SIO 223A, Geophysical Data Analysis
Problem Set 1
due 01/23/2020

1.1 Consider the model for the magnetic field reversals described in Chapter 1 of the notes: we consider time broken up into blocks Δ long, and in each block assume that there is a probability p of a reversal, independently of whatever happened before. What is the probability of getting a period $N\Delta$ with only one reversal at the end? What is the probability of getting no reversal between the times $N_1\Delta$ and $N_2\Delta$? What is the probability if N_1 is 1 and N_2 goes to infinity? Evaluate the probability of an unreversed interval longer than T for the case p/Δ constant, with Δ going to 0. You should be able to work these out from first principles; none of these answers should be given as unevaluated series.

1.2. Suppose we have a sine wave that is a function of time, t,

$$x(t) = A\cos(t)$$

and create a random variable by sampling this at random times. What is the pdf of this random variable? Hint: what is the distribution of the argument t? Given this, transform it to give the distribution of the final random variable.

1.3 The Cauchy cumulative distribution function is

$$F(x) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}(x), \qquad -\infty < x < \infty$$

(a) Show that this is a cdf. (b) Find the density function. (c) Find x such that $P(X > x) = 0.1$.

1.4 Show that if A and B are independent, then

$$P(A \cup B) = P(A) + P(B) - P(A)P(B)$$

1.5 Sometimes a random variable comes from a mixture of two or more distributions. Consider the pdf of the random variable X distributed as

$$X \sim 0.75N(0,1) + 0.25N(1,.3)$$

where $N(m,s)$ is the Normal (Gaussian) distribution, with mean m and standard deviation s, for which the pdf is

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-m)^2/2\sigma^2}.$$

Plot the pdf, find the first four moments, the median, the interquartile distance, the mode, and the standard error in the mean. You may use whatever mix of analytical methods, numerical methods, and looking up values in tables you like, but be sure to explain whatever method you use in adequate detail. If you use a canned routine, assume I have never heard of it and explain what it does.