Map Projections to Represent Possible Effects of Surface Loading

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Abstract

Time-varying surface loads affect many geodetic quantities. To help in understanding this process, I describe map projections which show an area of the Earth's surface in proportion to the influence a load at that location would have on a measurement at a given place. Such maps illustrate the great importance of nearby regions compared with those more distant, and should be helpful in evaluating possible errors in the computation of loading effects.

1. Introduction

As geodetic measurements have become more precise, they have been able to detect ever more effects—of which some of the more troublesome arise from the motion of the earth caused by varying surface loads. Such loading effects were first seen in earth-tide studies, since the ocean tides are the largest such loads. Loading by the atmosphere and non-tidal sea-level changes is much smaller, but can produce measurable signals (Van Dam and Wahr 1998). It is therefore desirable to understand as thoroughly as possible how loads affect measurements; the better our understanding, the better we can correct data for loads, or interpret data in terms of loads.

Formally, the connection between loading and measurements is straightforward. For whatever quantity we measure, we can, for a spherically symmetric Earth, define a Green function $G(\Delta, \phi)$, which expresses the change in that quantity produced by a unit mass loading the surface at angular distance Δ and azimuth ϕ. The azimuth is taken from the horizontal direction of the quantity being measured, for those quantities (such as tilt or horizontal displacement) which have a direction; we use ϕ_0 for the azimuth of this direction relative to true North. The Green function is computed from a model of the Earth's elastic structure; the paper of Farrell (1972) remains the standard reference on this procedure, and also gives Green functions for most quantities. In this paper I use Farrell's Green functions for a Gutenberg-Bullen average Earth.

We also need to have a load, defined by a mass distribution H over the surface of the Earth. If we use the coordinates Δ and ϕ for H, the effect of the load is given by the convolution integral over the surface

$$\int_0^{2\pi} \int_0^{\pi} G(\Delta, \phi) H(\Delta, \phi) \sin \Delta \, d\Delta \, d\phi \tag{1}$$

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Starting with Farrell (1973) a variety of programs have been written to compute (1), usually for ocean tide models; for example Sato and Hanada (1984), and Agnew (1997). Such programs are necessary to compute the effects of a specified load. But they provide little insight into the relative importance of different regions of the load in producing the final result; such relative importances are useful for deciding if errors in the mass distribution in particular regions are important to the overall answer. We can compute the convolution integral (1) over a series of subareas, and sum the results to get the final answer (e.g., Baker, 1980); but the division into subareas must be somewhat arbitrary, and a division appropriate to one place of measurement will not be appropriate for another.

2. "Equal-Area" Load Maps

I propose that one way to judge the relative importance of different loads (albeit only qualitatively) is to construct a map in which area is distorted so that regions of equal area on the map would contribute equally to the measurement if the loads on them were the same. Such a map immediately shows which regions of loading are unimportant, and which are possibly significant, though the true significance of any area depends on the load actually there.

Such a map shows how a measurement at some place "views" the world of loads, so we use an azimuthal projection, in which the coordinates (Δ, ϕ) map to radial coordinates (r, ϕ). The condition that an infinitesimal unit of map area be equal to one of surface area weighted by the contribution to the load is

$$r\,dr\,d\phi \propto |G(\Delta, \phi)| a^2 \sin \Delta\, d\Delta\, d\phi \qquad (2)$$

with a the radius of the Earth. If we integrate this, we find that the radial map coordinate is

$$r(\Delta, \phi) = \left[k \int_0^{\Delta} |G(\Delta, \phi)| \sin \Delta\, d\Delta \right]^{\frac{1}{2}} \qquad (3)$$

where k is a normalizing factor discussed below. Note that in both equation (2) and equation (3) we use the absolute value of G: since the purpose of the map is to show in which areas loads could contribute the most, the sign of G is irrelevant. More importantly, taking $|G|$ assures that the mapping described by (3) is one-to-one, as it must be for a proper map projection.

The map-projection literature contains a few examples similar to this, although not many, since the usual aim of a projection is to reduce distortion, not impose it deliberately. The usual term for a map in which areas are made proportional to something other than area is a cartogram; for recent approaches to developing these in a general way, see Keahey (1999) and House and Kocmoud (1998), both of which focus on the case in which G is some sort of statistical distribution. Tobler (1963) is one of the earliest treatments of this subject; for an azimuthal projection his more general development leads to an equation equivalent to (3). Snyder (1987) treats the case in which G has two values for different ranges of Δ, which produces a "magnifying-glass" effect for small distances.

In equation (3), k is chosen to set the scale of the final map; it has absorbed both the

a^2 and a factor of two gotten in going from equation (2) to equation (3). The most useful normalization comes from setting

$$k^{-1} = \int_0^\pi |G(\Delta, \phi_0)| \sin \Delta d\Delta \qquad (4)$$

in which case r runs from 0 to 1 along the azimuth of measurement (which makes this quantity always nonzero), or for all azimuths if G has no azimuthal dependence. Of course, both equations (3) and (4) suppose that $G(\Delta) \sin \Delta$ is integrable over the range 0 to π; this is true for displacement (horizontal and vertical), gravity, areal strain, and induced potential, all of which vary as Δ^{-1} for small Δ, a singularity canceled by the $\sin \Delta$.

Figure 1 shows $r(\Delta)$, with the normalization (4), for the five quantities for which the integral in (4) exists. Since much of the change in r occurs at small Δ, a logarithmic scale is used for Δ. The similarity between the curves for gravity and vertical displacement shows why tidal gravity measurements have sometimes been proposed as a proxy method for determining loading effects for vertical displacement; that the two curves differ shows the limitations of this method. The curve for areal strain shows a greater response to local loads than any of the others; this is because, as Farrell (1972) shows, this quantity has a response to loads about $1°$ away, and then a response close to zero for loads from roughly $3°$ to $30°$.
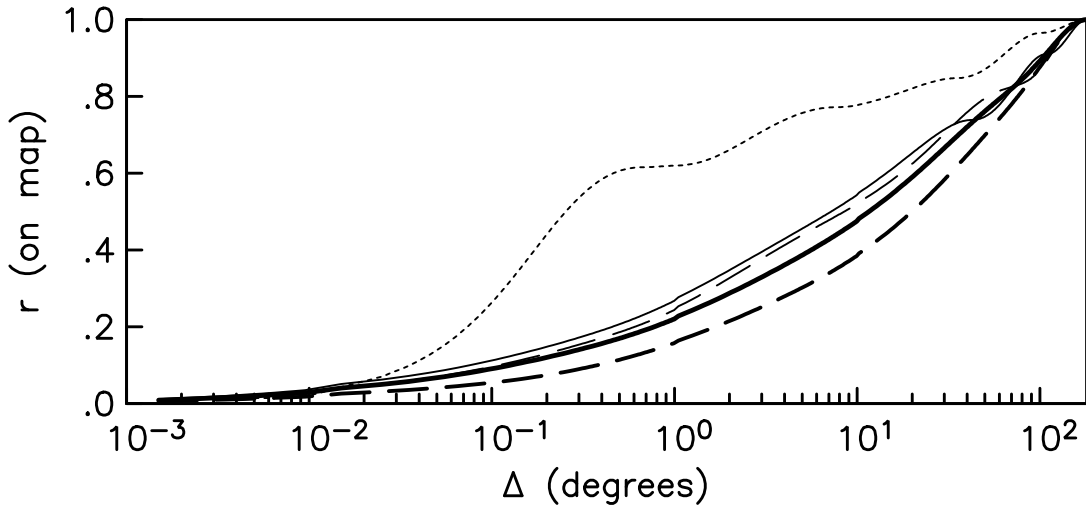

Fig. 1. The mapping functions $r(\Delta)$ for the nonsingular cases. The dotted line is for areal strain, the thin lines are for displacement (horizontal dashed), the thick solid line is gravity, and the thick dashed line is the induced tide-raising potential.

The curve for horizontal displacement is the response along the azimuth of measurement, but including azimuthal dependence is easy: since the Green function is

$$G(\Delta, \phi) = G(\Delta) \cos(\phi - \phi_0) \qquad (5)$$

the mapping is

$$r(\Delta, \phi) = r(\Delta)\sqrt{|\cos \theta|} \qquad (6)$$

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with $\theta = \phi - \phi_0$. Lines of equal distance Δ thus map, not into circles as in the azimuthally independent case, but rather into a two-lobed curve.

2.1. Singular Green Functions

The discussion so far has assumed that the integrals (3) and (4) exist, but for tilt and linear strain they do not, because these Green functions vary as Δ^{-2} for small Δ. In order to construct similar maps, we need to remove this singularity, which can be done by modifying the Green functions.

The first way to modify the Green functions is to recognize that in the real world these functions are not actually singular, for two reasons. The first reason is that all tiltmeters and strainmeters actually measure differential displacement over a finite distance; the relevant Green function is thus the difference of two Green functions for displacement. The second reason is that most tilt and strain measurements are made with buried sensors, and even a concentrated load at the surface will produce only finite deformations at depth. However, corrected for these two effects will modify $G(\Delta)$ only for distances of 10^2 m or less (a typical baselength or depth), so that most of the area of the map will be just the region within a few kilometers. This is, strictly, correct, but not very useful if there are no loads within this distance.

A second way to modify $G(\Delta)$ is to allow for the non-point nature of the load. This is not possible for ocean loading, which has a step-function behavior at the coast. However, for atmospheric pressure, the frequency-wavenumber spectrum shows that at low frequencies most power is confined to long wavelengths. If our interest is in atmospheric loading only below some frequency, we can thus choose a length L to be the (rough) cutoff point for the wavelength spectrum at that frequency, and remove the singularity as was done by Farrell's (1973) use of "disk factors:" we replace the point-load Green function $G(\Delta, \phi)$ by the Green function for a load spread uniformly over a disk of diameter L centered at (Δ, ϕ). This new Green function can be integrated in (3) and (4).

The final way of modifying the Green function recognizes that, for ocean loading, we do have a minimum Δ: from the place of measurement to the nearest load (the coast); we again call this L. (If we wish to make maps to compare measurements in different places, we can set L to be the smallest coastal distance amongst them.) Now suppose we choose this distance to be, on the map, a fraction α of the map distance to the antipode. We can then continue to use (3) and (4), provided that we make $G(\Delta, \phi) = G_c(\phi)$, independent of Δ for $\Delta < L$. This function is given by

$$G_c(\phi) = \frac{\alpha^2}{(1 - \alpha^2)(1 - cosL)} \int_L^\pi G(\Delta, \phi) \sin \Delta d\Delta \tag{7}$$

The above derivation works easily for tilt, for which the Green function has the same form, equation (5), as the one for horizontal displacement. The case of linear strain is more complicated, since its Green function is

$$G(\Delta, \phi) = G_{\Delta\Delta}(\Delta) \cos^2 \theta + G_{\theta\theta}(\Delta) \sin^2 \theta \tag{8}$$

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where $G_{\Delta\Delta}(\Delta)$ is the Green function for a load in the direction of strain, $G_{\theta\theta}(\Delta)$ that for loads perpendicular to this direction, and $\theta = \phi - \phi_0$. As there is no simple relation between $G_{\Delta\Delta}$ and $G_{\theta\theta}$, we have to evaluate (7), (4), and (3) over a range of both Δ and θ, although because of the symmetries of (6) only one quadrant of θ is required. It is most useful to take, as for tilt, the direction of normalization to along the direction of strain, since this corresponds to the maximum effect from loading.
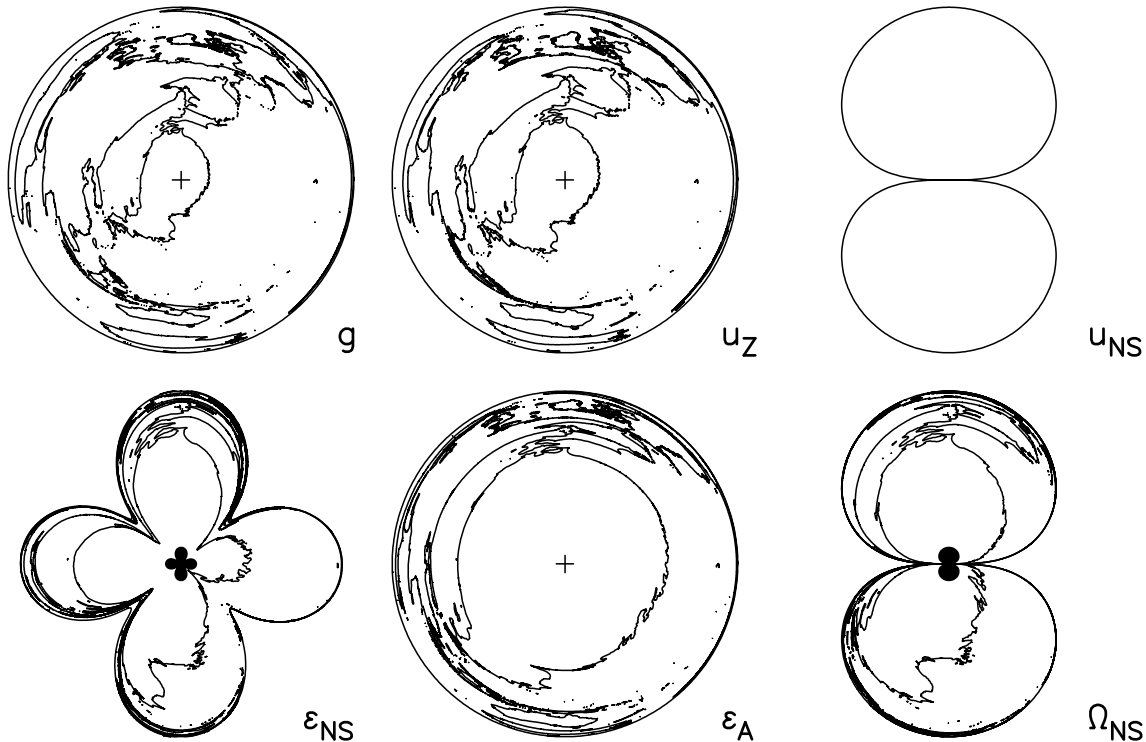

Fig. 2. Loading maps for Esashi; from top left, for gravity, vertical displacement, North-South displacement, North-South linear strain, areal strain, and North-South tilt. All maps use the high-resolution coastline of Wessel and Smith (1996), with points removed for the more distant locations. For the singular cases the cutoff distance L was taken to be 30 km (the distance to the nearest coast) and α was set to 0.1; the black areas near the center of the tilt and strain maps are the regions omitted.

3.   Conclusion

I have presented a method of drawing a map that shows how the world of applied surface loads "looks" to a measurement of gravity, displacement, or deformation. For quantitative studies of possible errors in loading such maps cannot replace simulations done by varying the applied loads and computing the integral (1), not least because it is difficult to make accurate judgements of the relative areas of irregular regions (Cleveland, 1985). However, for providing a good qualitative impression of the contribution of different regions, and thus serving as a guide to more quantitative studies, such maps would seem to be a valuable tool. In particular, for ocean-loading computations, they can show if local areas, possibly not well covered by global tide models, could be important.

As an example, Figure 2 some of the maps for the geophysical observatory at Esashi, in northern Honshu (Tsubokawa *et al.*, 1981). The gravity and vertical displacement maps are similar, though nearby loads have a slightly greater effect for displacement. Comparing the areal strain with either of these, or the NS displacement and tilt maps, shows the much greater importance of very local loads for tilt and strain; essentially, the loading signal will be dominated by tidal loads close to the coast of Japan. It is clear, for example, that tides in Ishinomaki Bay could contribute significantly to the NS tilt and strain, but not to areal strain.

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References

Agnew, D. C. (1997): NLOADF: a program for computing ocean-tide loading, J. Geophys. Res., **102**, 5109-5110.

Baker, T. F. (1980): Tidal gravity in Britain: tidal loading and the spatial distribution of the marine tide, Geophys. J. Roy. Astron. Soc., **62**, 249-267.

Cleveland, W. S. (1985): The Elements of Graphing Data, Monterey, Wadsworth.

Farrell, W. E. (1972): Deformation of the earth by surface loads, Rev. Geophys. Space Phys., **10**, 761-797.

Farrell, W. E. (1973): Earth tides, ocean tides, and tidal loading, Phil. Trans. Roy. Soc., Ser. A., **274**, 253-259.

House, D. H., and C.J. Kocmoud (1998): Continuous cartogram construction, IEEE 1998 Visualization Conference; URL http://www-viz.tamu.edu/faculty/house/cartograms/Vis98.html.

Keahey, T. A. (1999): Area-normalized thematic views, Proc. Internat. Cartograph. Assoc.; URL http://www.cs.indiana.edu/hyplan/tkeahey/research/papers/ica.99.html

Sato, T. and H. Hanada (1984): A program for the computation of oceanic tidal loading effects "GOTIC," Publ. Internat. Lat. Obs. Mizusawa, **18**, 29-47.

Snyder, J. P. (1987): "Magnifying-glass" azimuthal map projections, Amer. Cartographer, **14**, 61-68.

Tobler, W. R. (1963): Geographic area and map projections, Geograph. Rev., **53**, 59-78.

Tsubokawa, I., C. Sugawa, M. Ooe, S. Nakai, H. Sasaki, T. Sato, T. Tsubokawa, and E. Chiba (1979): Tidal Observations at the Esashi Earth Tides Station, Ann. Rep. Internat. Lat. Obs. Mizusawa.

Van Dam, T. M, and J. Wahr (1998): Modeling environment loading effects; a review, Phys. Chem. Earth, **23**, 1077-1087.

Wessel, P., and W. H. F. Smith (1996): A global, self-consistent, hierarchical, high-resolution shoreline database, J. Geophys. Res., **101**, 8741-8743.