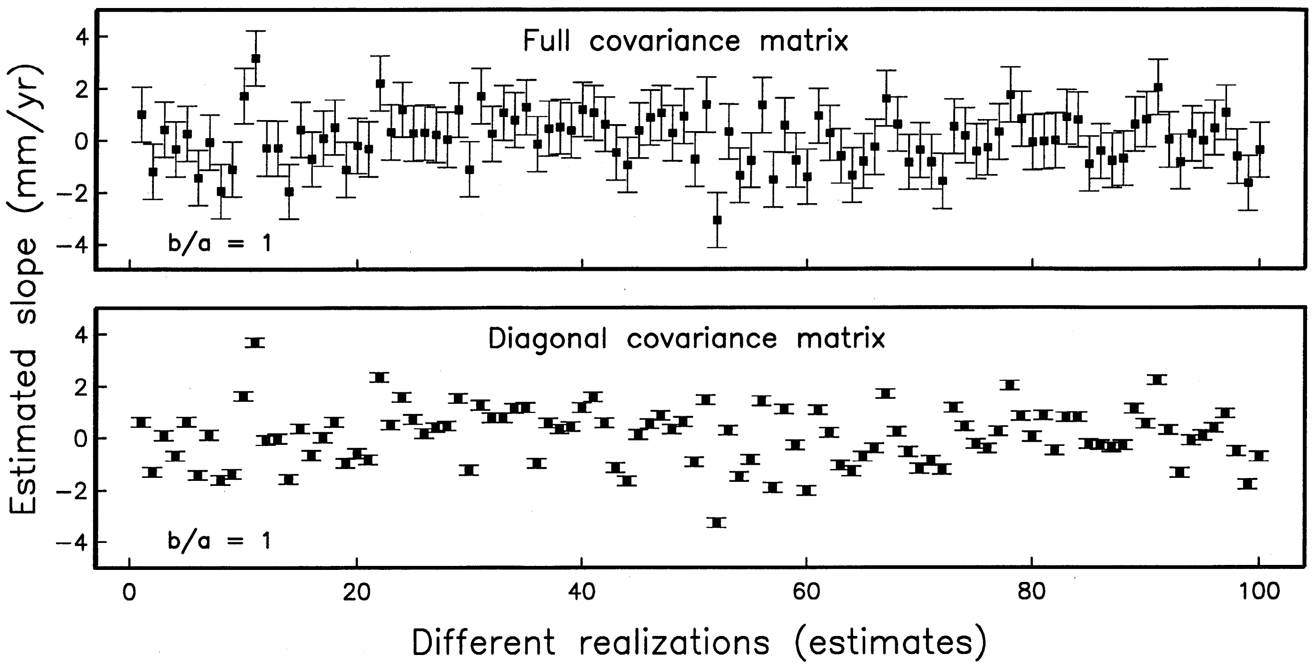

Figure 2. Slope estimates for 100 realizations of the bottom time series of Figure 1. In the top panel the correct covariance matrix is used in the calculations, including the correlations due to the random walk error term, while in the lower panel these correlations are incorrectly ignored. The error bars on the individual slope estimates in the lower panel are far too small (due to this omission) despite the fact that the correlation in the bottom time series of Figure 1 is not at all evident by eye.

ratios of b/a the estimated velocities $\hat{r}$ are nearly the same; but the estimated uncertainties, $\sigma_{\hat{r}}$, differ substantially, especially as the relative amount of monument instability increases. If the proper covariance matrix for correlated data is used then none of the estimates for $\hat{r}$ are significantly different from zero, however the estimates of uncertainty derived by ignoring the correlation in the data would lead one to believe otherwise. Figure 2 makes this same point in a different way; each panel presents the estimated deformation rate and its uncertainty for 100 simulations of one year of daily measurements with a and b equal to 1. (Again, the same sequence of pseudo-random numbers was used for each panel.) The lower panel shows how much the uncertainty in velocity is underestimated if the effects of temporal correlation are ignored; the error bars in the top panel, however, appear quite reasonable, serving to validate the methodology used. In fact, the size of each individual error bar in the top panel is statistically the same as the standard deviation of the ensemble of 100 velocity estimates.

Figure 2 demonstrates that we must include off-diagonal terms in the covariance matrix when correlated errors are present. Given such errors, how does the sampling frequency affect the uncertainty in velocity? Figure 3 (see also Table 2) presents this information for an experiment lasting 5 years and a sampling frequency which varies from once per year up to every other day (6 to 916 total measurements). We assume that the only source of correlation is the random-walk component, although in actual geodetic measurements with GPS there are other correlations present with periods of a few days (T. Herring, personal communication). The y-axis is the ratio $\sigma_{\hat{r}}/a$, and each solid curve in Figure 3 corresponds to different relative levels of error. When the random walk error dominates over the independent error ($b/a = 10.0$) $\sigma_{\hat{r}}$ is quite insensitive to the sampling frequency; if the random walk error level is much smaller than the independent error ($b/a = 0.1$) $\sigma_{\hat{r}}$ diminishes with more frequent sampling (more measurements) roughly as the $1/\sqrt{N}$ rule for completely independent errors.

Table 2. Deformation rate uncertainty: 5 year experiment

	Time between surveys			
	2 days	1 week	1 month	1 year
b/a	$\sigma_{\hat{r}}/a$			
10.0	4.477	4.479	4.480	4.481
3.0	1.348	1.352	1.359	1.370
1.0	0.454	0.459	0.471	0.514
0.3	0.141	0.148	0.168	0.276
0.1	0.053	0.065	0.100	0.246

Figure 3 shows that to make the most precise velocity estimates over a given time span it is crucial to reduce the amount of correlated error present. Unless the ratio between error sources, b/a, is kept below 1, making position determinations annually will give velocities with nearly the same uncertainty as daily measurements would. Current estimates of the horizontal precision of regional-scale GPS are 2 to 3 mm (Y. Bock, personal communication); if, as current estimates indicate, the typical level of random-walk error in geodetic data is between about 1 and 3 mm/$\sqrt{\text{yr}}$ then the b/a ratio would be between 0.3 and 1.5: only if we can hold the random-walk error to the lower of these values is there likely to be much benefit to frequent sampling.

It may be felt that there is another motivation for more frequent measurements: to provide enough data to identify outliers. J. Savage and M. Lisowski (personal communica-

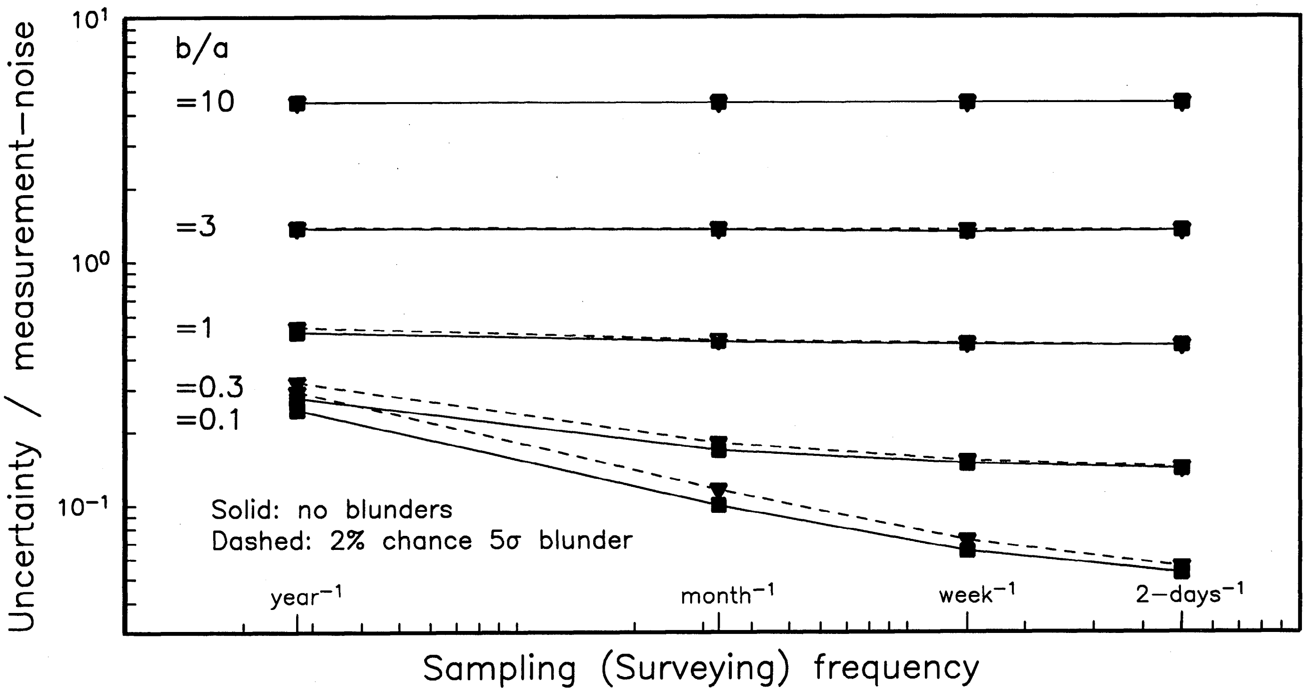

Figure 3. Rate uncertainty (normalized) as a function of sampling frequency for an experiment lasting five years. Each curve gives the results for a different ratio of the random walk to independent error terms. For example, yearly measurements made with $a = b = 1$ give $\sigma_{\hat{r}}$ of 0.52 mm/yr; if the amount of each noise source is doubled to $a = b = 2$ then $\sigma_{\hat{r}}$ also doubles to 1.05 mm/yr. The simulations resulting in the dashed curves include a 2% chance of a 5-σ blunder in the measurements; the largest difference between the solid and dashed curves is only 20%.

tion) have concluded, from an analysis of the complete southern California Geodolite data set, that such "blunders" occur in actual field measurements (probably from setup errors) about 1% of the time. To test the effect of outliers on our estimates of rate uncertainty we have modified equation (2) so that there is a small chance of a relatively large independent error for each synthetic datum created. To simulate a worst case situation we allow a 2% chance that a in equation (2) is 5 times its usual value at each point. Figure 3 also presents the results for this case, as the dashed curves, where each velocity uncertainty was calculated directly from a Monte Carlo analysis of 60,000 simulations. Since the dashed curves in Figure 3 are very nearly the same as the solid curves (the maximum difference is 20%, for very small monument motions and very few measurements), we conclude that the presence of outliers does not change our previous conclusions.

Conclusion

Long-term temporal correlations must be taken into account when geodetic data are analyzed to determine deformation rates (velocities). Random-walk noise, say due to the instability of geodetic monuments, can have a large effect on the uncertainty in these velocity estimates if it is comparable in size to the measurement system noise. In particular, there is then very little improvement in the velocity uncertainty as the frequency of measurement increases. This result implies that for continuous GPS networks to achieve high precision velocity estimates in a reasonable period of time (e.g., 5 years) they will have to be constructed so as to hold the monument instability to a small fraction of the measurement system precision.

Acknowledgments. The authors would like to thank F. Wyatt for his critical reviews of early versions of the text along with J. Langbein, S. Stein, and R. Snay whose reviews helped clarify several points. This research was supported by the Southern California Earthquake Center.

References

Arnadottir, T., P. Segall, and M. Matthews, Resolving the discrepancy between geodetic and seismic fault models for the 1989 Loma Prieta, California, earthquake, *Bull. Seism. Soc. Am*, 82, 2248-2255, 1992.

Beran, P., Statistical methods for data with long-range dependence, *Statistical Sci.*, 7, 404-427, 1992.

Jeffreys, H. *Theory of Probability*, 380 pp., The Clarendon Press, Oxford, 1939.

Johnson, H. O., and F. K. Wyatt, Geodetic network design for fault-mechanics studies, *Manuscripta Geodaetica*, 19, 309-323, 1994.

Langbein, J., E. Quilty, and K. Breckenridge Sensitivity of crustal deformation instruments to changes in secular rate, *Geophys. Res. Lett.*, 20, 85-88, 1993.

Wyatt, F. Displacement of surface monuments: horizontal motion, *J. Geophys. Res.*, 87, 979-989, 1982.

Wyatt, F. K. Displacement of surface monuments: vertical motion, *J. Geophys. Res.*, 94, 1655-1664, 1989.

D. C. Agnew and H. O. Johnson, U. C. San Diego, IGPP-0225, 9500 Gilman Drive, La Jolla, CA 92093-0225. (e-mail: hjohnson@ucsd.edu)

Received: June 14, 1995 Accepted: July 27, 1995