SIO 231 Geoelectromagnetism
Geomag Homework # 3,
Due February 13, 2024

Q1 On the class web set you will find a file called chaos_7_9_av_lm containing Schmidt normalized Gauss coefficients for the CHAOS7-9 internal geomagnetic field averaged over the time interval 1997.0 to 2022.1.

Calculate and plot the geomagnetic spatial power spectrum R_l at Earth's surface (radius a= 6371.2km), at two additional radii of your choice one above and one below $r = a$, and at the core mantle boundary (r= c=3485 km). Use the spectrum at Earth's surface and its rate of fall off to estimate the radius of Earth's outer core. Redo the calculation excluding the $m = 0$, zonal Gauss coefficients and comment on the origin of the differences.

Q2 In class we discussed the possibility that paleomagnetic secular variation could be described by a Giant Gaussian Process where time variations are ascribed a statistical representation in terms of each Gauss coefficient for degree and order $l = 1 - 14$. One GGP published by Lisa Tauxe and Dennis Kent in 2004 has the feature that the variances for the g_l^m and h_l^m depend on whether $l - m$ is even or odd which correspond to SH functions that are respectively symmetric or antisymmetric about the equator. Slide 27 gives

$$\sigma_l^{m2} = \frac{(c/a)^{2l}\alpha^2}{(l+1)(2l+1)} \qquad \text{for } l-m \text{ even}$$

$$\sigma_l^{m2} = \frac{(c/a)^{2l}\alpha^2\beta^2}{(l+1)(2l+1)} \qquad \text{for } l-m \text{ odd.}$$

with $\alpha = 7.5\mu$T and $\beta = 3.8$. All the mean values $\bar{g}_l^m$ and $\bar{h}_l^m$ are zero except for the axial dipole $\bar{g}_1^0 = -18\mu$T. Calculate and plot the spatial power spectrum up to degree and order 14 for this GGP. How do things change if the parameter β is modified to have value $\beta = 1$? And what would you expect if the σ_l^{m2} are allowed to vary within each SH degree l?

Q3 Also on the class web set you will find a file containing MAGSAT observations of B_θ and B_r as a function of colatitude and radial distance from Earth's center. Plot the measurements as a function of θ. Use these to make a least squares estimate of i_1 and e_1 for this pass as described in Lecture 9. Plot the predictions of internal and external contributions to B_r and B_θ as a function of colatitude. How accurate are your results?