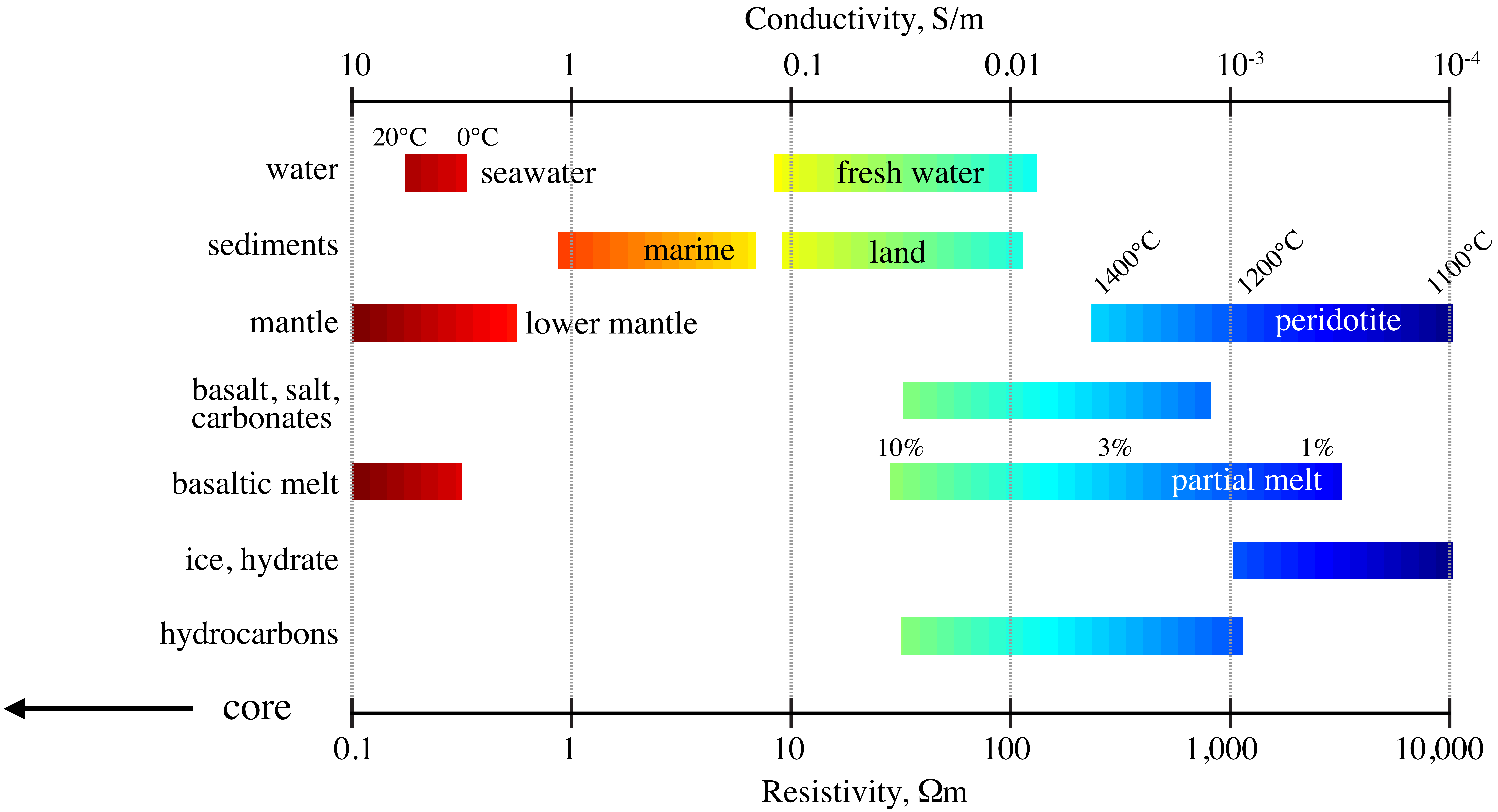
SIOG 231
GEOMAGNETISM AND ELECTROMAGNETISM

Lecture 18
Electrical Conductivity of Rocks, Crust, Mantle, and Core
3/7/2024

Conductivity varies by orders of magnitude:



Ionic conduction:

seawater at 20°C	5 S/m	0.2 Ωm
seawater at 0°C	3 S/m	0.3 Ωm
marine sediments	1 S/m	1 Ωm
land sediments	0.001–0.1 S/m	10–1000 Ωm
igneous rocks	10^{-5}–10^{-2} S/m	100–100,000 Ωm

Semiconduction and metallic conduction:

Graphite	10,000–1,000,000 S/m	10^{-4}–10^{-6} Ωm
Galena	10–100,000 S/m	10^{-1}–10^{-5} Ωm
Pyrite	1–1,000 S/m	10^{-3}–1 Ωm
Magnetite	20,000 S/m	5×10^{-5} Ωm
Copper	5.8×10^{7} S/m	1.7×10^{-8} Ωm
Silver	6.2×10^{7} S/m	1.6×10^{-8} Ωm
Gold	4.5×10^{7} S/m	2.2×10^{-8} Ωm

Defect conduction and electrolytic conduction in melt:

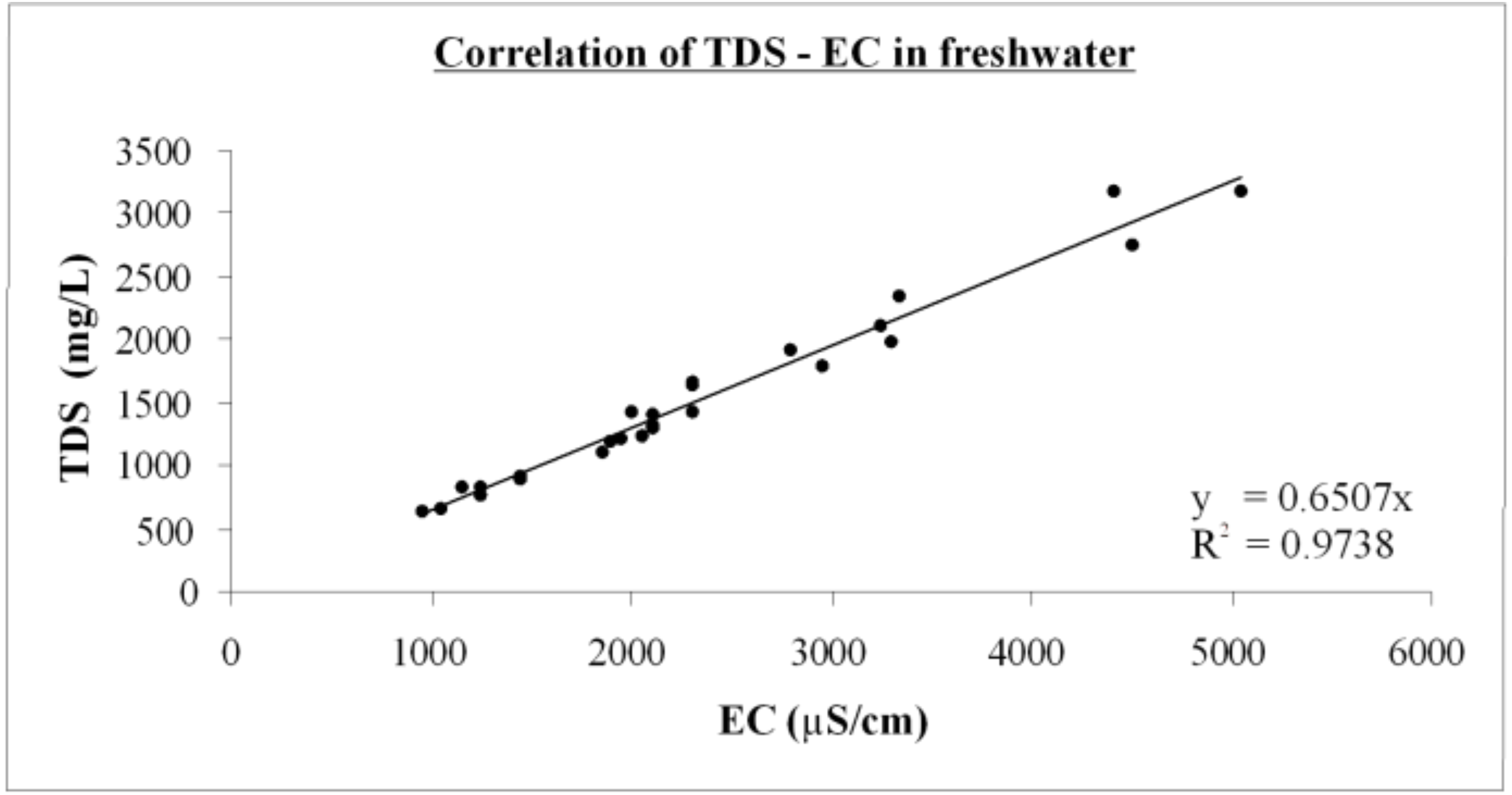
Olivine at 1000°C	0.0001 S/m	10,000 Ωm
Olivine at 1400°C	0.002 S/m	500 Ωm
Theoleite melt	3 S/m	0.3 Ωm
Silicate perovskite	1 S/m	1 Ωm

Ionic conduction:

Conductivity and total dissolved salts (TDS):

$$\sigma = a \times \mathrm{TDS}$$

where TDS is in g/l and a is between 0.13 and 0.20, generally being lower for higher salinity. For seawater $a = 0.143$. Temperature 20°C.



Rusydi, 2018 doi :10.1088/1755-1315/118/1/012019

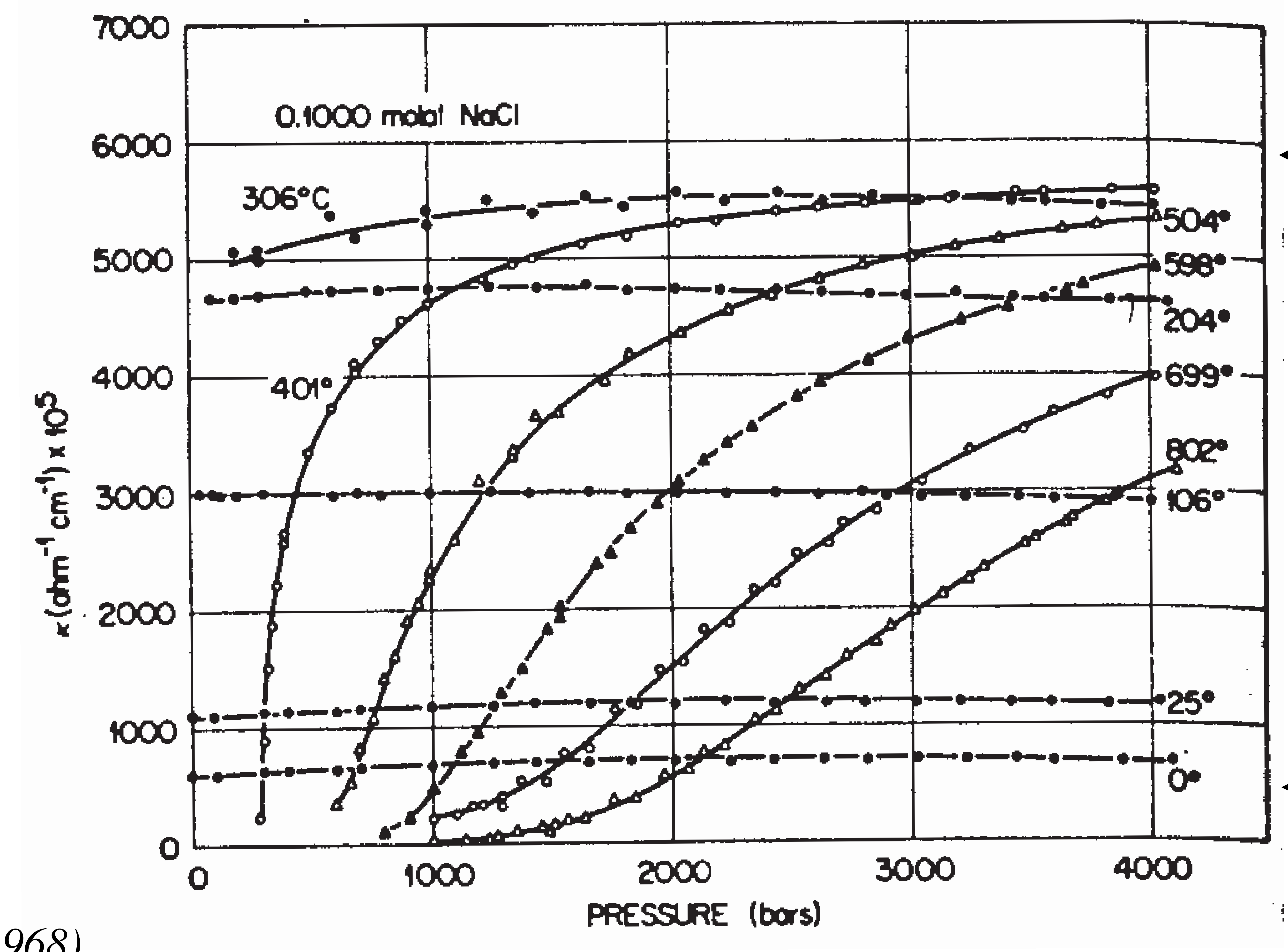
Ionic conduction:

$$\sigma = 3 \quad \mathrm{S/m} \quad T = 0°\mathrm{C}$$
$$\sigma = 5 \quad \mathrm{S/m} \quad T = 20°\mathrm{C}$$

Electrical conductivity of seawater: $\sigma = 3 + T/10 \quad \mathrm{S/m}$

$$\sigma_f = 2.903916(1 + 0.0297175T + 0.00015551T^2 - 0.00000067T^3) \quad \mathrm{S/m.}$$

High P & T:



50 S/m

seawater equivalent

3 S/m

Quist and Marshall (1968)

Binary mixing laws:

There are various ways to model porosity and water/melt content. The most popular and venerable is Archie's Law:

$$\sigma = \sigma_s + (\sigma_f - \sigma_s)\beta^m$$

where the exponent *m* is between 1.5 and 2.0 (determined empirically) and

$\sigma_s =$ conductivity of the mineral grains

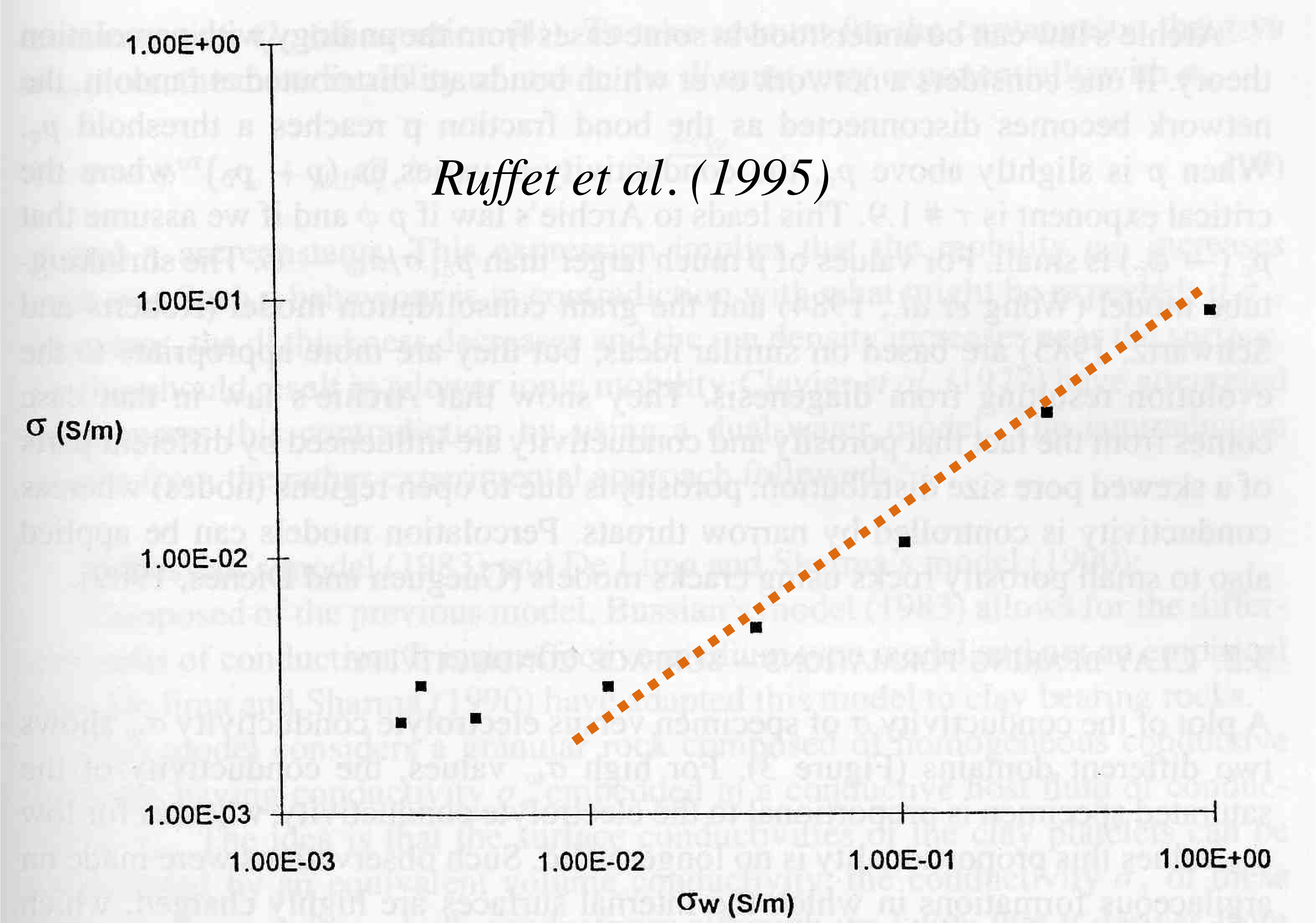
$\sigma_f =$ conductivity of the fluid

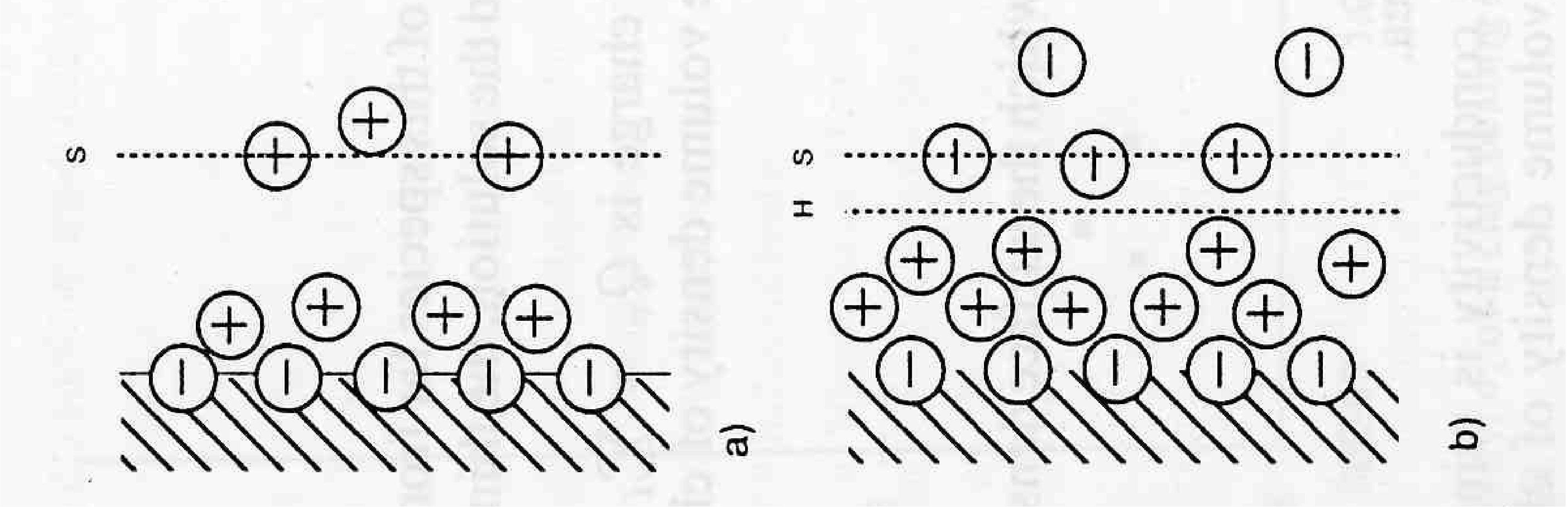
$\beta =$ fluid fraction (or porosity if saturated).

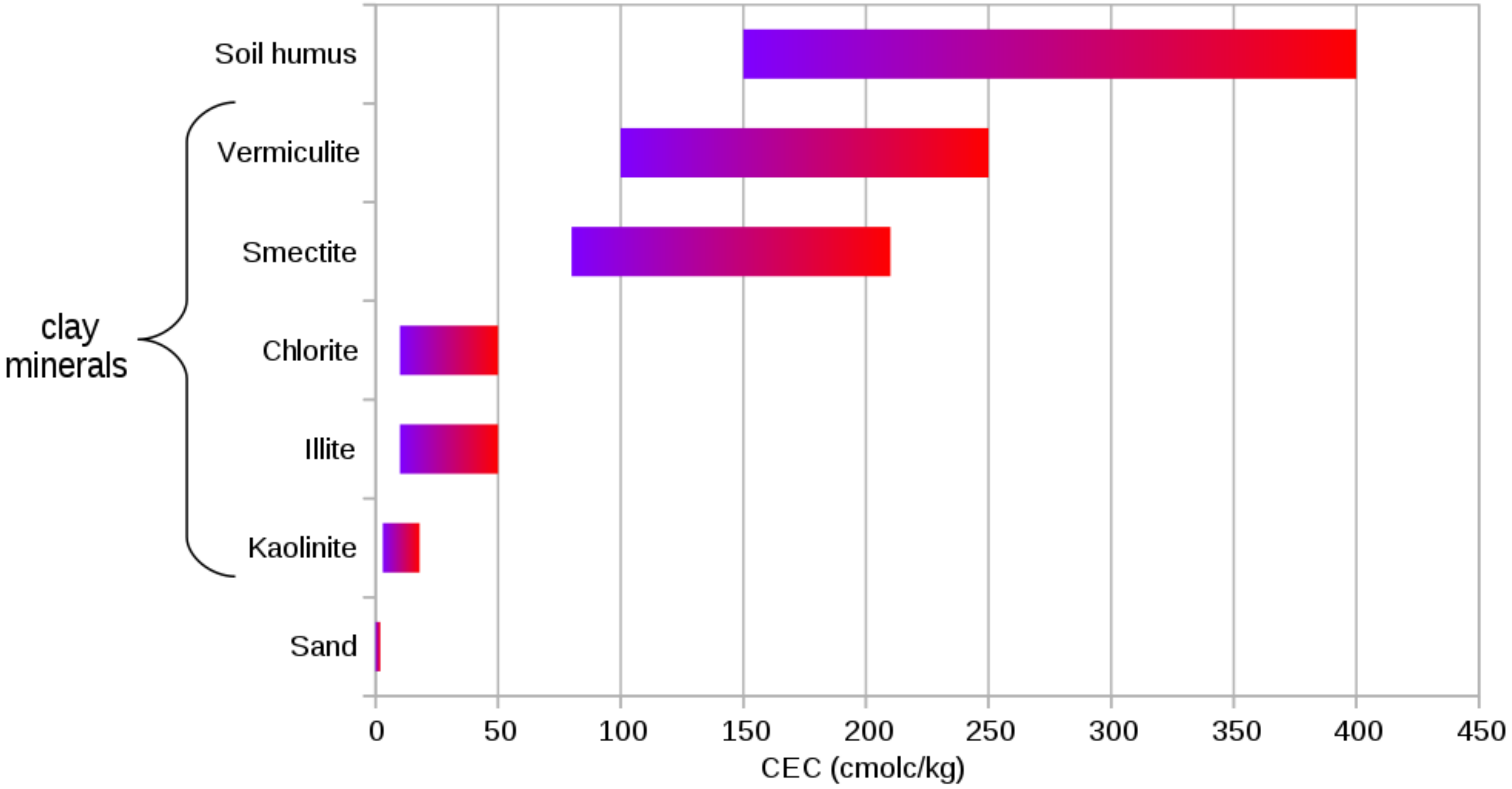
There are other models on which mixing laws can be based, for example fluid filled tubes:

$$\sigma = \frac{1}{3}\beta\sigma_f + (1 - \beta)\sigma_s)$$

Clay-bearing sandstone (silty sandstone): For low pore water salinities, the layer of absorbed charge on the surface of clay minerals increases the conductivity. The density of this layer is measured by the *cation exchange capacity*.







Wikipedia

Waxman-Smits (1968):

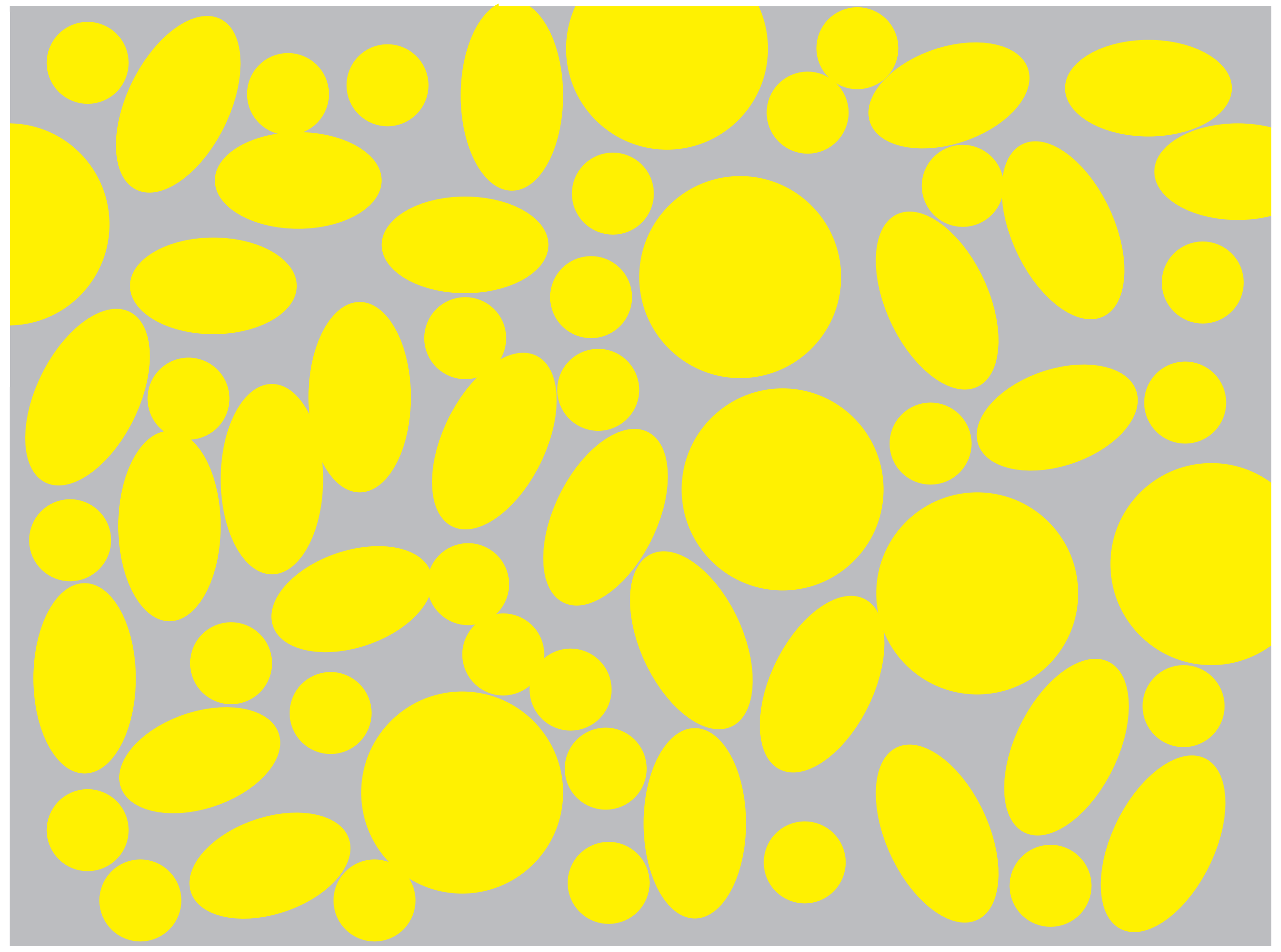
$$\sigma = \frac{1}{F*}(BQ_V + \sigma_w)$$

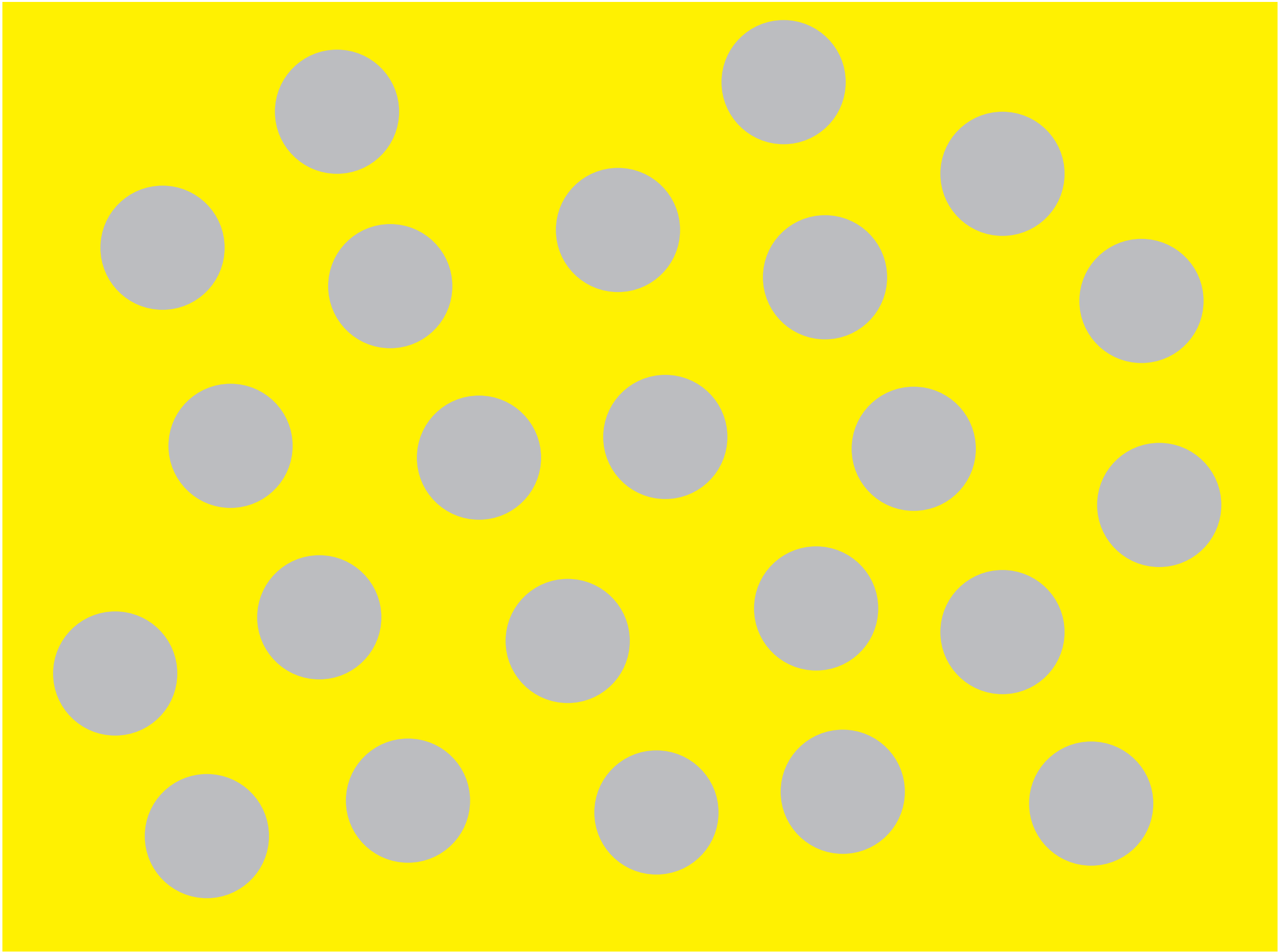
$$B = [1 - ae^{-\sigma_w/\gamma}]\lambda^e_{Na} \qquad a = 1 - \frac{(\lambda^e_{Na})_{\sigma_W=0}}{\lambda^e_{Na}}$$

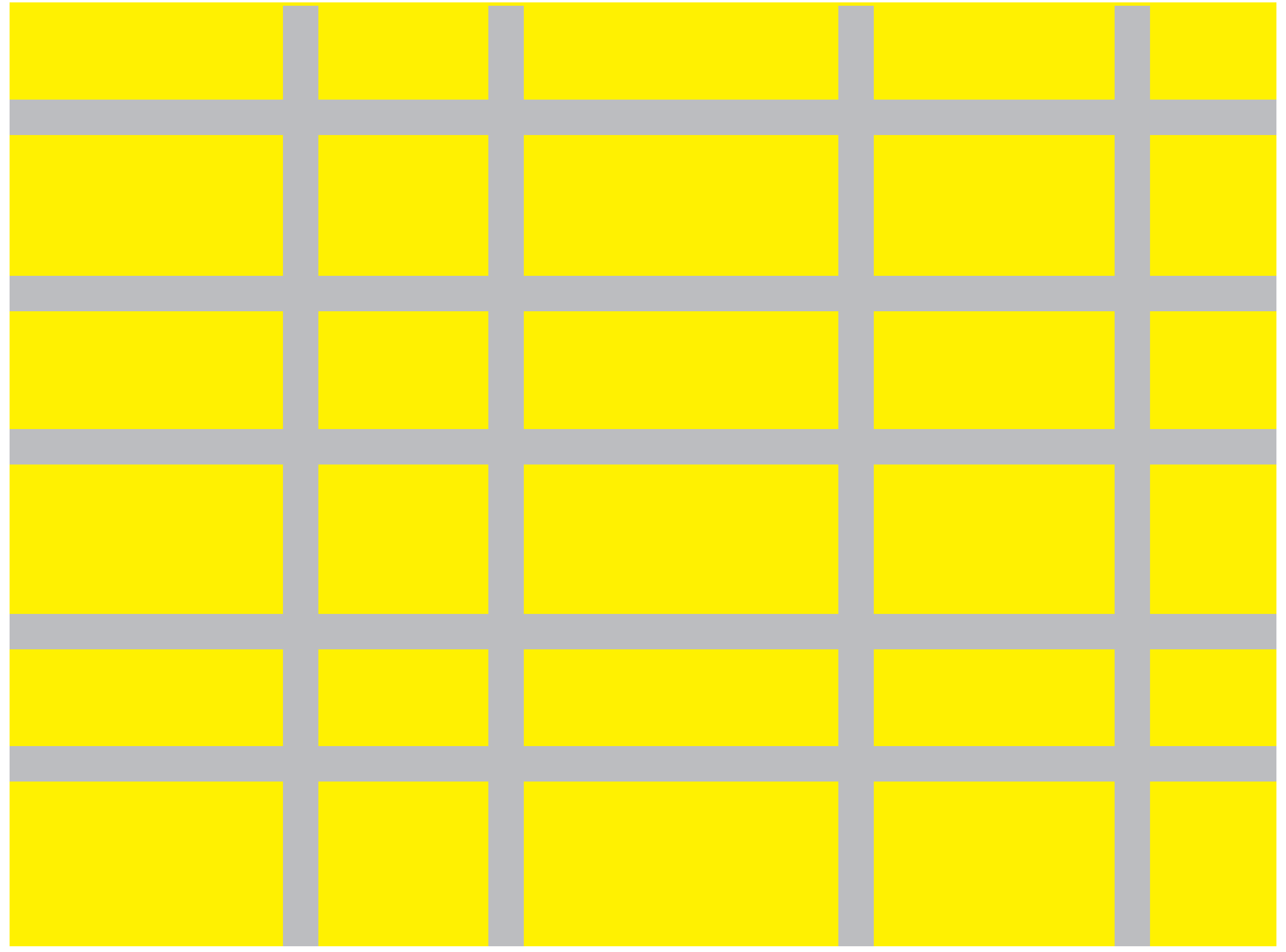
All possible isotropic models must lie between the Hashin-Shtrikman (HS) bounds:

$$HS^{-} = \sigma_s + \beta \left(\frac{1}{\sigma_f - \sigma_s} + \frac{1-\beta}{3\sigma_s} \right)^{-1}$$

$$HS^{+} = \sigma_f + (1-\beta) \left(\frac{1}{\sigma_s - \sigma_f} + \frac{\beta}{3\sigma_f} \right)^{-1}$$

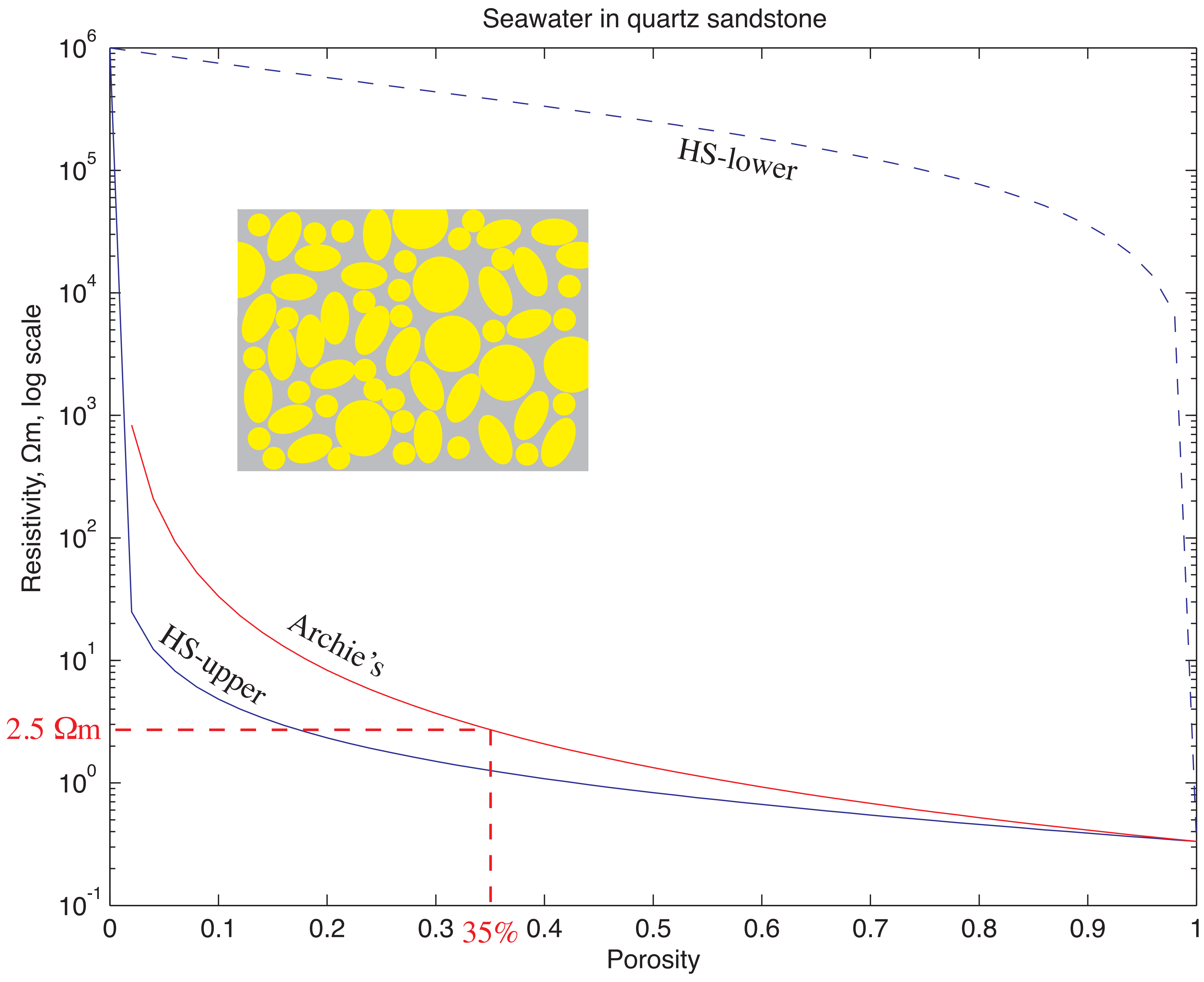

Archie's


HS - lower

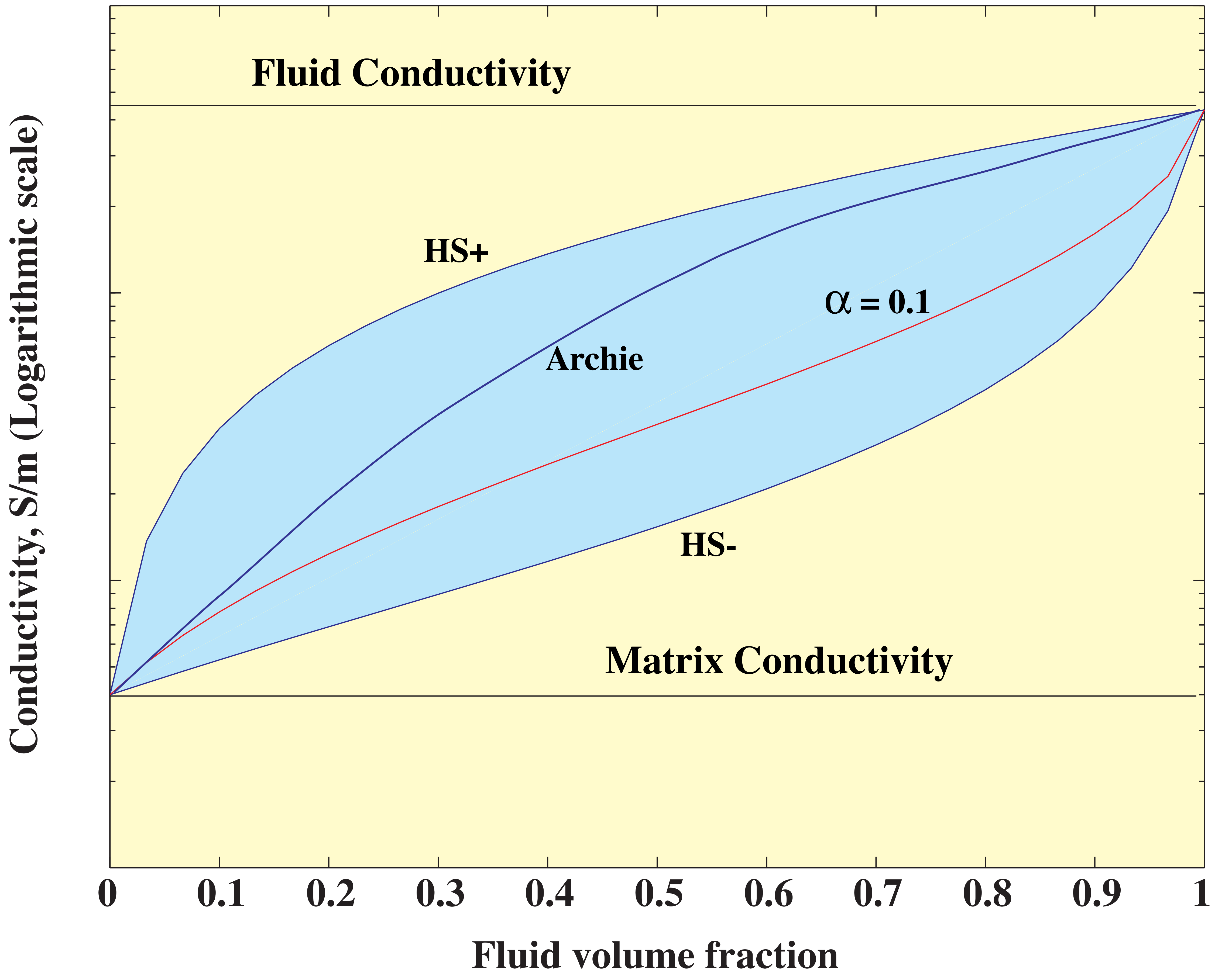

HS - upper

grey = conductor (melt, water)   yellow = resistor (sand grains, sub-solidus mineral grains)

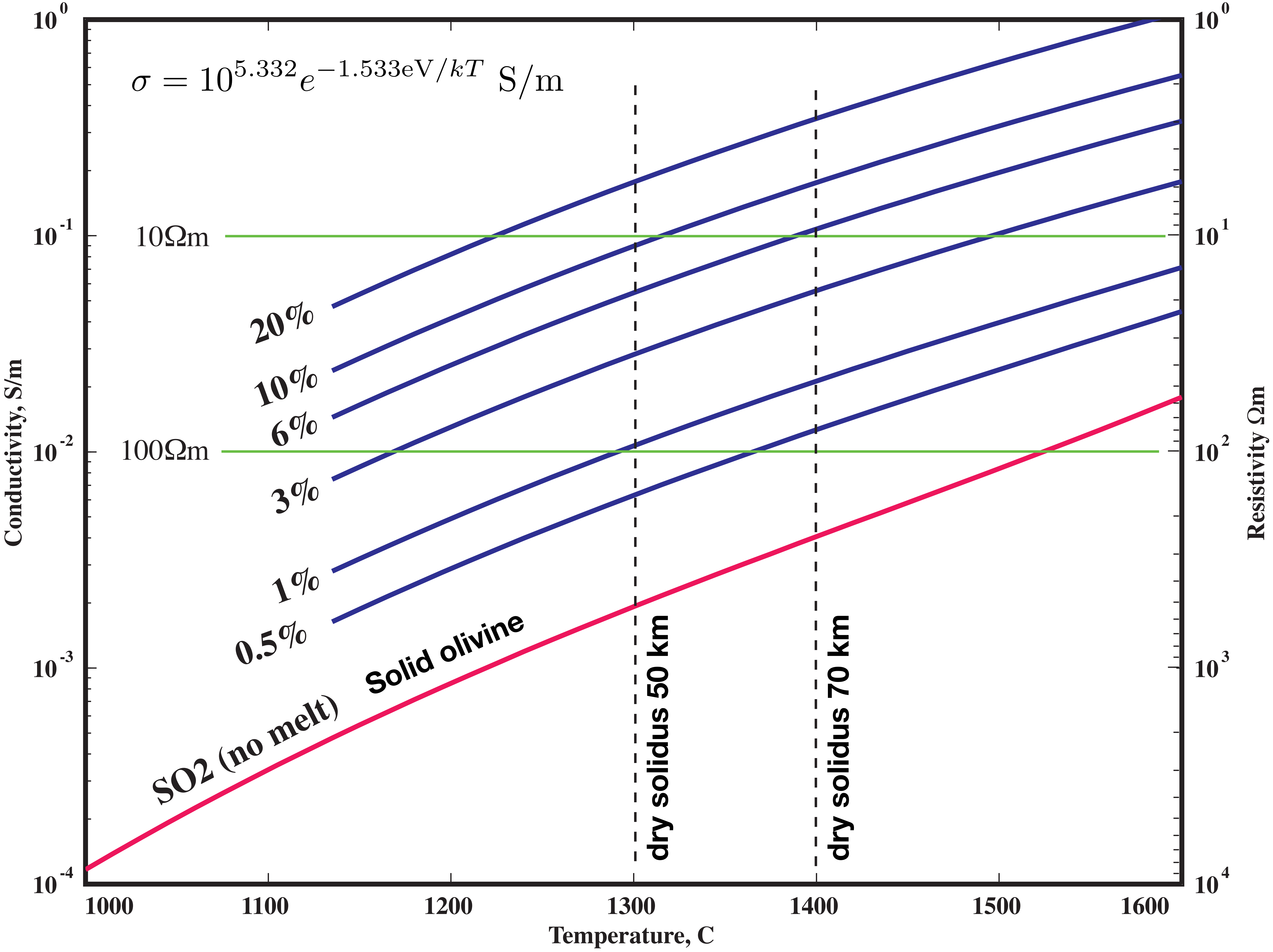
Seawater in sandstone:



Melt and hot minerals:



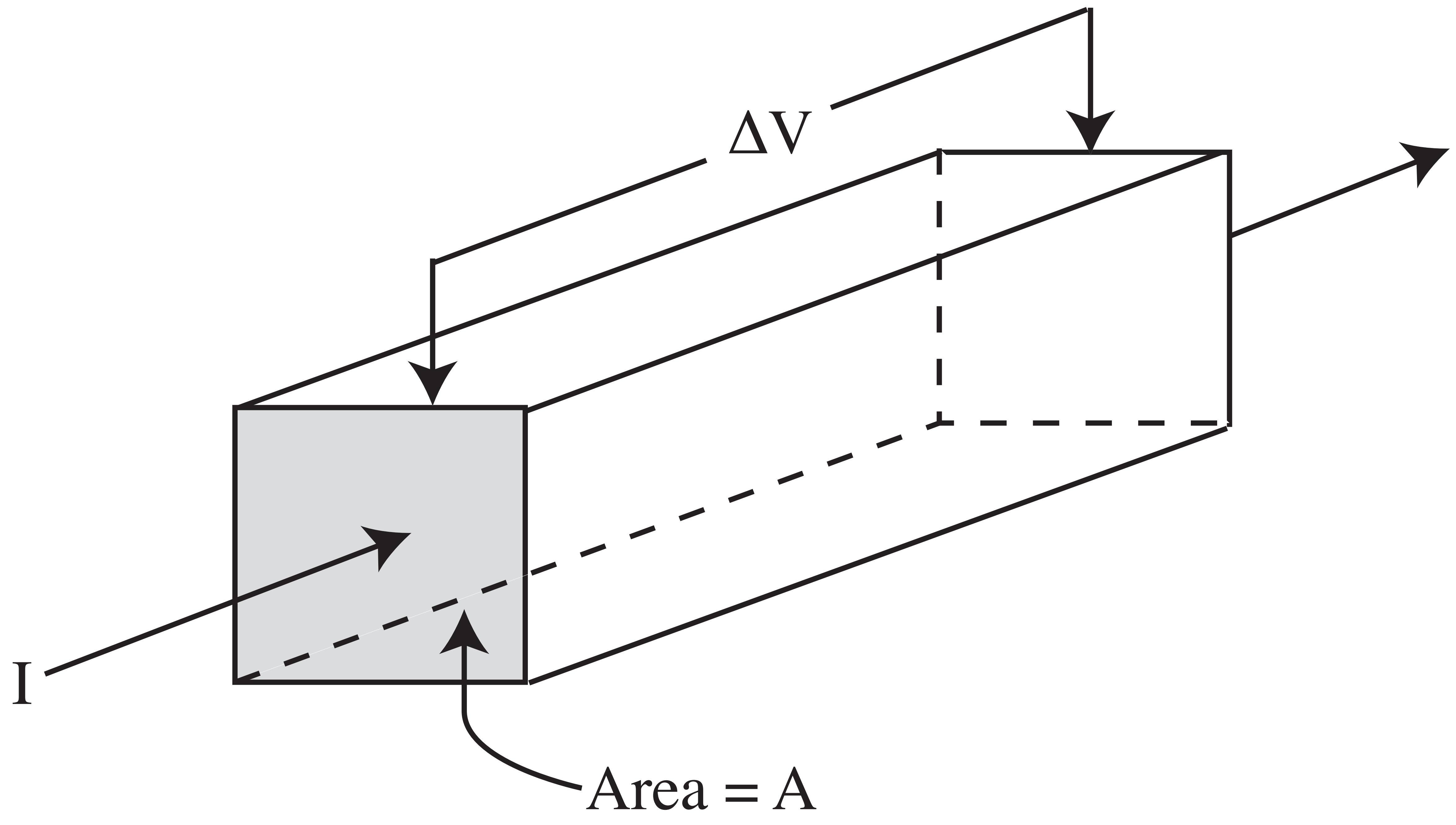
Conductivity Versus Melt Fraction and Temperature

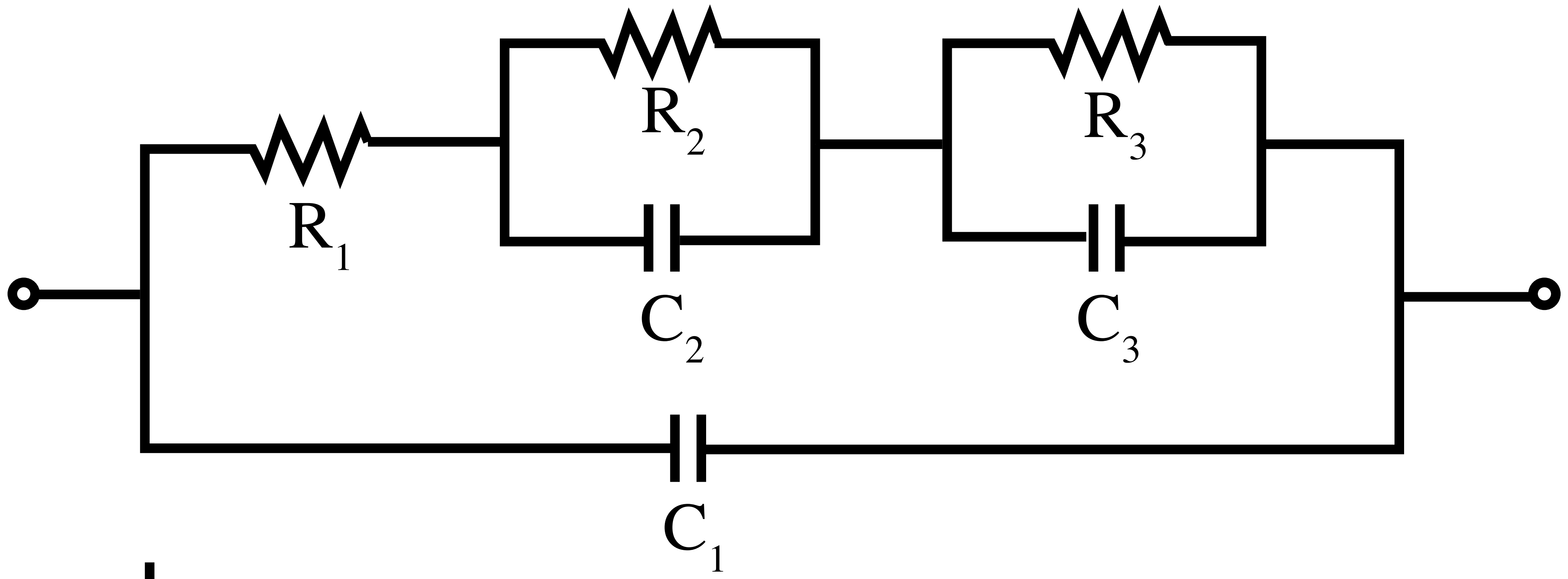


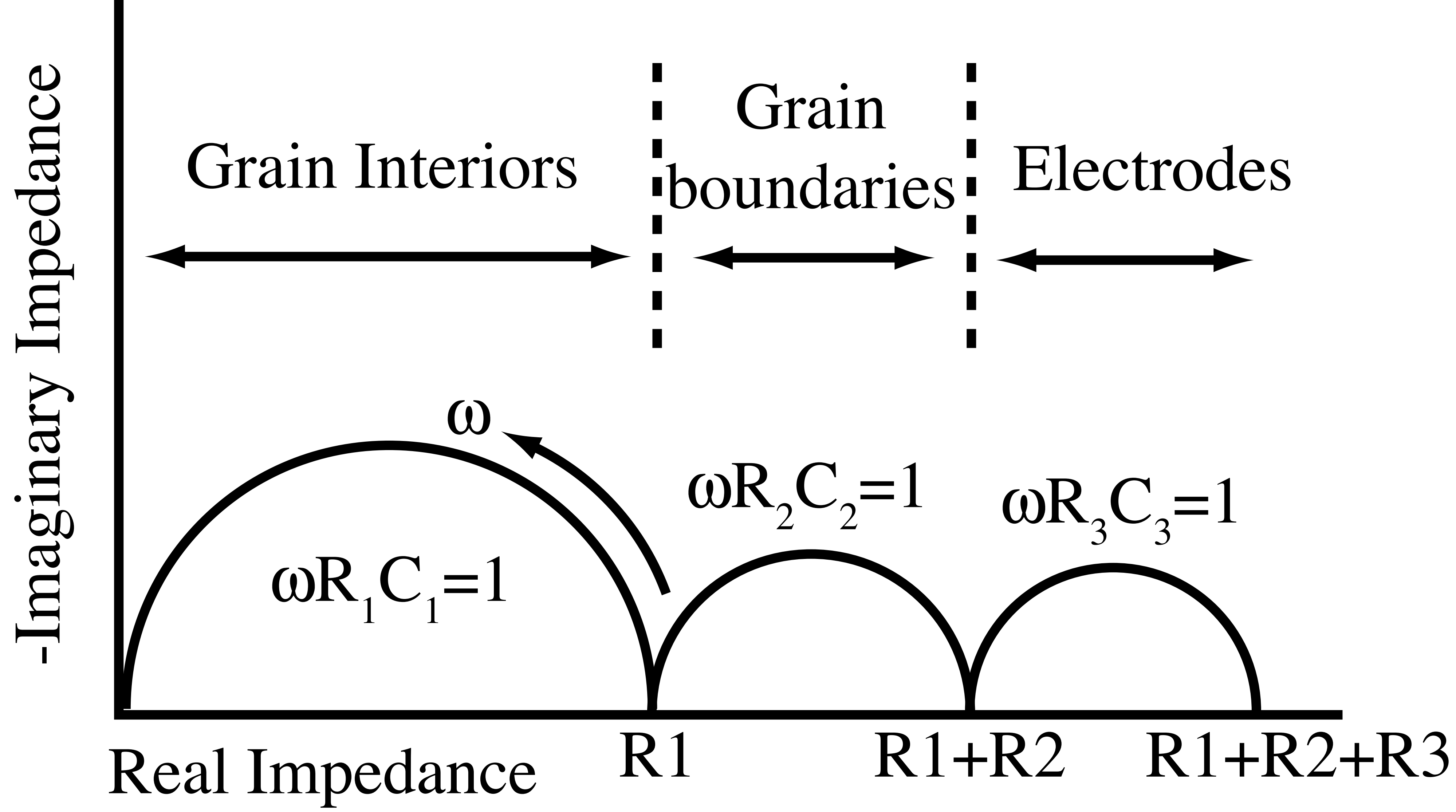
Measuring conductivity is simple in principle, but impedance spectroscopy is used to separate electrode polarization from sample conductivity, and even grain boundary effects in a multi-grain sample.

$$R = \Delta V / I$$

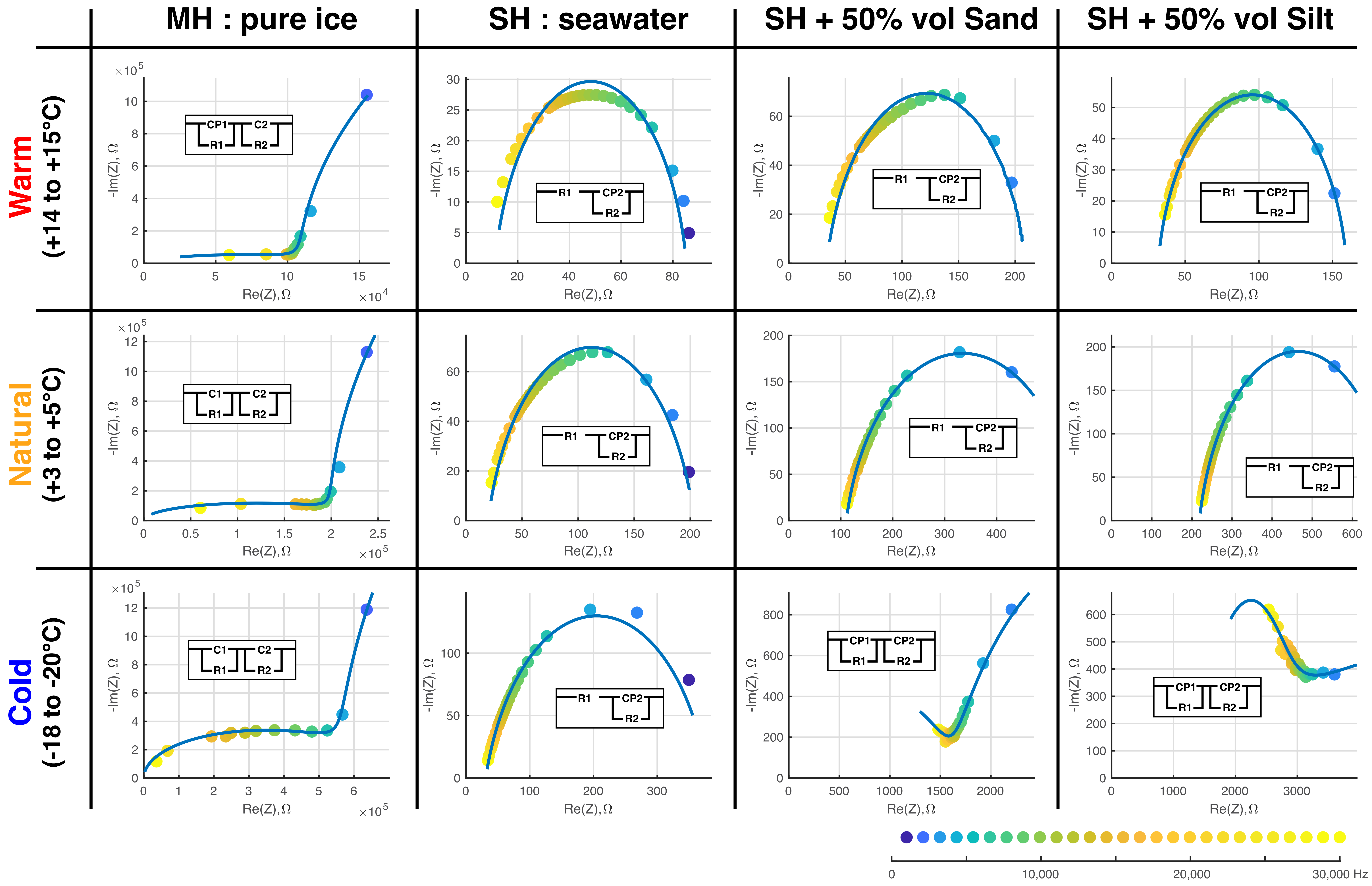
$$R = \rho L / A$$







Examples from methane hydrate + seawater + sand/silt:



Conductivity in metals

Moving an electric charge: $\sigma = nq\mu$

q is charge, *n* is concentration, μ is mobility, or drift velocity *v* per electric field $\left|\frac{v}{E}\right|$

Drift velocity of electrons in metals is $v = -eE\tau/m$

where τ is the mean free time, *m* is mass. So, conductivity is $\sigma = \dfrac{ne^2\tau}{m}$

This can be though of as an acceleration in an electric field (*e/m*) of charge density (*ne*) for a time τ. Drift velocity can be thought of as current density divided by charge density:

$$v = \frac{J}{c}$$

Typical values:

$$n \approx 10^{28} – 10^{29}\ /\mathrm{m}^3 \qquad v \approx 10^{-4}\ \mathrm{m/s} \qquad \sigma \approx 10^{7}\ \mathrm{S/m}$$

$$v\tau \approx 10^{-2}\mathrm{m\ at\ 4\ K} \qquad v\tau \approx 10^{-8}\mathrm{m\ at\ 20^\circ C}$$

Semiconduction:

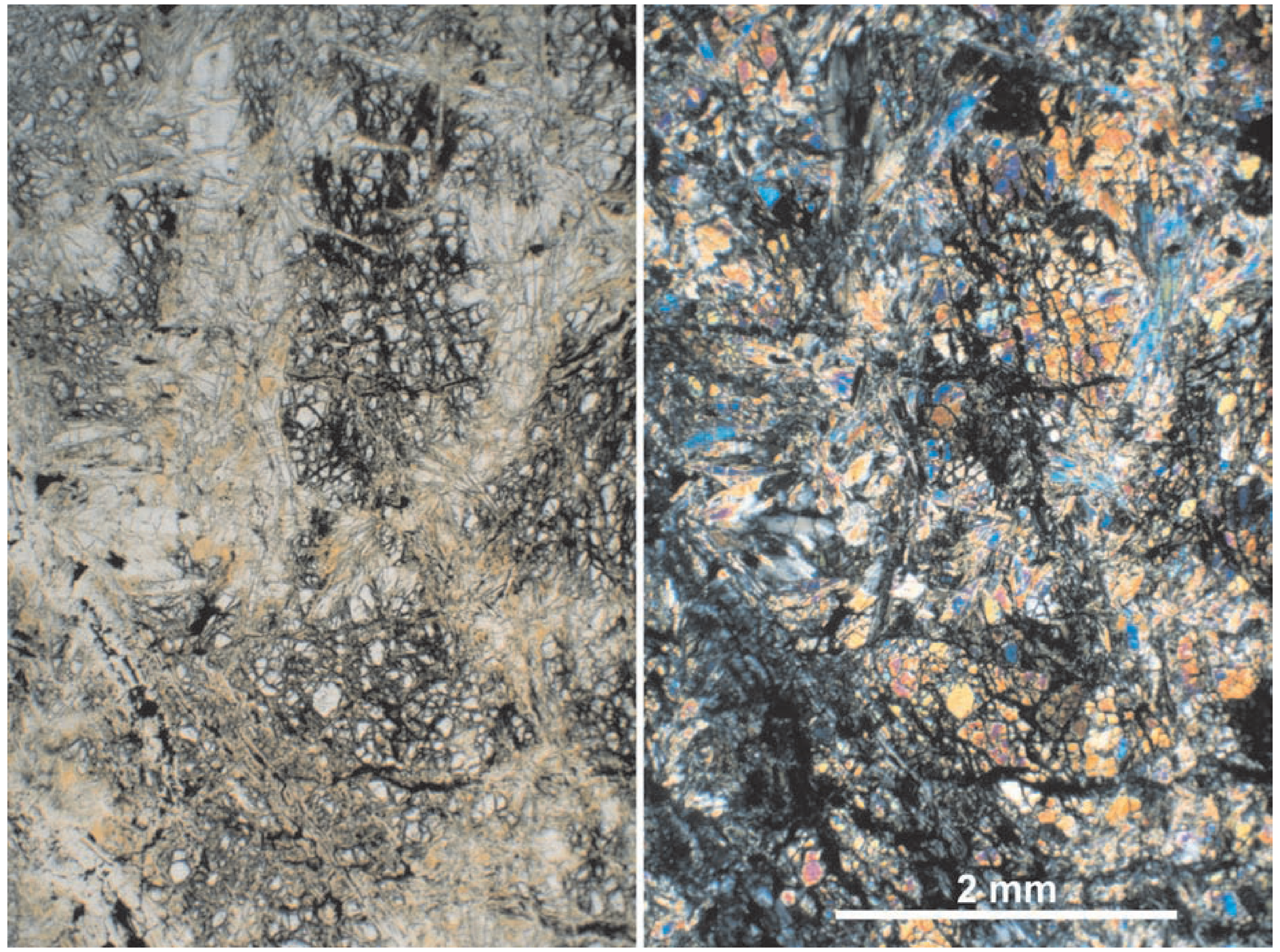
Ore minerals are generally semiconductors.

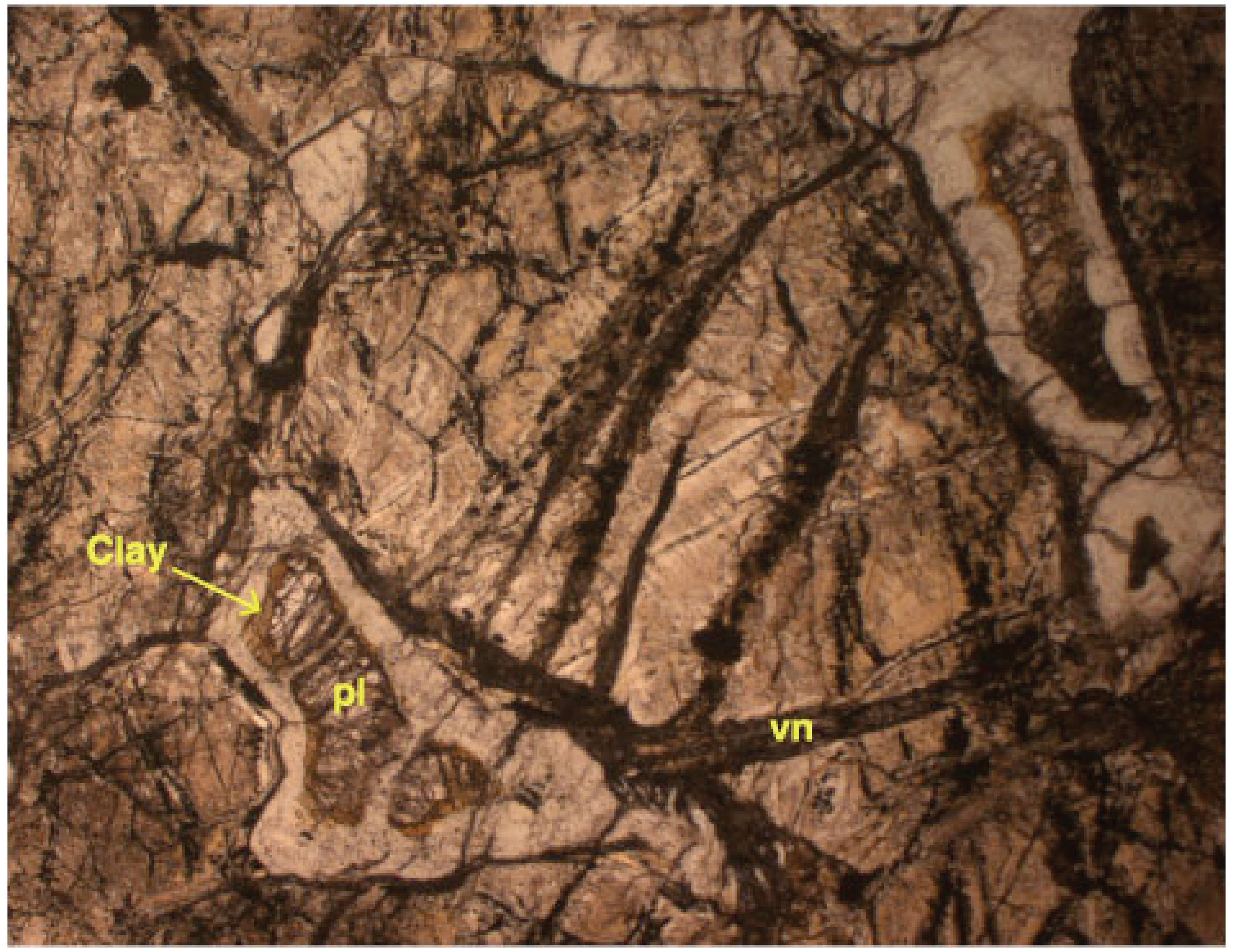
Other conductive mineral phases can form a connected path though rocks. Serpentinization can create halos of connected magnetite around relic olivine grains.

It has been suggested that cumulate magnetite in layered intrusions is responsible for lower crustal conductors in rifted margins.

It has also been suggested that sulfides on grain boundaries of mantle minerals increases the conductivity beneath the Sierra Nevada.

Graphite precipitated from CO2 fluids in the crust has also been implicated to explain high conductivity.





Serpentinite.

Mineral electrical conductivity as a function of pressure and temperature really is parameterized.

Each conduction mechanism is described by a thermally-activated Boltzmann process:

$$\sigma = \sum_i \sigma_i e^{-E_i/kT}$$

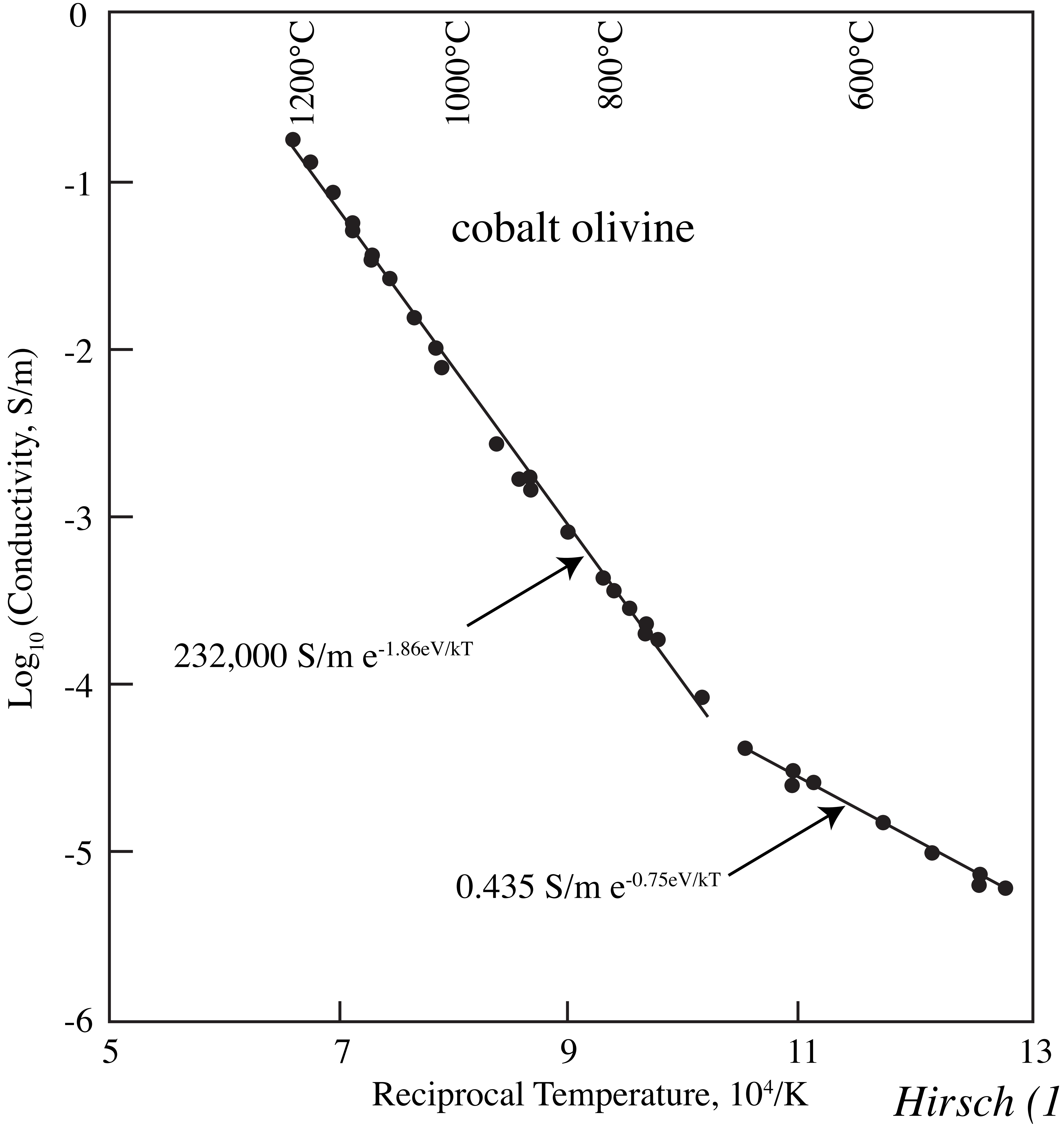
σ is conductivity, S/m

T is absolute temperature, Kelvin

E is activation energy

k is Boltzmann's constant

These are called Arrhenius relationships, and can be linearized by plotting log(sigma) against 1/T. The slope gives E_A



Hirsch (1990)

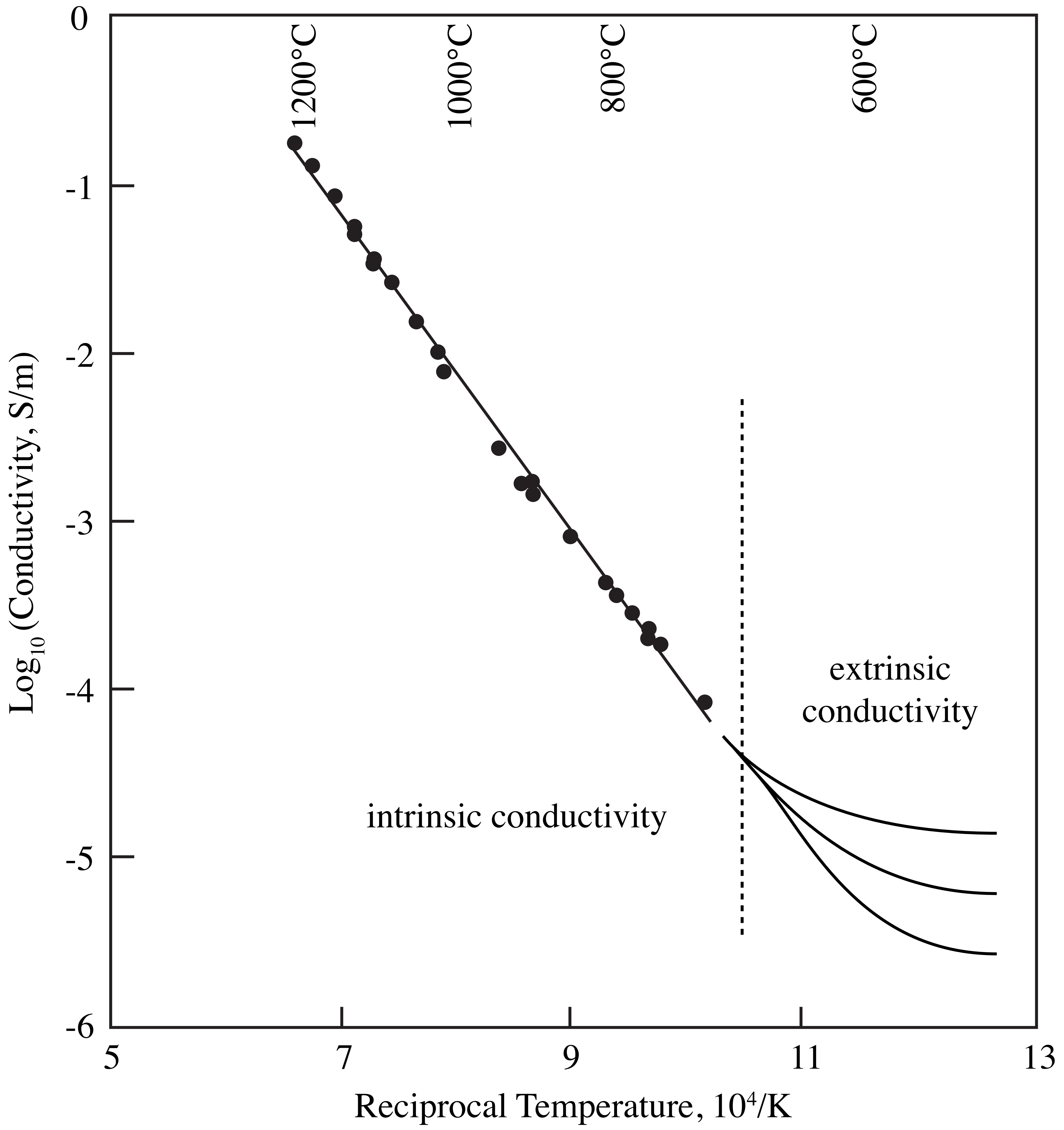
Semiconductors

$$\sigma = ne\mu_e + pe\mu_h$$

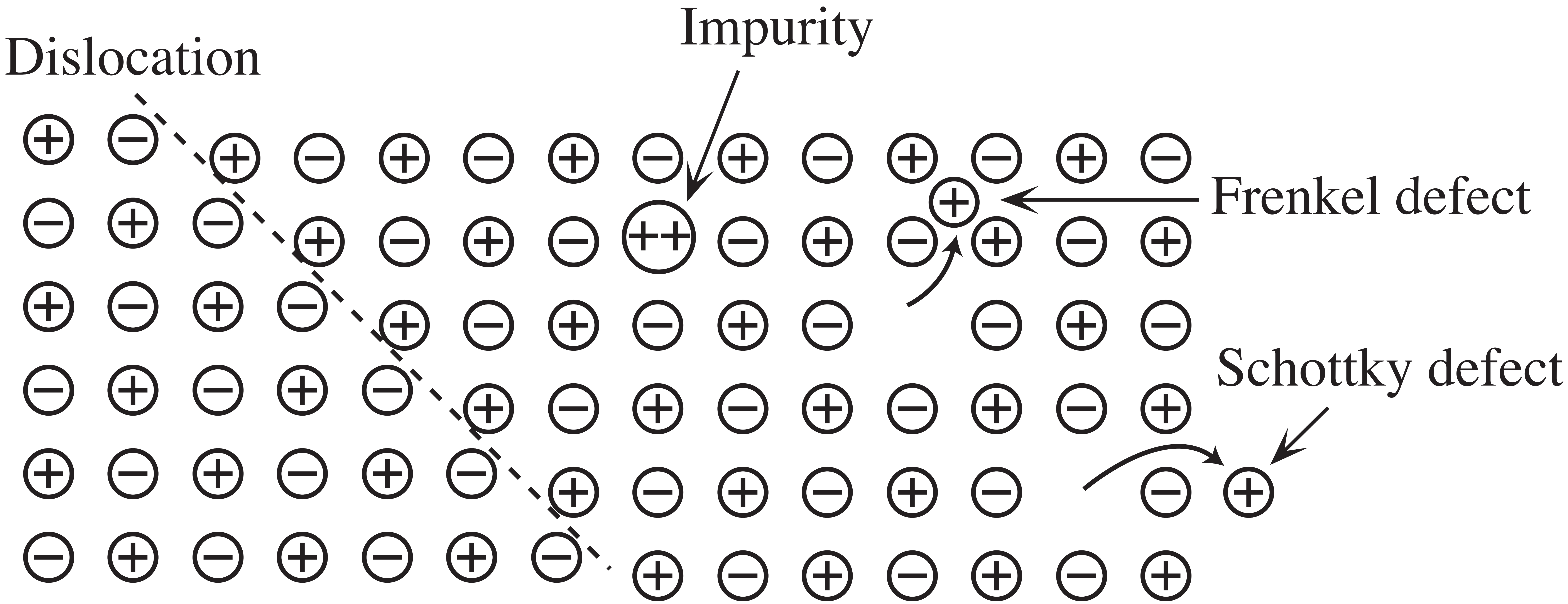
$$n \propto T^{3/2} e^{-E_g/2kT}$$

Intrinsic conduction is a result of thermally activated electrons and holes. All samples of a given material behave the same.

Extrinsic conduction is a result of impurities which donate either electrons or holes to conduction. The amount of impurity will vary from sample to sample.



Defect conduction:



$$n_s = Ne^{-E_v/kT}$$ Schottky defects or vacancies

$$n_f = (NN')^{1/2}e^{-E_i/2kT}$$ Frenkel defects or interstitials

Fick's Law:

$$\mathbf{J}_N = -D\nabla N$$

$\mathbf{J}$ is the flux of a species and D is diffusivity or diffusion constant

$$D = D_o e^{-E_d/kT}$$

The Nernst-Einstein equation relates mobility μ to diffusivity

$$kT\mu = qD$$

Using $\sigma = nq\mu$ and substituting for the mobility and the concentration for Schottky defects we get

$$\sigma = Nq^2 D_o/kT e^{-E_v/kT} e^{-E_d/kT}$$

concentration **mobility**

But the exponentials dominate, and so to a good approximation

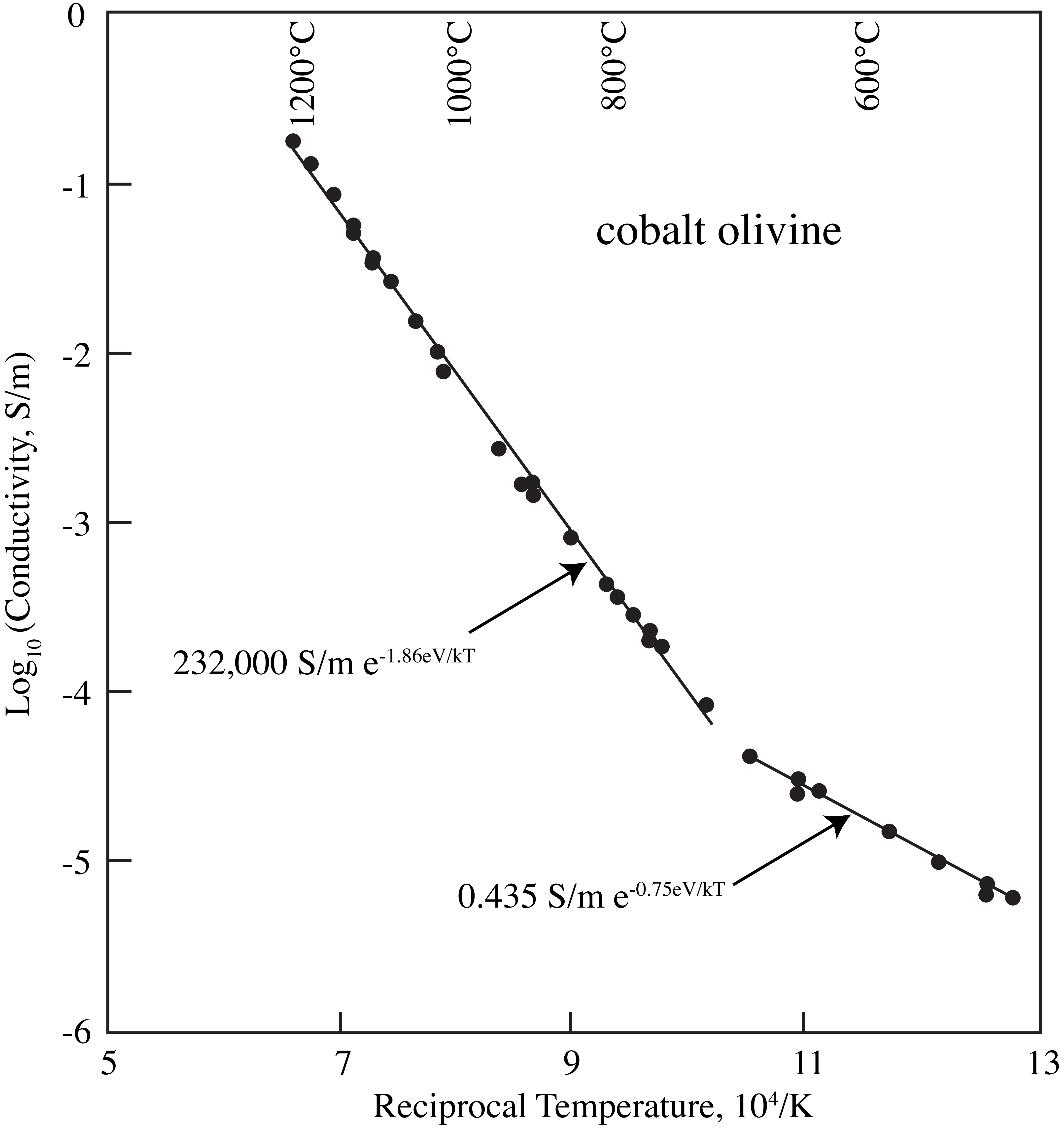
$$\sigma = \sigma_o e^{-E/kT}$$

If we have multiple conduction species then conductivity is just the sum. But exponentials dominate at particular temperatures.

$$\sigma = \sum_i \sigma_i e^{-E_i/kT}$$

Sometimes one has to include a pressure effect:

$$\sigma = \sigma_o e^{-(E+P\Delta V)/kT}$$

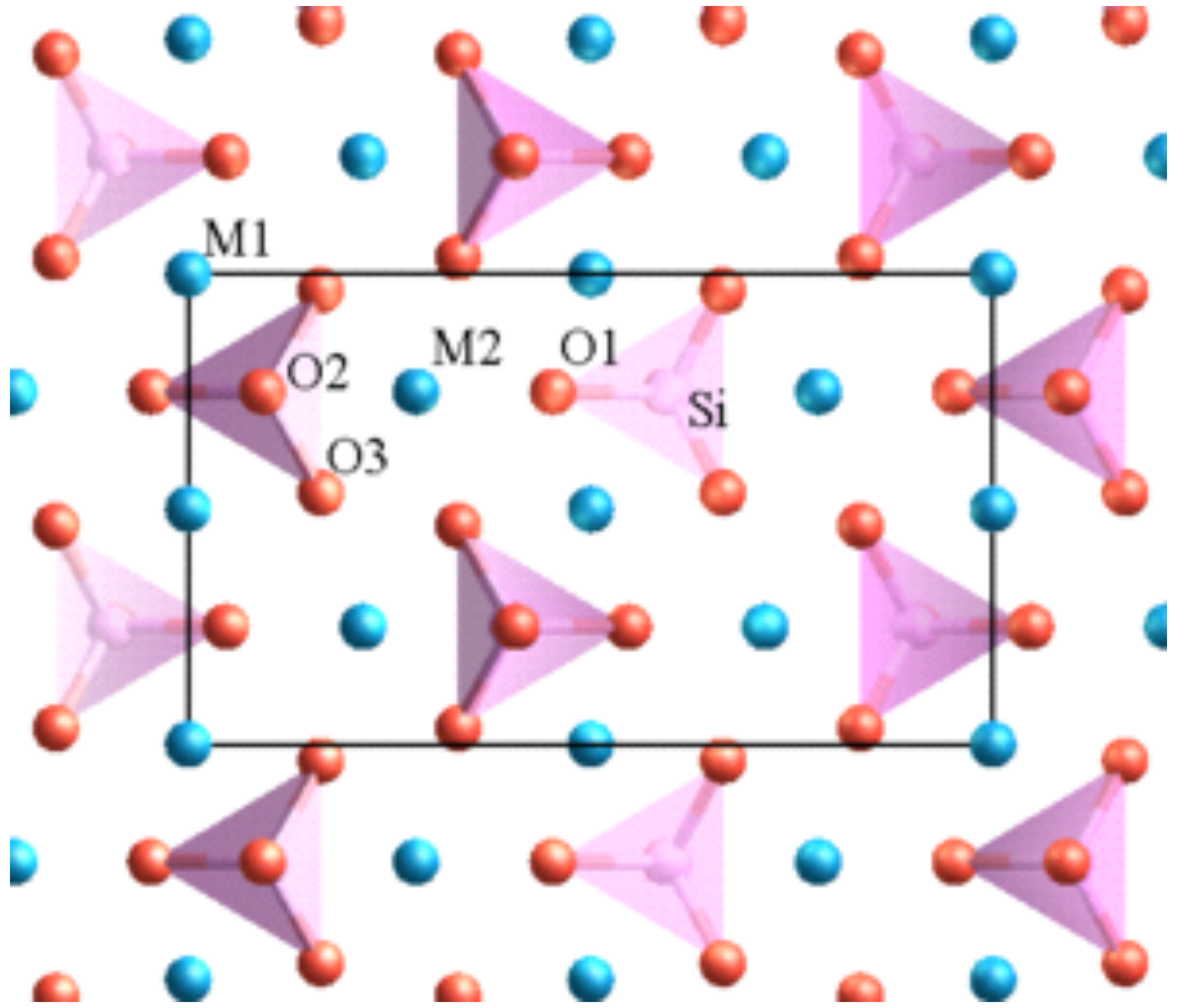


Hirsch (1990)

Oxidation of iron:

$$Fe^{++} \rightleftharpoons Fe^{+++} + e'$$

$$\mathrm{Fe}^{\mathrm{x}}_{\mathrm{Mg}} \rightleftharpoons \mathrm{Fe}^{\bullet}_{\mathrm{Mg}} + \mathrm{e}'$$



Oxidation of olivine ($Mg_{0.9}Fe_{0.1}SiO_4$):

$$8Fe^{\times}_{Mg} + 2O_2 \rightleftharpoons 2V''_{Mg} + V''''_{Si} + 4O^{\times}_{O} + 8Fe^{\bullet}_{Mg}$$

V''_{Mg}	magnesium vacancies
$Fe^{\bullet}_{Mg}$	“small polaron”
e'	electron
V''''_{Si}	silicon vacancies

Forsterite Mg_2SiO_4

$$\begin{array}{cccccc}
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} \\
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} \\
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} \\
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times}
\end{array}$$

Olivine $Mg_{0.9}Fe_{0.1}SiO_4$

$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Fe}_{\mathrm{Mg}}^{\times}$
$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$
$\mathrm{Fe}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Fe}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$
$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$

Oxidation of iron:

$$\mathrm{Fe}_{\mathrm{Mg}}^{\mathrm{x}} \rightleftharpoons \mathrm{Fe}_{\mathrm{Mg}}^{\bullet} + \mathrm{e}'$$

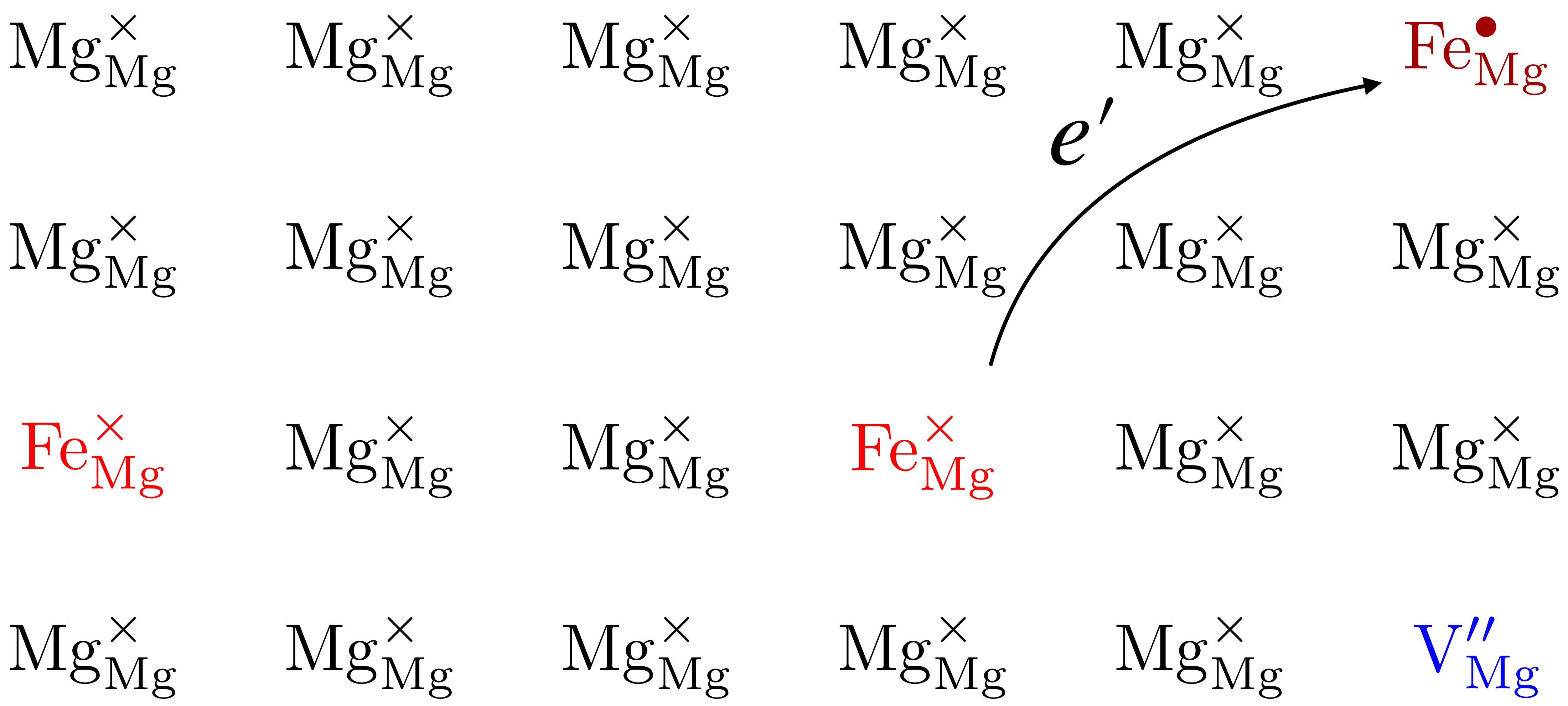
Olivine $Mg_{0.9}Fe_{0.1}SiO_4$ with defects

$$\begin{array}{cccccc}
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Fe}_{\mathrm{Mg}}^{\bullet} \\
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} \\
\mathrm{Fe}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Fe}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} \\
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{V}_{\mathrm{Mg}}''
\end{array}$$

Oxidation of iron:

$$\mathrm{Fe}_{\mathrm{Mg}}^{\mathrm{x}} \rightleftharpoons \mathrm{Fe}_{\mathrm{Mg}}^{\bullet} + \mathrm{e}'$$

Polaron hopping



$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Fe}_{\mathrm{Mg}}^{\bullet}$

e'

$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$

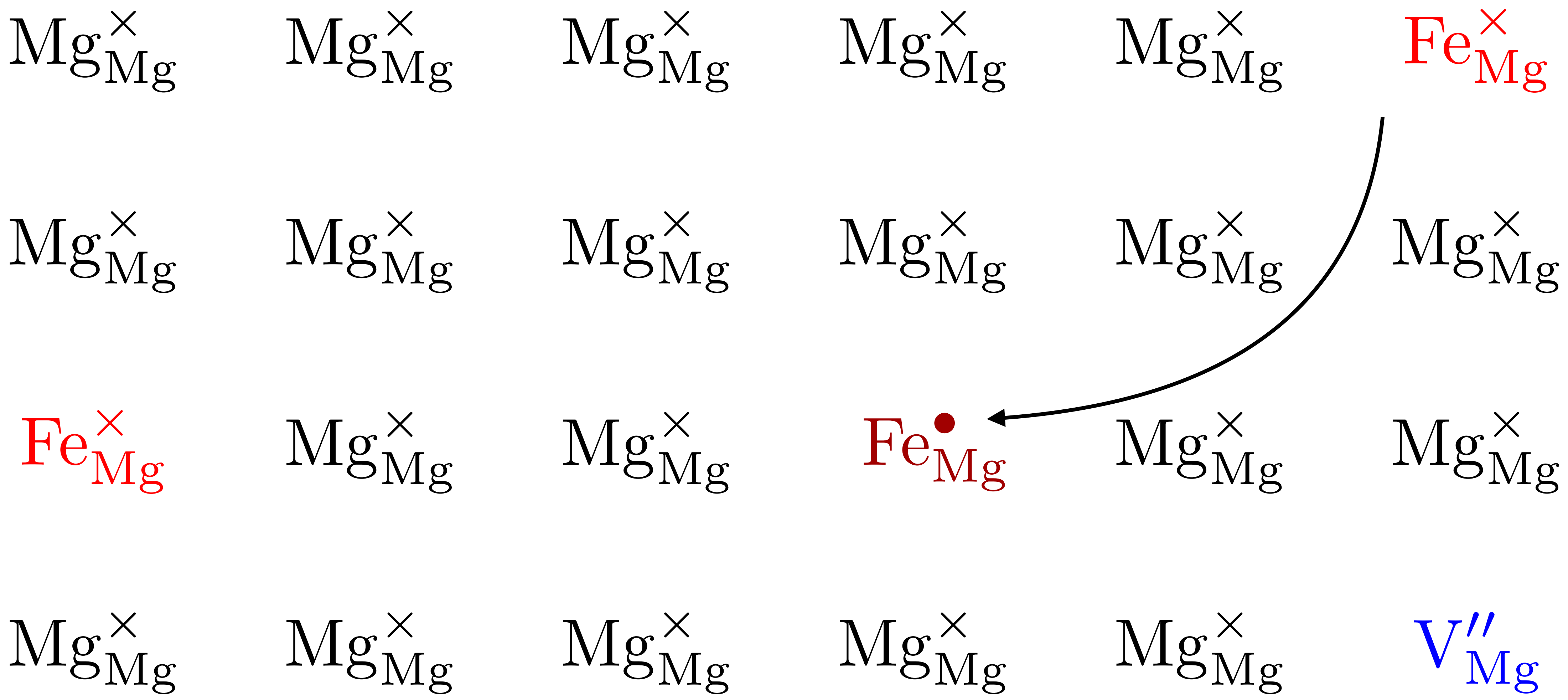
$\mathrm{Fe}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Fe}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$

$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{Mg}_{\mathrm{Mg}}^{\times}$ $\mathrm{V}_{\mathrm{Mg}}''$

Oxidation of iron:

$$\mathrm{Fe}_{\mathrm{Mg}}^{\mathrm{x}} \rightleftharpoons \mathrm{Fe}_{\mathrm{Mg}}^{\bullet} + \mathrm{e}'$$

This effectively is movement of positive charge

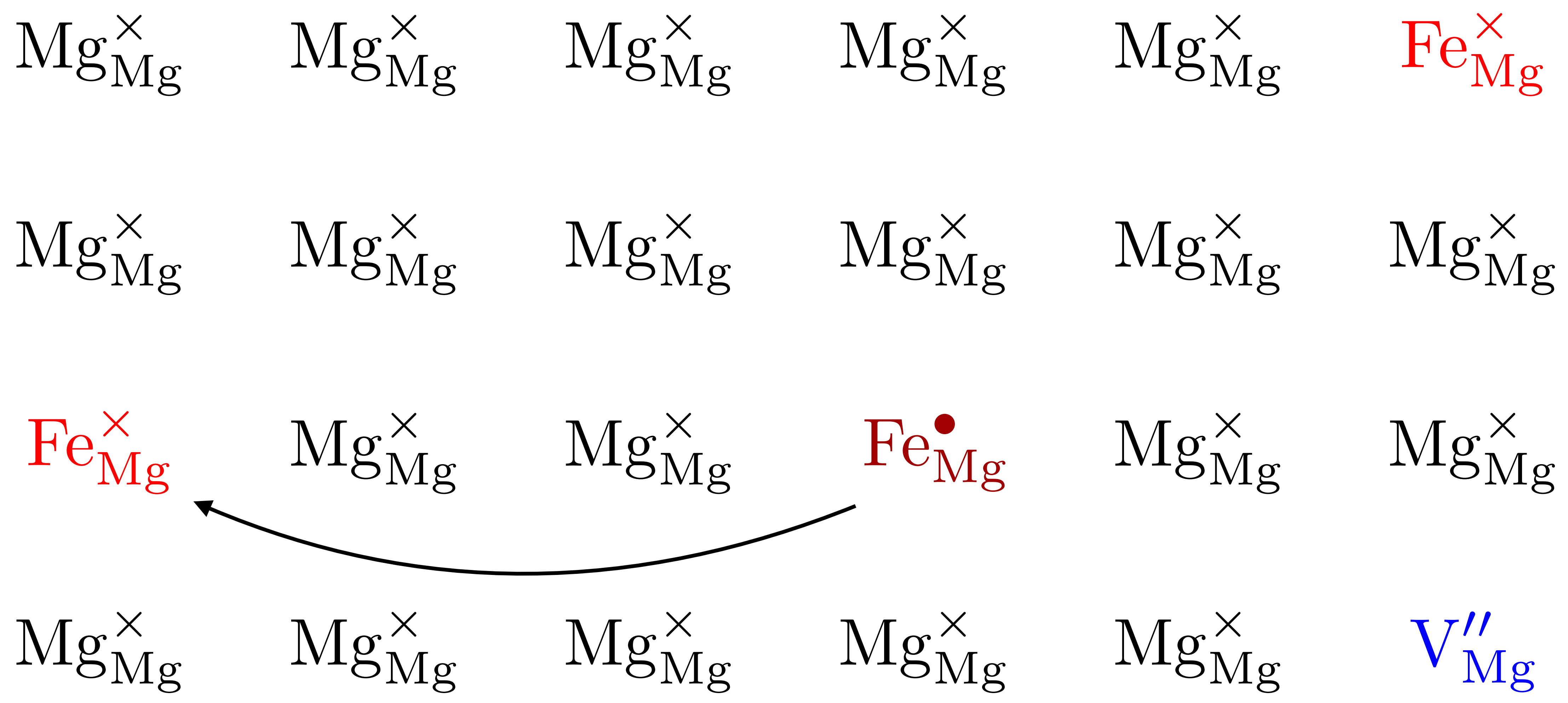


$$\begin{array}{cccccc}
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Fe}_{\mathrm{Mg}}^{\times} \\
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} \\
\mathrm{Fe}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Fe}_{\mathrm{Mg}}^{\bullet} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} \\
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{V}_{\mathrm{Mg}}''
\end{array}$$

Oxidation of iron:

$$\mathrm{Fe}^{\mathrm{x}}_{\mathrm{Mg}} \rightleftharpoons \mathrm{Fe}^{\bullet}_{\mathrm{Mg}} + \mathrm{e}'$$

Polaron hopping



$$\begin{array}{cccccc}
\mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Fe}^{\times}_{\mathrm{Mg}} \\
\mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} \\
\mathrm{Fe}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Fe}^{\bullet}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} \\
\mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{Mg}^{\times}_{\mathrm{Mg}} & \mathrm{V}''_{\mathrm{Mg}}
\end{array}$$

Oxidation of iron:

$$\mathrm{Fe}_{\mathrm{Mg}}^{\mathrm{x}} \rightleftharpoons \mathrm{Fe}_{\mathrm{Mg}}^{\bullet} + \mathrm{e}'$$

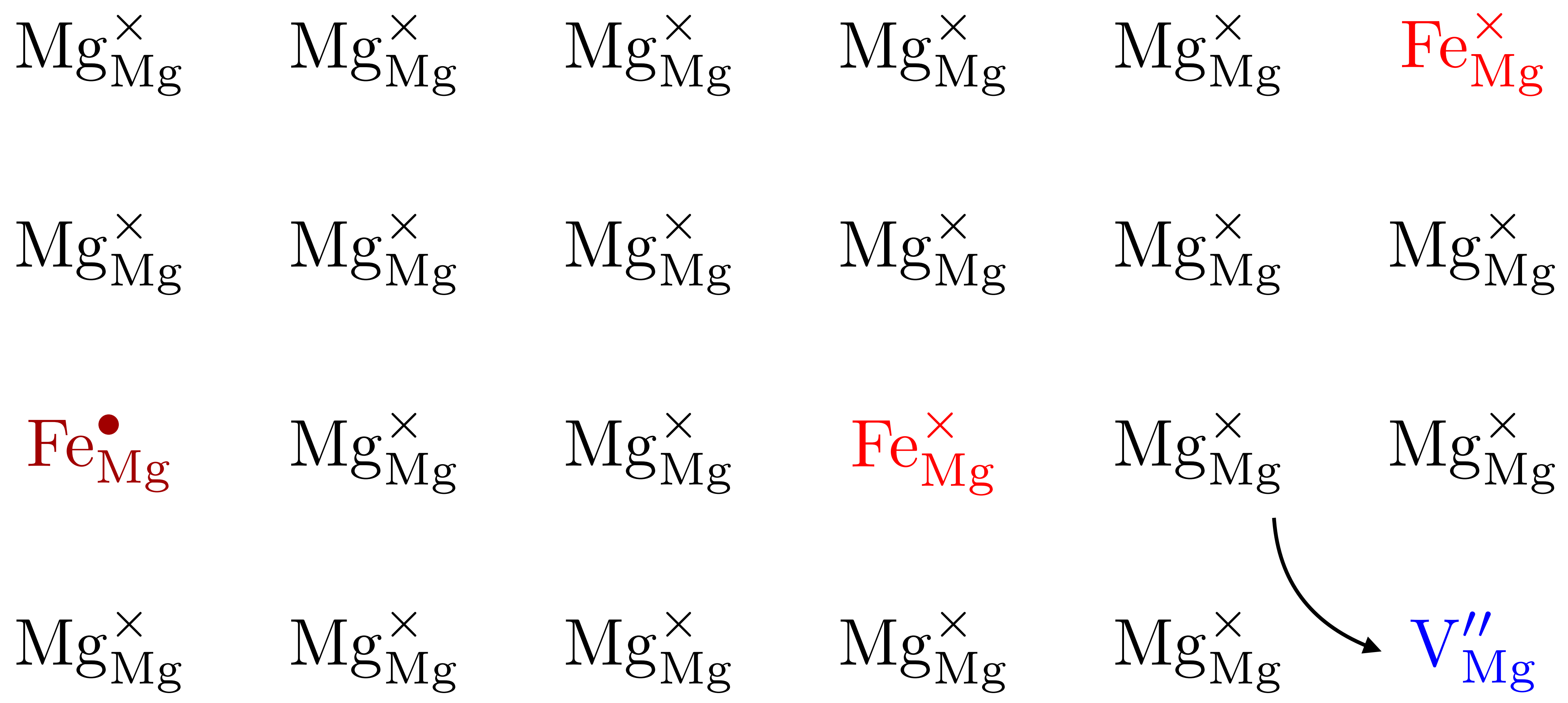
This effectively is movement of positive charge

$$\begin{array}{cccccc}
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Fe}_{\mathrm{Mg}}^{\times} \\
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} \\
\mathrm{Fe}_{\mathrm{Mg}}^{\bullet} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Fe}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} \\
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{V}_{\mathrm{Mg}}''
\end{array}$$

Oxidation of iron:

$$\mathrm{Fe}_{\mathrm{Mg}}^{\mathrm{x}} \rightleftharpoons \mathrm{Fe}_{\mathrm{Mg}}^{\bullet} + \mathrm{e}'$$

Similarly vacancies (Schottky defects) can move

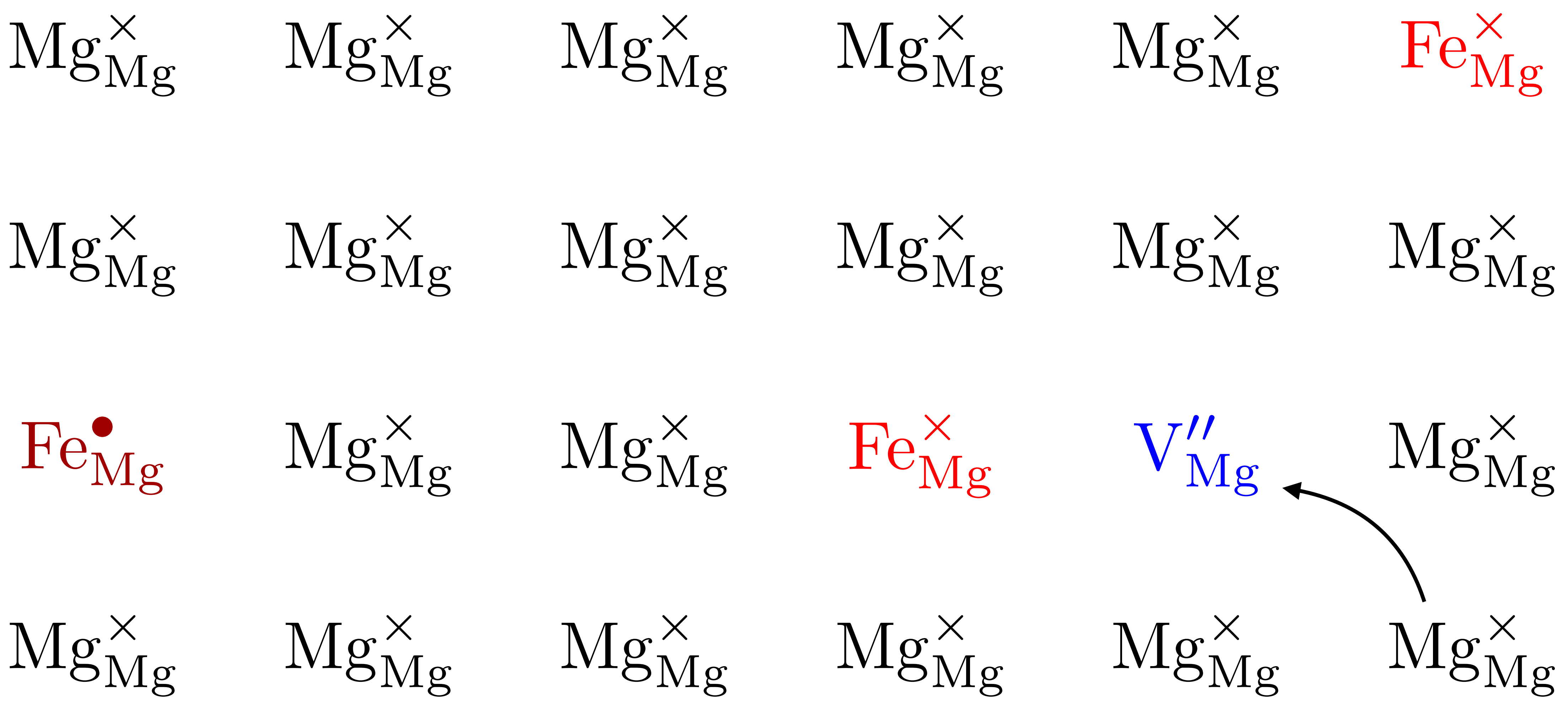


$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Fe}_{\mathrm{Mg}}^{\times}$
$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$
$\mathrm{Fe}_{\mathrm{Mg}}^{\bullet}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Fe}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$
$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{Mg}_{\mathrm{Mg}}^{\times}$	$\mathrm{V}_{\mathrm{Mg}}''$

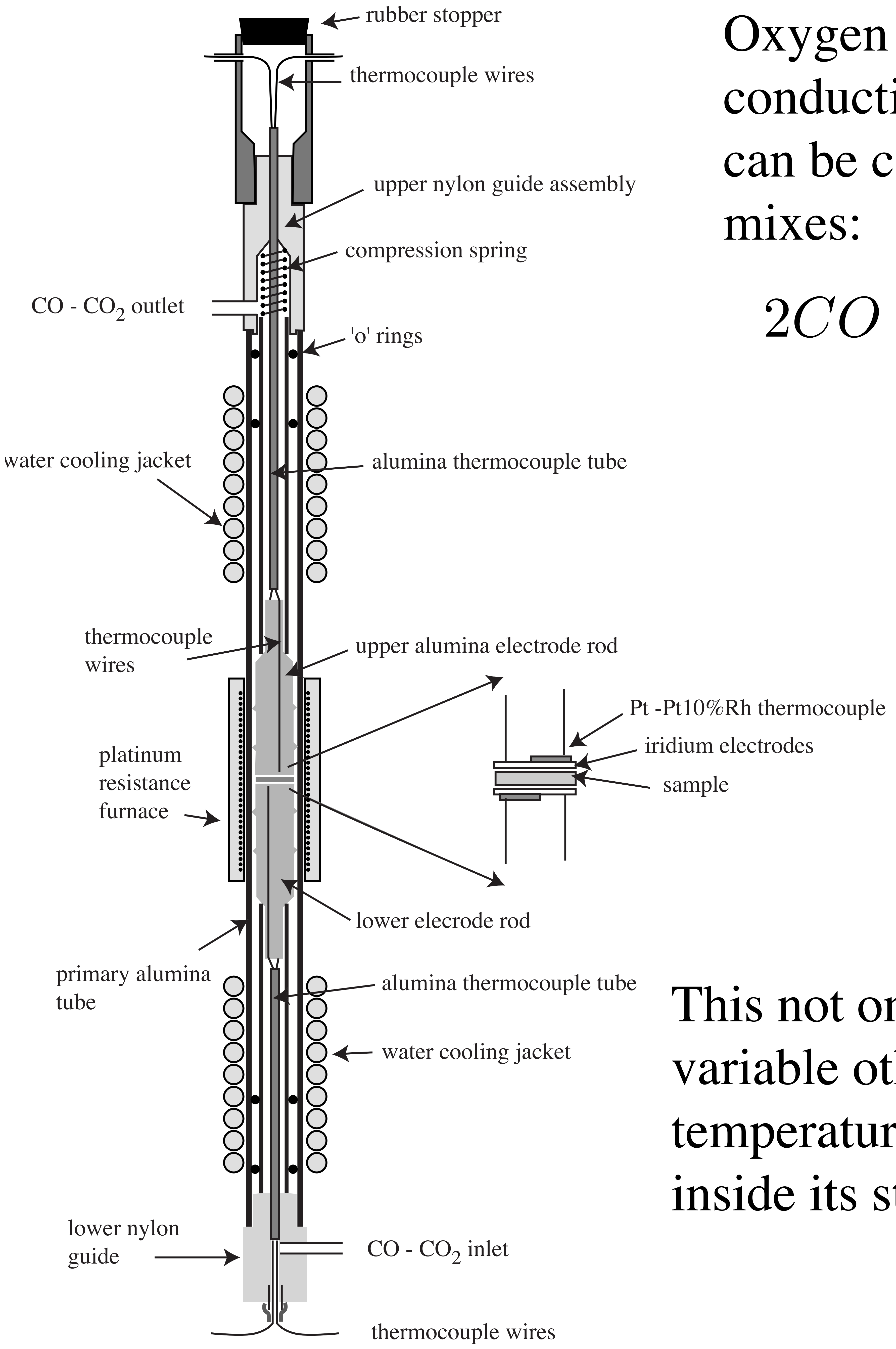
Oxidation of iron:

$$\mathrm{Fe}_{\mathrm{Mg}}^{\mathrm{x}} \rightleftharpoons \mathrm{Fe}_{\mathrm{Mg}}^{\bullet} + \mathrm{e}'$$

Effectively a movement of negative charge



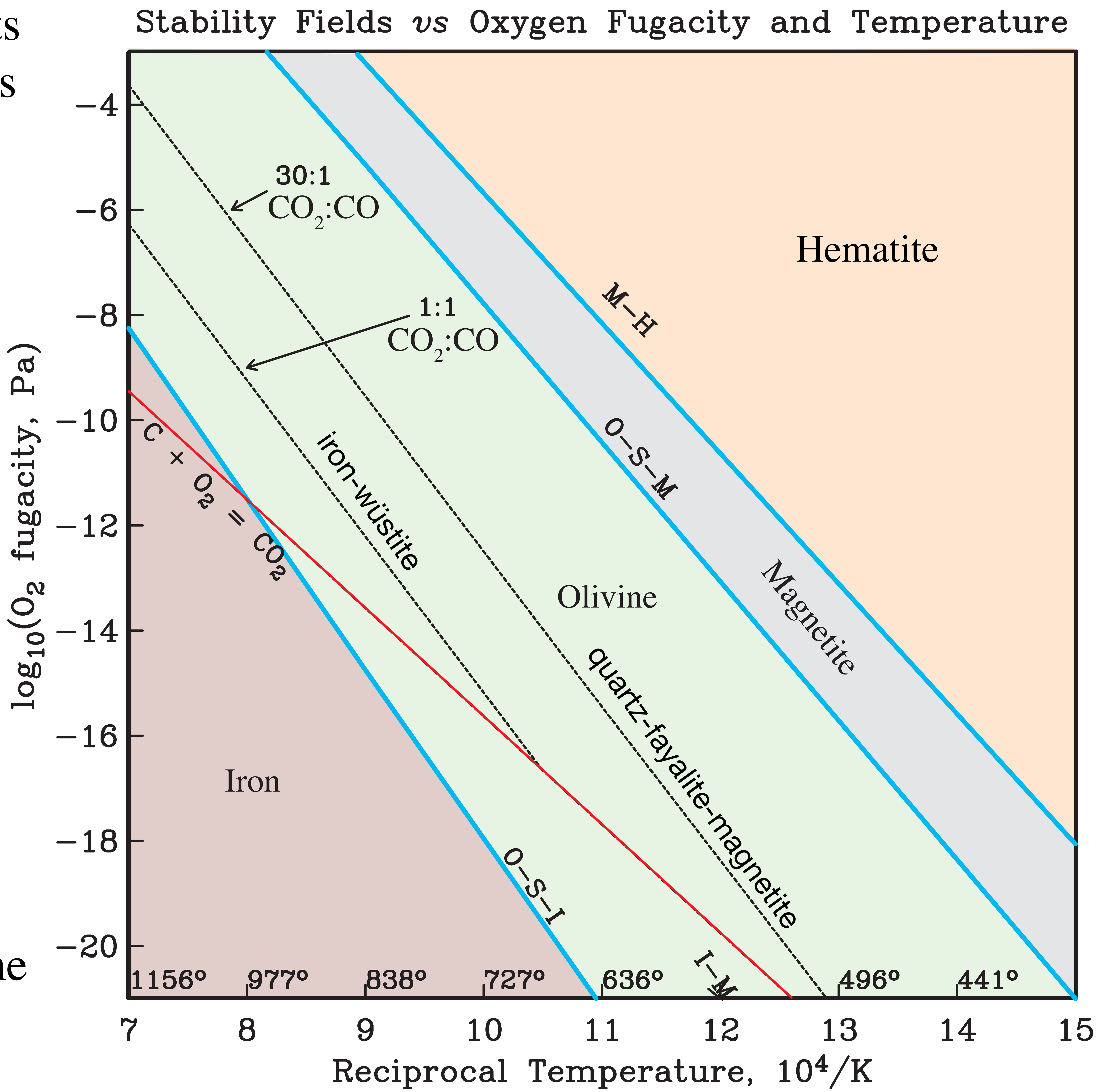
$$\begin{array}{cccccc}
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Fe}_{\mathrm{Mg}}^{\times} \\
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} \\
\mathrm{Fe}_{\mathrm{Mg}}^{\bullet} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Fe}_{\mathrm{Mg}}^{\times} & \mathrm{V}_{\mathrm{Mg}}'' & \mathrm{Mg}_{\mathrm{Mg}}^{\times} \\
\mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times} & \mathrm{Mg}_{\mathrm{Mg}}^{\times}
\end{array}$$



Oxygen activity during conductivity measurements can be controlled using gas mixes:

$$2CO + O_2 \rightleftharpoons 2CO_2$$

This not only provides a variable other than temperature, but keeps olivine inside its stability field.



Fe^o

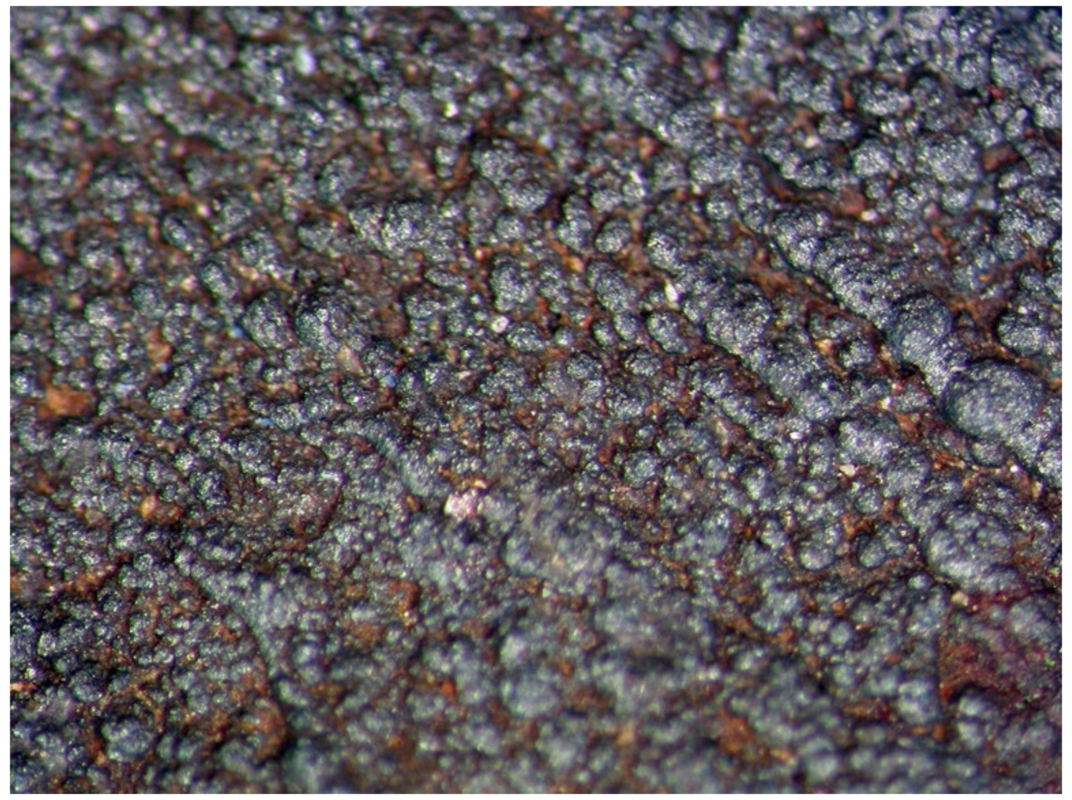
$\mathbf{Fe}$

metallic iron

Fe^{++}

$\mathbf{FeO}$

wüstite



$Fe^{++}Fe^{+++}$

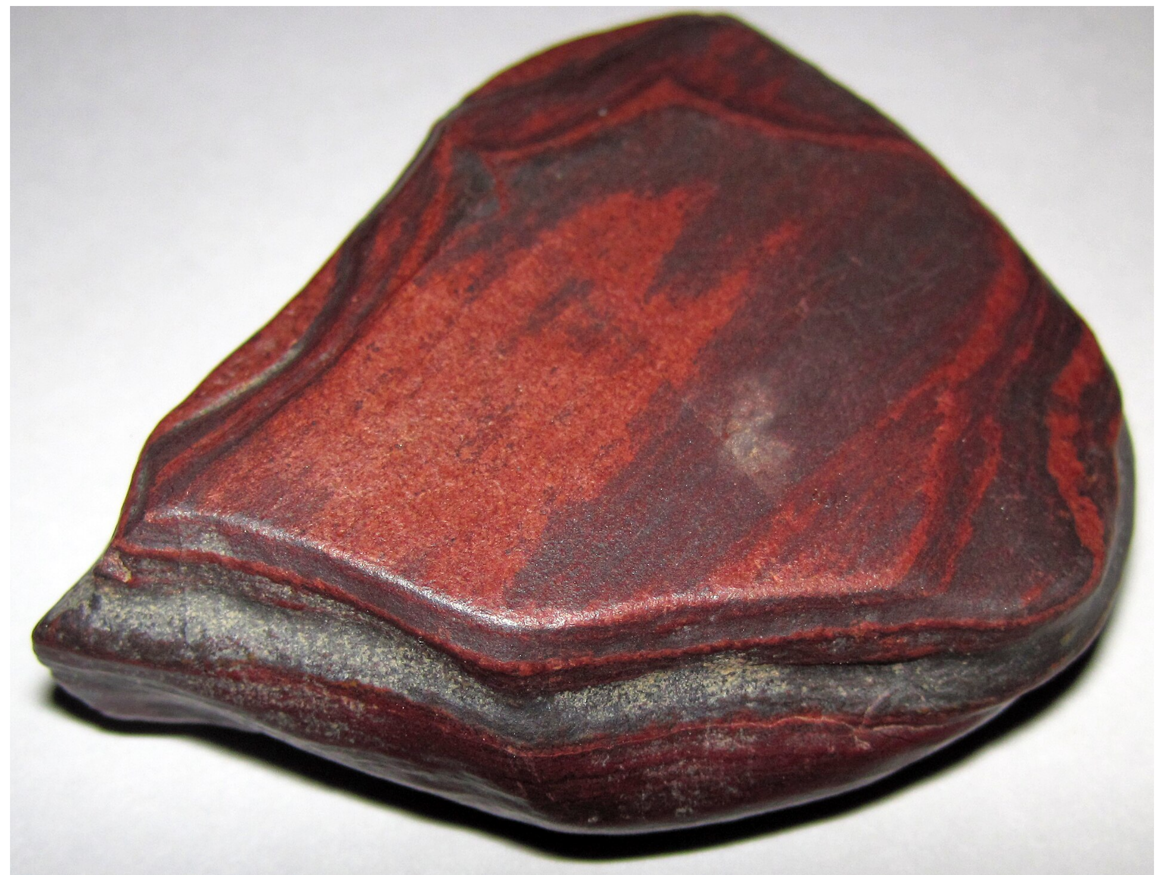
$\mathbf{FeO{\bullet}Fe_2O_3}$

magnetite

Fe^{+++}

$\mathbf{Fe_2O_3}$

hematite

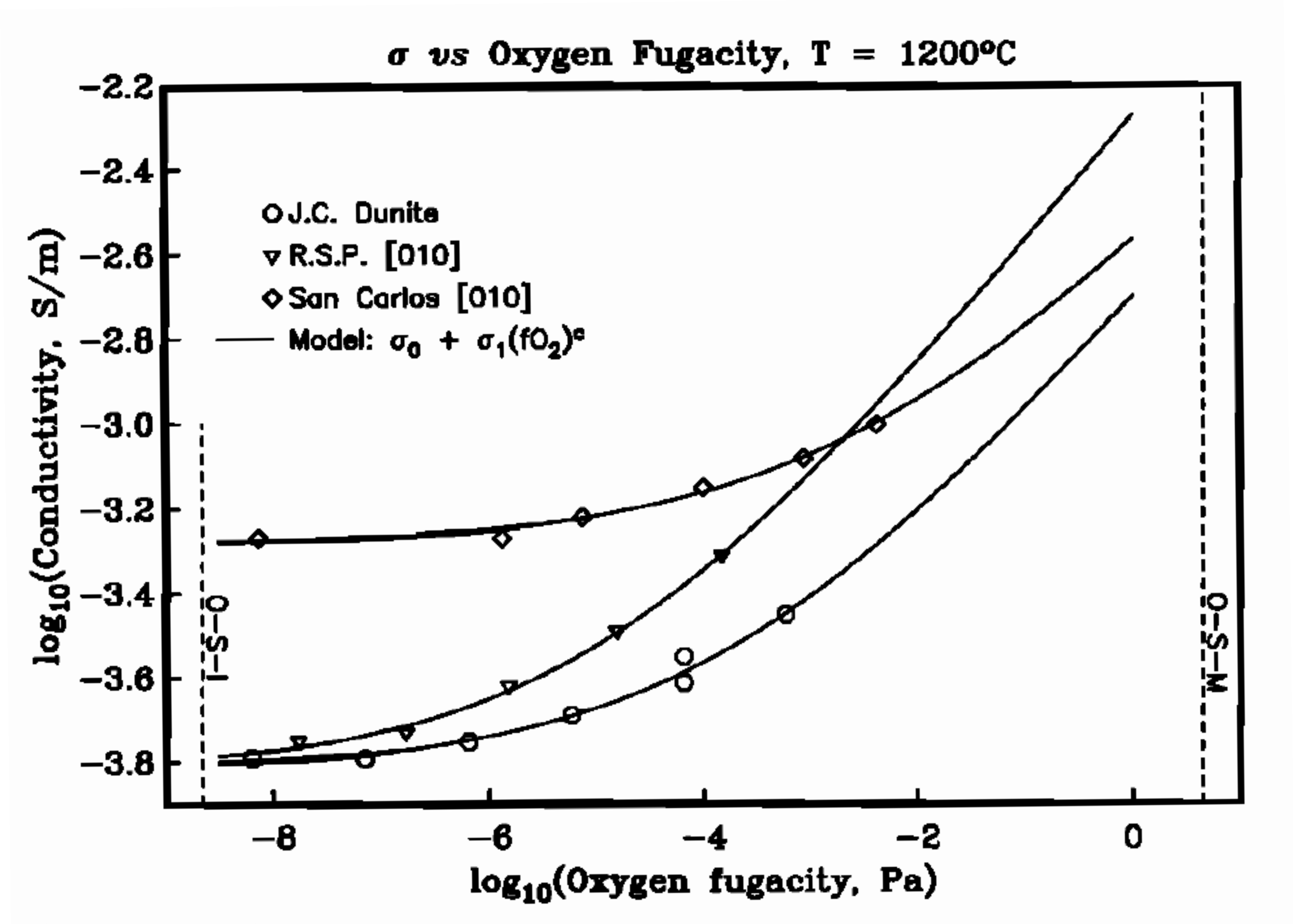


Images: Wikipedia

The oxidation of olivine ($Mg_{0.9}Fe_{0.1}SiO_4$)

$$8Fe_{Mg}^{\times} + 2O_2 \rightleftharpoons 2V_{Mg}^{''} + V_{Si}^{''''} + 4O_O^{\times} + 8Fe_{Mg}^{\bullet}$$

predicts that defect concentrations have a 1/6 power in oxygen fugacity $$[\mathrm{X}] = \mathrm{a_x f_{O_2}^{1/6}}$$



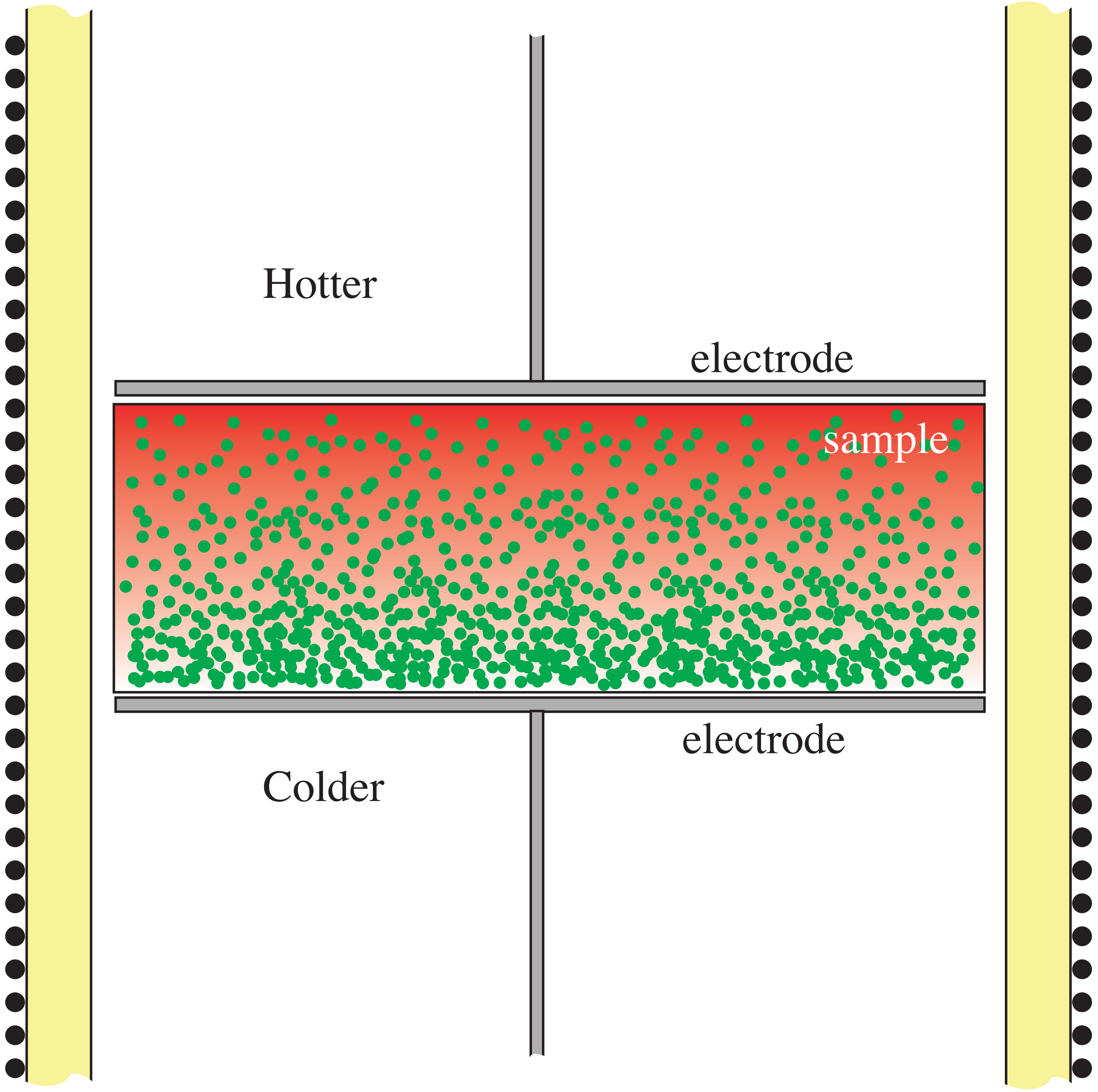
However, observed conductivity data require an additional term related to impurity defects and similar to doped conductivity

$$[\mathrm{X}] = \mathrm{b_x} + \mathrm{a_x f_{O_2}^{1/6}}$$

(this could result from charge imbalances associated with impurity defects such as Al, Cr, Ti).

Constable and Duba (1990)

Thermo-electric power can also be used to estimate defect concentrations.



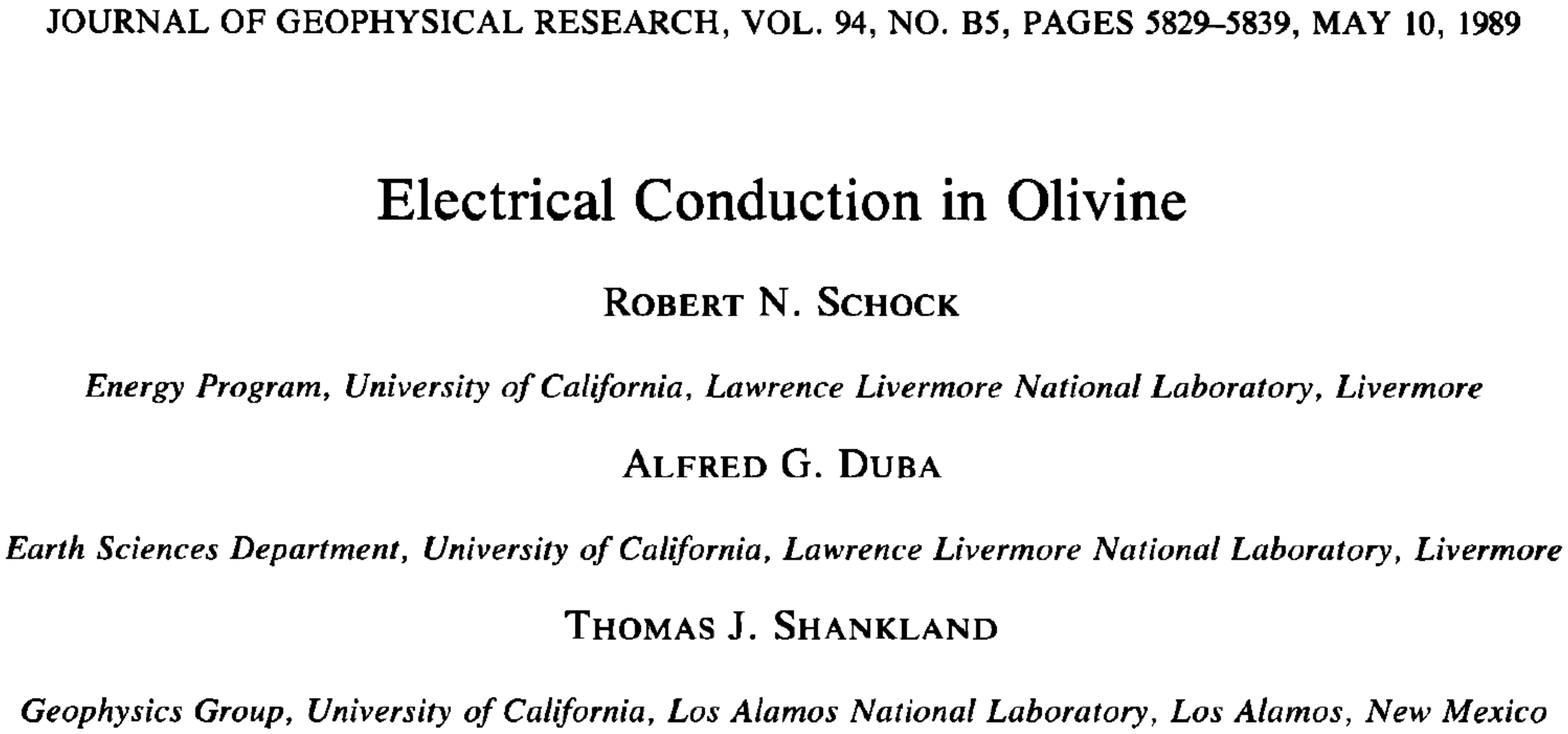
A temperature gradient ΔT across a mineral sample produces a potential difference ΔV as the charge carriers "condense" on the cold side. Thermopower, or Seebeck coefficient is defined as

$$Q = -\lim_{\Delta T \to 0} \frac{\Delta V}{\Delta T}$$

The sign of the thermopower is the same as the sign of the charge carriers, and depends on relative concentration, c:

$$Q = \frac{k}{q}\left[\ln \beta \frac{(1-c)}{c} + \frac{S}{k}\right]$$

A change in sign in the Seekbeck coefficient at 1400° C was documented in Schock, Duba, and Shankland's seminal 1989 paper.



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Electrical Conduction in Olivine

ROBERT N. SCHOCK

Energy Program, University of California, Lawrence Livermore National Laboratory, Livermore

ALFRED G. DUBA

Earth Sciences Department, University of California, Lawrence Livermore National Laboratory, Livermore

THOMAS J. SHANKLAND

Geophysics Group, University of California, Los Alamos National Laboratory, Los Alamos, New Mexico

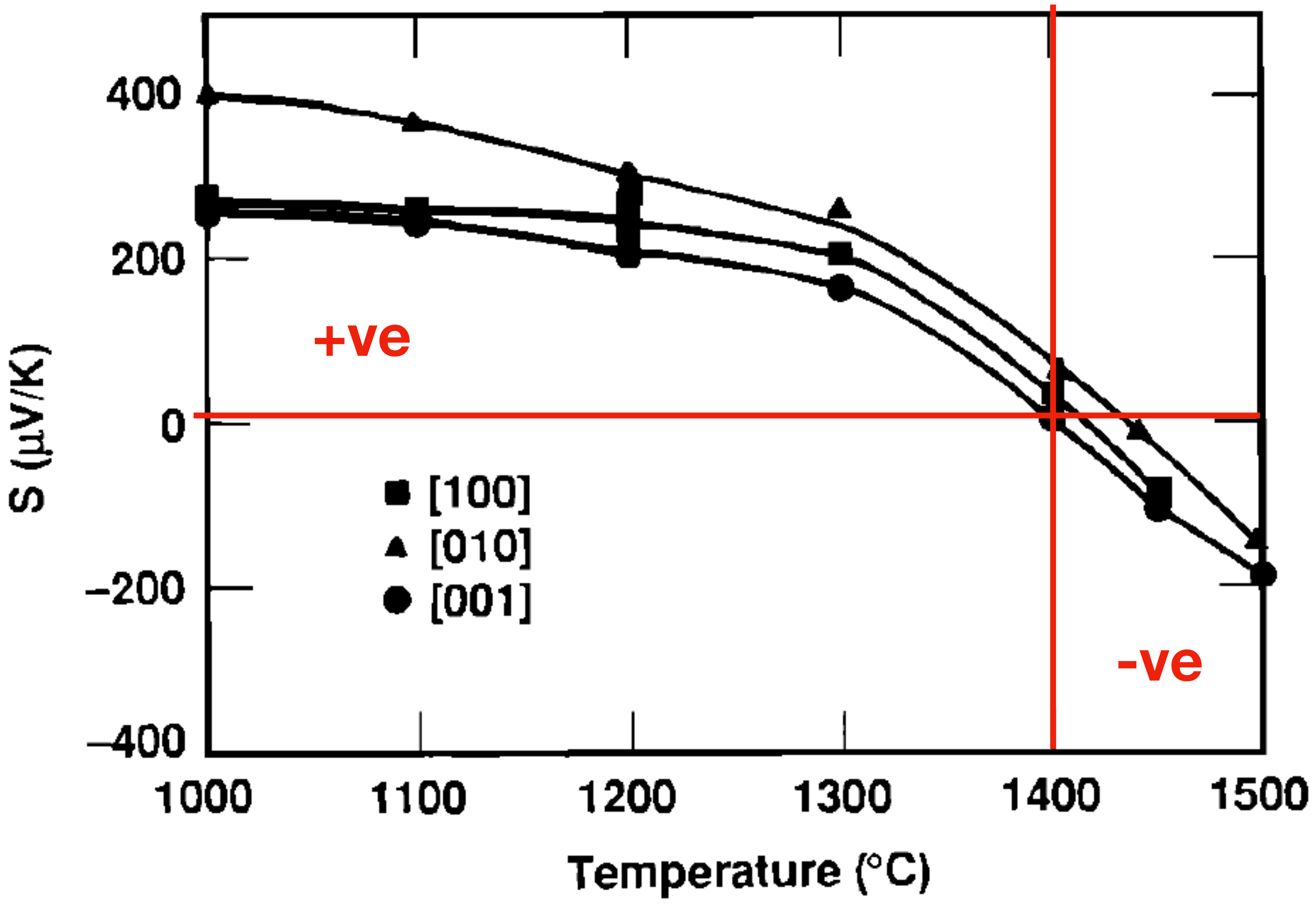
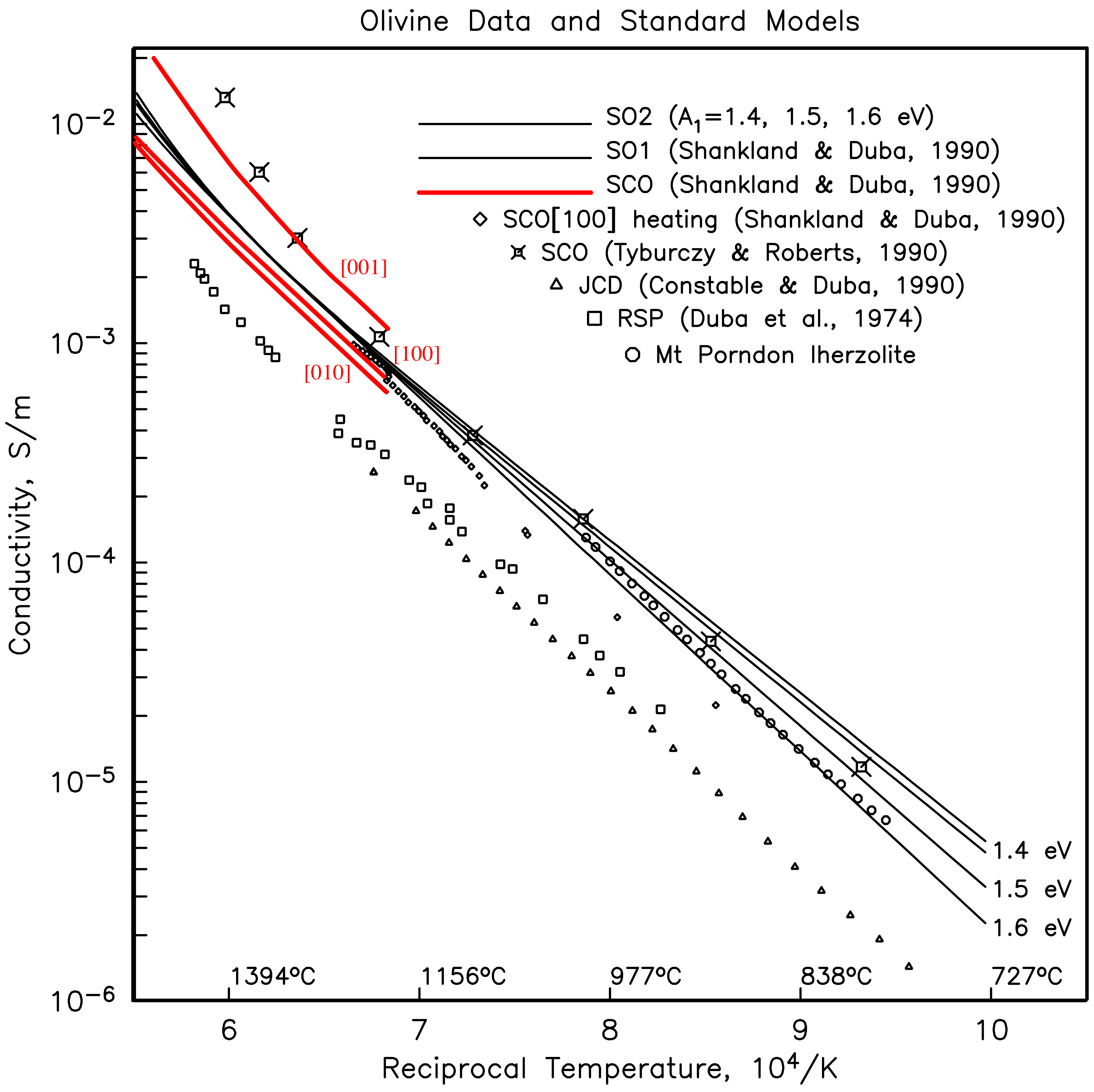


Fig. 9. Thermoelectric coefficient S of San Carlos olivine as a function of temperature along the three principal crystallographic axes. $P_{O_2} = 10^{-4}$ Pa (1200°C).

They interpreted this a change in conduction from electron holes at low temperature to magnesium vacancies at high temperature.

Standard olivine models:



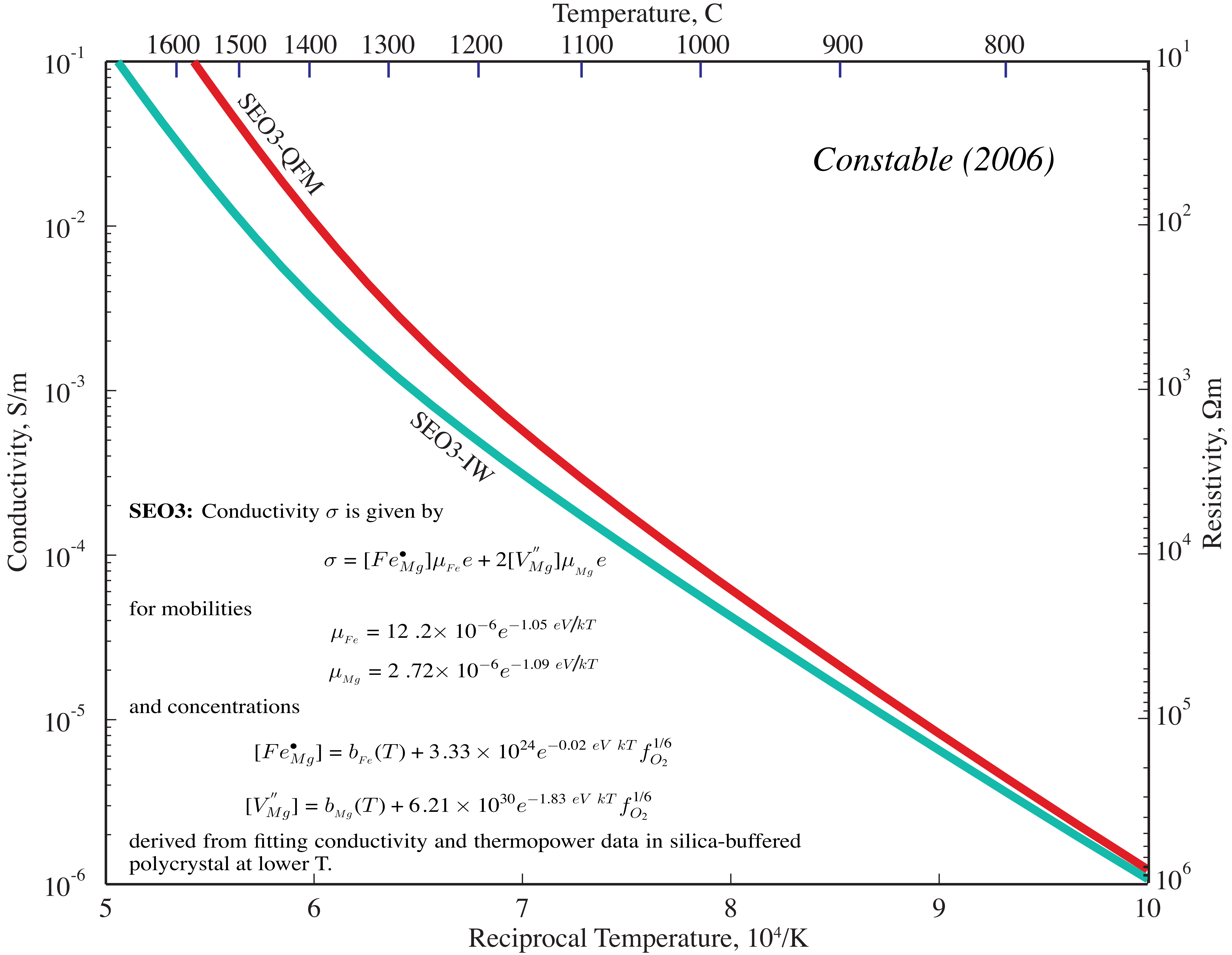
SO1: *Shankland and Duba (1990)*

$$\sigma = 10^{1.67} e^{-1.38\mathrm{eV}/kT} + 10^{8.7} e^{-3.90\mathrm{eV}/kT}\ \mathrm{S/m}$$

SO2: *Constable, Shankland and Duba (1992)*

$$\sigma = 10^{2.40} e^{-1.6\mathrm{eV}/kT} + 10^{9.17} e^{-4.25\mathrm{eV}/kT}\ \mathrm{S/m}$$

SEO3 (Standard Electrical Olivine #3):



SEO3: Conductivity σ is given by

$$\sigma = [Fe^{\bullet}_{Mg}]\mu_{Fe}e + 2[V''_{Mg}]\mu_{Mg}e$$

for mobilities

$$\mu_{Fe} = 12.2 \times 10^{-6} e^{-1.05\ eV/kT}$$

$$\mu_{Mg} = 2.72 \times 10^{-6} e^{-1.09\ eV/kT}$$

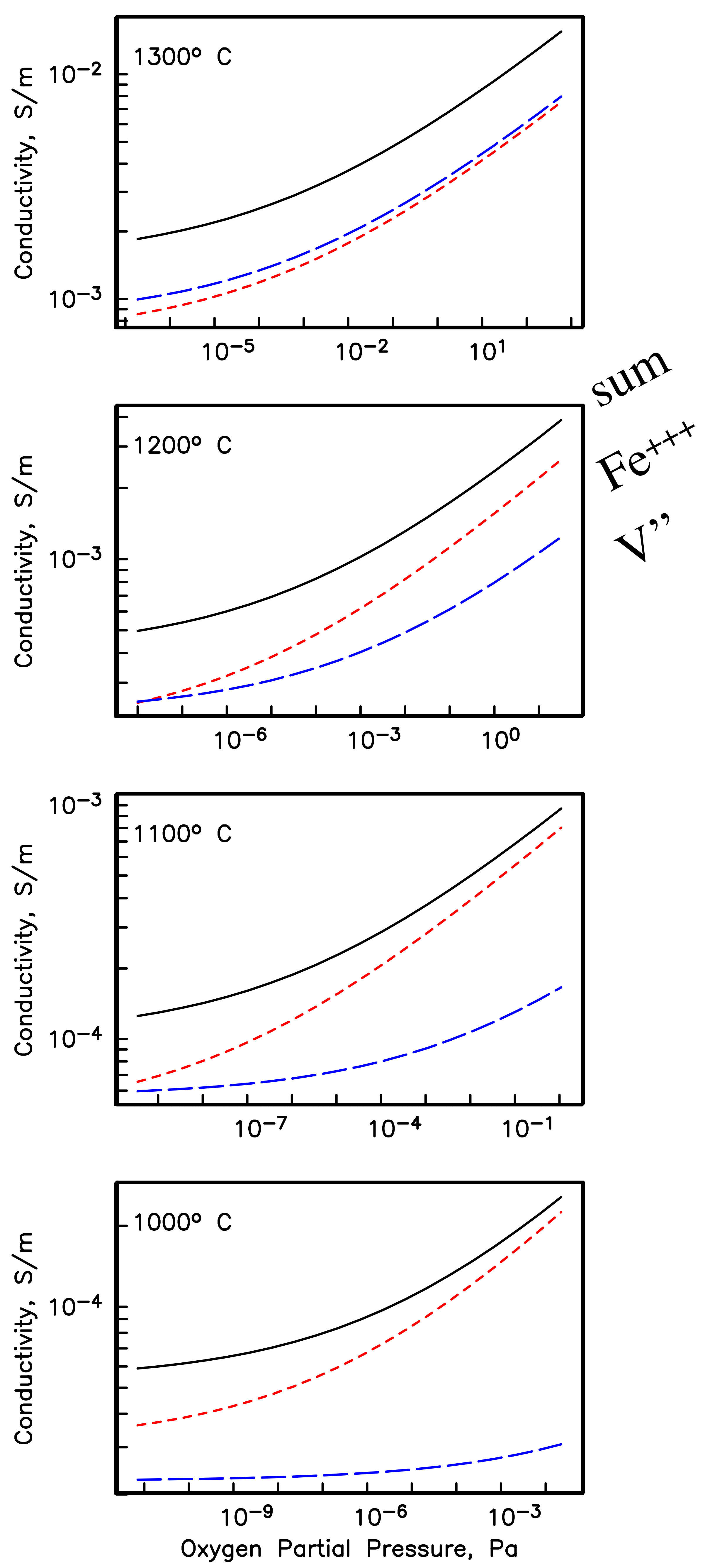
and concentrations

$$[Fe^{\bullet}_{Mg}] = b_{Fe}(T) + 3.33 \times 10^{24} e^{-0.02\ eV\ kT} f_{O_2}^{1/6}$$

$$[V''_{Mg}] = b_{Mg}(T) + 6.21 \times 10^{30} e^{-1.83\ eV\ kT} f_{O_2}^{1/6}$$

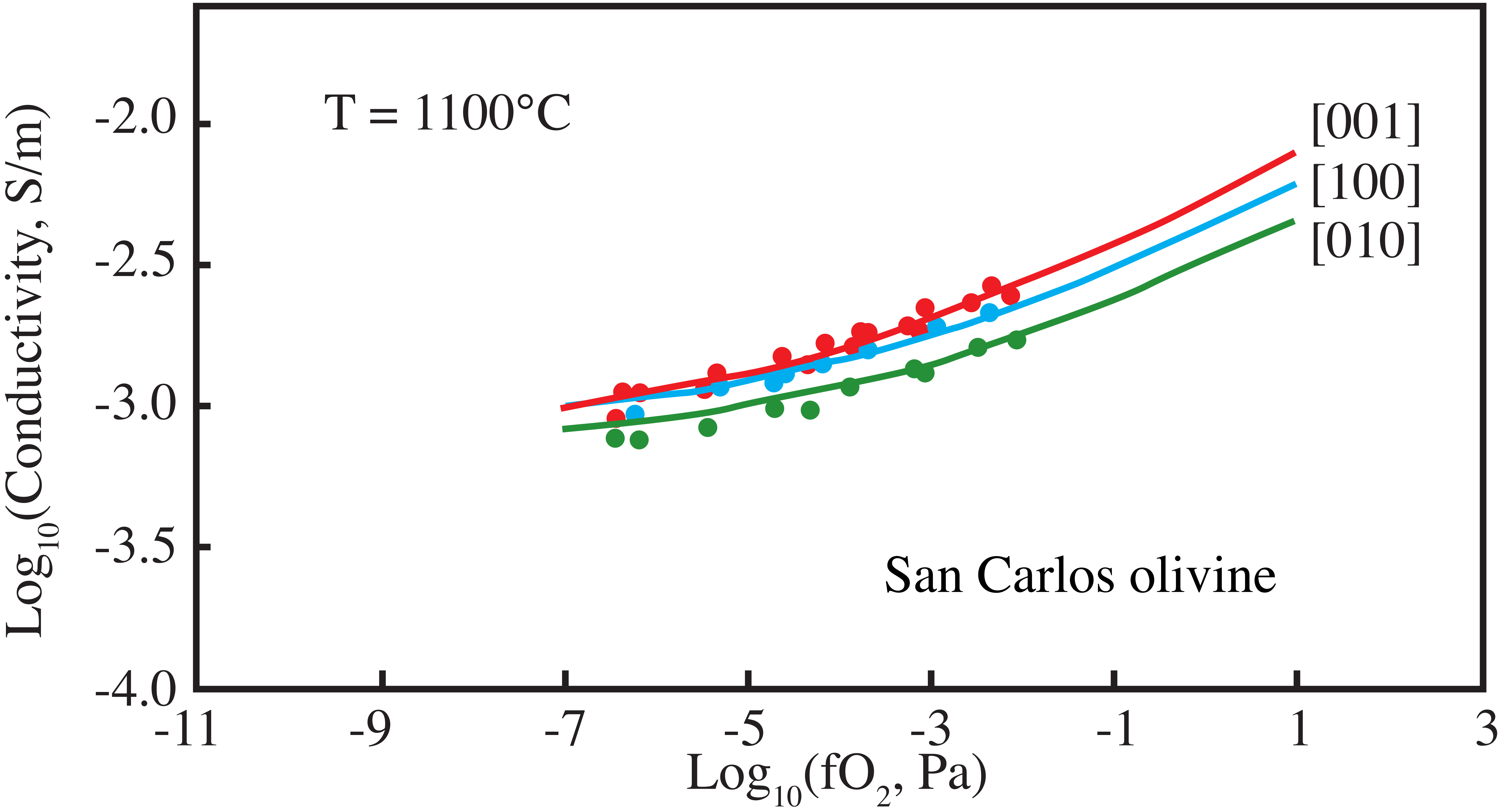
derived from fitting conductivity and thermopower data in silica-buffered polycrystal at lower T.

Constable and Roberts (1997)



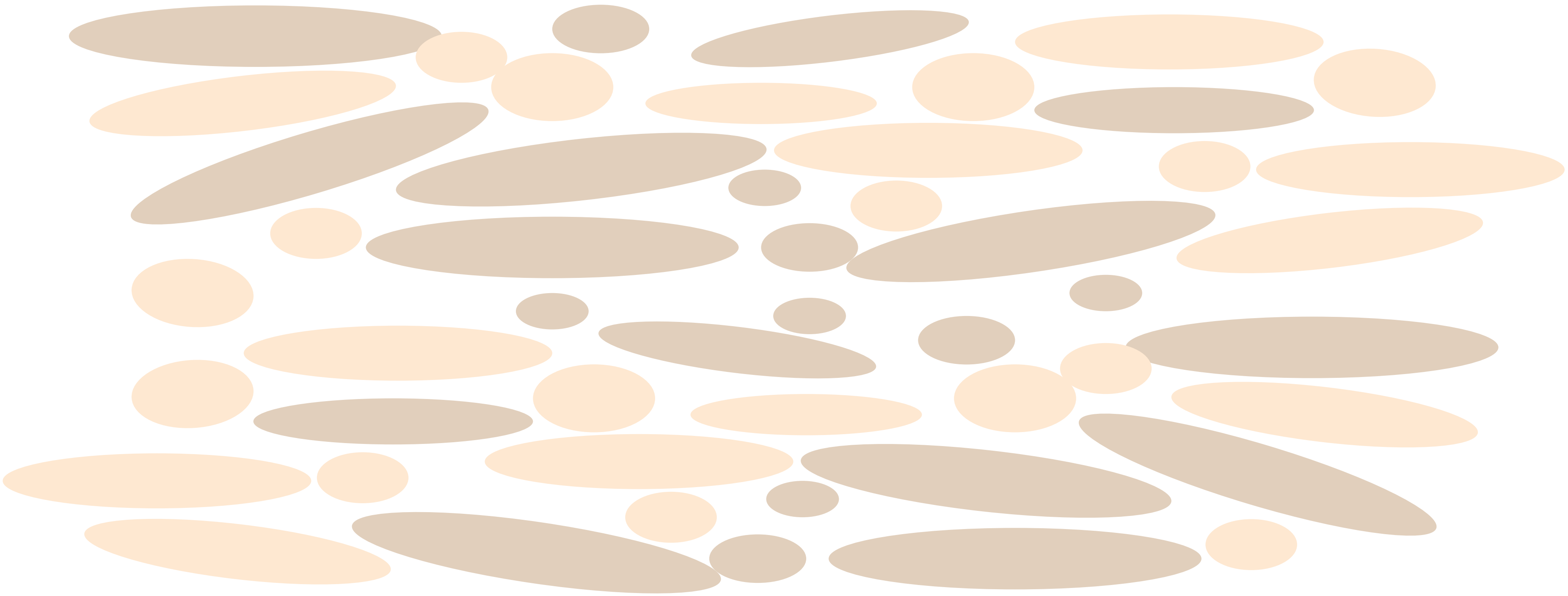
Anisotropy results when conductivity varies with direction. It can occur at the crystal level.

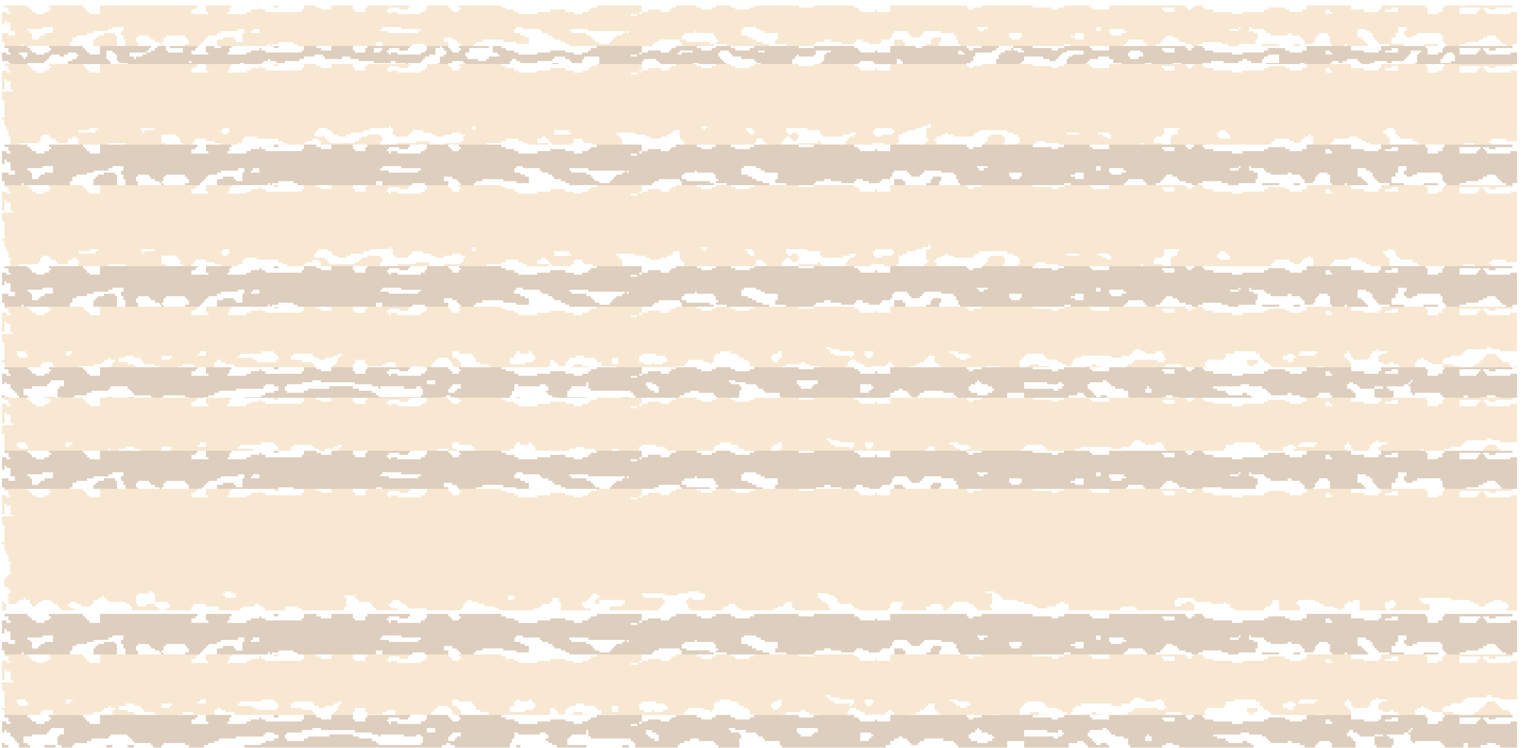

olivine ($Mg_{0.9}Fe_{0.1}SiO_4$)

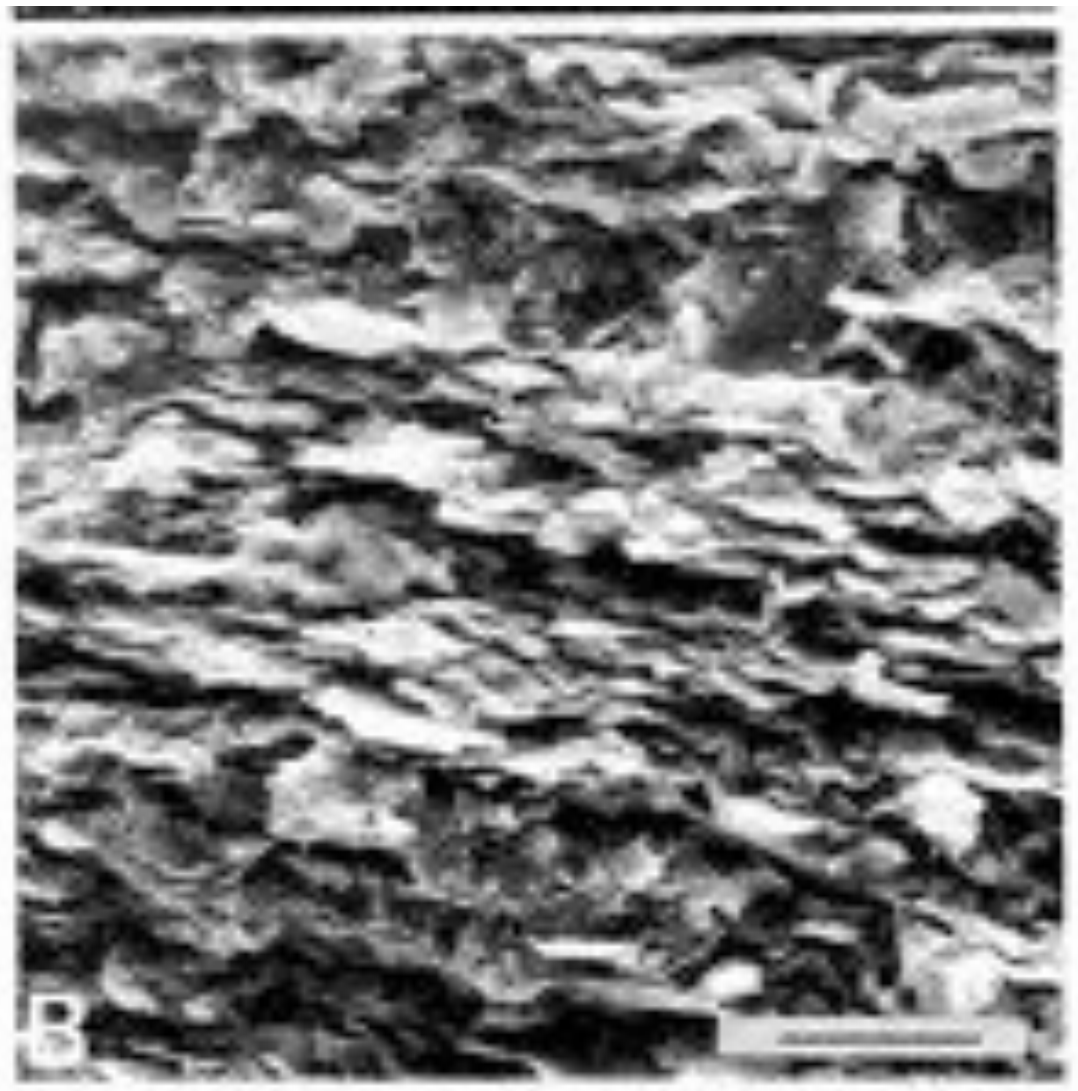


Anisotropy: Aligned crystals and interbedding can produce anisotropy on microscopic to macroscopic scales. Because of gravity, in undeformed sedimentary rock the orientation is sub-horizontal, resulting in higher resistivities in the vertical direction than the horizontal direction.

EM cannot tell micro from macro anisotropy until the layers get very thick.



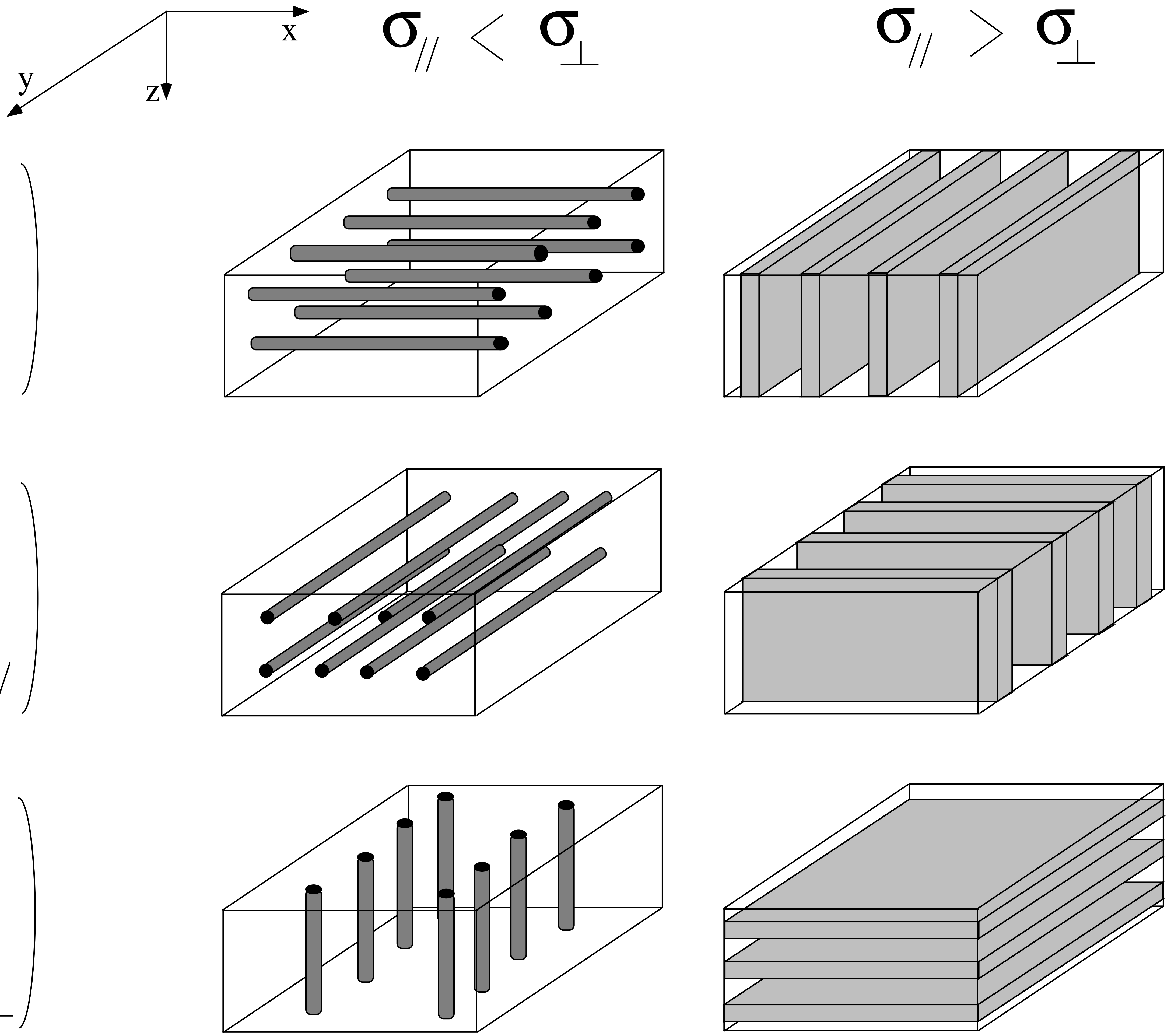




NR O'Brien, J. Sed. Res. 1987

http://www.sleepingdogstudios.com/

In general all three axes can be different (and not even orthogonal), but many geological systems are *transversely isotropic:*



$$\begin{pmatrix} \sigma_{\perp} & & \\ & \sigma_{//} & \\ & & \sigma_{//} \end{pmatrix}$$

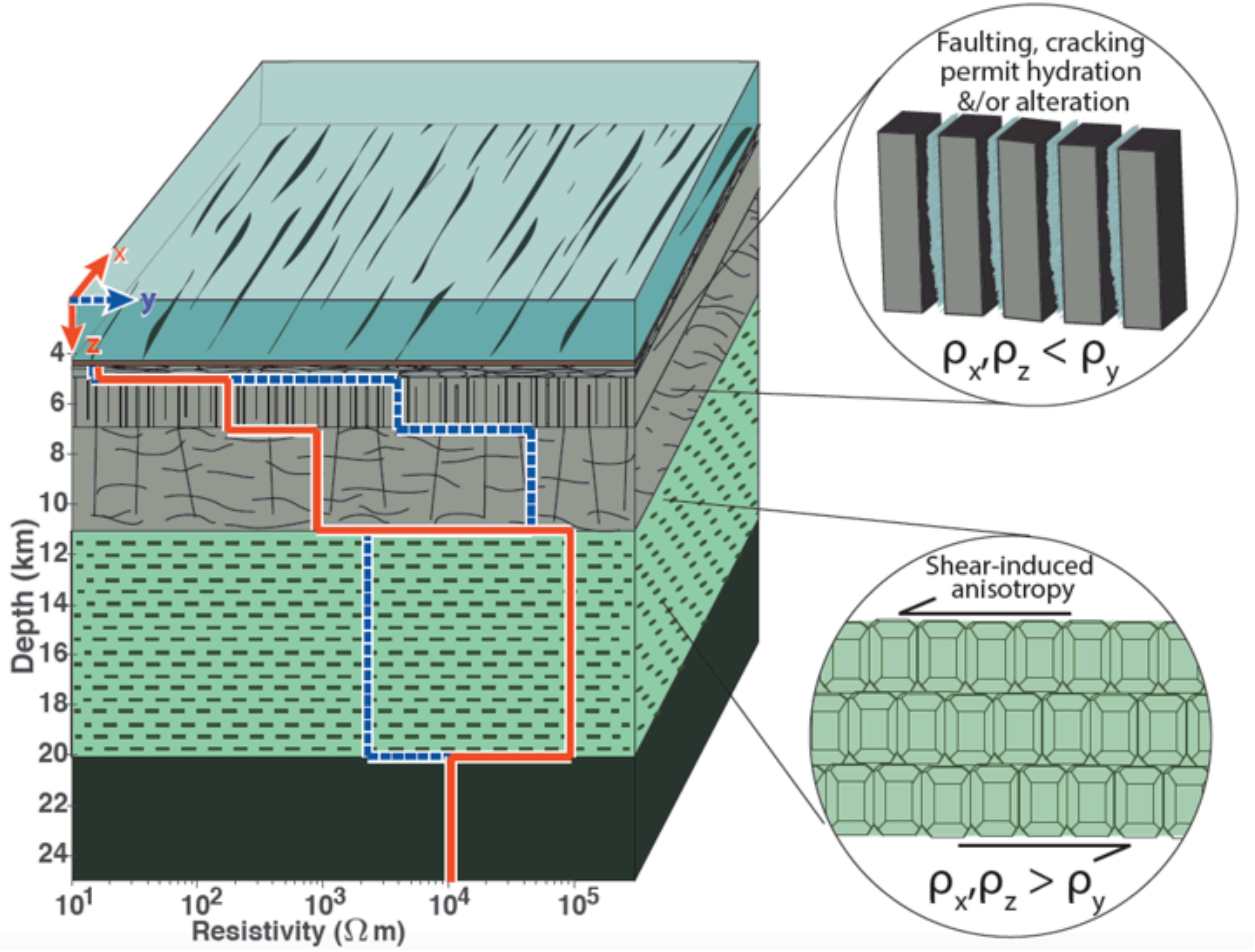
$$\begin{pmatrix} \sigma_{//} & & \\ & \sigma_{\perp} & \\ & & \sigma_{//} \end{pmatrix}$$

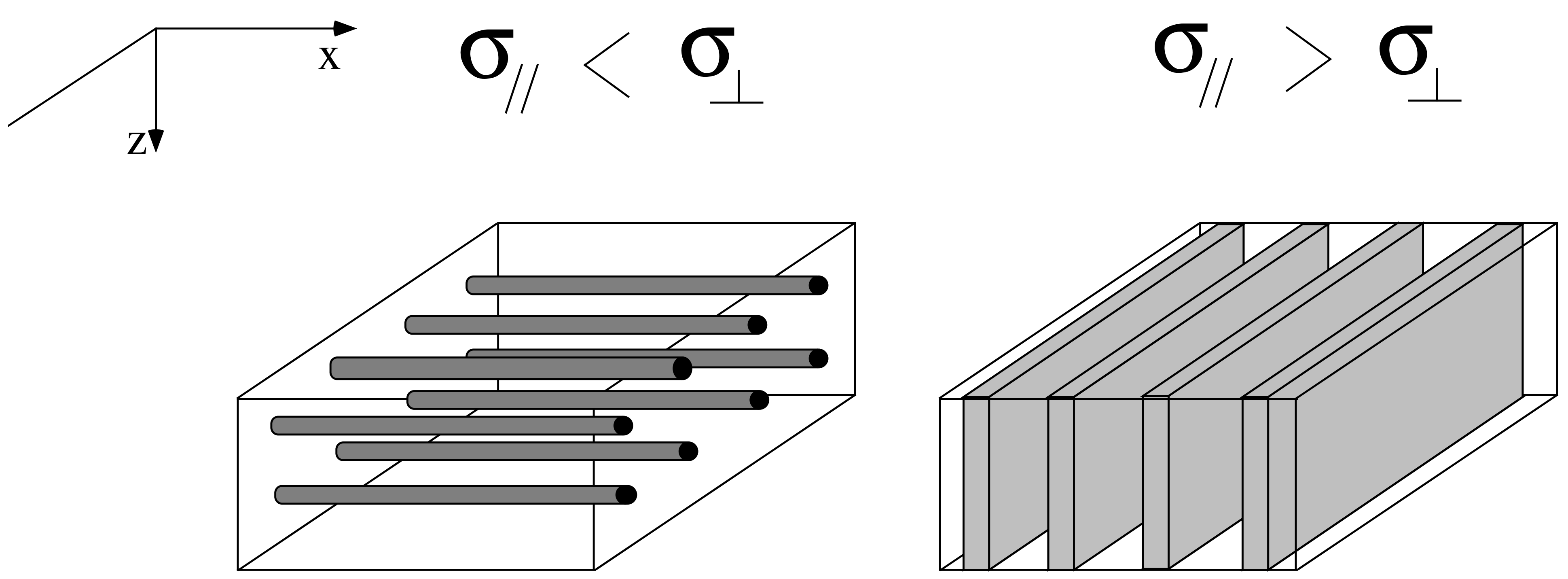
$$\begin{pmatrix} \sigma_{//} & & \\ & \sigma_{//} & \\ & & \sigma_{\perp} \end{pmatrix}$$

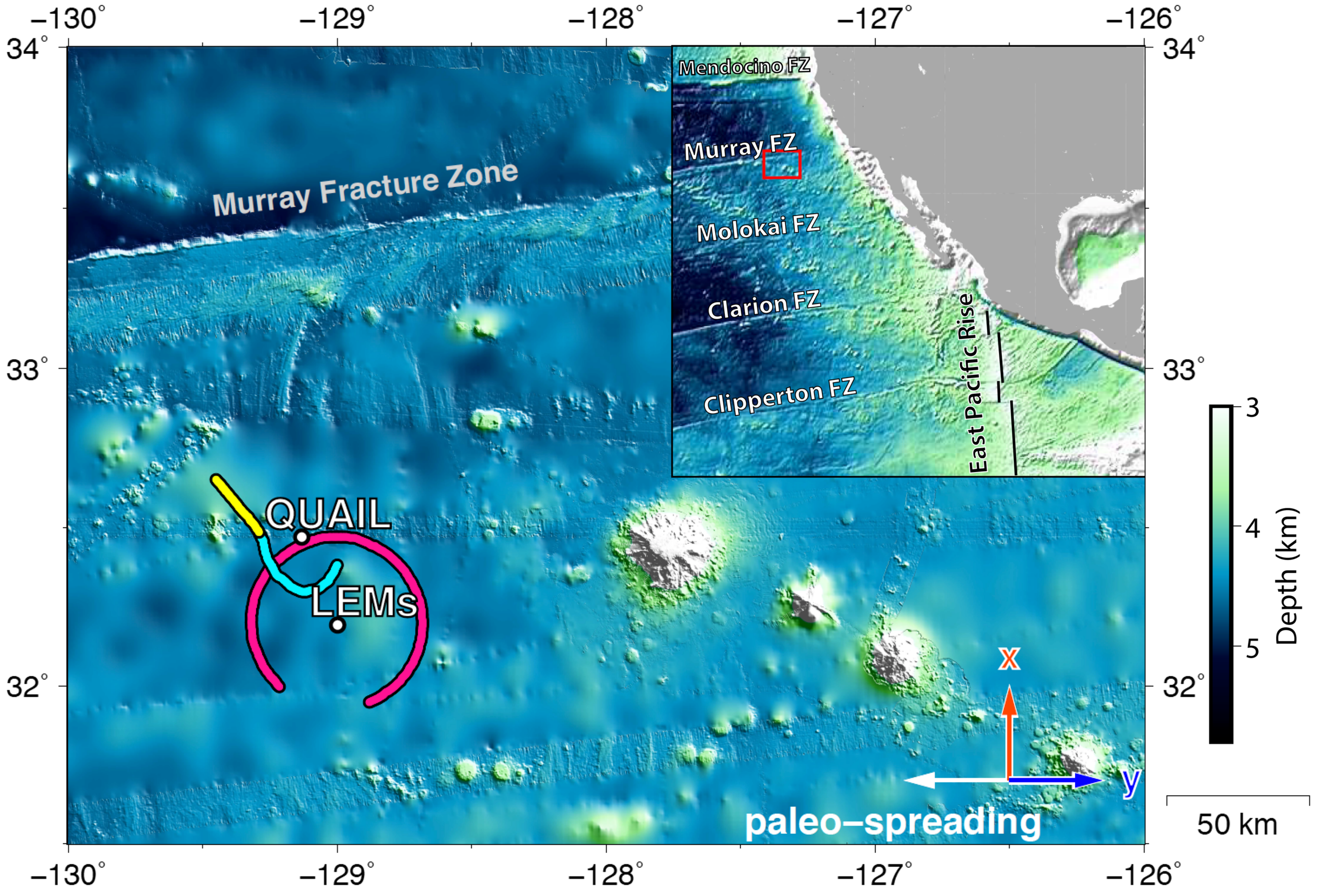
Everett and Constable (1999)

Conductors = grey.

Some recent work:

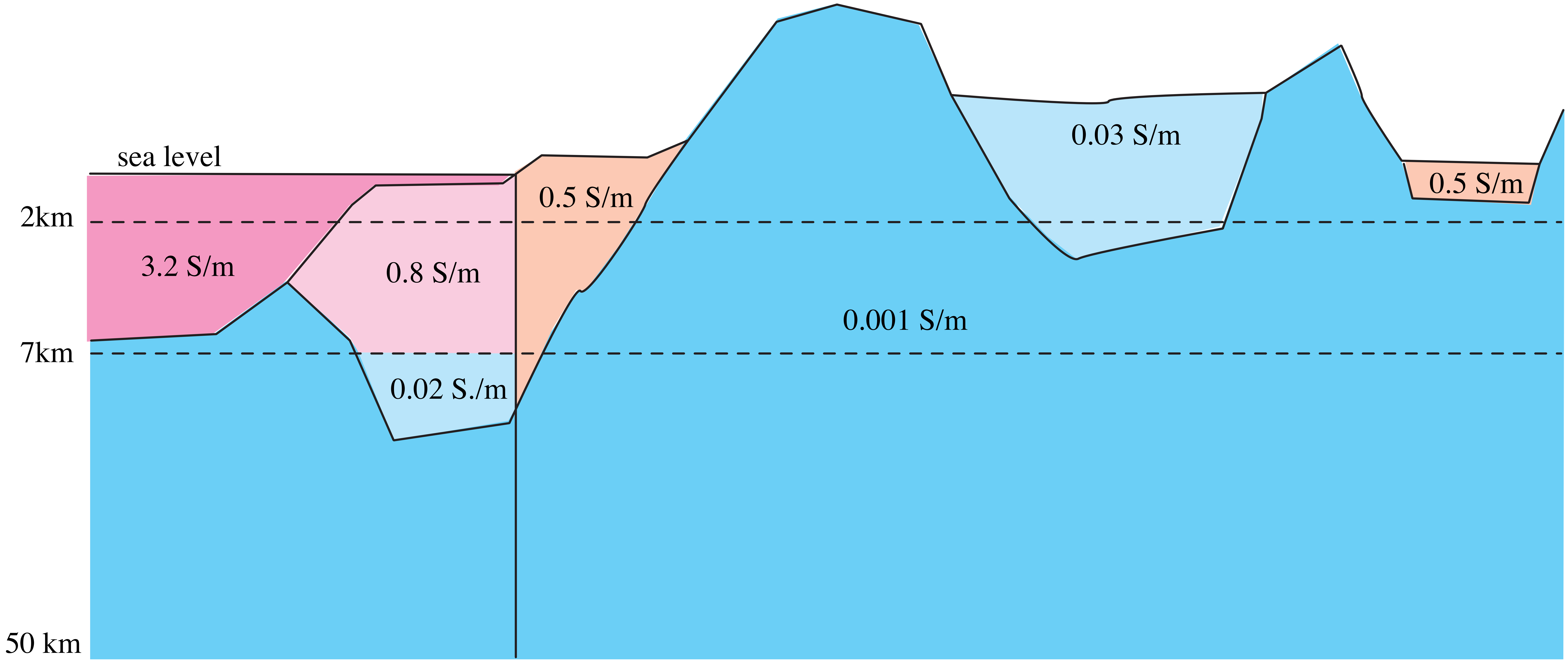




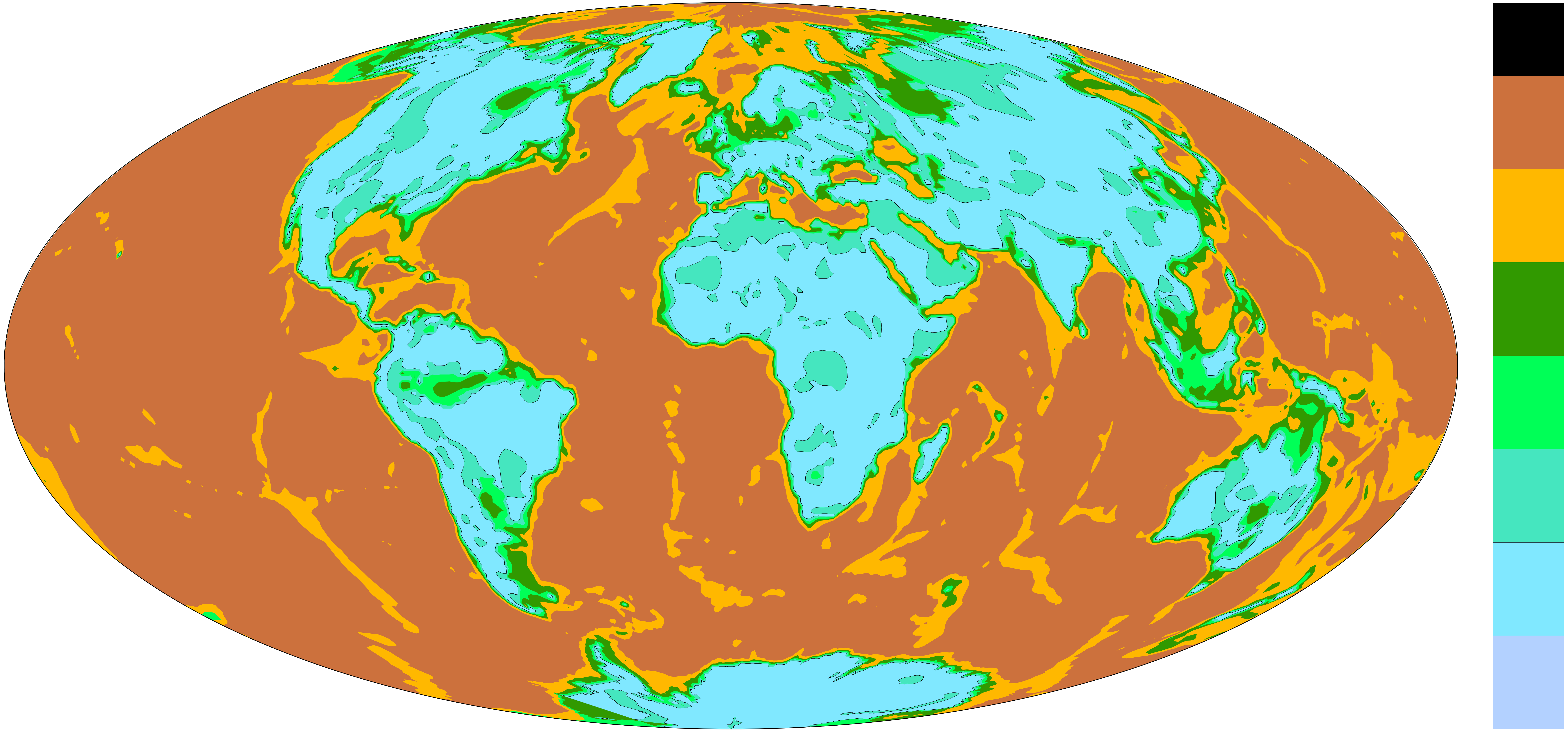


Chesley et al. (2019)

A crustal conductivity model:

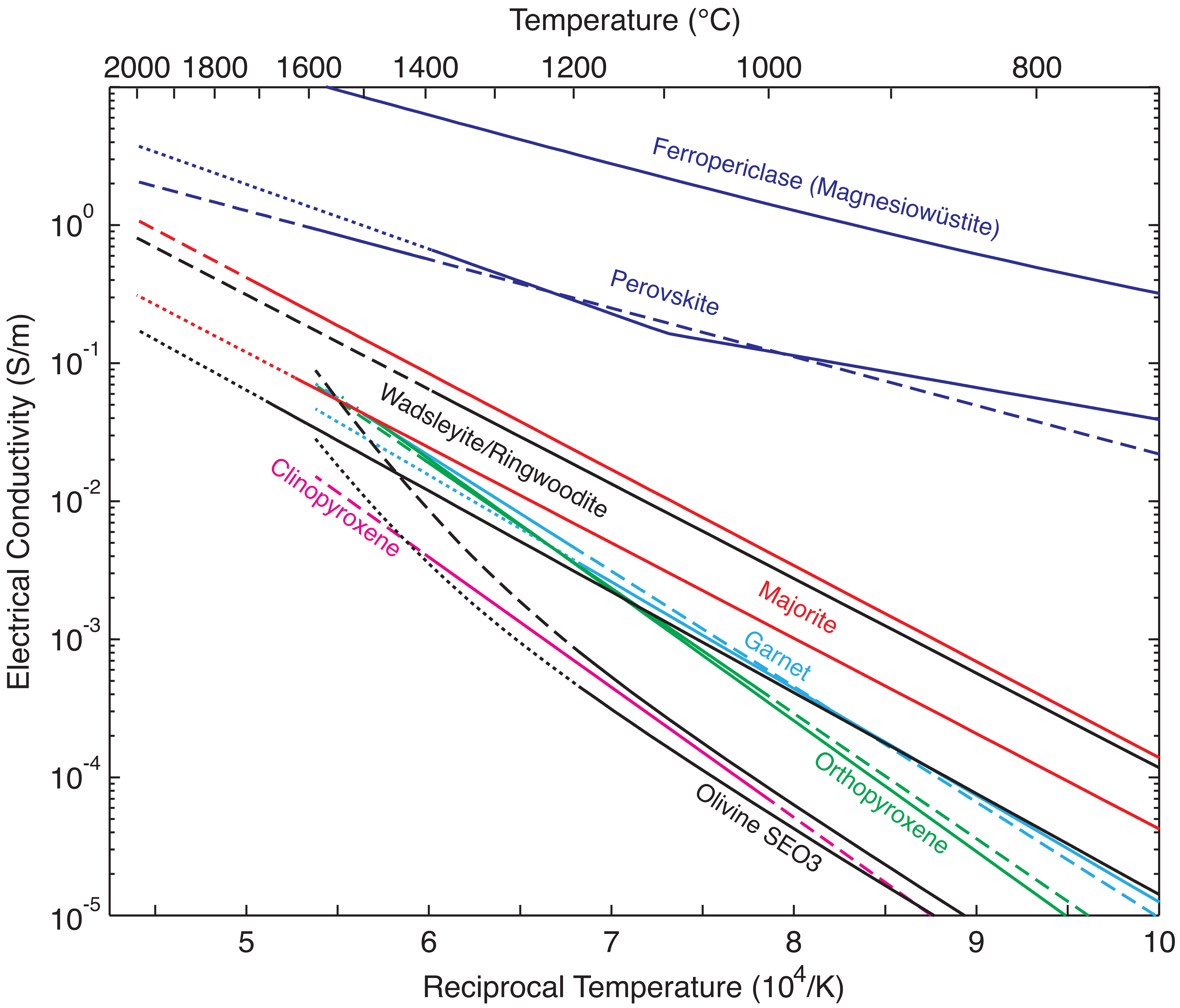


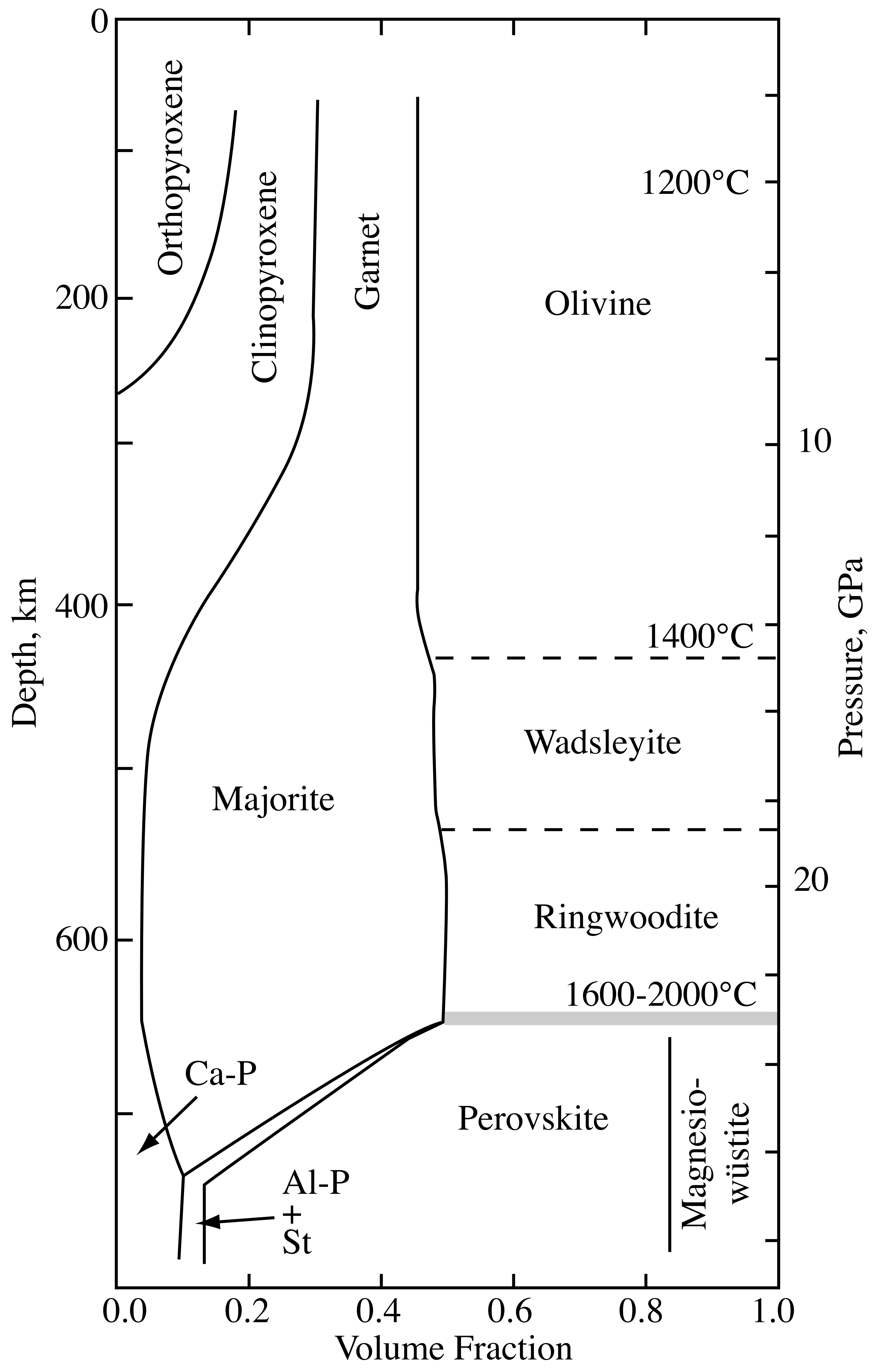
global surface conductance map, logarithmic scale [log10 S]

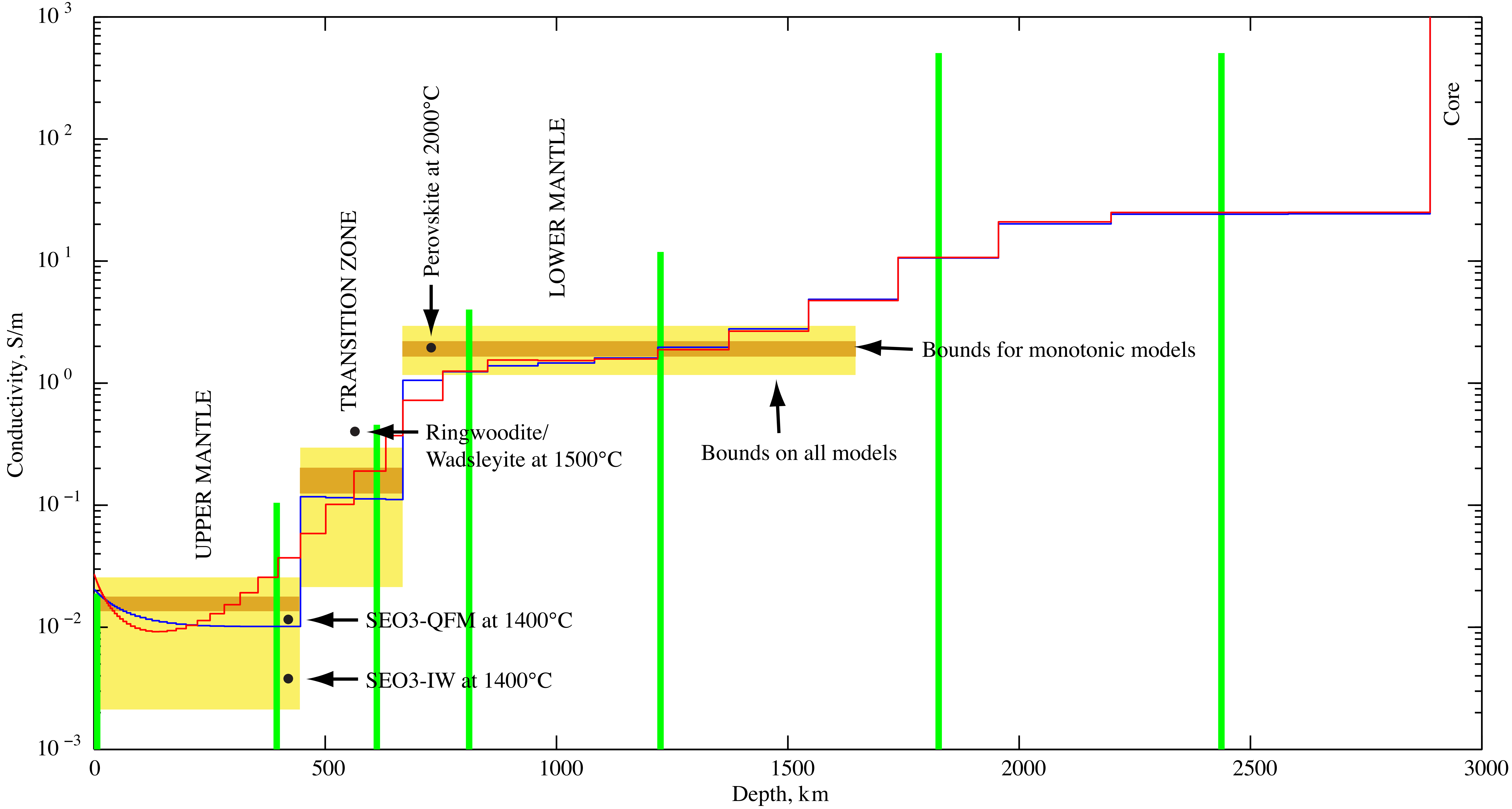


Everett and Constable^2 (2003)

A dry mantle model:

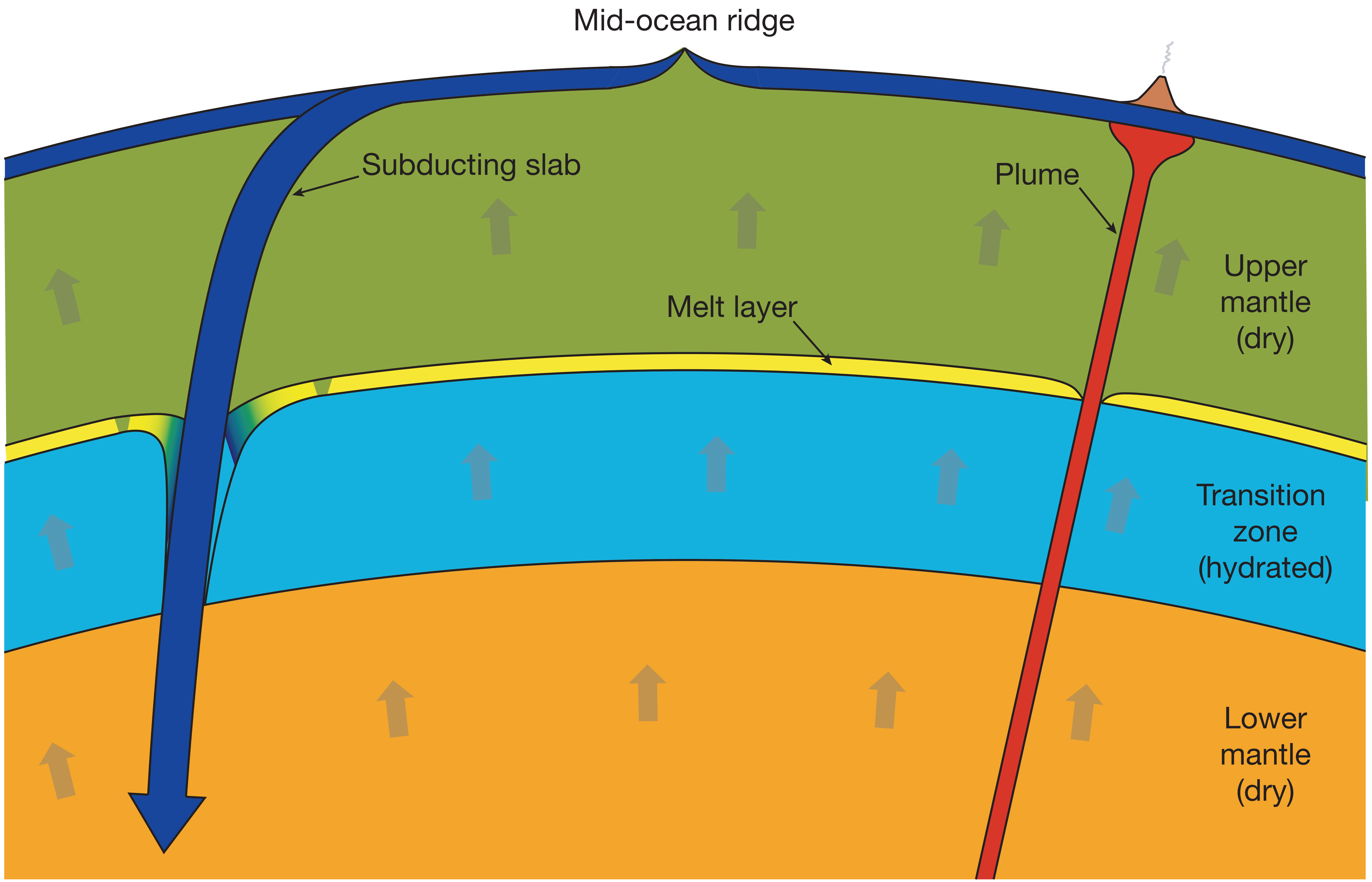




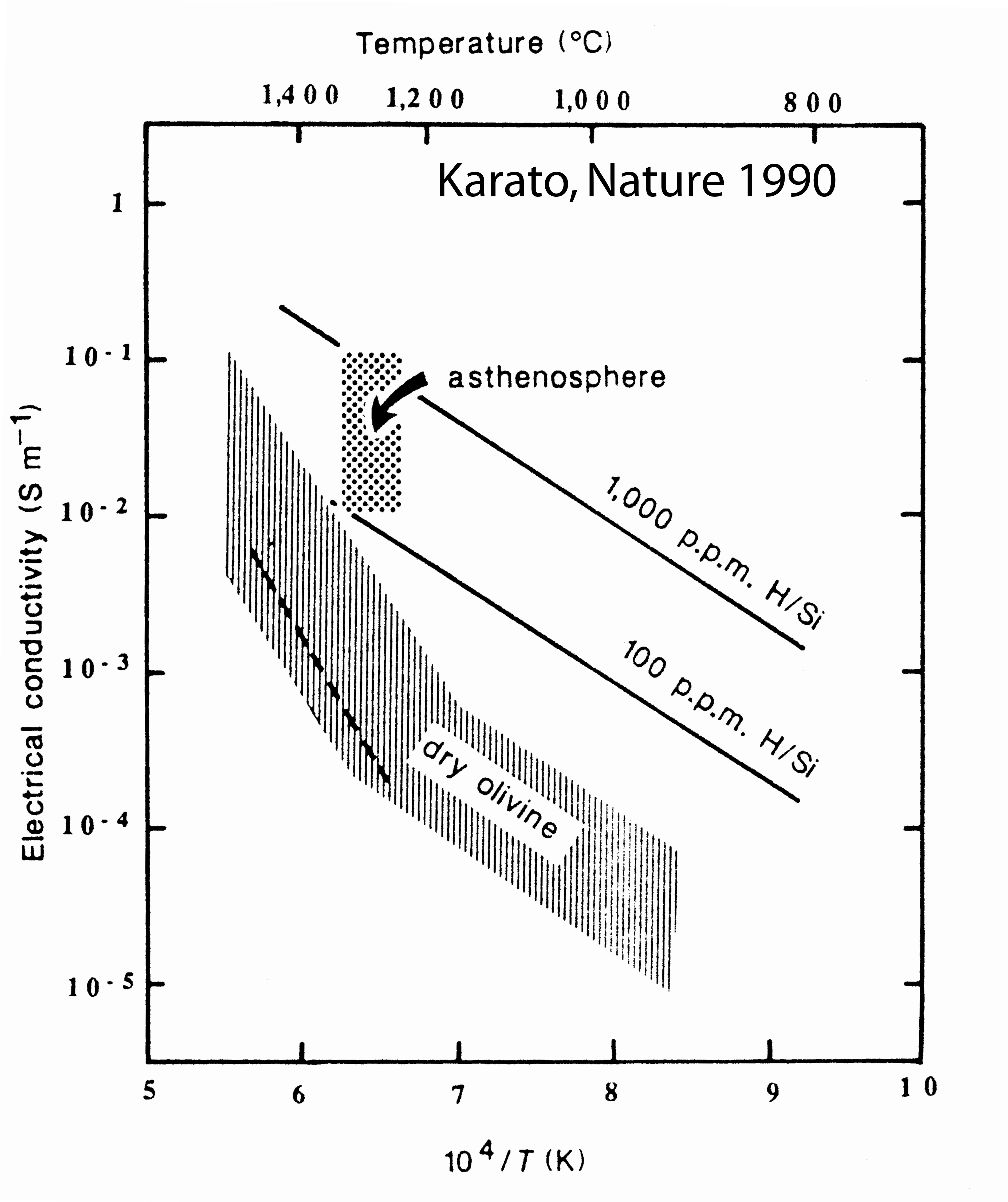


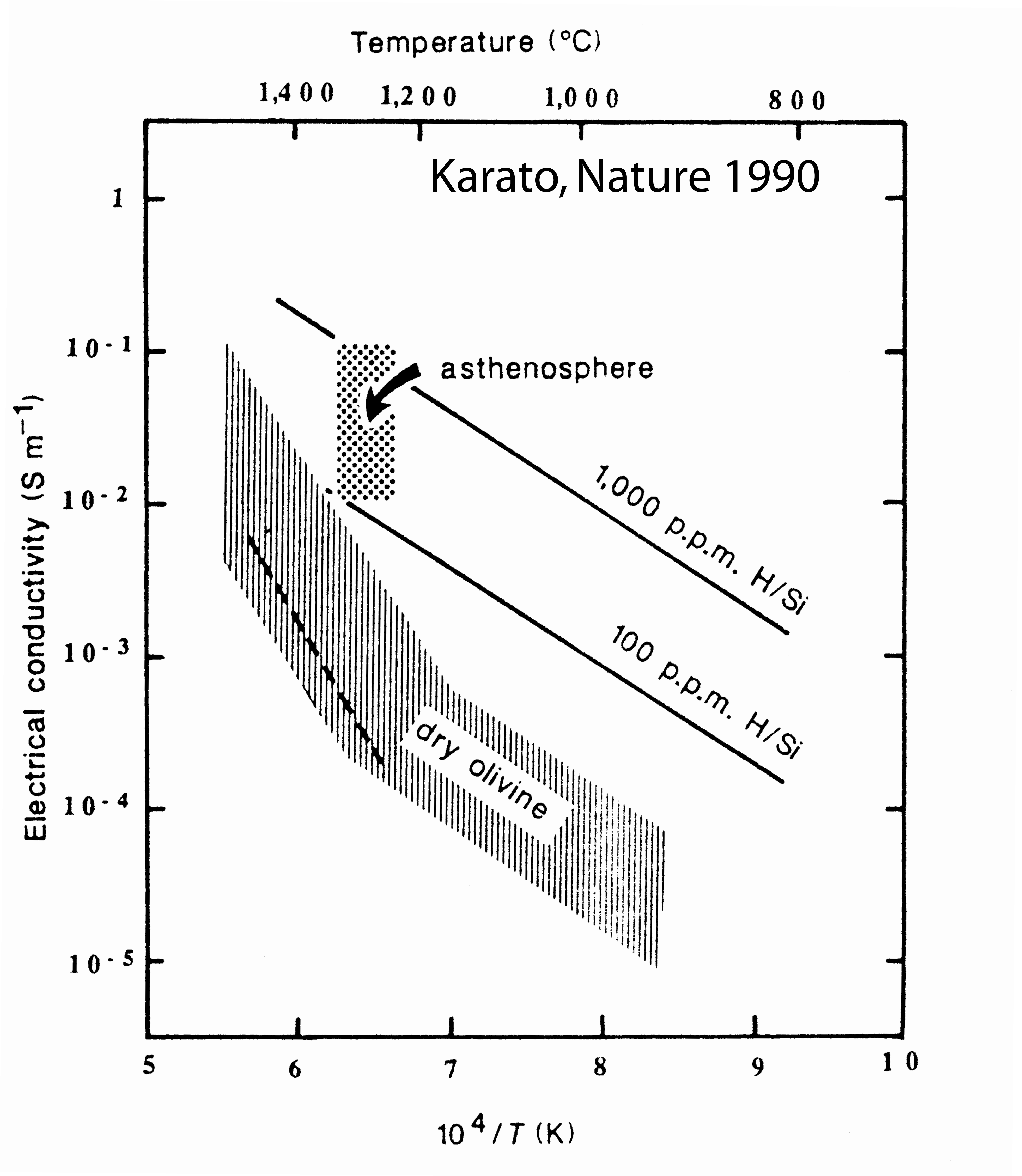
Conductivity, S/m
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
500
1000
1500
2000
2500
3000
Depth, km
UPPER MANTLE
TRANSITION ZONE
Perovskite at 2000°C
LOWER MANTLE
Core
Bounds for monotonic models
Bounds on all models
Ringwoodite/
Wadsleyite at 1500°C
SEO3-QFM at 1400°C
SEO3-IW at 1400°C

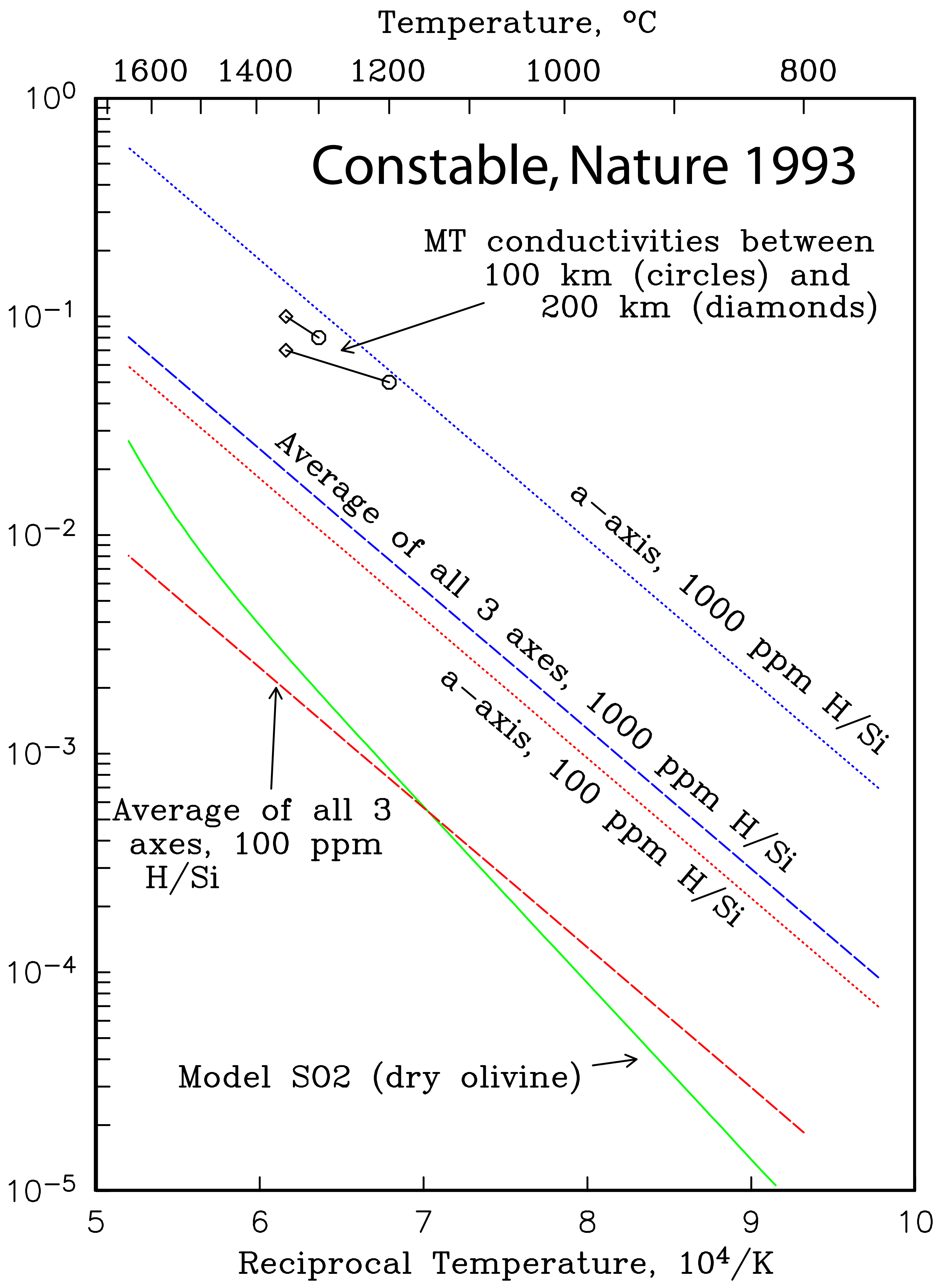
A wet mantle.

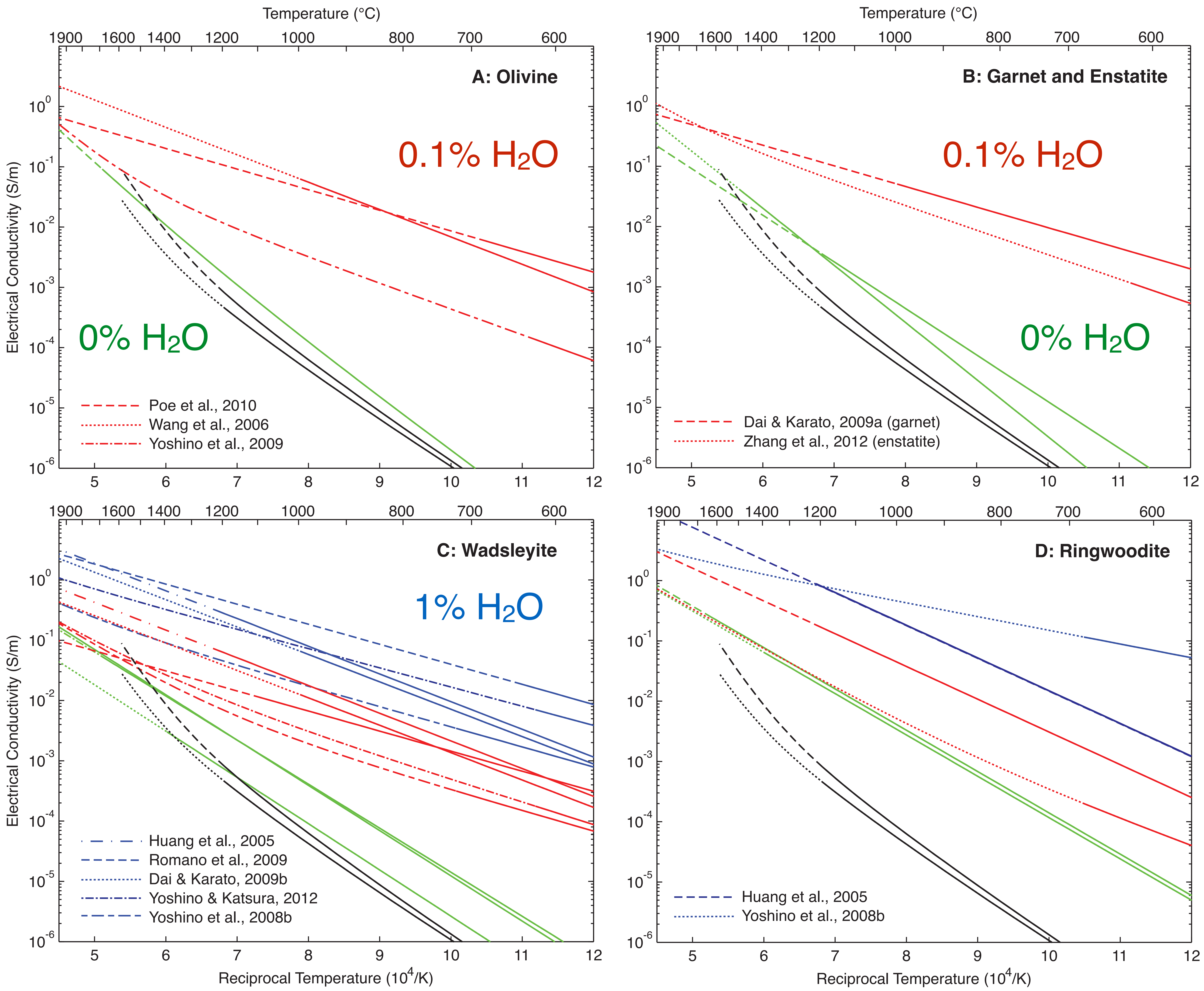


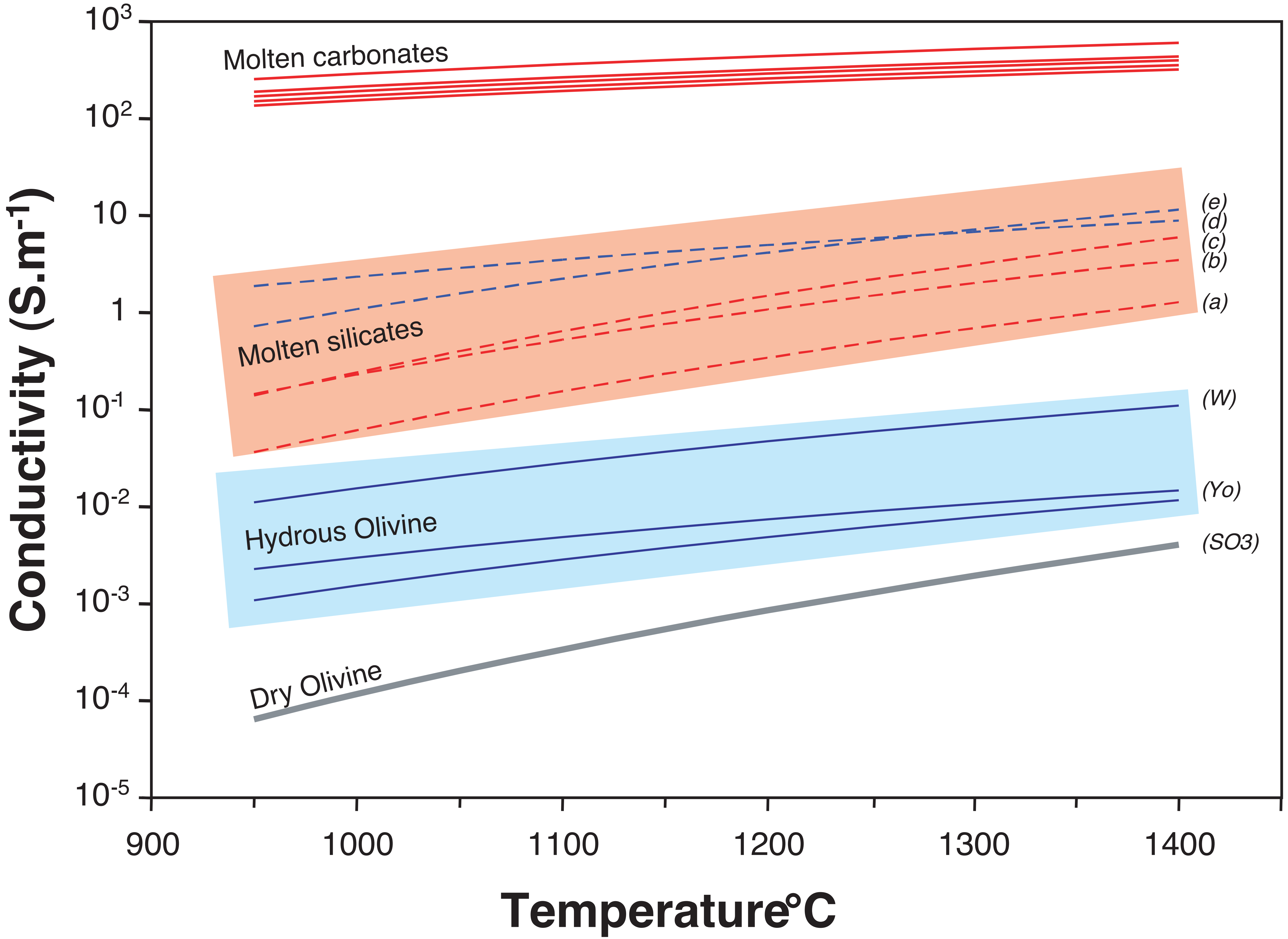
Bercovici and Karato (2003)


Temperature (°C)
1,400
1,200
1,000
800
Karato, Nature 1990
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Electrical conductivity (S m^{-1})
asthenosphere
1,000 p.p.m. H/Si
100 p.p.m. H/Si
dry olivine
5
6
7
8
9
10
$10^4/T$ (K)

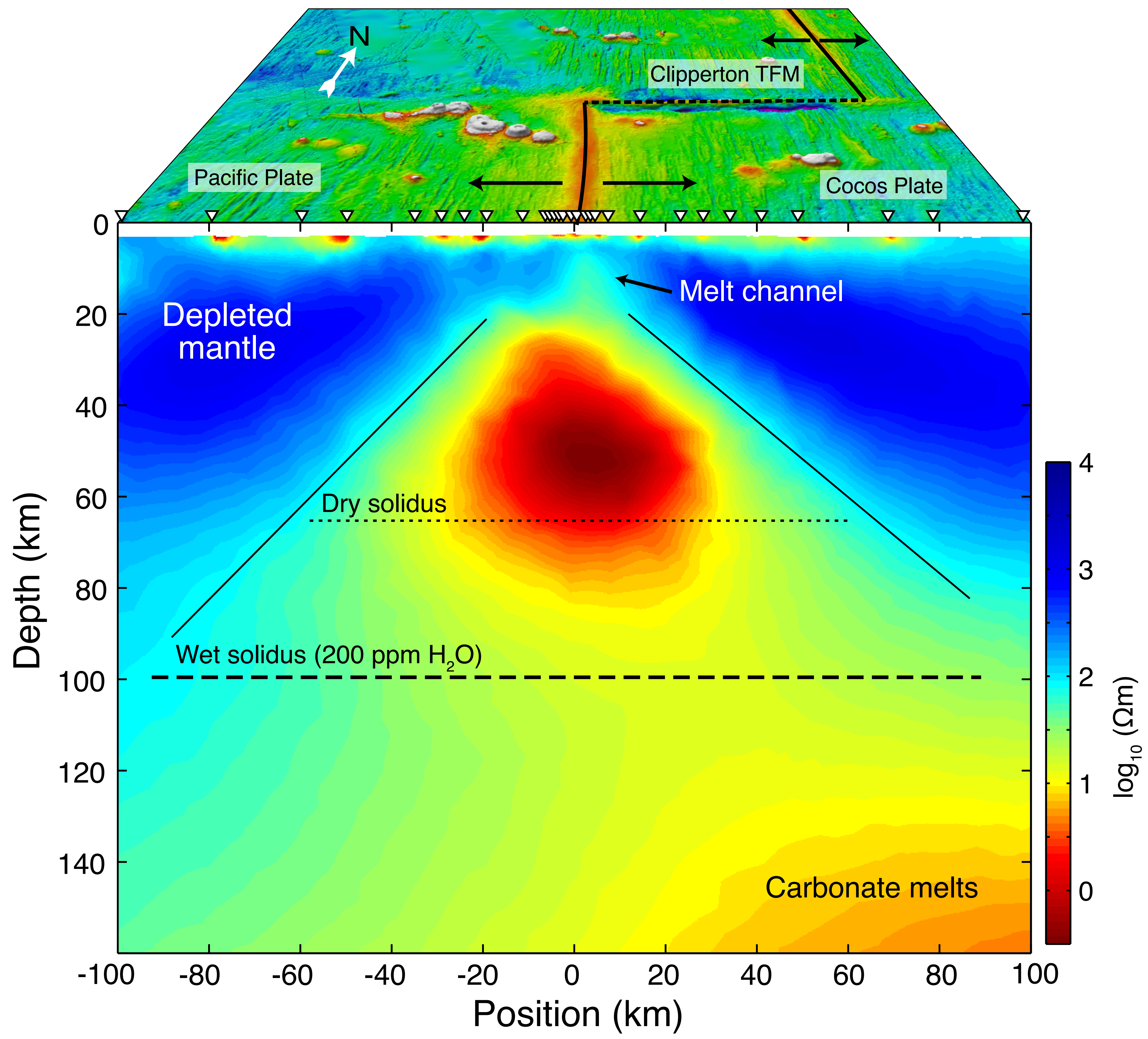

Karato, Nature 1990


Constable, Nature 1993


Temperature (°C)
1900
1600
1400
1200
1000
800
700
600
A: Olivine
0.1% H₂O
0% H₂O
Poe et al., 2010
Wang et al., 2006
Yoshino et al., 2009
B: Garnet and Enstatite
0.1% H₂O
0% H₂O
Dai & Karato, 2009a (garnet)
Zhang et al., 2012 (enstatite)
C: Wadsleyite
1% H₂O
Huang et al., 2005
Romano et al., 2009
Dai & Karato, 2009b
Yoshino & Katsura, 2012
Yoshino et al., 2008b
D: Ringwoodite
Huang et al., 2005
Yoshino et al., 2008b
Electrical Conductivity (S/m)
10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
Reciprocal Temperature (10⁴/K)
5
6
7
8
9
10
11
12

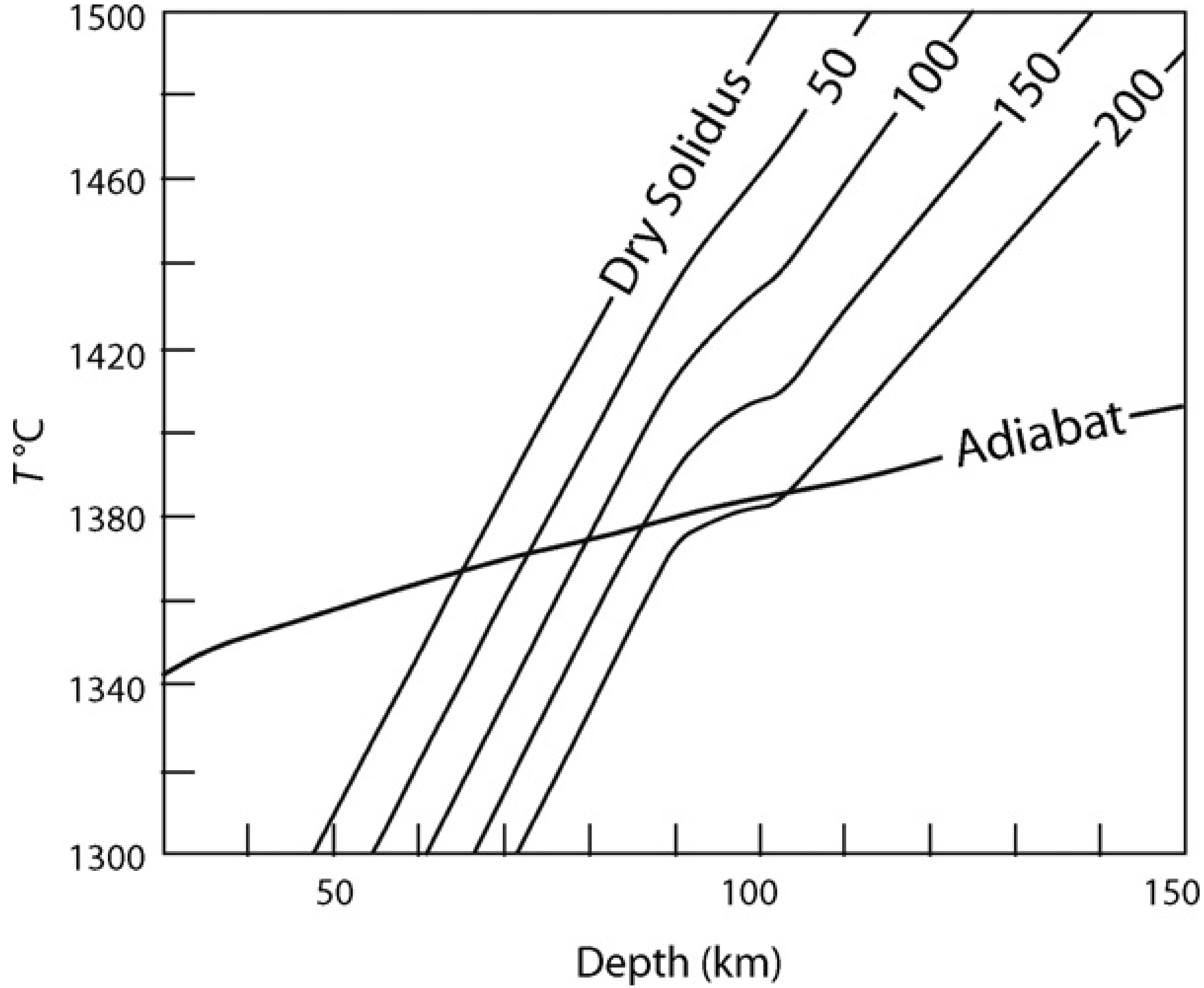


Gaillard et al. (2008)



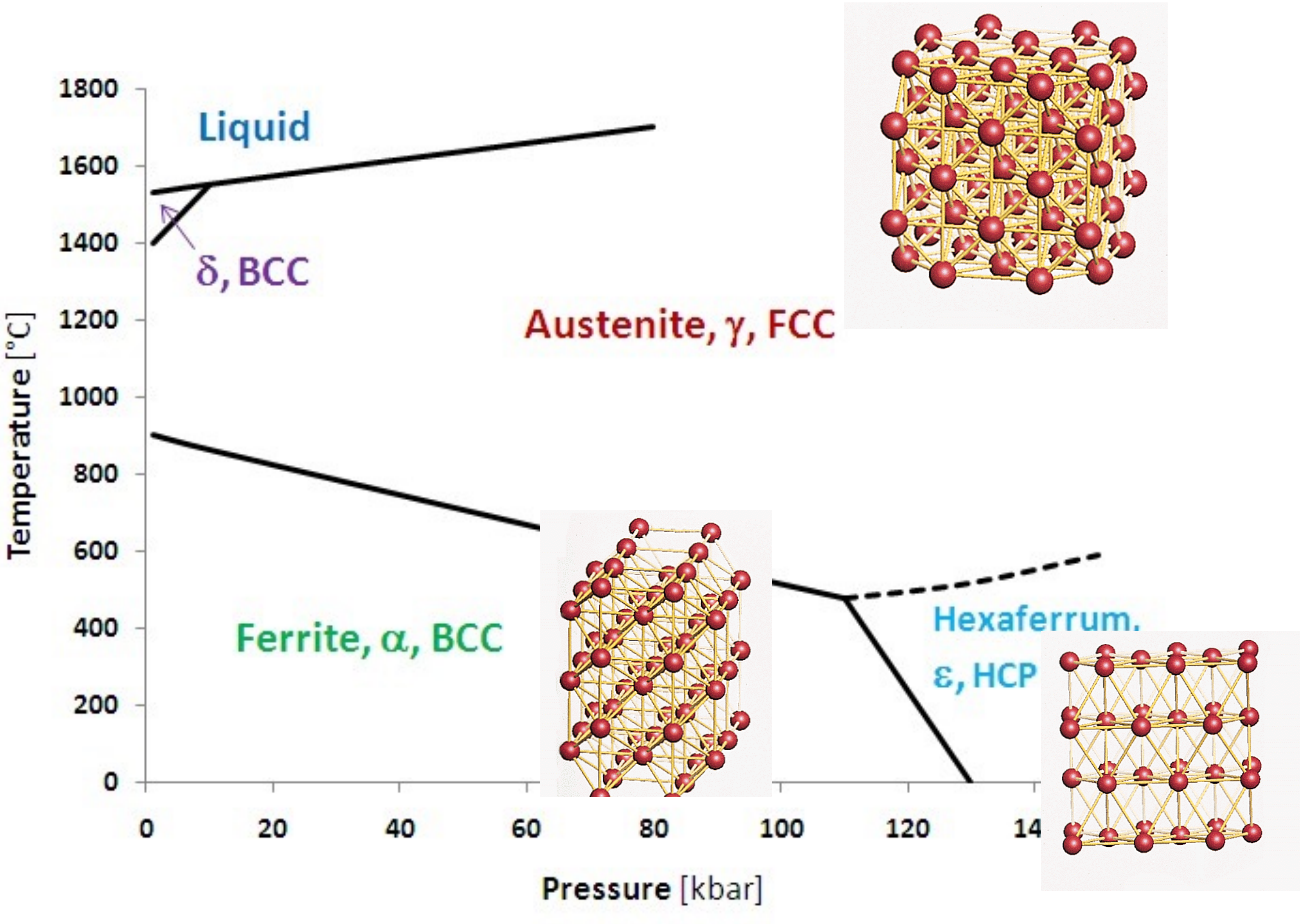
Key et al. (2013)

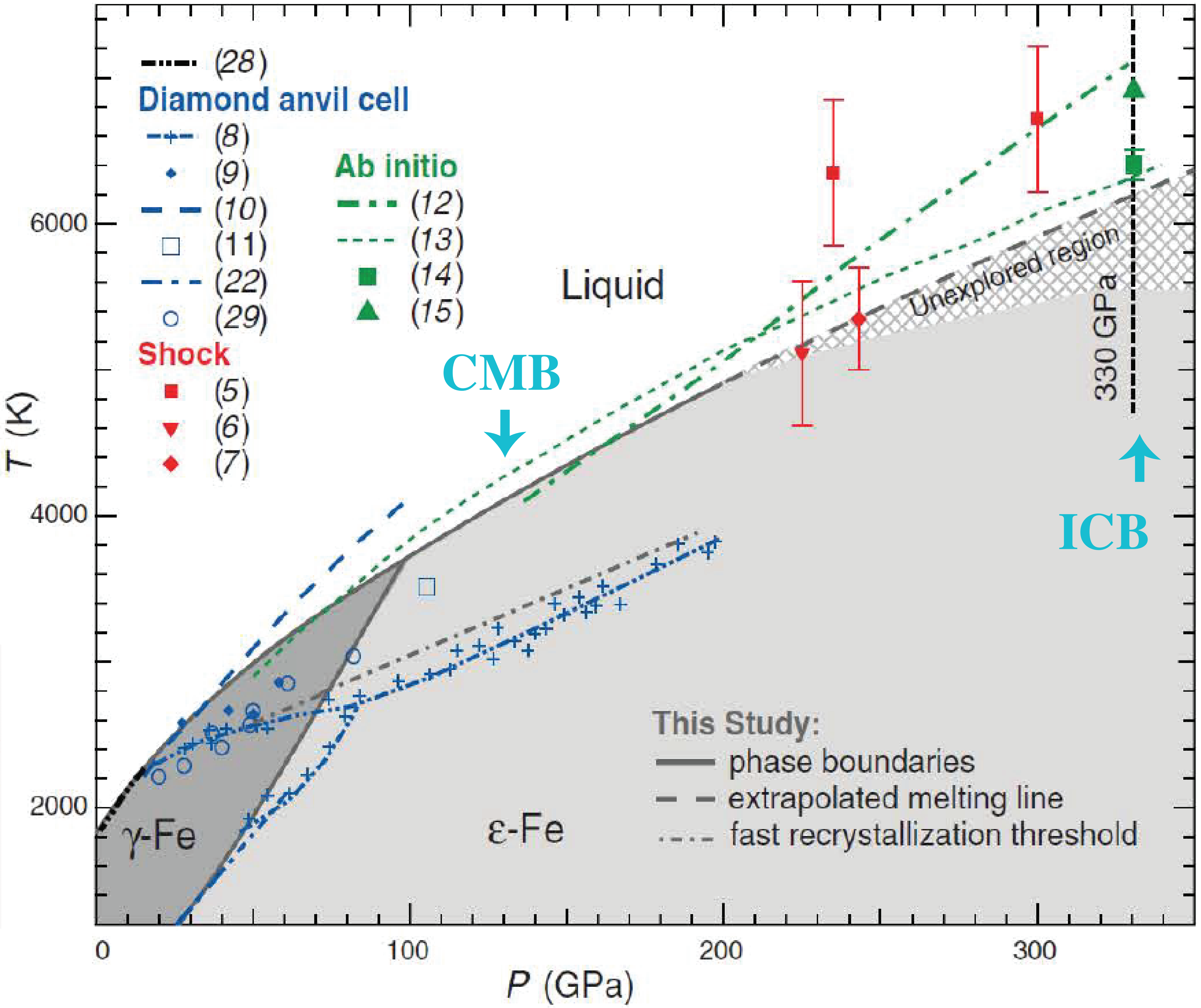
A few hundred parts per million water lowers mantle solidus by 100°C



Hirschmann et al. (2008)

The core:

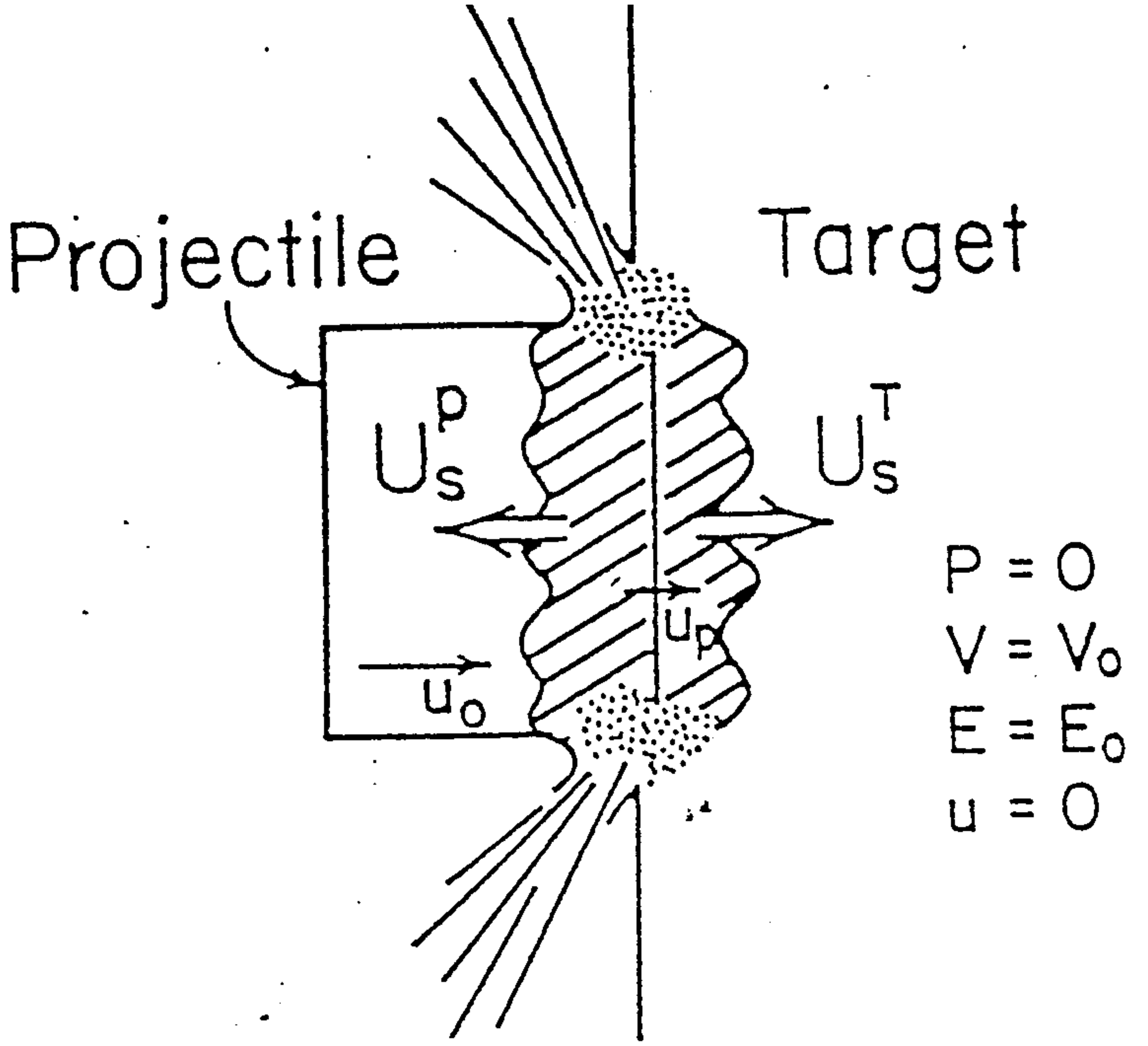


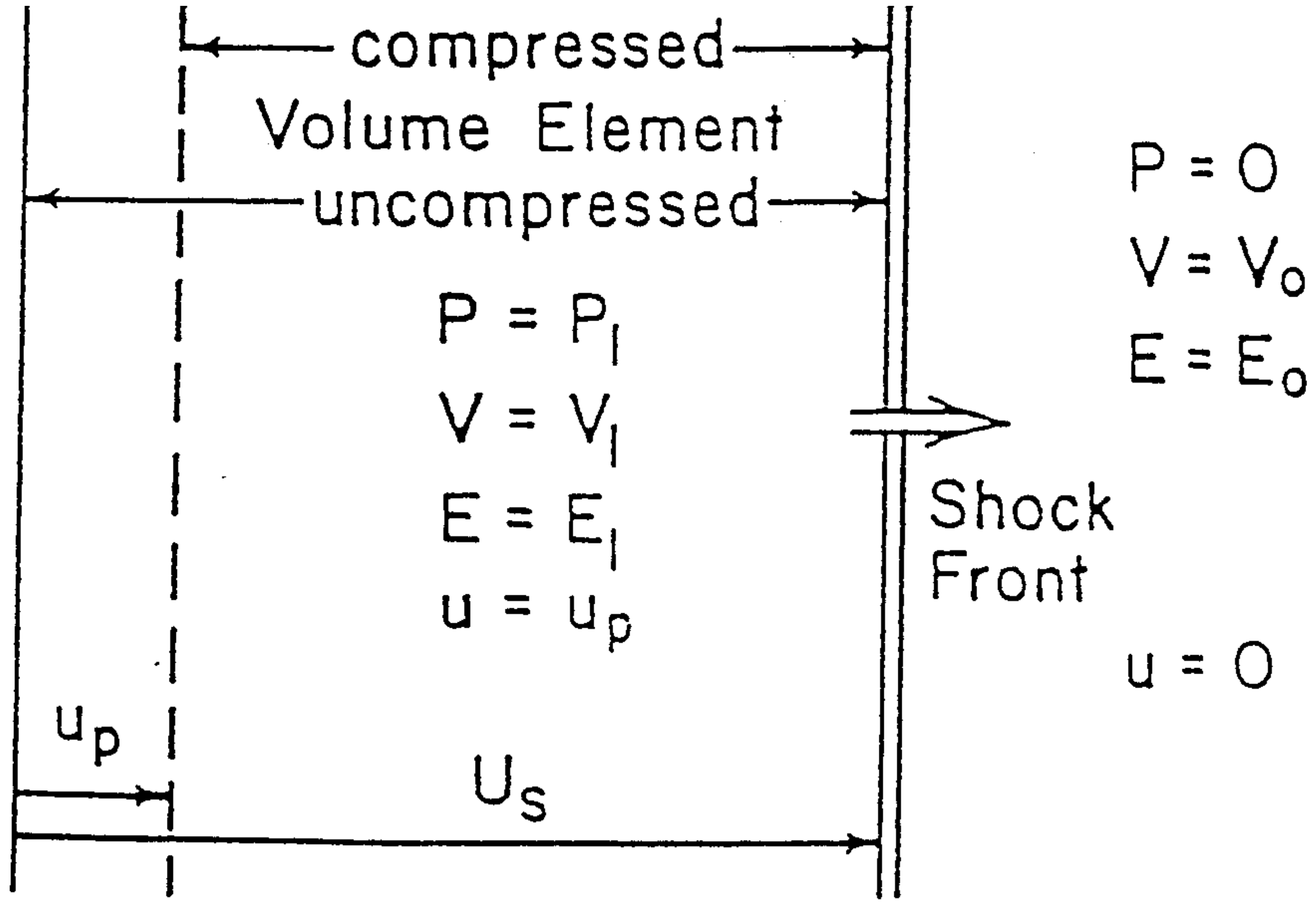


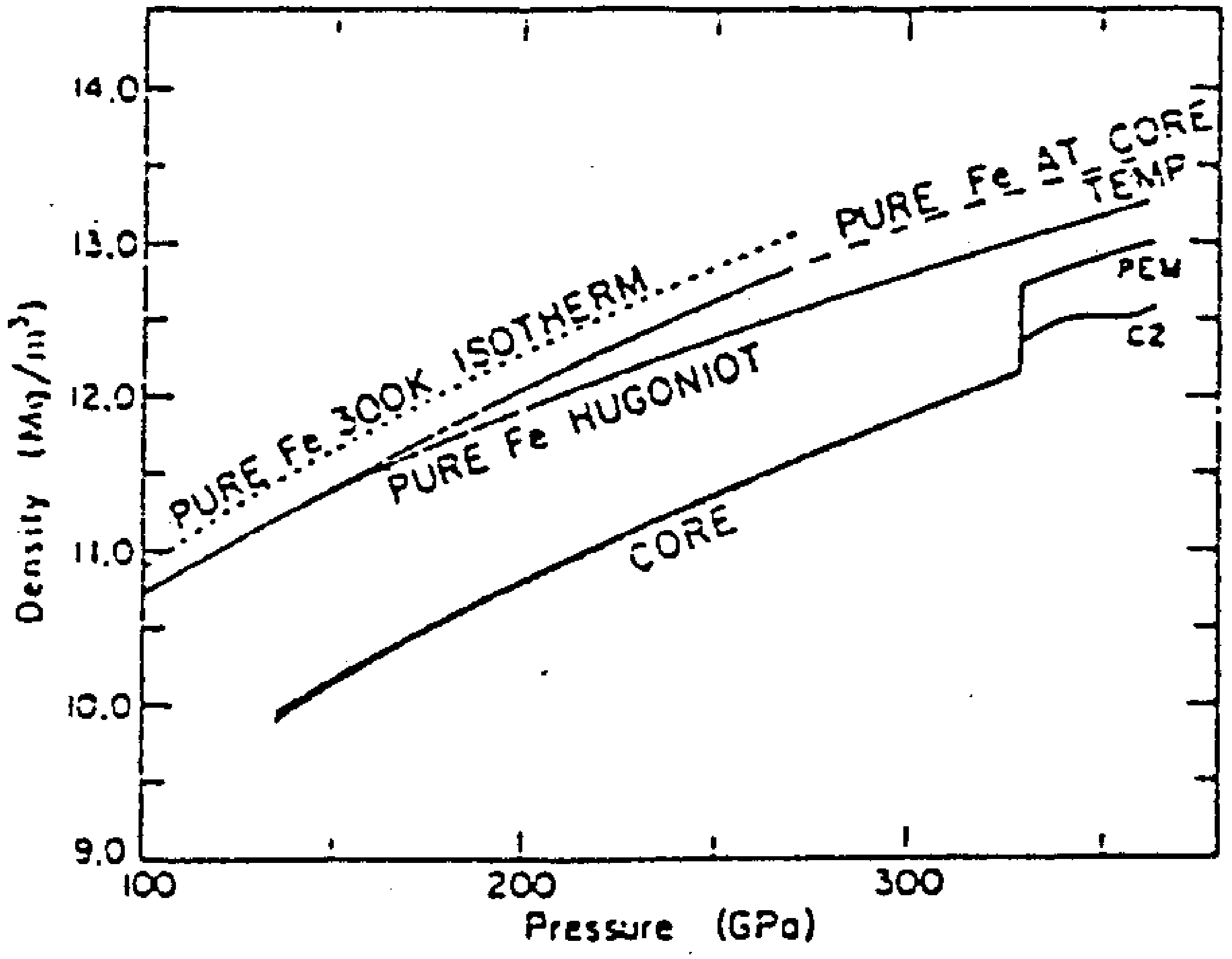
Anzellini et al. (2013)

Shock wave experiments

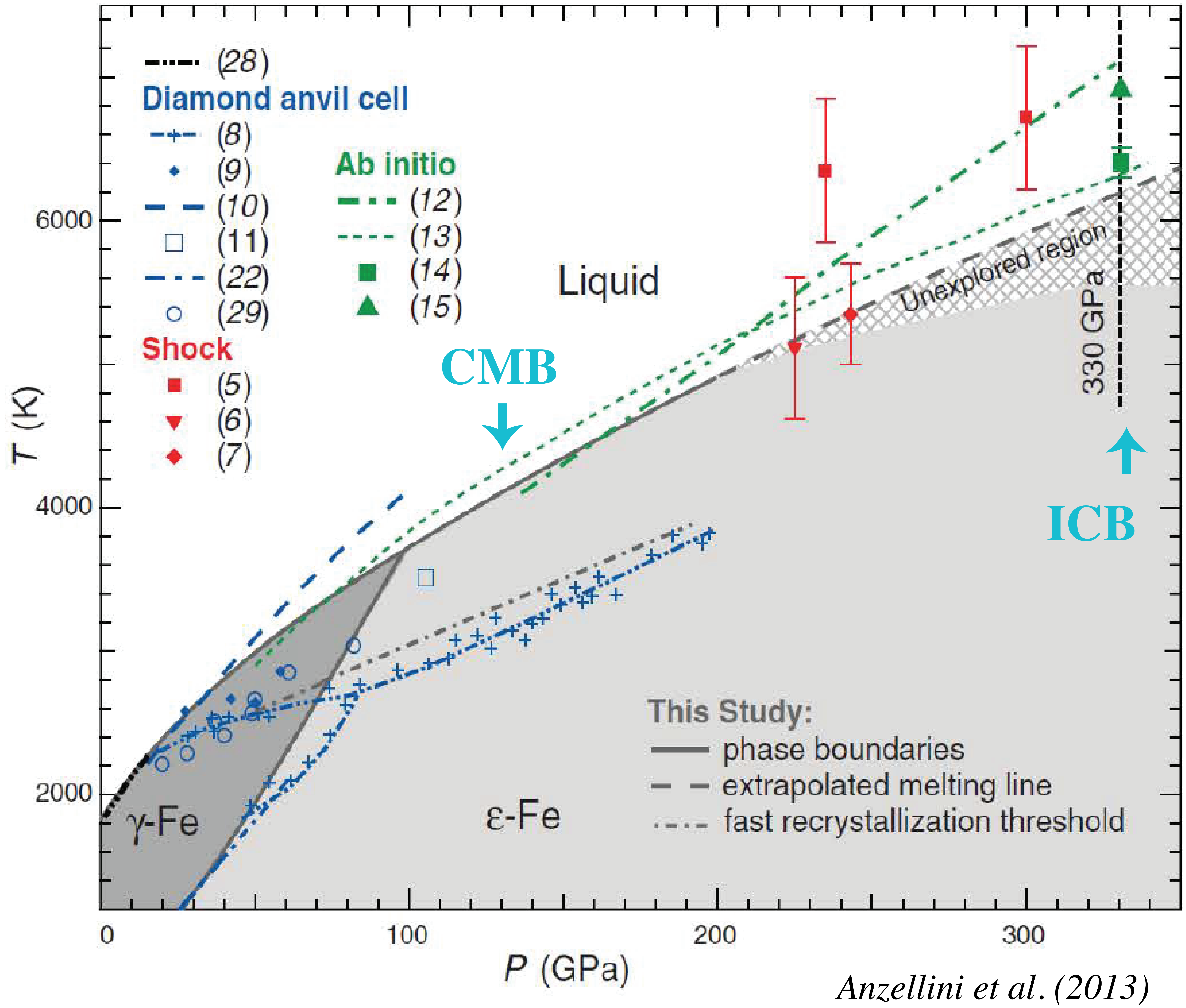
Stacey and Anderson (2001, PEPI) estimated the resistivity of the outer core to be 2.1x10^{-6} Ωm and inner core 1.6x10^{-6} Ωm, based on shock wave experiments of iron with Si and Ni impurities and the idea that for an "ideal" metal the conductivity is constant on the pressure-temperature melting curve (shock wave experiments showed that outer core density was lower than Fe).



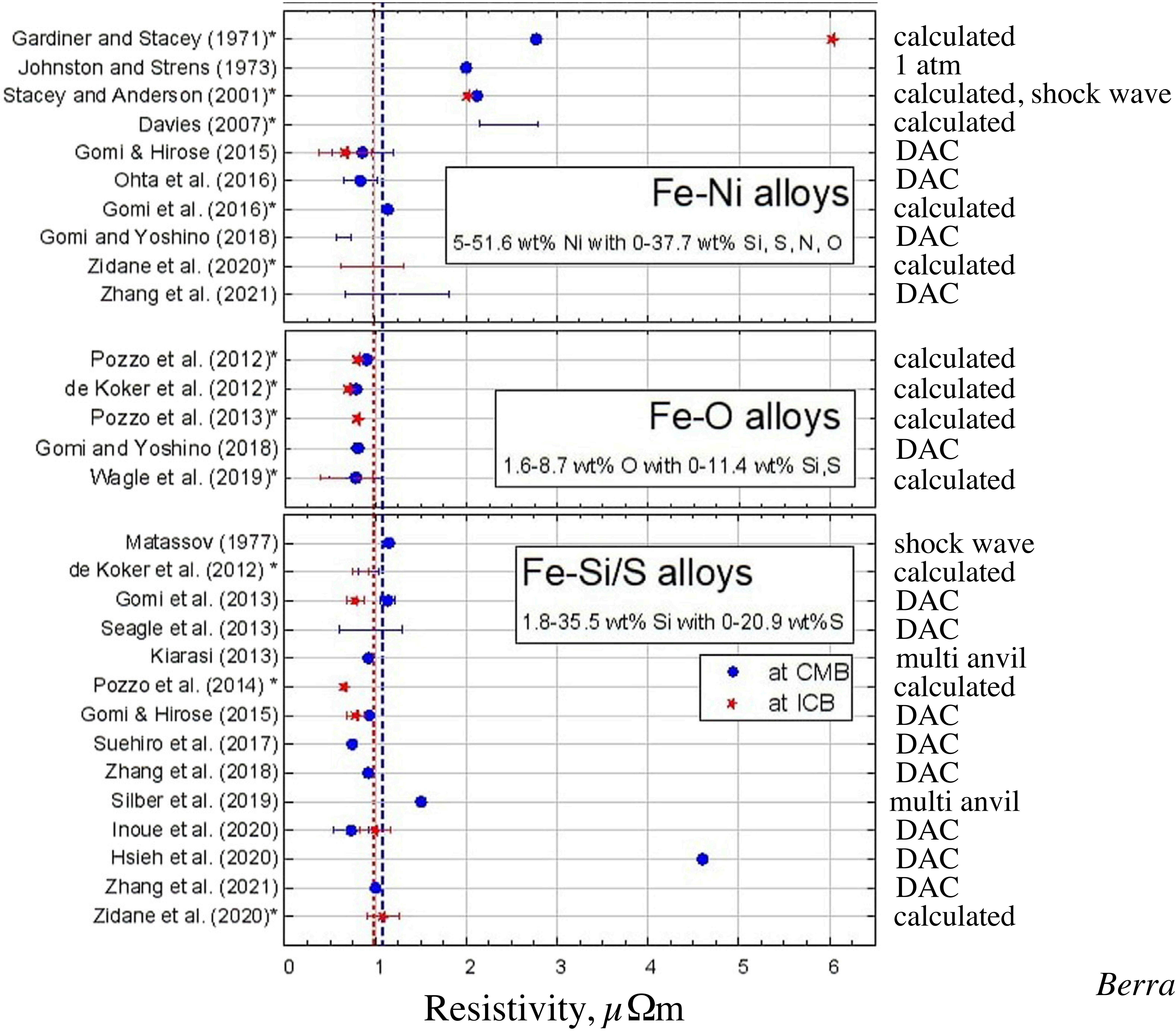




However, more recent measurements using ab initio calculations and direct diamond anvil measurements put the outer core resistivity at $6\text{-}7\times10^{-7}$ Ωm, or three times the shock wave values.



Anzellini et al. (2013)



Berrada and Secco (2021)