1. For \( x \in \mathbb{R}^n \) show that the functional
\[
N(x) = \left( |x_1|^{1/2} + |x_2|^{1/2} + \cdots + |x_n|^{1/2} \right)^2
\]
is not a norm for the space when \( n > 1 \).
Hint: treat the case \( n = 2 \) first.

2. Consider the linear vector space of symmetric matrices \( S^{n \times n} \). The trace of the matrix \( A \in S^{n \times n} \) is given by
\[
\text{tr}(A) = \sum_{j=1}^{n} A_{jj}
\]
Show that \( \text{tr}(AB) = (A, B) \) is an inner product on \( S^{n \times n} \). Show the norm associated with the inner product is the Frobenius norm.

3(a). The linear operator \( O \) maps the normed linear vector space \( \mathcal{V} \) onto itself; in symbols \( O : \mathcal{V} \rightarrow \mathcal{V} \). Linearity of \( O \) is the property
\[
O(\alpha f + \beta g) = \alpha Of + \beta Og
\]
for \( f, g \in \mathcal{V} \) and \( \alpha, \beta \) scalars. Show that the functional \( \|Of\| \) provides a seminorm for \( \mathcal{V} \). When is it a true norm, rather than a seminorm? Give an example.

3(b). Equip the space \( C^\infty[0, 1] \), the space of infinitely differentiable functions, with a norm under which \( \|f_1\| > \|f_2\| \) where
\[
f_1(x) = e^x; \quad f_2(x) = e^{2x}
\]