

SIO 231 Geoelectromagnetism
Homework Set 1, due 1/18/2024

1. What are the SI units for \mathbf{B} , \mathbf{H} , \mathbf{E} , \mathbf{D} , \mathbf{J} , σ , and ϵ_0 ? Show that the unit Tm A^{-1} is equivalent to H m^{-1} for μ_0 , and express the units in terms of another pair of SI quantities.

Estimate and rank in decreasing order: (a) the magnetostatic energy stored in space around a 10g permanent magnet; (b) the chemical energy stored in 10g of cornflakes; (c) the gravitation potential energy in a 10g pencil sitting on your desk; (d) the kinetic energy of a 10g bullet moving at the speed of sound; (e) the mass energy released by fission of 10g of ^{235}U .

The status of the kilogram was redefined in 2019. What are the advantages?

See https://en.wikipedia.org/wiki/2019_redefinition_of_the_SI_base_units. Provide a concise synopsis of the changes and explain why this results in a change in μ_0 , while the relationship $1/(\mu_0\epsilon_0) = c^2$ still holds.

2. Also in Lecture 2 we learned that within a source free region like Earth's atmosphere the vector magnetic field \mathbf{B} can be represented as the gradient of a scalar $\mathbf{B} = -\nabla V$, and that V is a harmonic function, satisfying Laplace's equation, namely $\nabla^2 V = 0$. Let's approximate Earth's surface by a sphere of radius ($r = a$) and call it $S(a)$. The vector \mathbf{r} is a radius vector extending outward from the center of $S(a)$.

The radially outward component of the magnetic field at position \mathbf{r} is $B_r = \hat{\mathbf{r}} \cdot \mathbf{B}$, with $\hat{\mathbf{r}}$ a unit radial vector in the same direction as \mathbf{r} . The magnitude of \mathbf{r} is written as r . Show that $\nabla^2(rB_r) = 0$, and thus $\Omega = rB_r$ is also harmonic outside of $S(a)$. You may find it easiest to exploit Einstein summation notation to do this.