SIO 231 Geoelectromagnetism
Problems set 1, due 1/18/2022

1. *What are the SI units for* **B**, **H**, **E**, **D**, **J**, σ, *and* ϵ_0*? Show that the unit Tm* A^{-1} *is equivalent to H* m^{-1} *for* μ_0*, and express the units in terms of another pair of SI quantities.*

Estimate and rank in decreasing order: (a) the magnetostatic energy stored in space around a 10g permanent magnet; (b) the chemical energy stored in 10g of cornflakes; (c) the gravitation potential energy in a 10g pencil sitting on your desk; (d) the kinetic energy of a 10g bullet moving at the speed of sound; (e) the mass energy released by fission of 10g of ^{235}U.

As we learned in Lecture 1, the status of the kilogram was recently redefined. Explain why this results in a change in μ_0, and show that the relationship $1/(\mu_0\epsilon_0) = c^2$ still holds.

My rankings are below.

(e) From (http://www.kayelaby.npl.co.uk/atomic_and_nuclear_physics/4_7/4_7_1.html), the fission of one atom of ^{235}U generates 202.5 MeV = 3.24 $\times 10^{-11}$J, which translates to 19.54 TJ/mol, or 83.14 TJ/kg. Yield for 10g is 0.8314 TJ, aka 8.314$\times 10^8$ kJ. Here T stands for Terra or 10^{12} not Tesla.

(b) chemical energy in 10g of cornflakes: Courtesy Google, 10g of cornflakes yield 36 calories. Actually, by long standing convention, these are kcal and conversion to Joules gives 150kJ.

(d) the kinetic energy of a 10g bullet moving at the speed of sound; $KE = \frac{1}{2}mv^2 \quad m = 0.01\ kg,\ v = 343\ m/s\ =$ 588.25 Joules or 0.6 kJ.

(a) magnetostatic energy in 10g magnet $\quad U = \frac{1}{2\mu_0}\int_V B^2 dV = \frac{1}{2\mu}\int_V H^2 dV$. Taking a simplistic approach (for more see e.g., Lovett & Patterson, IEEE Transactions on Magnetics, Vol. 35(1), 1999) I used an Alnico magnet (strongest available before development of rare earth magnets), with densities around 7000 kg/m^3, maximal energy density expressed in terms of BH_{max} of 44 T.A/m. Volume of 10g is 1.43$\times 10^{-6}$ m^3, yielding energy of $U_B = \frac{BH_{max}V}{2\mu_0} = 25J = .025\ kJ$.

(c) the gravitation potential energy in a 10g pencil sitting on your desk at height h; $U = mgh$ Assume $h = 0.7m,\ g = 9.8\ m.s^{-2},\ m = 0.01\ kg$ yields gravitational energy of 0.065 Joules aka 6.5$\times 10^{-5}$ kJ.

2. *The radial component of the magnetic field is* $B_r = \hat{\mathbf{r}} \cdot \mathbf{B}$. *Show that* $\nabla^2(rB_r) = 0$*, that is* rB_r *is also harmonic outside of* $S(a)$.

We know already that we can write $\mathbf{B} = -\nabla V$ and $\nabla^2 V = \nabla \cdot \mathbf{B} = 0$. Using the ever useful and compact summation notation we write

$$\begin{aligned}
\nabla^2(rB_r) &= \partial_i\partial_i(r_j B_j) \\
&= \partial_i(\partial_i r_j B_j + r_j \partial_i B_j) \\
&= \partial_i(\delta_{ij} B_j + r_j \partial_j B_i) \\
&= \partial_i B_i \\
&= \nabla \cdot \mathbf{B} = 0
\end{aligned}$$