SIOG 231
Homework Assignment #4
An Introduction to Finite Difference Calculations

Referring to Lecture 12, for a 1D earth of $\sigma(z)$ and harmonic excitation of frequency ω, the complex vertical electric field obeys

$$\frac{d^2E}{dz^2} = i\omega\mu_0\sigma(z)E(z) \tag{1}$$

If we discretize $\sigma(z)$ and $E(z)$ over evenly spaced intervals Δz of z such that

$$z = i\Delta z, \quad i = 1, N \tag{2}$$

then to a first approximation for any i

$$\frac{d^2E}{dz^2} \approx \frac{E_{i+1} - 2E_i + E_{i-1}}{\Delta z^2} \tag{3}$$

If you substitute (3) into (1), with a little care you can cast the problem into the linear form

$$\mathbf{A}x = b \tag{4}$$

where x is a vector of E_i, and which, conveniently, can be solved in MATLAB by typing

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x = A\b
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To get a unique solution you will have to add some 'boundary conditions' to (4), specifying E at the top and bottom of the model.

Choose some sensible boundary conditions, and set up the system of linear equations in MATLAB. Solve for a 3 Hz excitation of a 0.5 S/m half-space. Check your results against the known skin depth relationships. Compute c and ρ_a at the surface of the model, as well as the phase angle (which is the same for both).

Now solve for the excitation of a model in which σ increases linearly with depth from 1 S/m at the surface to 10 S/m at 1,000 m depth, then is terminated with a 10 S/m half-space. Compute the response at 1 Hz, but then extend this to create sounding curves of apparent resistivity and phase and components of c.

Think of tricks to improve numerical accuracy for a given N, and ways to assess whether N is large enough and Δz is small enough.