Prediction of silicate melt viscosity from electrical conductivity: A model and its geophysical implications

Anne Pommier
School of Earth and Space Exploration, Arizona State University, Tempe, Arizona, USA

Rob L. Evans
Department of Geology and Geophysics, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts, USA

Kerry Key
Scripps Institution of Oceanography, La Jolla, California, USA

James A. Tyburczy
School of Earth and Space Exploration, Arizona State University, Tempe, Arizona, USA

Stephen Mackwell
Lunar and Planetary Institute, Houston, Texas, USA

Jimmy Elsenbeck
Department of Geology and Geophysics, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts, USA

[1] Our knowledge of magma dynamics would be improved if geophysical data could be used to infer rheological constraints in melt-bearing zones. Geophysical images of the Earth’s interior provide frozen snapshots of a dynamical system. However, knowledge of a rheological parameter such as viscosity would constrain the time-dependent dynamics of melt bearing zones. We propose a model that relates melt viscosity to electrical conductivity for naturally occurring melt compositions (including H2O) and temperature. Based on laboratory measurements of melt conductivity and viscosity, our model provides a rheological dimension to the interpretation of electromagnetic anomalies caused by melt and partially molten rocks (melt fraction ~ >0.7).

Components: 4,200 words, 3 figures, 1 table.
Keywords: electrical conductivity; magnetotellurics; viscosity; silicate melts; magma mixing.
Index Terms: 3914 Mineral Physics: Electrical properties; 1749 History of Geophysics: Volcanology, geochemistry, and petrology; 1036 Geochemistry: Magma chamber processes (3618).
Received 1 October 2012; Revised 8 February 2013; Accepted 24 February 2013; Published XX Month 2013.

1. Introduction

Large volcanoes represent a significant societal risk because many are located in densely populated areas. Risk assessment is improved by monitoring (e.g., magma chamber inflation), but further improvements would be made if we could probe magma composition and properties (e.g., viscosity). An important clue as to the likelihood for explosive volcanism involves the rheological properties of silicate melts. Yet to date, geophysical constraints on magma chambers rarely go beyond rough estimates of temperature and, in some cases, melt fractions.

Electrical conductivity is a tool with the potential for bridging different scales of observations because it is measured both in the laboratory and in the field. The dependence of conductivity on parameters such as temperature and composition make electrical measurements in the laboratory critical to probe the Earth’s interior, and therefore to interpret field-based electromagnetic surveys. However, magnetotelluric (MT) profiles are not generally interpreted in terms of the dynamics that characterize the geological process they probe. In a high-melt fraction magma reservoir, such characterization would allow determination of the time-scale for buoyancy-driven motion and fractionation of crystals, melt segregation, and magma mixing. Because viscosity governs any convective system, solid and/or fluid [Shaw, 1965; Vetere et al., 2006; Karki and Stixrude, 2010], relating melt electrical conductivity to melt viscosity represents an opportunity to improve the interpretation of electromagnetic field results.

Some attempts have been made to relate melt electrical conductivity to viscosity, principally in materials science because both physical parameters are critical for the design of industrial smelters [e.g., Zhang and Chou, 2010; Zhang et al., 2011]. However, further investigation is warranted, principally because existing models generally consider only simple synthetic melt compositions [e.g., Grandjean et al., 2007; Zhang et al., 2011]. Therefore, because these models do not embed the compositional dependence of conductivity for more complex compositions, they do not satisfactorily reproduce the physical properties of natural melts.

This study proposes a semi-empirical model that relates these two physical properties of melt (electrical conductivity and viscosity). Our model can be used to convert melt conductivity into viscosity and vice versa for a defined melt composition and temperature. On this basis, our model should significantly aid in the interpretation of field-based data sets.

2. Relating Electrical Conductivity to Viscosity for Geophysical Purposes

2.1. Electrical Conductivity vs. Viscosity of Silicate Melts

The dependence of electrical conductivity and viscosity on melt composition is illustrated in Figure 1 for four kinds of natural melts (basaltic, andesitic, phonolitic, and rhyolitic compositions) as a function of temperature. Electrical conductivity values are from...
previous experimental measurements by Rai and Manghnani [1977], Tyburczy and Waff [1983], Gaillard [2004], and Pommier et al. [2008]. Viscosity values are from experimental studies by Bottinga and Weill [1972], Giordano and Dingwell [2003], and Ardia et al. [2008]. Melt electrical conductivity and viscosity are both dependent on chemical composition at the same temperature. Melt viscosity has a much higher dependence on chemical composition than does electrical conductivity; for instance, at 1400°C, there is a difference of more than 4 log-units between the viscosity of a dry rhyolitic melt and that of a basaltic melt, whereas the conductivities of similar melts differ only by ~0.1 log-unit (Figure 1).

The effect of water is also greater on viscosity than on electrical conductivity: adding 5.6 wt % H2O to a phonolitic melt increases its conductivity by ~0.2 log-unit [Pommier et al., 2008], but will decrease its viscosity by 1 to more than 2 log-units, according to the model by Giordano et al. [2008] (Figure 1). Viscosity is a critical property controlling the behavior of magmas, and its strong dependence on melt composition highlights the need for independent constraints on magma composition within the crust.

2.2. Electrical Conductivity-Viscosity Model

[7] Electrical conductivity and viscosity are both thermally activated transport properties and their temperature-dependence can be described by the empirical Vogel-Fulcher-Tammann equation [e.g., Vogel, 1921]

\[ X = A \exp\left[\frac{B}{(T - C)}\right] \]  

(1)

where \( X \) is either the conductivity or the viscosity of melt, \( T \) is temperature, and \( A, B, \) and \( C \) are adjustable parameters specific to a given material. However, conductivity and viscosity are controlled by, and provide information about, different characteristics of the melt. Electrical conductivity is essentially controlled by charge carrier diffusivity (i.e., mostly alkalis in natural silicate melts [Tyburczy and Waff, 1983; Gaillard, 2004; Pommier et al., 2008]), involving ion displacements within the melt structure that require rearrangement of near-neighbor ions (local or short range dynamics). Electrical conductivity \( \sigma \) is related to the diffusion coefficient \( D \) for ions in liquids through the Nernst-Einstein equation

\[ \sigma \propto D/T \]  

(2)

[5] The silica framework of the melt controls the viscosity and its deformation requires further change of the configuration (long-range dynamics) [ Singh et al., 2005]. Using the Stokes-Einstein equation, shear viscosity \( \eta \) is related to \( D \) as

\[ \eta \propto T/D \]  

(3)

[9] Although diffusion processes in equations (2) and (3) are governed by different species (alkali and Si/O, respectively), it is possible to define, as a first approximation, a unique coefficient \( D \) that expresses bulk diffusion processes in melt. Furthermore, as discussed by Grandjean et al. [2007], melt conductivity and viscosity can be related through the so-called modified Stokes-Einstein equation

\[ \sigma T = a(T/\eta)^b \]  

(4)

with \( a \) and \( b \) constants. As shown in Figure 2a, different trends in the proportionality between \( \sigma \) and \( \eta \) exist, which are likely explained by compositional differences and the resulting variations in interactions between ionic species in the melts.

[10] Equation (4) provides a relationship between \( \eta \) and \( \sigma \). However, to be well suited to geophysical applications, it needs to clearly express the dependence on melt composition, water content, and temperature. Therefore, we propose a model that allows conversion of melt electrical conductivity into viscosity for defined composition and temperature. This model is based on laboratory studies of silicate melt conductivity and viscosity. The conductivity data set corresponds to the SIGMELTS experimental database [Pommier and Le Trong, 2011, and references therein], updated with the recent data from Ni et al. [2011]. The viscosity of these melts is calculated using the model of Giordano et al. [2008] that spans the entire compositional range of the melts in the SIGMELTS database. Melt compositions range from rhyolite to basalt to latite, water contents range from 0 to ~6 wt % H2O, and temperatures range from 900 to 1600°C [Pommier and Le Trong, 2011].

[11] In our model, melt composition (including its water content) is expressed through the parameter of silicate melt optical basicity (OB) [e.g., Duffy and Ingram, 1976; Duffy, 1993; Zhang and Chou, 2010; Mathieu et al., 2011], which is a semiempirical estimation of oxide ion activities, as detailed in the auxiliary material. Based on the current databases of silicate melt conductivity (S/m) and viscosity (Pa·s), we propose the following expression that best reproduces the existing data set of measured melt conductivity. This model is obtained from simple

1 Additional supporting information may be found in the online version of this article.
multiple linear regression for dry and hydrous silicate melts ranging from basaltic to rhyolitic compositions

\[
\log \sigma = 1.315 - 4.8 \times 10^5 / T + 9.1 \times 10^{-2} H_2O + 4.46 OB - 3.2 \times 10^{-2} \log \eta
\]  

(5)

Or conversely

\[
\log \eta = 41.09 - 1.50 \times 10^5 / T + 2.84 H_2O + 139.4 OB - 31.25 \log \sigma
\]  

(6)

in which \( T \) is temperature (K), \( H_2O \) is water content (wt %), and OB is optical basicity. Experimental conductivity data (\( \log \sigma \)) are reproduced with a standard error of estimate <0.029 (Figure 2b), and modeled viscosity data (\( \log \eta \)) are reproduced with a standard error of estimate <0.42 (Figure 2c). This equation is valid in the \( T \) range from 1173 to 1773 K and for water contents up to \( \sim 6 \) wt %. Extrapolation of the model to water contents exceeding 6 wt % is not recommended because of a lack of appropriate experimental data. Efforts to fit conductivity-viscosity data to expressions of the form of equation (4) with compositionally dependent parameters \( a \) and \( b \) yielded poorer fits than equations (5) and (6).

[12] The effect of pressure is not directly taken into account. To a first approximation, the pressure dependence of both viscosity and conductivity can be considered to be similar (between 10 and 20 cm\(^3\)/mol, depending on melt composition (e.g., Tinker et al. [2004] and Ardia et al. [2008] for viscosity measurements, and Gaillard [2004] and Pommier et al. [2008] for conductivity measurements). Both properties decrease in dry melts with

Figure 2. Electrical conductivity and viscosity of silicate melts. (A) Experimentally determined electrical conductivity versus calculated viscosity (represented as \( \log \eta / T \) versus \( \log 1 / (\sigma T) \)) of natural dry and hydrous (0 to 6 wt % H2O) silicate melts and comparison with simple synthetic compositions [Grandjean et al., 2007; Zhang and Chou, 2010; Zhang et al., 2011]. Temperature ranges from 800 to 1600°C, and pressure is 1atm for all dry melt compositions, 50MPa to 300MPa for hydrous melts from Gaillard [2004] and Pommier et al. [2008] and 2GPa for hydrous melt from Ni et al. [2011]. Conductivities of natural melts are from experimental studies [Pommier and Le Trong, 2011 and references therein; Ni et al., 2011]. Melt viscosities are calculated using the model of Giordano et al. [2008] for melt compositions that have been investigated as part of conductivity measurements from the literature. (B) Electrical conductivity of silicate melts: comparison between modeled (Eq. 5) and experimental data from previous studies from the literature. The viscosity term in Eq. 5 is calculated using the model by Giordano et al. (C) Viscosity of silicate melts: comparison between model (Eq. 6) and viscosity calculated using Giordano et al. for melt compositions considered in this study and defined from the conductivity dataset. The conductivity term in Eq. 6 corresponds to measured conductivity from the literature.
increasing pressure. A similar pressure dependence allows relative comparison of both properties at pressures to perhaps 1–2 GPa. At crustal depths, pressure has a smaller effect on melt conductivity compared to temperature [Gaillard, 2004; Pommier et al., 2008] and viscosity [Allwardt et al., 2007], allowing our model to be used over crustal depths without requiring a pressure correction. This model is appropriate for high melt fractions ($X_{\text{melt}} > 0.7$), i.e., where bulk conductivity is controlled by melt conductivity and bulk viscosity is controlled by melt viscosity. For lower melt fractions, computation of bulk conductivity or bulk viscosity necessitates the use of two-phase formalisms, such as the Hashin-Shtrikman bounds (see ten Grotenhuis et al. [2005] for partial melt electrical conductivity and Costa et al. [2009] for partial melt viscosity) and accounting for changes in melt composition with melt fraction [Roberts and Tyburczy, 1999; Gaillard and Iacono Marziano, 2005].

3. Application to the Field: Imaging Magma Chambers With Magnetotellurics

Magnetotellurics surveys have been used to image the structure of volcanic edifices [e.g., Müller and Haak, 2004; Aizawa et al., 2005; Hill et al., 2009]. The resulting images of shallow magma reservoirs consist of conductive bodies having an essentially homogeneous bulk conductivity value; this is true even where petrological knowledge suggests substantial chemical heterogeneity. This is explained by the fact that electrical conductivity is not strongly dependent on melt composition (Figure 1a), but also reflects the resolution of MT data, which primarily constrain the total conductance of a magma reservoir rather than smaller-scale heterogeneity within the reservoir.

Our goal in developing the conductivity-viscosity model for silicate melts is to improve the interpretation of electromagnetic field data. Therefore, the usefulness of the model depends on the capacity for field data to detect magma reservoirs at crustal depth. Here we explore the potential for detecting and characterizing magma chambers using forward modeling of electrical conductivity. We consider a set of chemical and physical parameters (melt composition and temperature, melt viscosity and conductivity, volume, and structure of the reservoir) that are representative of typical magma reservoirs (Table 1). Each scenario is expressed as a hypothetical conductivity structure, to which we apply a forward modeling to simulate the corresponding field electrical response, and therefore estimate the magnitude of the conductivity anomaly.

### Table 1. Parameters for Forward Conductivity Models. Conductivities are Calculated From Viscosity Values Using Equation (5). See Text for Details

<table>
<thead>
<tr>
<th>Model</th>
<th>Melt Storage Conditions</th>
<th>Melt Viscosity (Pa·s)</th>
<th>Corresponding Cond. (S/m)</th>
<th>Vol. of Reservoir (km³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous reservoir</td>
<td>Rhyolite at 900°C, 3 wt % H₂O</td>
<td>4.6·10⁴</td>
<td>2.2</td>
<td>10 or 20</td>
</tr>
<tr>
<td></td>
<td>Anhydrous basalt at 1200°C</td>
<td>7.6·10⁵</td>
<td>1.5</td>
<td>10 or 20</td>
</tr>
<tr>
<td>Layered reservoir</td>
<td>Rhyolite at 1000°C*, 2 wt % H₂O (top)</td>
<td>3.2·10⁴</td>
<td>3.6</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Basalt at 1000°C*, 2 wt % H₂O (bottom)</td>
<td>1.9·10⁴</td>
<td>0.60</td>
<td>20</td>
</tr>
<tr>
<td>Magma mixing</td>
<td>Hybrid melt at 1000°C*, 2 wt % H₂O</td>
<td>2.5·10⁴</td>
<td>1.0</td>
<td>20</td>
</tr>
<tr>
<td>Magma mingling</td>
<td>Rhyolite and basalt both at 1000°C*, 2 wt % H₂O (HS upper bound)</td>
<td>2.5·10⁴</td>
<td>1.9</td>
<td>20</td>
</tr>
</tbody>
</table>

*Considered temperature of homogeneization.
Hypothetical conductivity properties of a volcanic edifice [e.g., Pommier et al., 2010] are presented in Table 1 and highlight the fairly small range of conductivity variations expected. The models consider three types of melt at different temperatures, including: (1) a basaltic andesite, (2) a rhyolitic melt, and (3) a hybrid melt resulting from mixing. Four different scenarios have been considered regarding the structure of the magmatic reservoir: a homogenous chamber, a zoned chamber (evolved melt on the top of mafic melt), magma mixing, and magma mingling (Table 1). The simulation of magma mingling corresponds electrically to a reservoir whose bulk conductivity can be estimated using the Hashin-Shtrikman upper bound [Hashin and Shtrikman, 1962] (the volume proportions of the two melts are assumed to be equal). The background conductivity for each simulation is 0.01 S/m.

Melt conductivities have been calculated using our model (equation (5)). It is worth noticing that our model requires precise constraints on each parameter, in turn allowing accurate calculations of conductivity (equation (5)) or viscosity (equation (6)). For instance, as shown in Figure 3, a change of 50°C and 1 wt % water will imply non-negligible viscosity variation. Sensitivity analysis on the model’s capacity to predict viscosity shows that if melt conductivity is known to within 8% (uncertainty on conductivity measurements in the laboratory ranges typically between ~5 and 10% [e.g., Pommier et al., 2008]), then an uncertainty on the calculated melt viscosity to within a factor of 10 would require an uncertainty in temperature of less than ~20°C, on water content of less than 35% and on melt composition (OB) of less than 0.7%.

Our forward modeling shows that for models based on chambers at 4 km depth and having volumes of either 10 or 20 km³ (parallelepiped geometry with dimensions of 5*2*1 km³ and 5*2*2 km³, respectively), the presence of the conductive magma chamber results in a detectable 2–5° phase shift in the MT responses at around 1 s period, but the phase differences between the reservoir types are less than 1°, making the discrimination between distinct models difficult for present-day MT acquisition systems. This is not surprising given the small conductivity variations for each magma type and the limited extent of the 3-D magma chambers considered here. Also, these models do not consider heterogeneity in the surrounding host rock. A particular and frequent issue in imaging magma chambers regards the effect of overlying hydrothermal fluids which themselves are conductive and inhibit the ability of MT surveys to well constrain the conductivities within the chamber [e.g., Manzella et al., 2004; Pommier et al., 2010]. As a consequence, typical MT measurements would be hard pressed to discriminate between these models, even if they imply strong chemical variations in melt composition. This statement is in agreement with previous findings by Pommier et al. [2010], who performed 3-D forward modeling considering different magma reservoirs (chamber volume up to 1000 km³ and magma resistivity varying from 2 to 2000 ohm-m) and showed that the electrical response was only slightly affected by magma conductivity.

The similarity of electrical responses between various forward models highlights the fact that, while MT is a useful tool, there is a strong need for additional geologic information such as melt composition to extend the interpretation to quantify viscosity. Chemical compositions of previous eruptions provide clues as to the likely chemistry of current melts. Electrical conductivity at shallow crustal depths can also be imaged using controlled-source transmitter-receiver systems that offer higher resolution, but little work has been conducted for studying crustal magma chambers with these methods. Seismology has been used to image chambers [e.g., Everson et al., 2011; Paulatto et al., 2011], and provides first-order estimates of temperature using heat flow data from boreholes adjacent to the caldera. At present, the usefulness of the MT technique in volcanic contexts lies in the fact that it provides an acceptable conductivity range for the magma reservoir. When combined with other information (from surface petrology, seismic data), conductivity data can significantly reduce uncertainties regarding melt temperature, chemical composition, and therefore viscosity. Our model allows the conversion of melt conductivity values into viscosity estimates for defined storage conditions (melt chemical composition, including water content, and temperature). We have not attempted to test how MT imaging would be improved through adding constraints such as limiting the boundaries of a seismically defined chamber, but such constraints are known to improve interpretations. The addition of further constraints would allow hypothesis testing regarding chemical heterogeneity of the reservoir and dynamic processes, such as magma mixing and comingling.

4. Conclusions

We propose a simple model that relates melt conductivity to viscosity to promote rheological considerations as part of electromagnetic data
interpretation in volcanic regions. This model is a function of temperature and composition, expressed through the concept of optical basicity of melt, and successfully reproduces experimental conductivity and viscosity data from the literature.

Although the resolution of the MT method does not allow discrimination between two compositionally different melts in a same reservoir, the conversion of melt conductivity (calculated for relevant compositions) into melt viscosity allows constraints to be placed on the dynamics of the reservoir and to test the hypothesis of magma mixing. It is a fair statement to say that advances in data density and quality are required before detailed images of the internal structure of magma chambers are achievable, but in instances where the potential risk is high, the investment is likely worthwhile.

Because density is, like viscosity, an important physical property of magmas that determines their migration in the Earth’s interior and constrains the occurrence of mixing [e.g., Sparks et al., 1980], future experimental and computational challenges could consist of including density-viscosity relationships for melts [e.g., Hack and Thompson, 2011] as part of a conductivity-viscosity-density-composition model for natural melts.

Acknowledgments

The authors thank Joe Baker for his careful editorial work and Kelly Russell for his comments that greatly improved the manuscript. A.P. thanks Ed Garnero, Michel Pichavant and Bernhard Charlier for thoughtful comments. We acknowledge partial support under NASA USRA subaward 02153-04, NSF EAR 0739050, and the ASU School of Earth and Space Exploration (SESE) Exploration Postdoctoral Fellowship Program. This paper is LPI Contribution number 1666.

References


Vogel, H. (1921), The temperature dependence law of the viscosity of fluids, Physikalische Zeitschrift, 22, 645–646.
