

SIO 223A, Geophysical Data Analysis

Problem Set 3

Due 2/18/2020

3.1 Let X and Y have the joint density

$$f(x, y) = \frac{6}{7}(x + y)^2, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

- By integrating over the appropriate regions, find (i) $P(X > Y)$, (ii) $P(X + Y \leq 1)$, (iii) $P(X \leq \frac{1}{2})$.
- Find the marginal densities of X and Y .
- Find the two conditional densities.

3.2 Suppose that (X, Y) is uniformly distributed over the region defined by $0 \leq y \leq 1 - x^2$ and $-1 \leq x \leq 1$.

- Find the marginal densities of X and Y .
- Find the two conditional densities.

3.2 Consider the pdf given by

$$\phi(x) = \begin{cases} e^{-(x-\mu)} & x \geq \mu \\ 0 & x < \mu \end{cases} \quad (1)$$

and find the estimate of μ for n data using (a) the method of moments and (b) maximum likelihood; in the second case, don't just differentiate without first thinking about what values of μ give a positive likelihood.

3.4 Estimate the parameter λ for a Poisson process that would model the Parkfield earthquakes, using the data in Chapter 1, and the maximum likelihood estimator given in the notes. Use your result in a parametric bootstrap simulation to see what the sampling distribution of $\hat{\lambda}$ would be, given the same number of events as we have now had at Parkfield (seven); estimate the 95% confidence limits.