From the Sun to Earth's core: The EM response of the sunspot cycle and the conductivity of the lowermost mantle

(with some extra stuff for SIOG 231)

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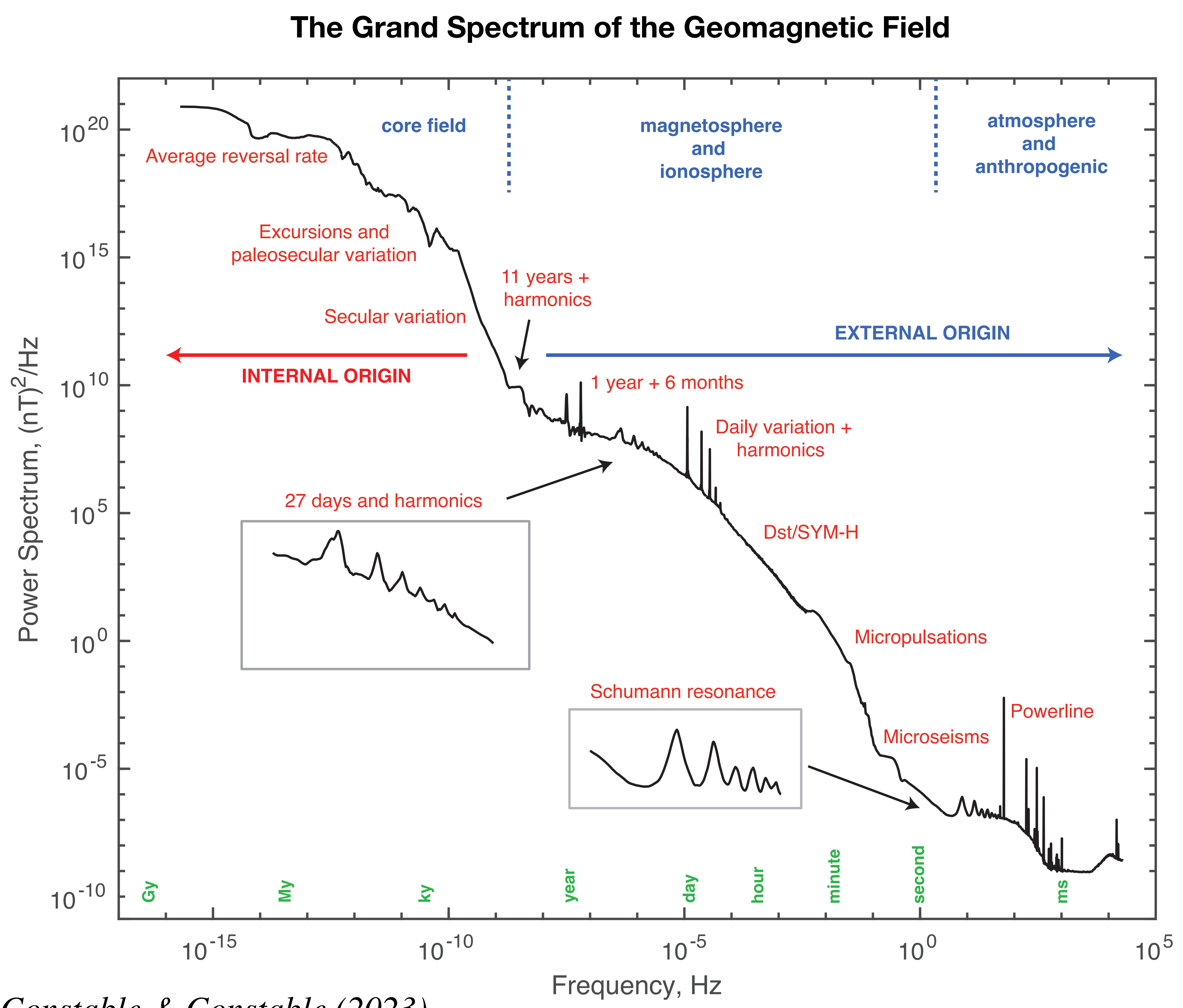


Constable & Constable (2023)

Earth's magnetic field varies on **all** time scales.

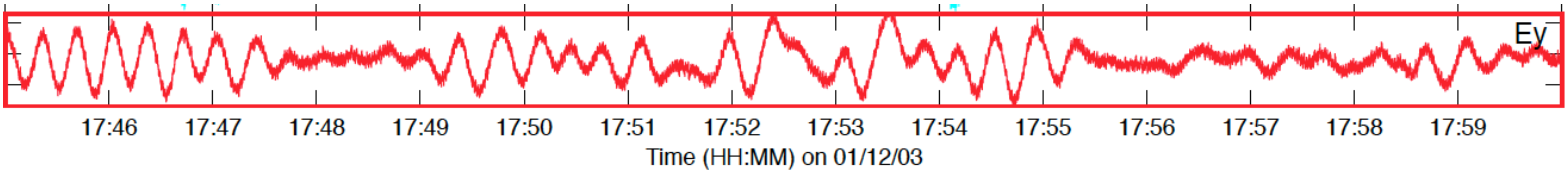
At periods > 11 years we can observe variations associate with the core geodynamo.

At periods < 11 years we can observe external magnetic field variations.

Electromagnetic induction driven by external field variations are used to probe electrical conductivity of the crust and mantle, typically to less than 1 year periods.

Here we try to extend the external field response out to 11 years.

Refresher of time series analysis:



To convert a **time series** into the **frequency domain** we use the discrete Fourier transform:

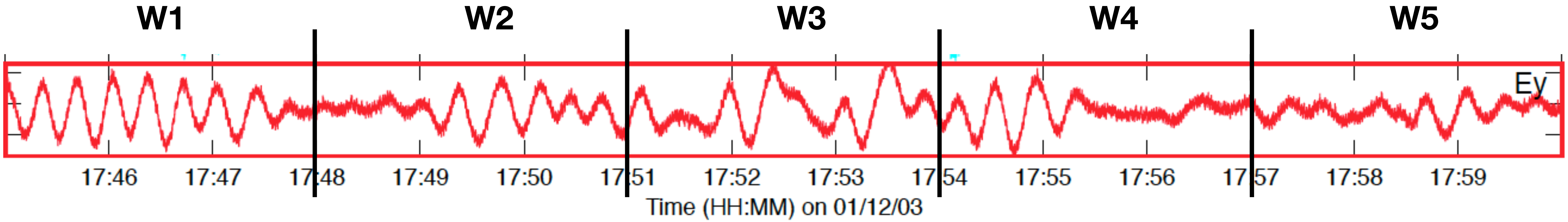
$$\tilde{X}(m\Delta f) = \Delta t \sum_{n=0}^{N-1} X_{n+1} e^{-2\pi i m n/N} \quad , \qquad m = 1, 2, \ \ldots \ N/2 - 1 \qquad \text{(complex numbers)}$$

Frequency bandwidth: $\Delta f = (N\Delta t)^{-1}$ Periodogram: $|\tilde{X}(m\Delta f)|^2$

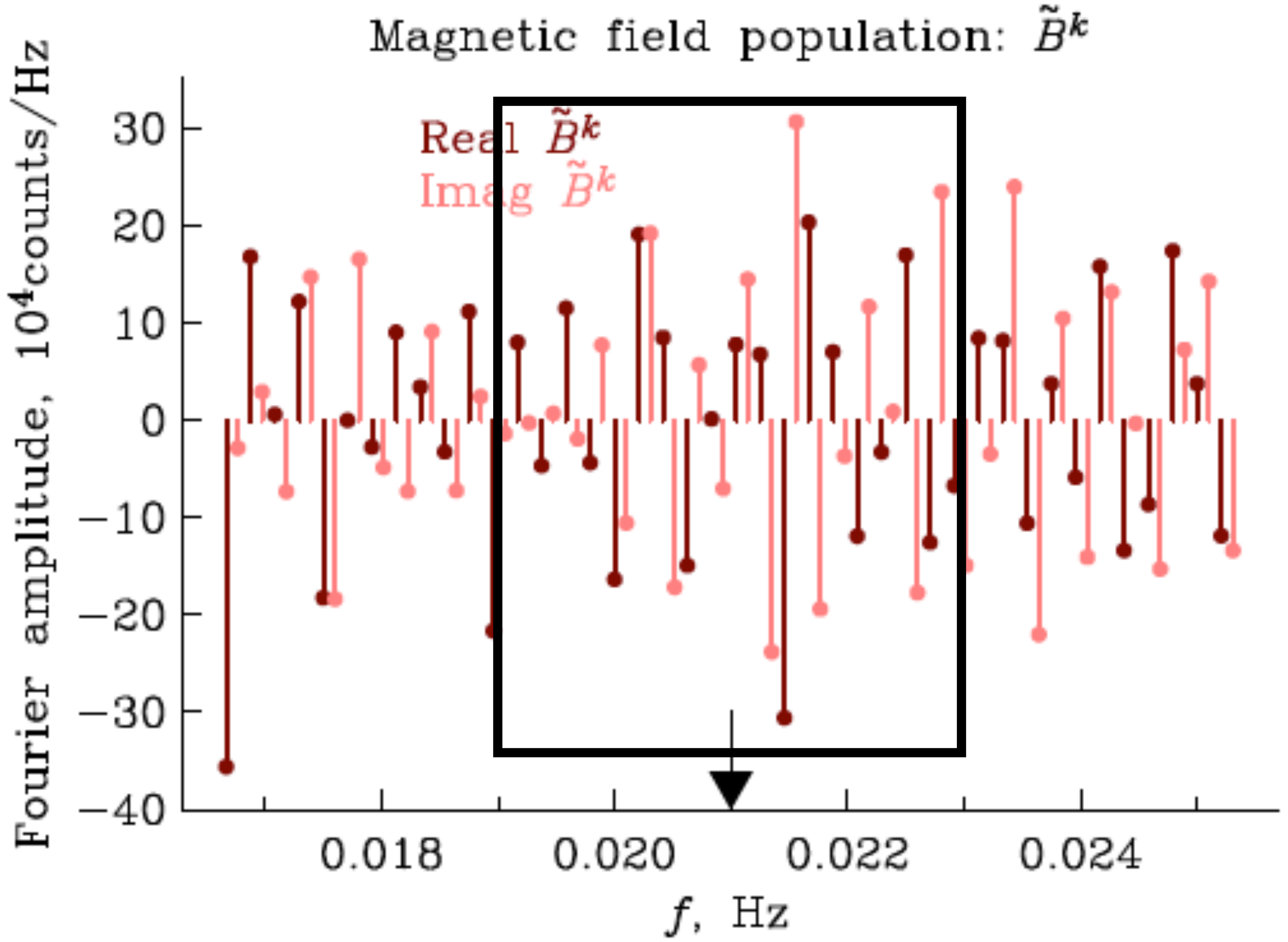
The periodogram obtained from a FT is **biased** (-> spectral leakage) and has **large variance**. The solution to the bias is to use a **taper**, but this doesn't improve the variance. For that we need averaging. There are three approaches to averaging:

- frequency averaging
- window averaging
- multi taper averaging

Window averaging: Chop the time series up into shorter bits, apply one taper to reduce bias, and average the FT of each bit. But we lose the longest periods this way.



Frequency averaging: Average over adjacent frequencies (in the complex domain) and assign the average to the average frequency.

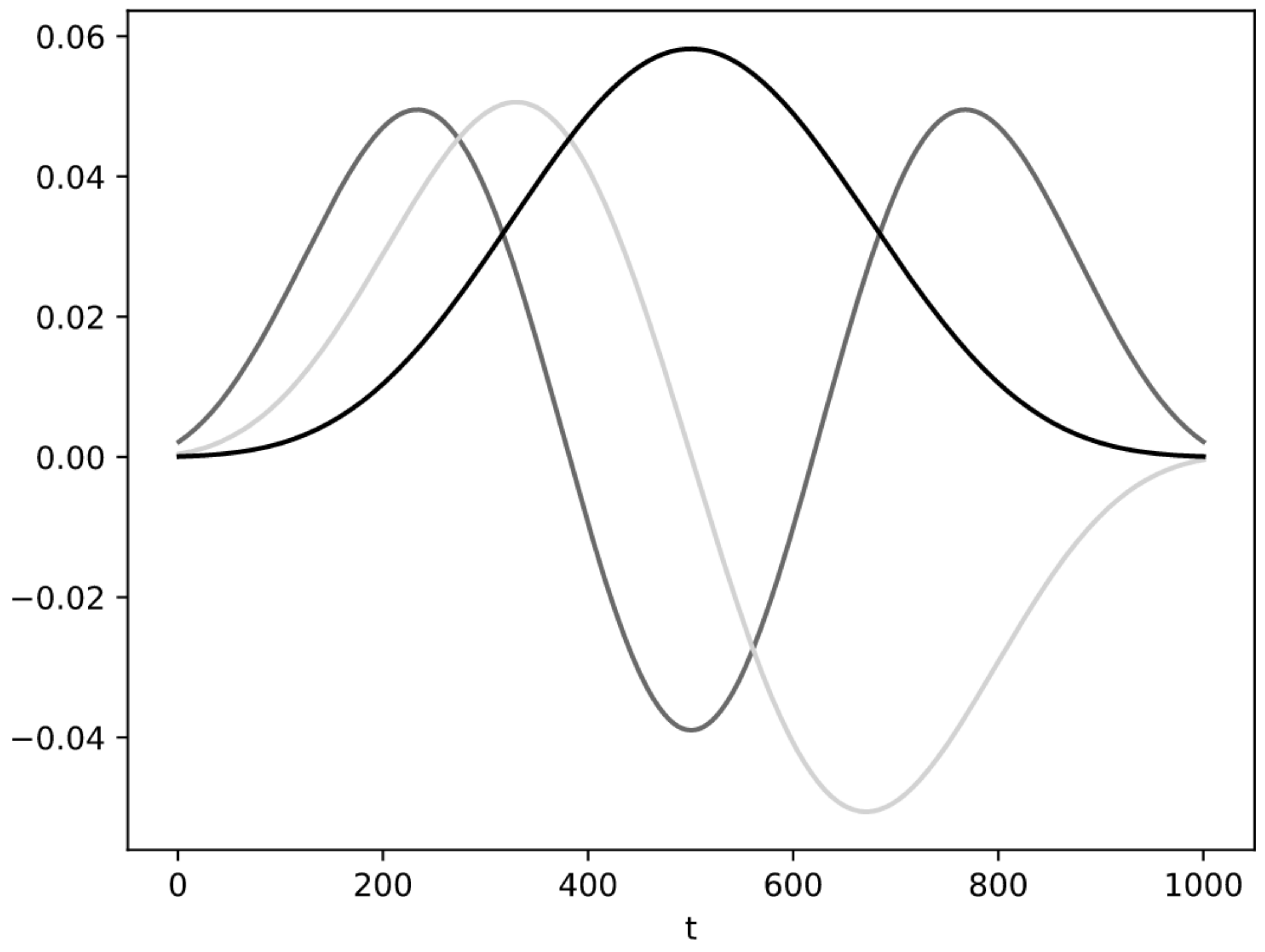


Tapering:

$$\tilde{X}(m\Delta f) = \Delta t \sum_{n=0}^{N-1} w_n X_{n+1} e^{-2\pi i m n/N}$$

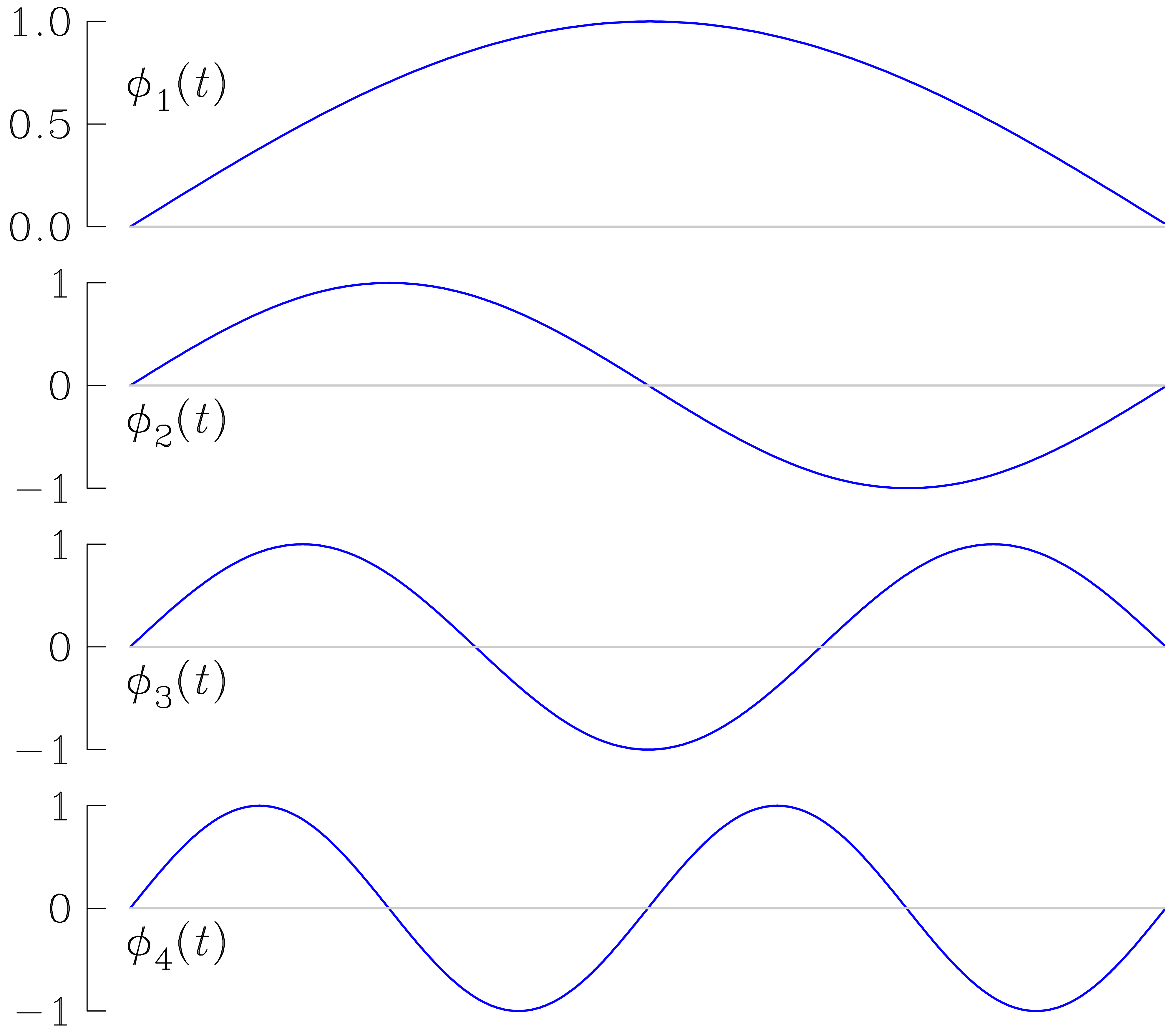
Here *w* is a taper - a smooth bunch of weights that usually go to zero at the ends of a series. The multitaper method uses an orthogonal set of tapers and averages the resulting Fourier coefficients.

First 3 Slepian tapers



Wikipedia

First 4 sine tapers



Bob Parker

The advantage of multi-taper spectral estimates is that you use the entire time series and obtain the lowest possible frequencies - important if your data series are limited in length. More tapers means more variance and bias reduction, but lowers frequency resolution. There is a trade-off. A clever adaptive sine multi-taper algorithm (*Riedel and Sidorenko, 1996*) varies the number of tapers with frequency — this was used for the Grand Spectrum.

In MT data processing we mentioned the cross spectrum but went on to develop MT processing using a covariance approach. For observatory records we don't have the luxury of extra data channels, remote references, or cutting up the data into windows. We need the **cross spectrum** S_{xy}

$$S_{xy}(f) = \int_{-\infty}^{\infty} R_{xy}(t) e^{-2\pi i f t} dt$$

where

$$R_{xy}(t) = \mathrm{cov}[X(s)Y(s+t)] = \mathrm{E}[X(s)Y(s+t)]$$

is the **cross-covariance** between X and Y. The cross-spectrum is complex. The coherence is given by

$$\gamma_{xy}^2 = \frac{|S_{xy}|}{\sqrt{S_{xx}S_{yy}}}$$

and is between 0 (no correlation) and 1 (perfect correlation).

In practice we don't use the convolution to compute the cross spectrum but use our (fast) discrete Fourier transform

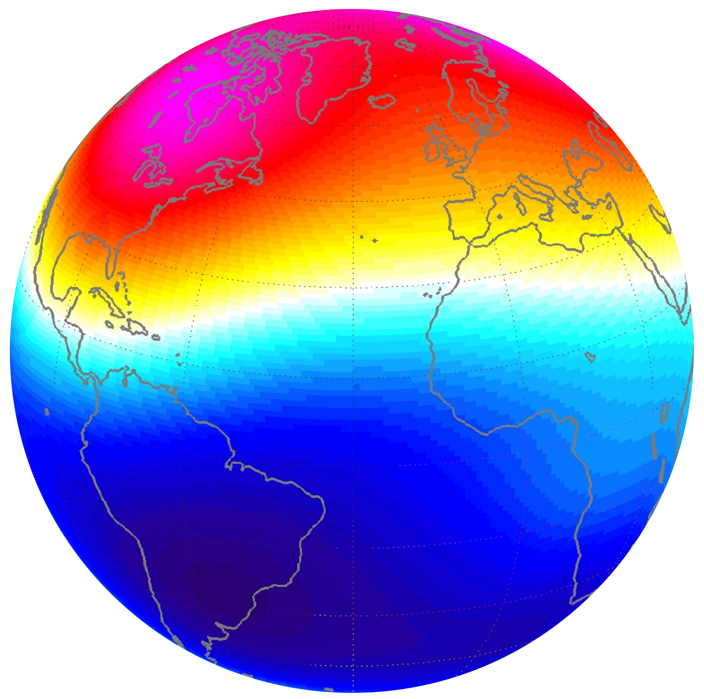
$$S_{xy}(m\Delta f) = \tilde{X}(m\Delta f)\tilde{Y}^*(m\Delta f)$$

where

$$\tilde{X}(m\Delta f) = \Delta t \sum_{n=0}^{N-1} X_{n+1} e^{-2\pi i m n/N} \quad , \qquad m = 1, 2, \; ... \; N/2 - 1$$

$$\tilde{Y}(m\Delta f) = \Delta t \sum_{n=0}^{N-1} w_n Y_{n+1} e^{-2\pi i m n/N} \quad , \qquad m = 1, 2, \; ... \; 1/2N - 1$$

Modeling magnetic field complexity:



Outside magnetic sources: $\nabla^2 V_m = 0$

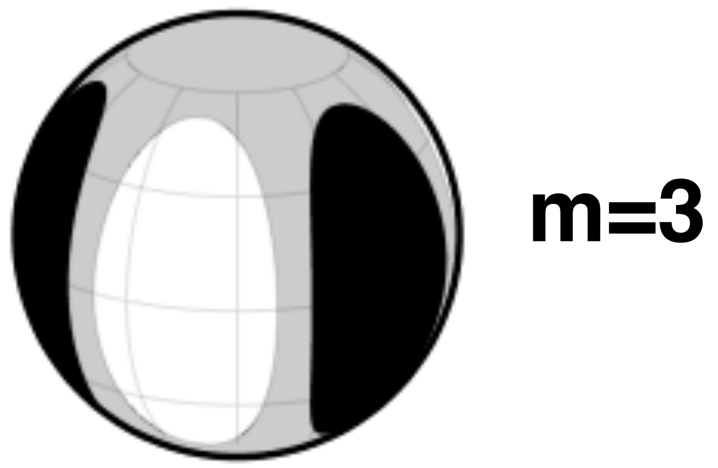
$$V_m(r,\theta,\phi) = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} P_n^m(\cos\theta) \left[\left((g_e)_n^m \left(\frac{r}{a}\right)^n + (g_i)_n^m \left(\frac{a}{r}\right)^{n+1} \right) \cos m\phi \right.$$

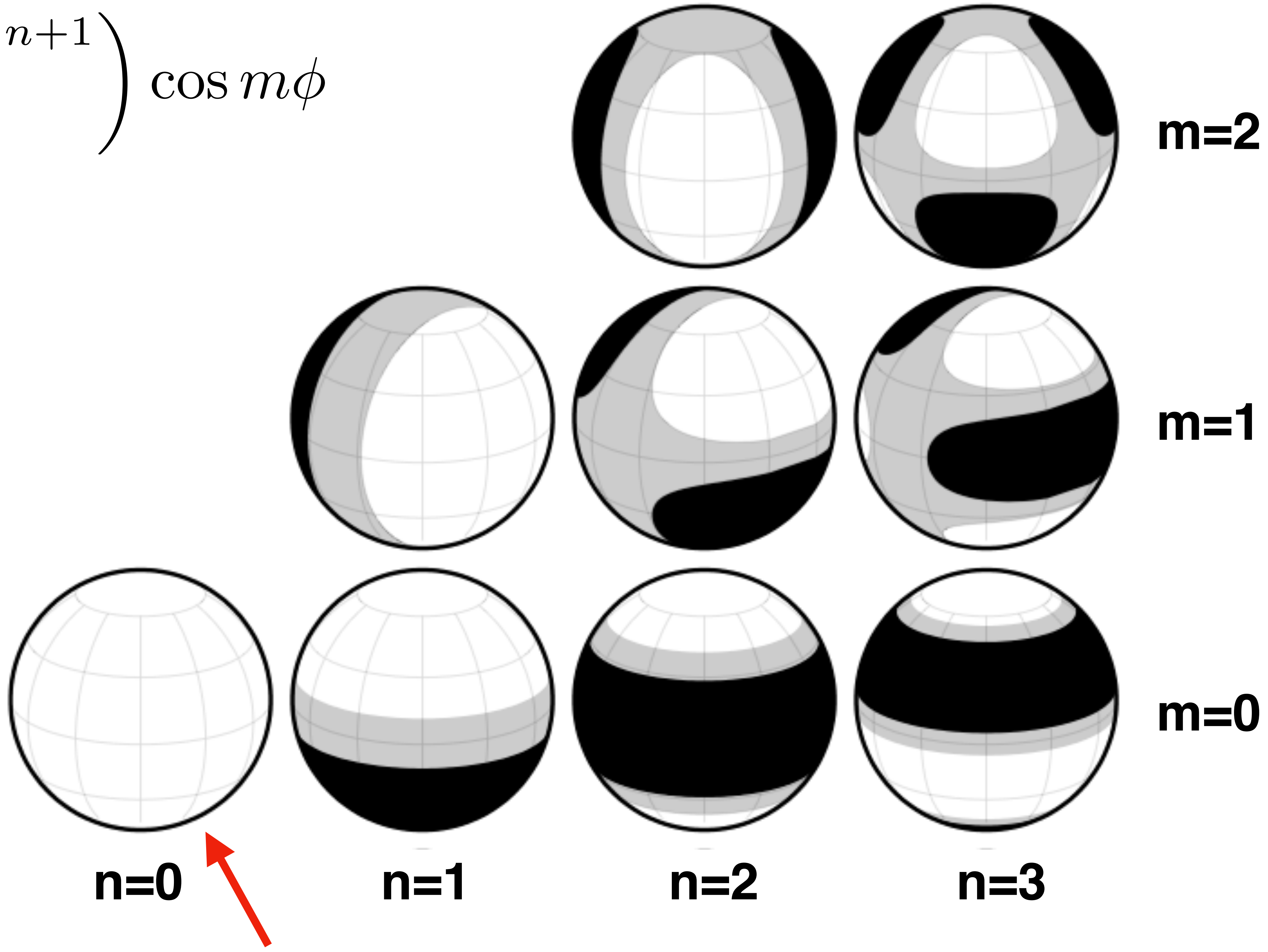
external internal

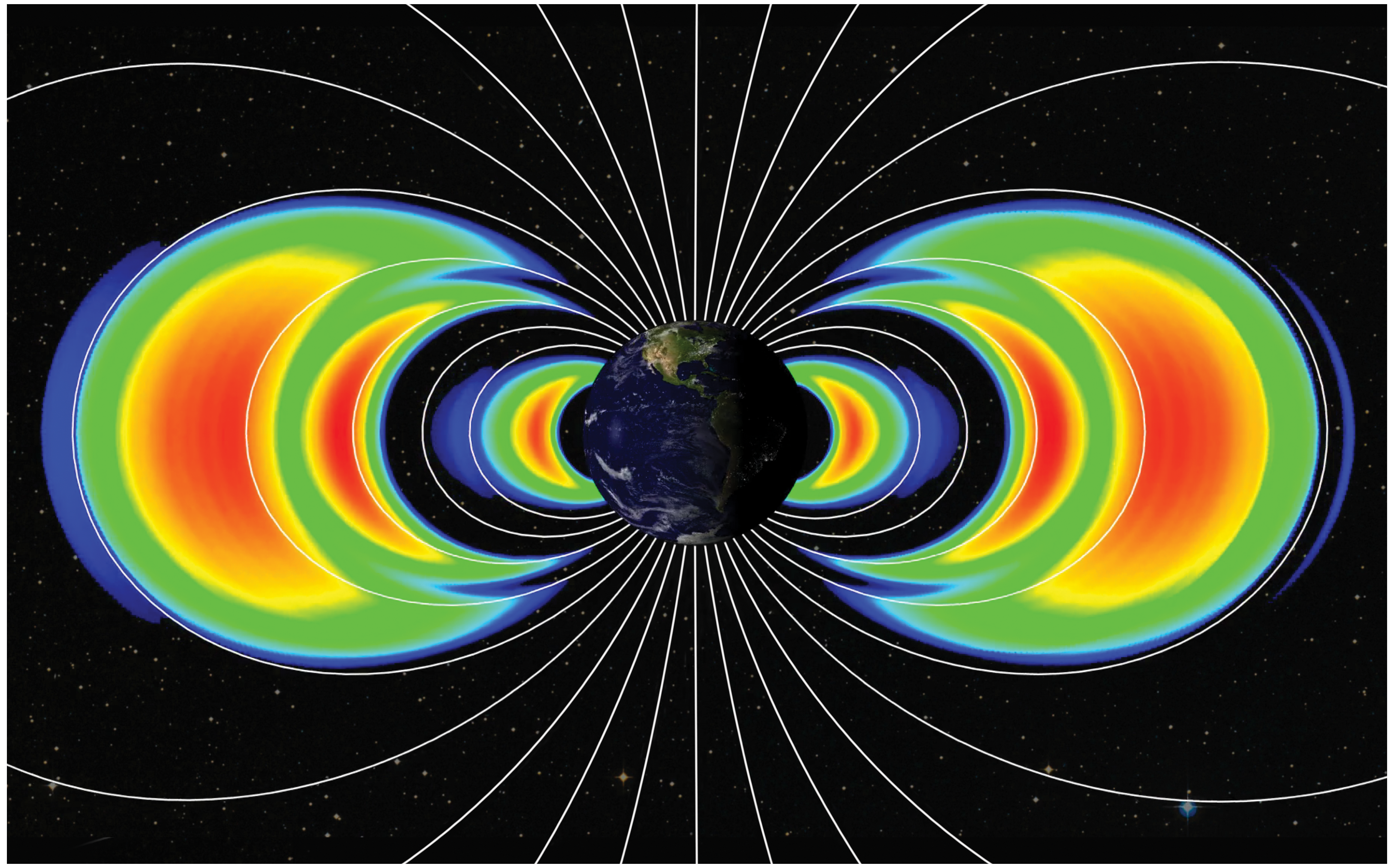
$$\left. + \left((h_e)_n^m \left(\frac{r}{a}\right)^n + (h_i)_n^m \left(\frac{a}{r}\right)^{n+1} \right) \sin m\phi \right]$$

Vector magnetic field is gradient of potential:

$$\mathbf{B} = -\nabla V_m$$







https://svs.gsfc.nasa.gov/hyperwall/index/download/a030000/a030400/a030470/van_allen_probes_discov_new_rad_belt_cal.png

At periods longer than one day fields are mainly generated by the equatorial ring current, with a P_1^0 spherical harmonic geometry.

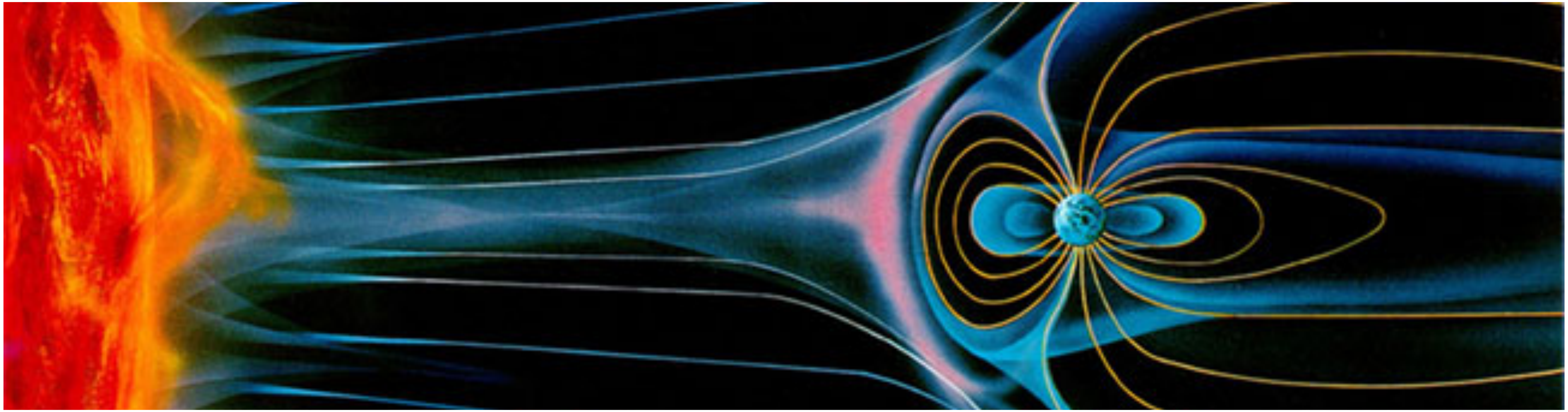
$$\Phi_1^0(r,\theta,\phi) = a_o\left\{ i_1^0(t)\left(\frac{a_o}{r}\right)^2 + e_1^0(t)\left(\frac{r}{a_o}\right)\right\} P_1^0(\cos\theta)$$

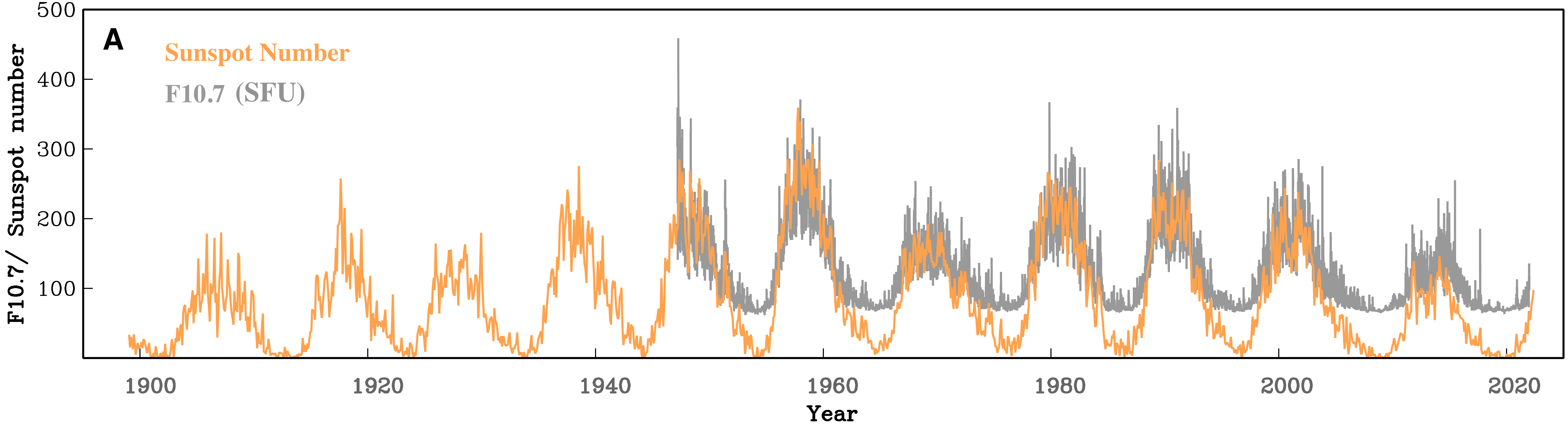
time

The ratio of induced to external field is proportional to electrical conductivity

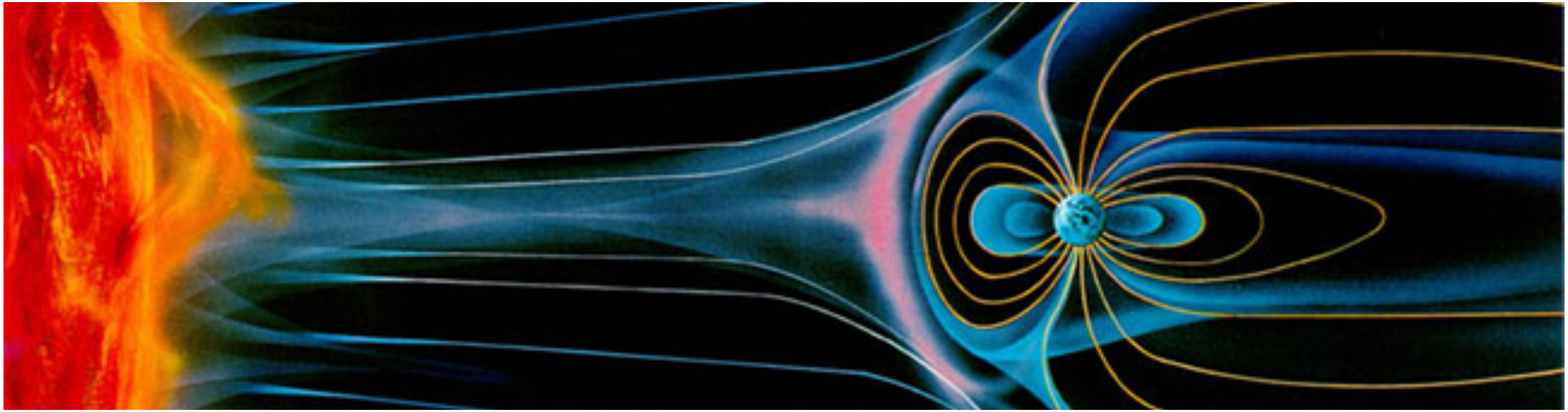
$$Q_1^0(\omega) = \frac{i_1^0(\omega)}{e_1^0(\omega)}$$

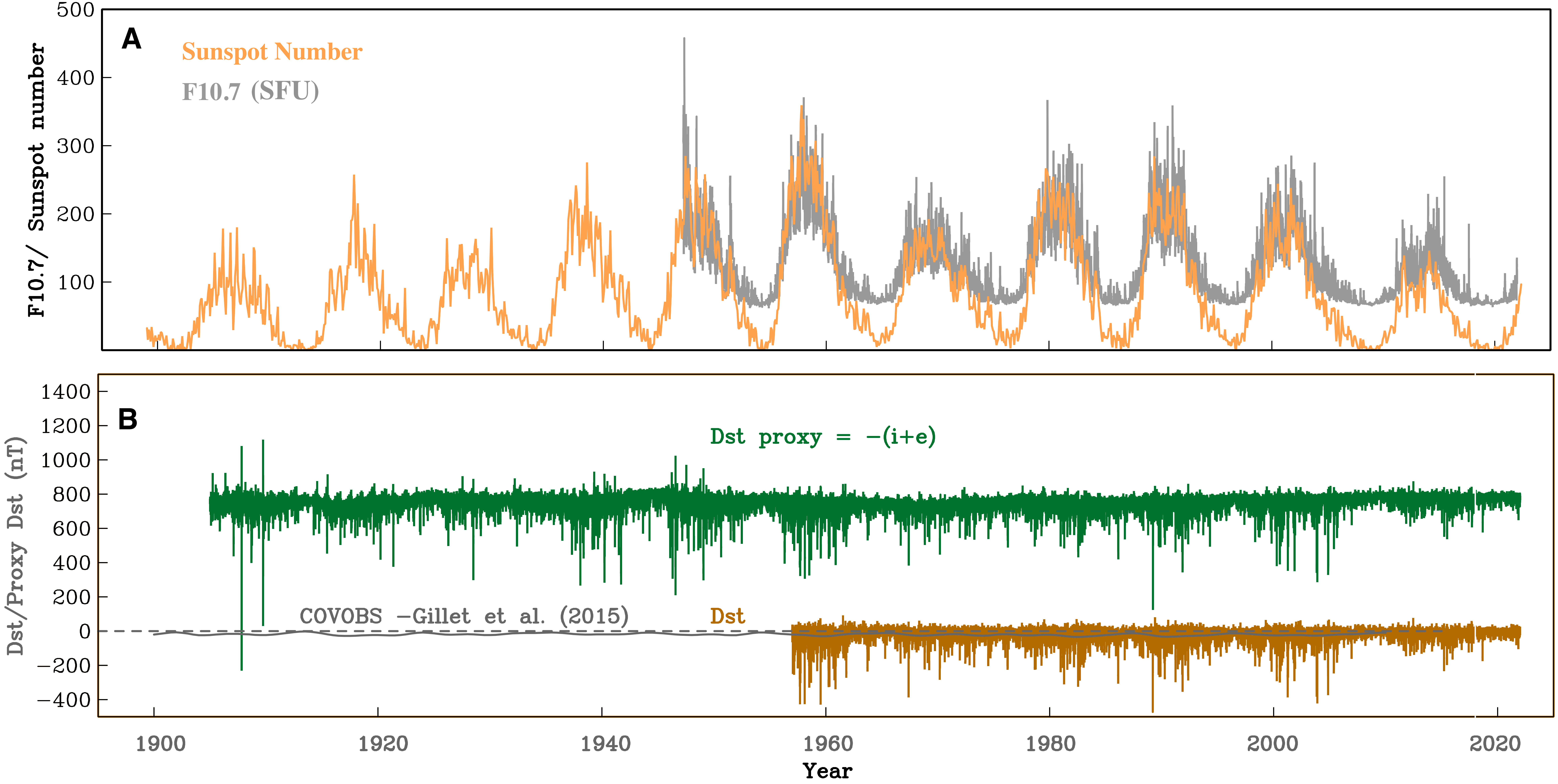
frequency





The solar magnetic field reverses approximately every 11 years, modulating sunspot number, radio flux, magnetic storms, and the strength of the ring current.





Since 1957 the strength of the ring current is tracked by the Dst index, but similar estimates can be made from observatory data back to 1900

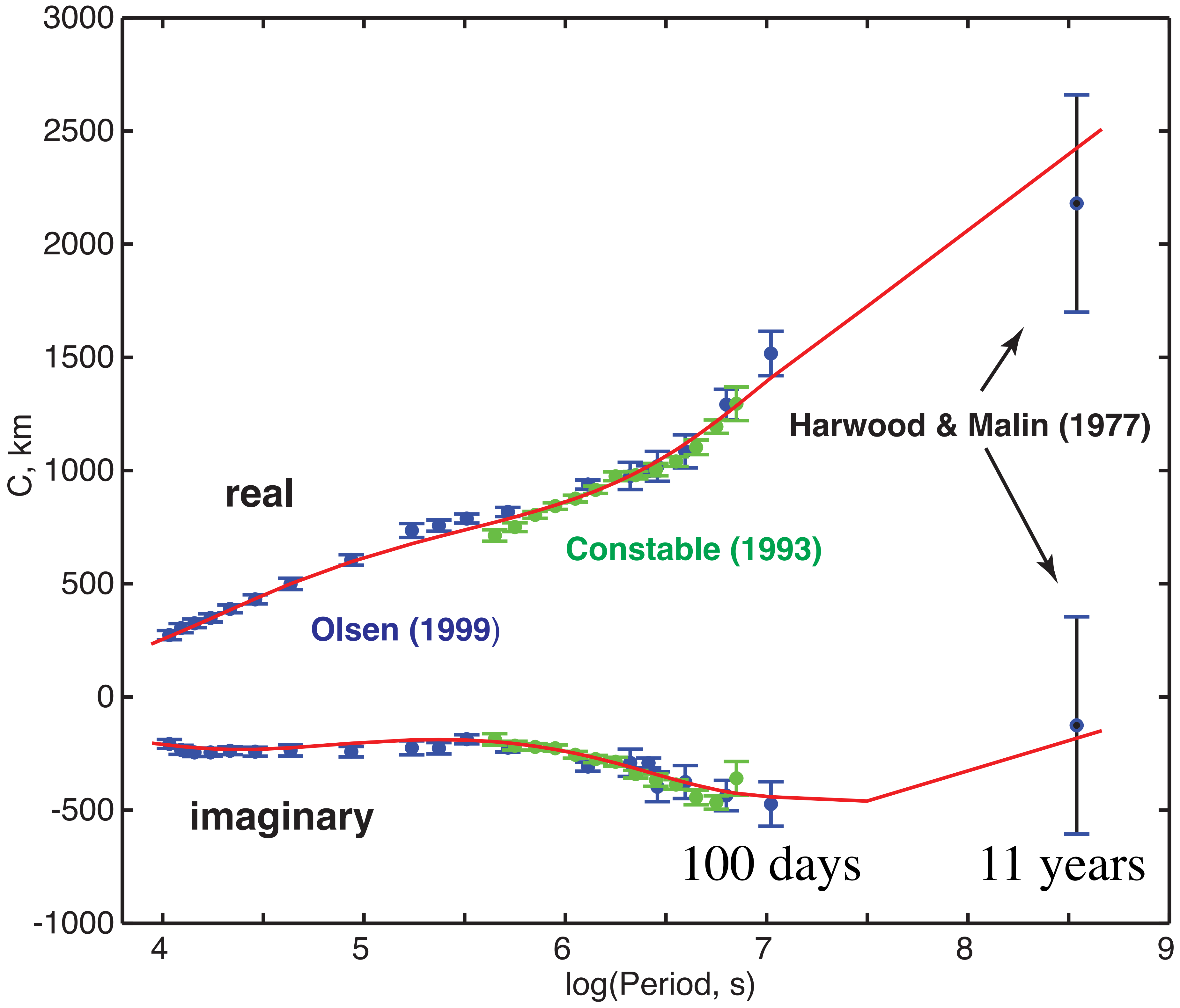
The electromagnetic response Q is mapped into the inductive scale length c, a complex number that is easily predicted from a conductivity-depth profile $\sigma(z)$ (a **forward problem**):

$$c(\omega) = a_o \frac{1-2Q}{2+2Q} \qquad c(\omega) = f(\omega, \sigma(z))$$

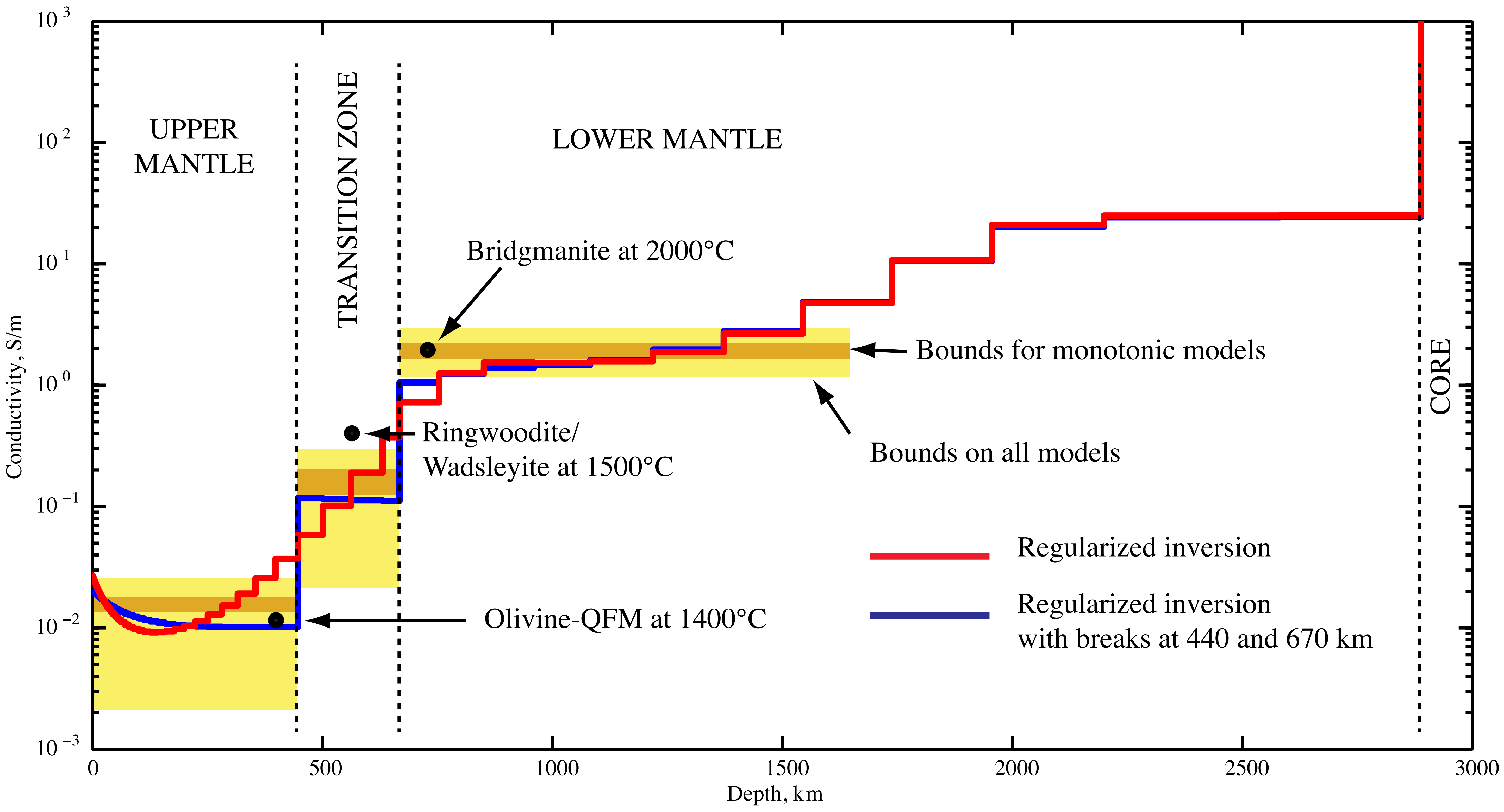
The **inverse problem** seeks to recover $\sigma(z)$ from $c(\omega)$.

$$\sigma(z) = f^{-1}(\omega, c)$$

Data

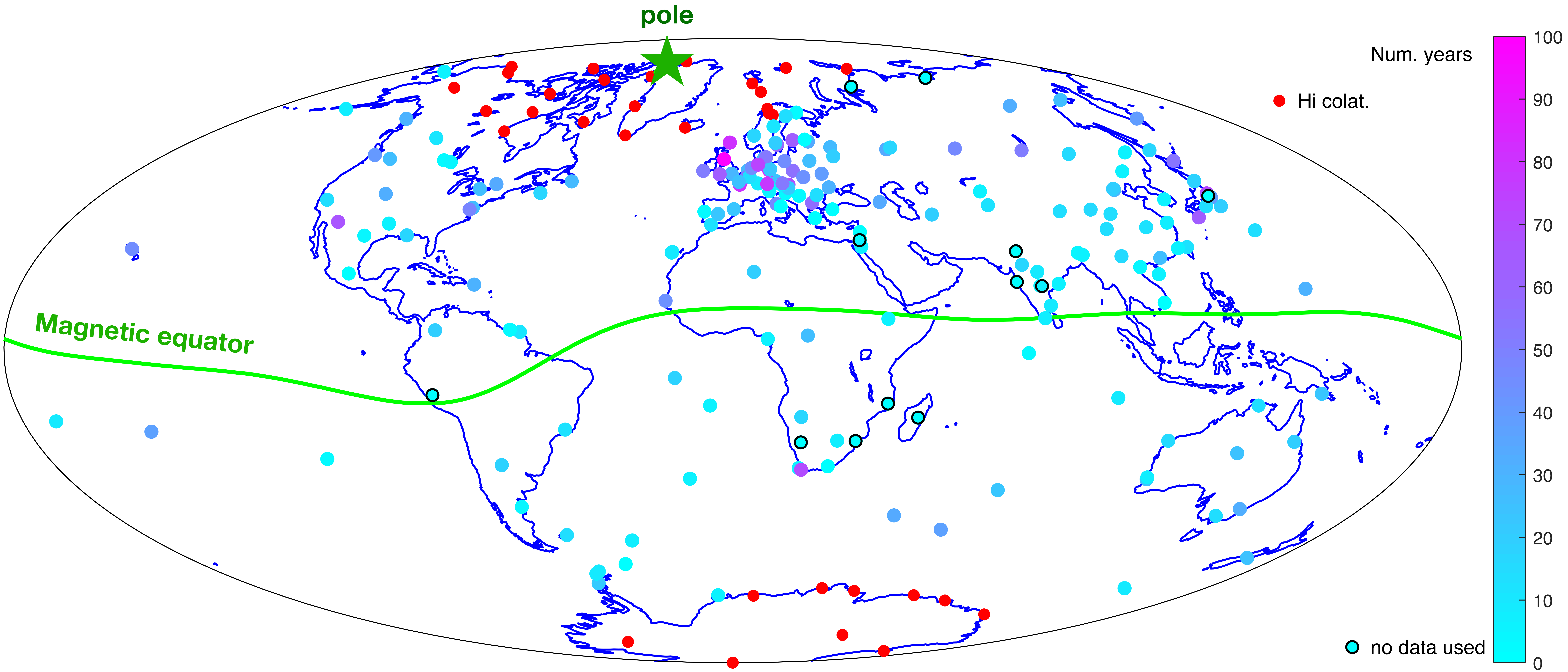


Models



Constable (Treatise on Geophysics, 2015); Medin et al. (2007)

BGS data distribution: Hourly data from 227 observatories between 1900 — 2022.
192 with magnetic colatitudes 25° < θ < 155°, 180 with usable data.
Baselines were corrected using the IGRF/Covobs, outliers removed, and gaps up to 48 hours filled with interpolation.

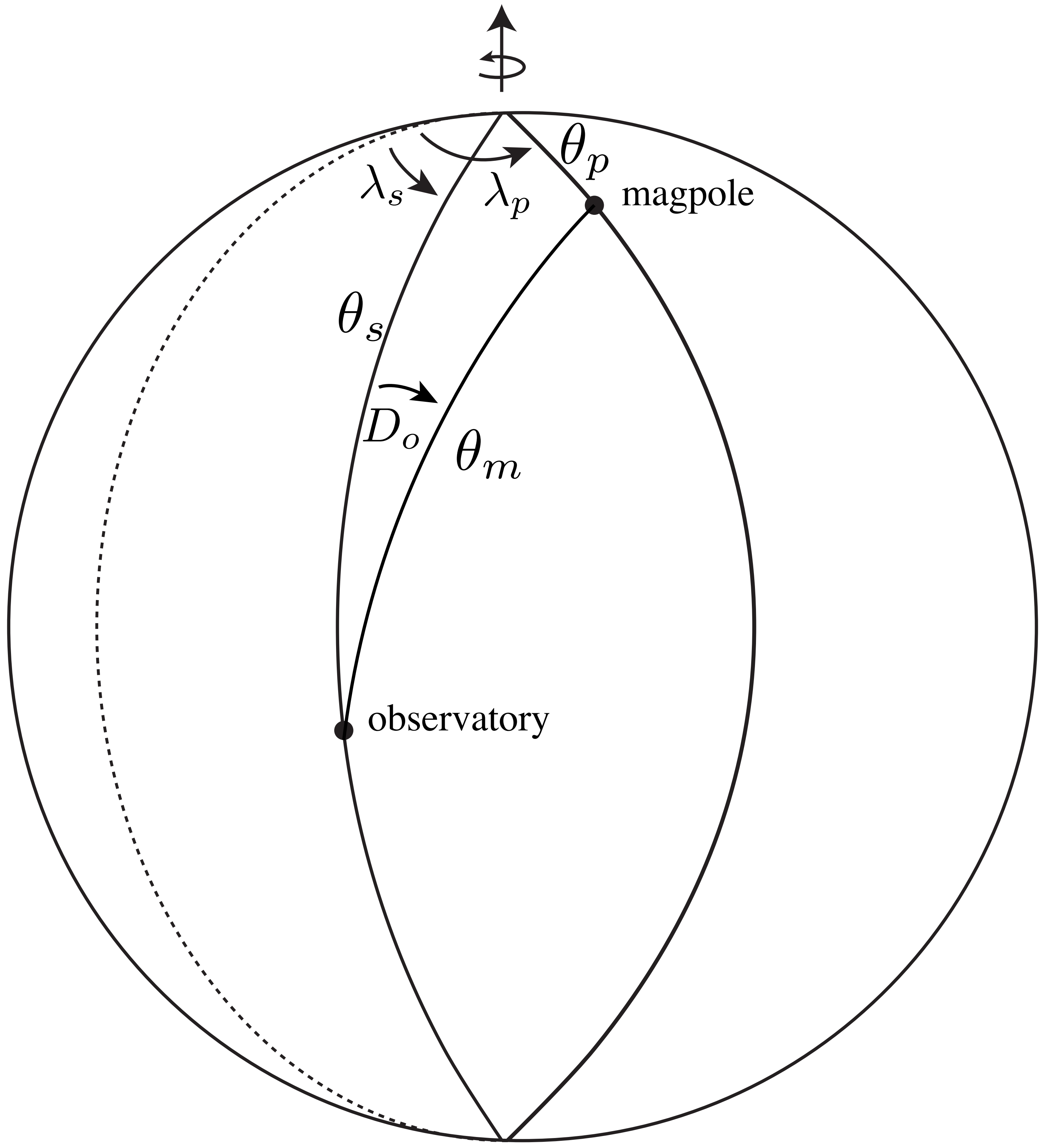


Data are rotated into geomagnetic coordinates year by year to track the movement of the poles

First compute pole positions from the IGRF n=1 terms:

$$\theta = 90 - \text{atan2}\left(-g_1^0, \sqrt{(h_1^1)^2 + (g_1^1)^2}\right)$$

$$\lambda = \text{atan2}\left(-h_1^1, -g_1^1\right) + 360$$



Then compute declination angle to the pole at every observatory

$$D_o = s\cos^{-1}\left(\frac{\cos\theta_p - \cos\theta_m\cos\theta_s}{\sin\theta_m\sin\theta_s}\right)$$

The standard rotation matrix is

$$\mathbf{R} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

but to rotate clockwise we need the transpose with negative declination

$$\mathbf{R} = \begin{bmatrix} \cos(-D_o) & \sin(-D_o) \\ -\sin(-D_o) & \cos(-D_o) \end{bmatrix}$$

- Data are rotated into geomagnetic coordinates year by year to track the movement of the poles
- International Geomagnetic Reference Field (IGRF) or Covobs removed from the time series
- Internal and external fields of P_1^0 geometry fit to the hourly radial and latitudinal fields

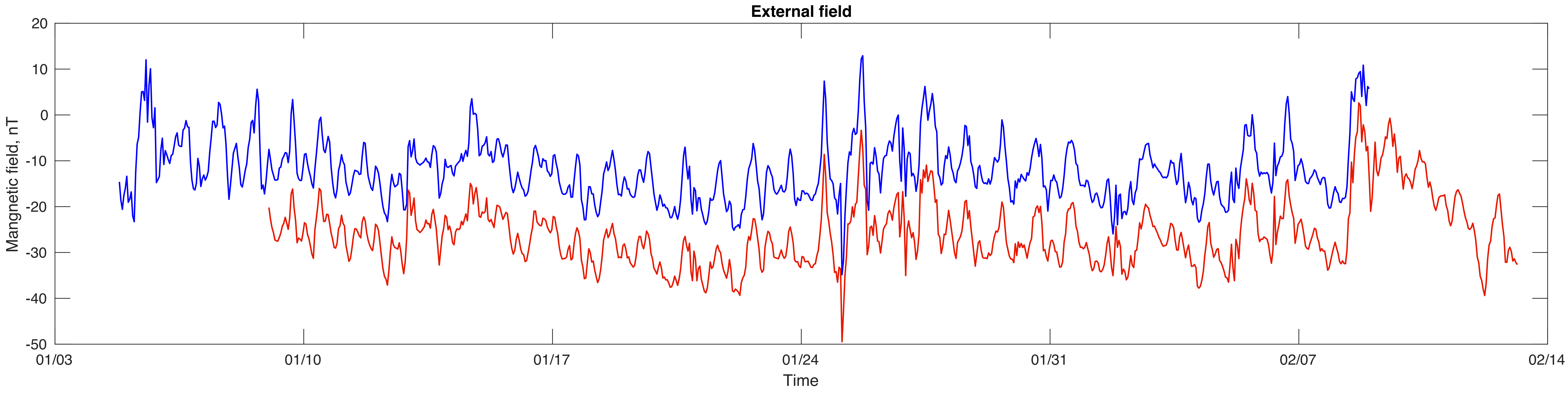
$$\Phi_1^0(r,\theta,\phi) \;=\; a_o\left\{ i_1^0(t)\left(\frac{a_o}{r}\right)^2 \;+\; e_1^0(t)\left(\frac{r}{a_o}\right)\right\} P_1^0\big(\cos\theta\big)$$

$$B_r = -\frac{\partial \Phi}{\partial r} = \left[-e_1^0 + 2 i_1^0 \left(\frac{a}{r}\right)^3\right]\cos(\theta)$$

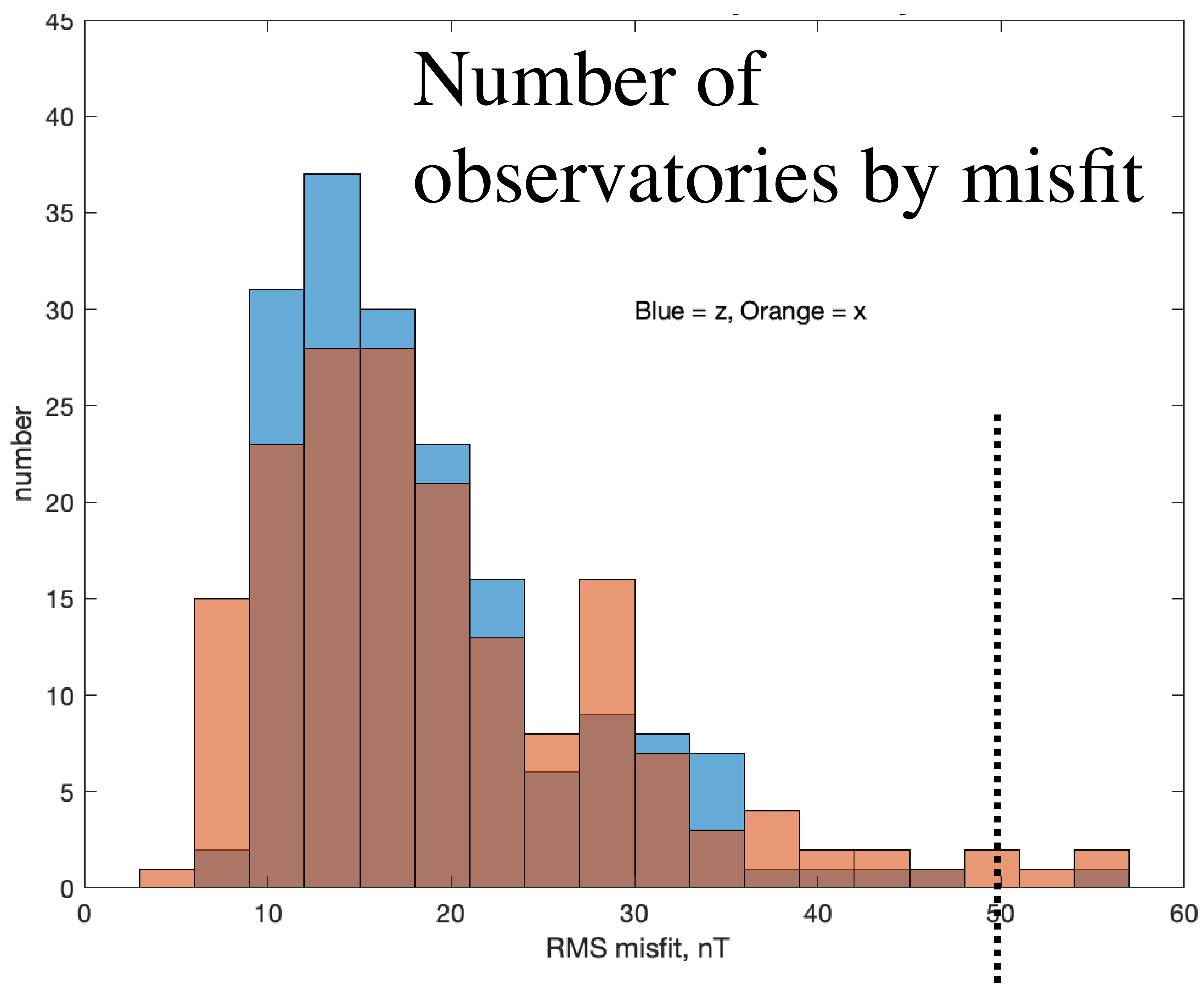
$$B_\theta = -\frac{1}{r}\frac{\partial \Phi}{\partial \theta} = \left[e_1^0 + i_1^0 \left(\frac{a}{r}\right)^3\right]\sin(\theta)$$

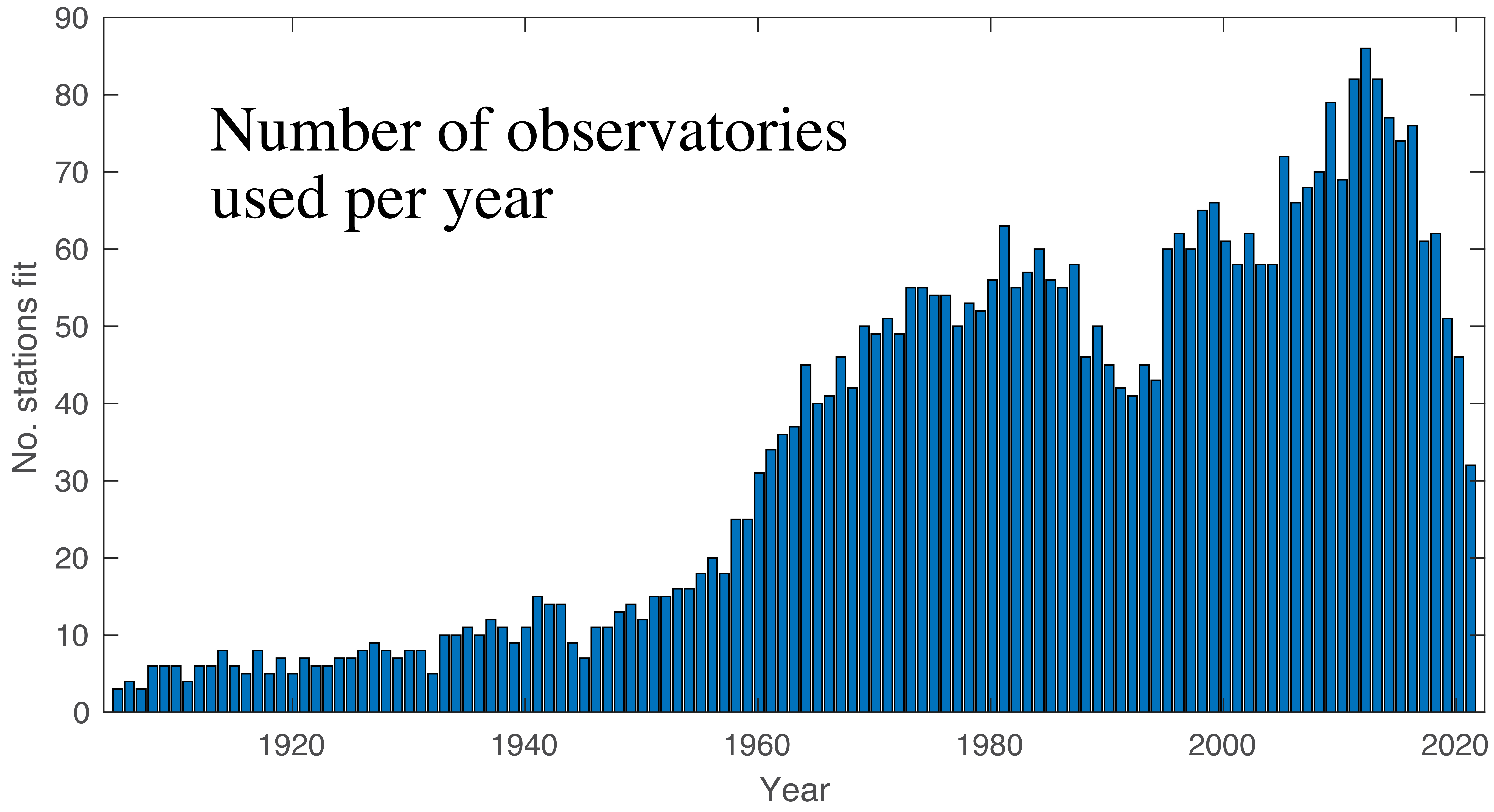
$$\Rightarrow \begin{bmatrix} -\cos(\theta) & 2(a/r)^3\cos(\theta) \\ \sin(\theta) & (a/r)^3\sin(\theta) \end{bmatrix} \begin{bmatrix} e_1^0 \\ i_1^0 \end{bmatrix} = \begin{bmatrix} B_r \\ B_\theta \end{bmatrix}$$

- Fits are made year by year using only observatories with data for that particular whole year
- One month overlaps between years gives baseline corrections from changes in observatory distribution

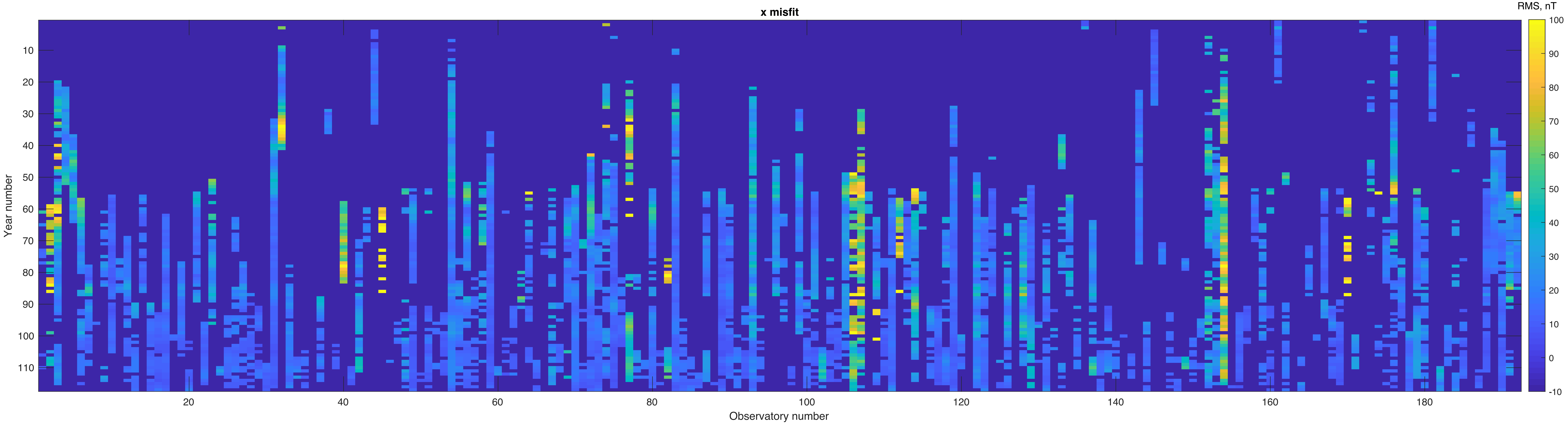


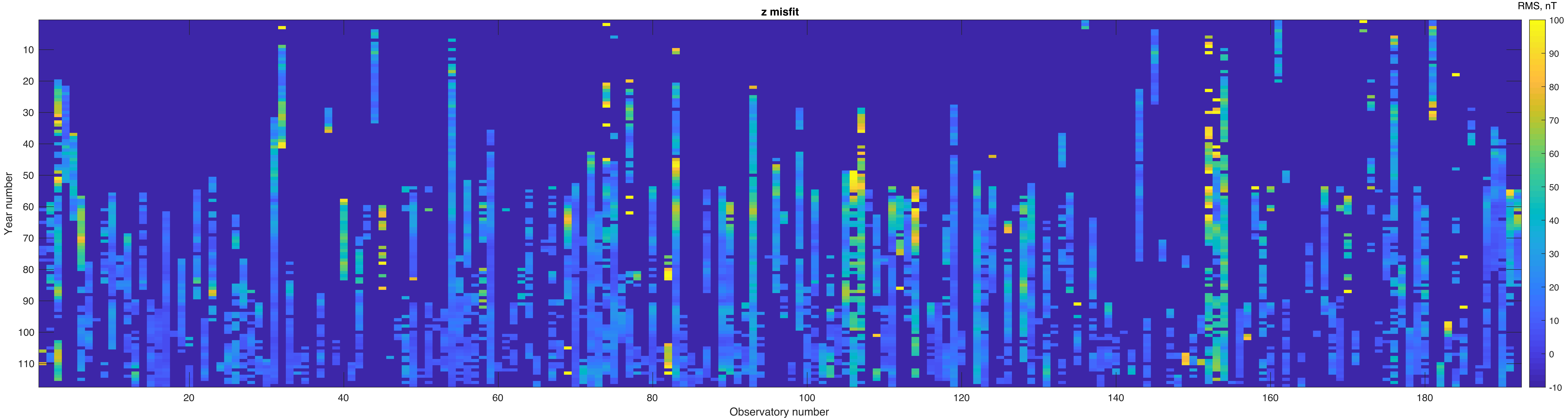
Average misfit is around 15 nT. Minimum number of observatories used per year is 3 — 86.





Any observatory-year with > 50 nT misfit was iteratively excluded


x misfit
RMS, nT
Year number
Observatory number
10
20
30
40
50
60
70
80
90
100
110
20
40
60
80
100
120
140
160
180
100
90
80
70
60
50
40
30
20
10
0
-10


z misfit
RMS, nT
Year number
Observatory number
10
20
30
40
50
60
70
80
90
100
110
20
40
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120
140
160
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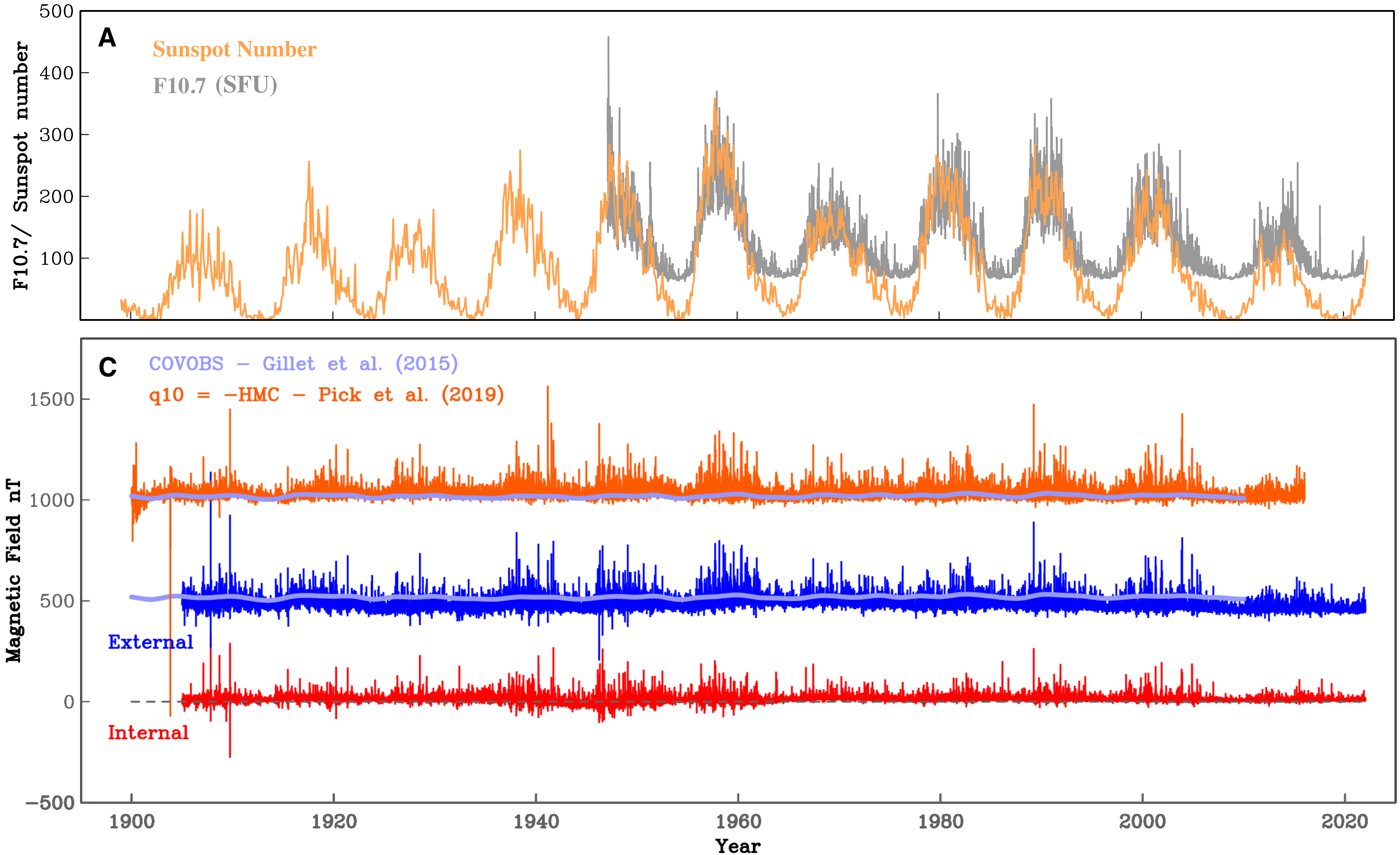
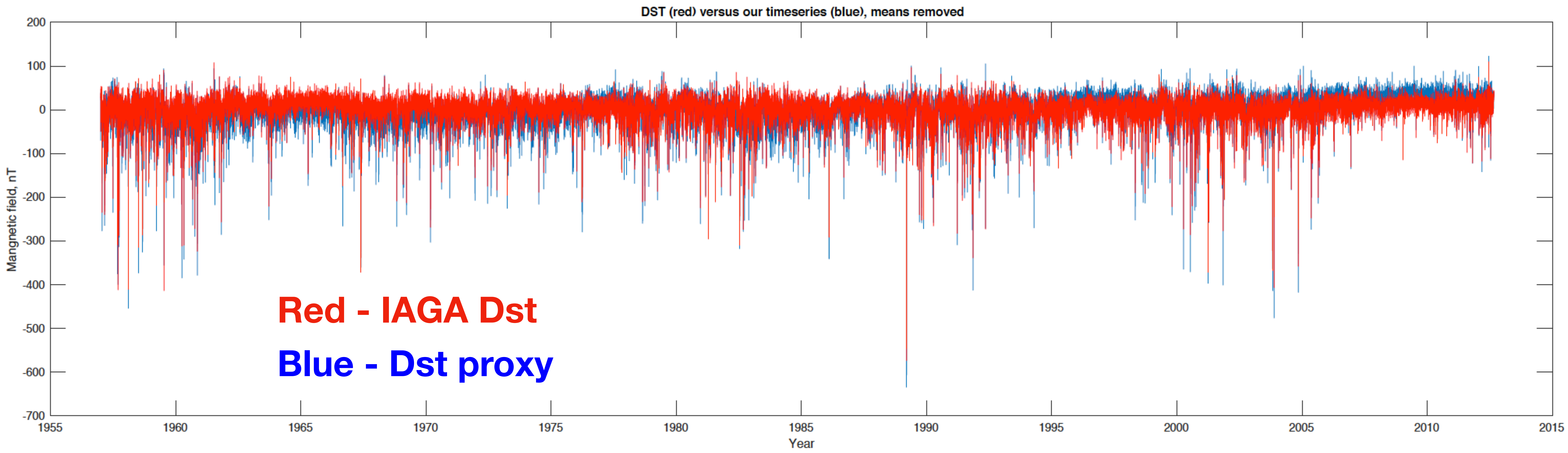
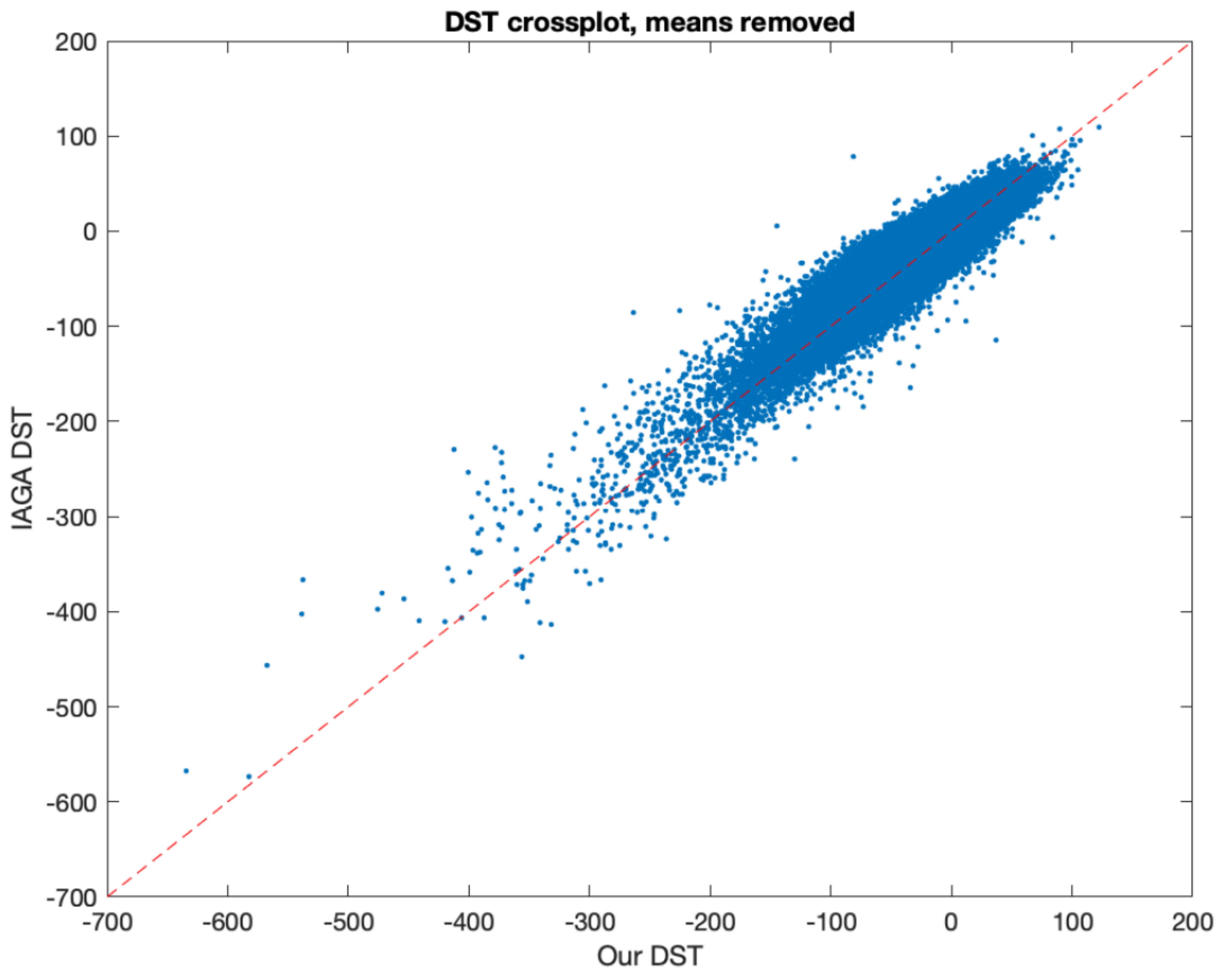
30
20
10
0
-10

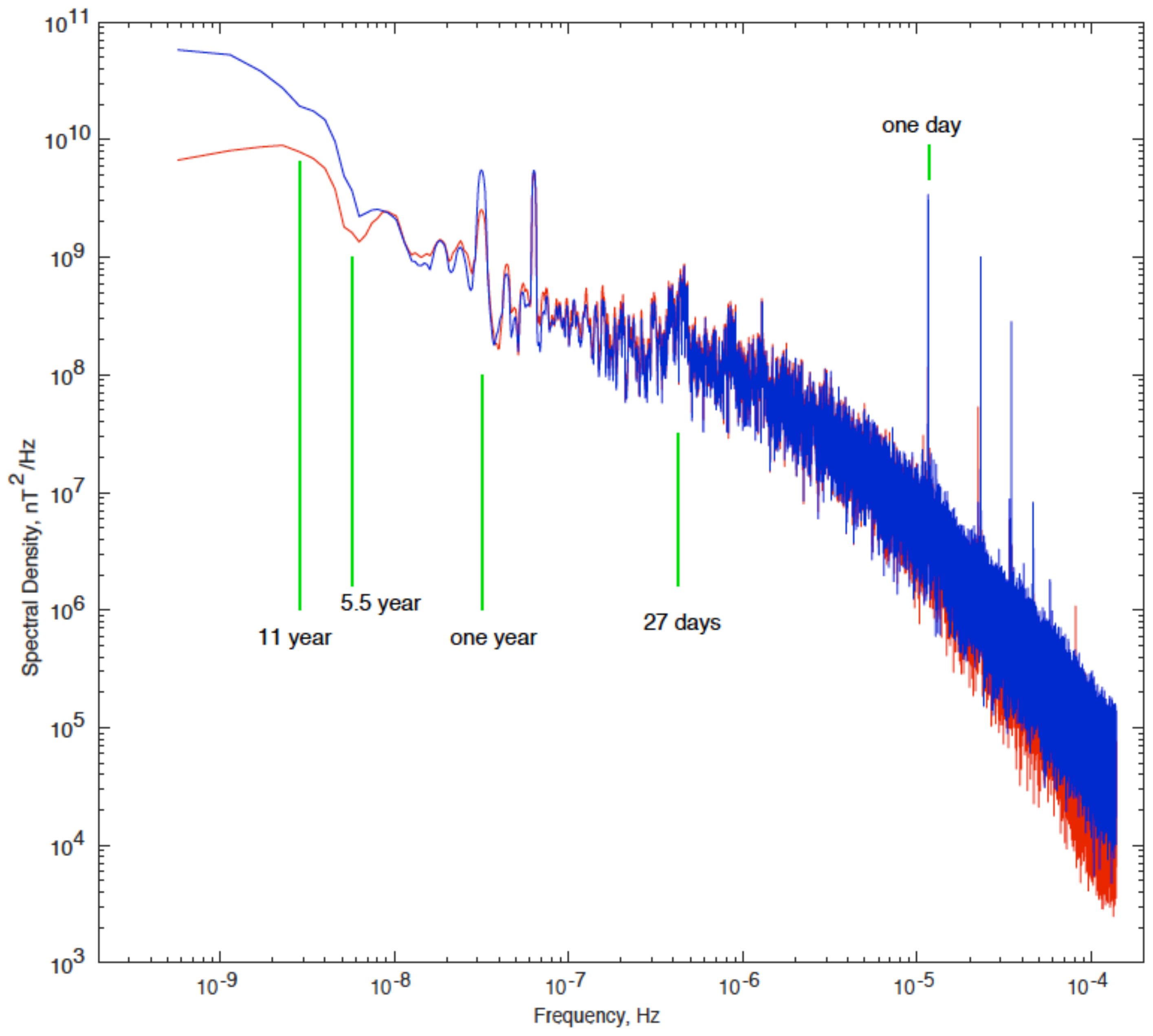


A
Sunspot Number
F10.7 (SFU)
F10.7/ Sunspot number
500
400
300
200
100
C
COVOBS – Gillet et al. (2015)
q10 = –HMC – Pick et al. (2019)
Magnetic Field nT
1500
1000
500
0
–500
External
Internal
1900
1920
1940
1960
1980
2000
2020
Year

If we add the internal and external time series we have a proxy for Dst, which we can compare with IAGA Dst (back to 1957)

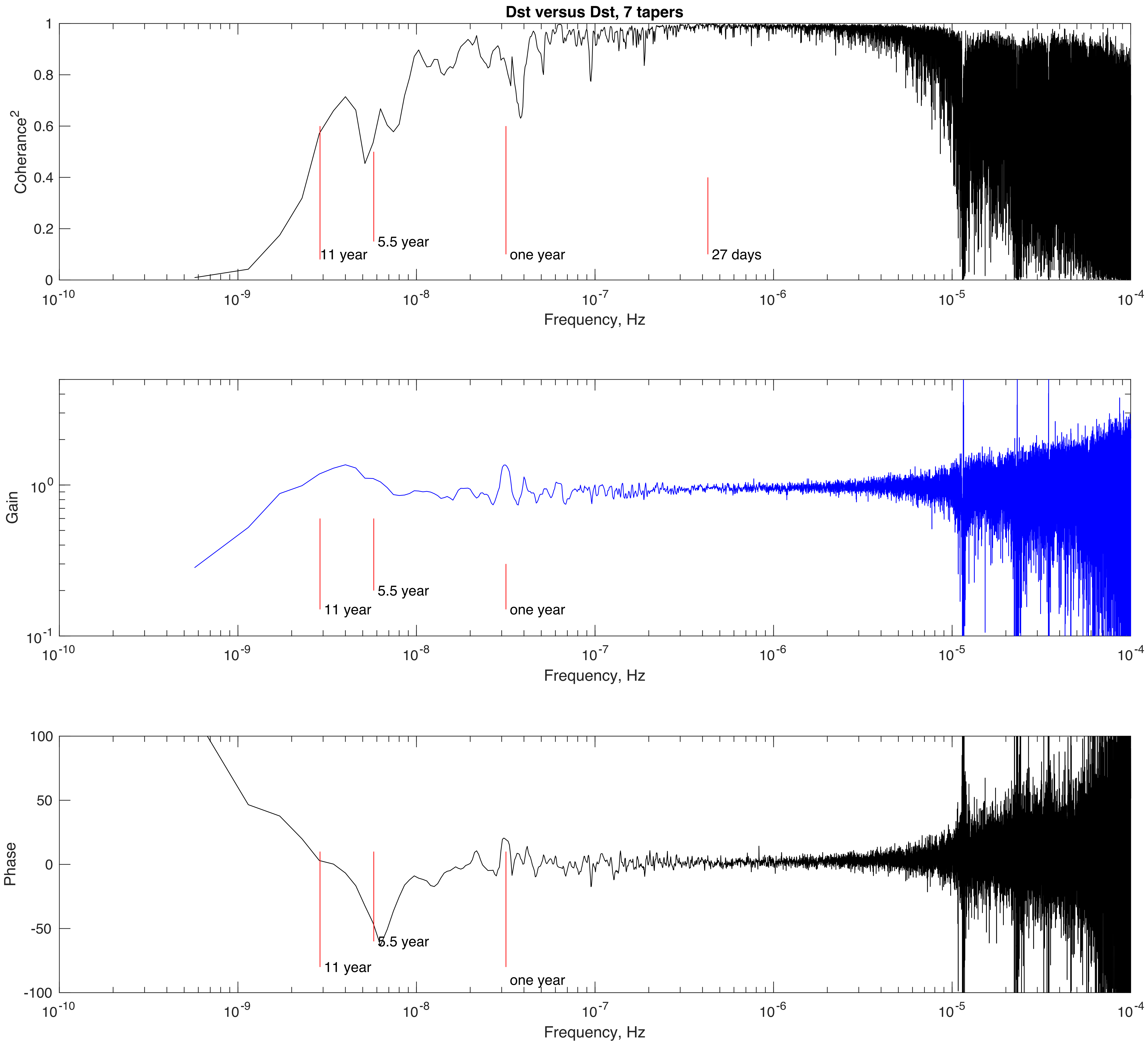




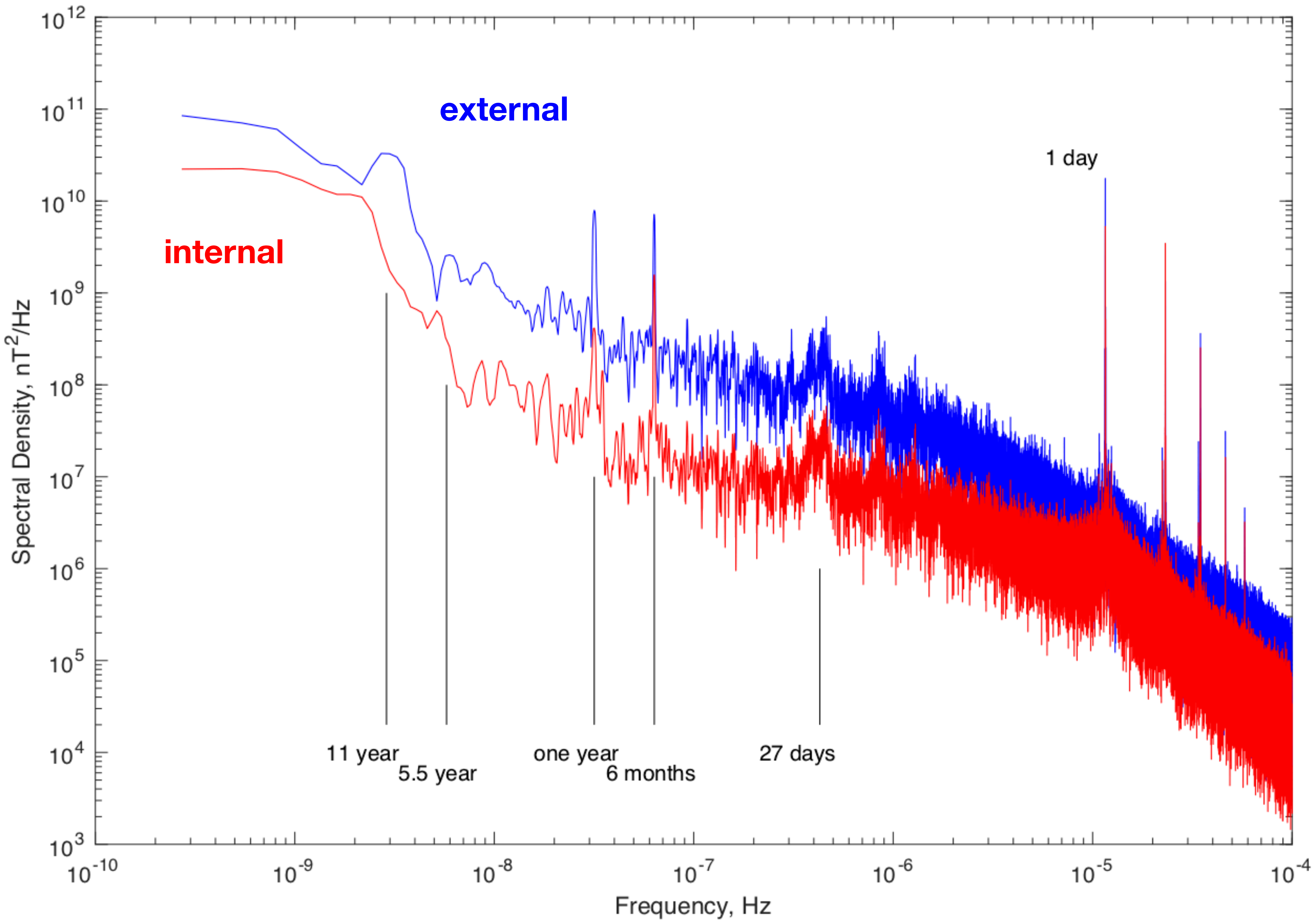


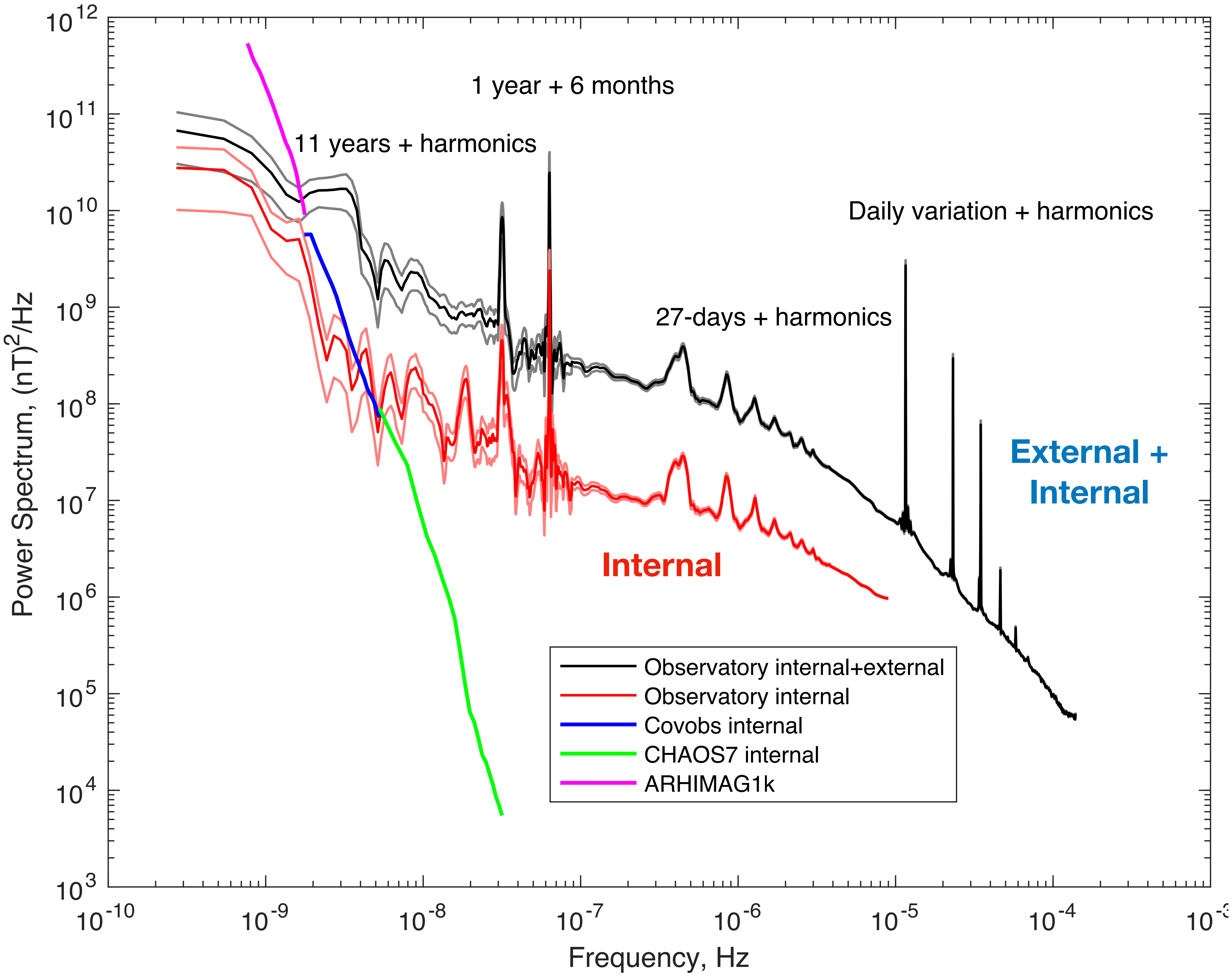
IAGA Dst baselines are corrected every year, so power falls off at longer periods

A cross spectrum between IAGA Dst and proxy Dst shows the long period differences.



Now we need to convert our **time series** to the **frequency domain.** We can compute the spectrum of the internal and external time series as well as the cross-spectrum.



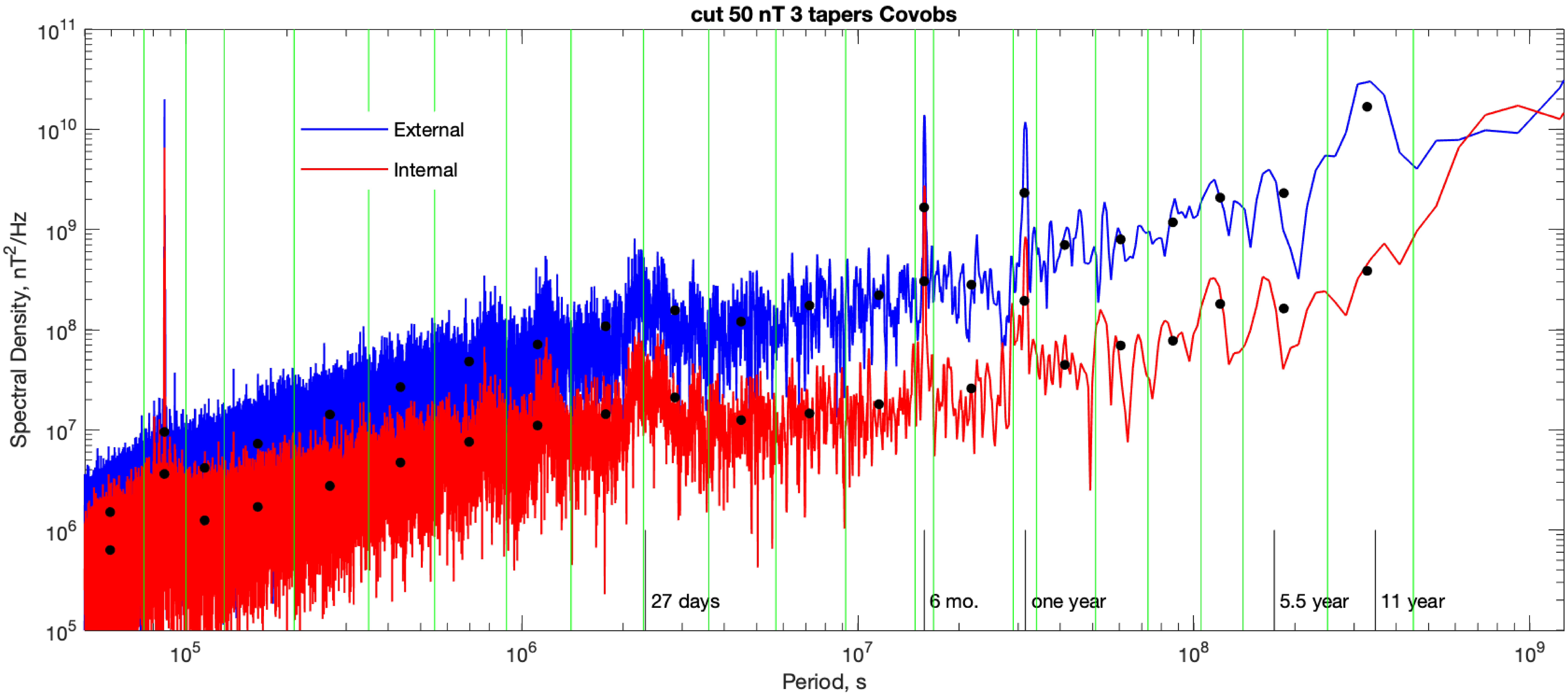


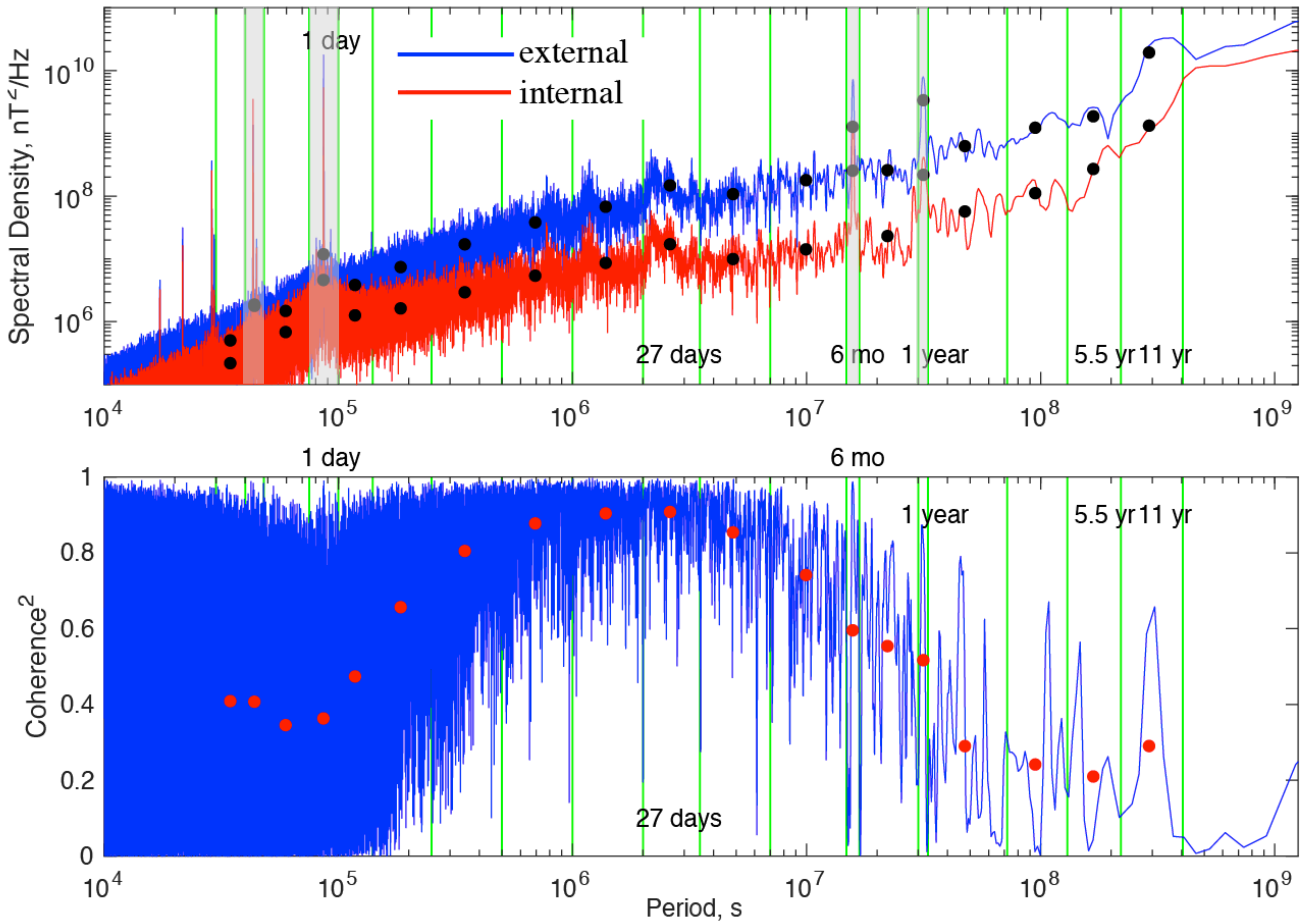
Spectra of the internal and external time series computed with the adaptive multi-taper estimates (which also give uncertainty estimates).

There is a clear peak in the external field at 11-year period, as well as the 5.5 year first harmonic (and the usual suspects at 1 year, 6 months, 27 days, 1 day etc.).

We don't have a code for adaptive cross-spectra so we use a fixed number of tapers (3-5).

From now on we will work in period, not frequency.





We only want 5-6 estimates of *c* per decade.

Further variance reduction is obtained by band averaging (green lines show bands). Periods of one year and one day (and harmonics) are not P_1^0 and so are excluded in narrow bands. Dots show averages in bands.

The **EM response function** Q is the frequency dependent complex ratio of internal to external fields, which in a noise-free world is the ratio of the cross spectrum to the external power spectrum:

$$Q(\omega) = \frac{i(\omega)}{e(\omega)} = \frac{S_{ei}}{S_e}$$

S_{ei} = complex cross spectrum

S_e = external power spectrum

ζ_e = noise to signal ratio

$\eta = \zeta_e/\zeta_i$ = ratio of noise ratios

γ = spectral coherency

r = Earth radius

However, noise in the spectra biases the estimate up and down

$$Q_{ie} = \frac{S_{ie}}{S_e} = Q\left[\frac{1}{1+\zeta_e}\right] \qquad Q_{ei} = \frac{S_{ei}}{S_i} = Q^{-1}[1+\zeta_i]$$

We can correct for the bias if we know the noise characteristics

$$Q_{corr} = \frac{Q_{ie}}{2}\left[1-\eta+\sqrt{(1-\eta)^2+4\eta/\gamma^2}\right]$$

Here we assume the absolute noise is the same in the internal and external field spectra. Finally, for P_1^0 source field geometry the inductive scale length is

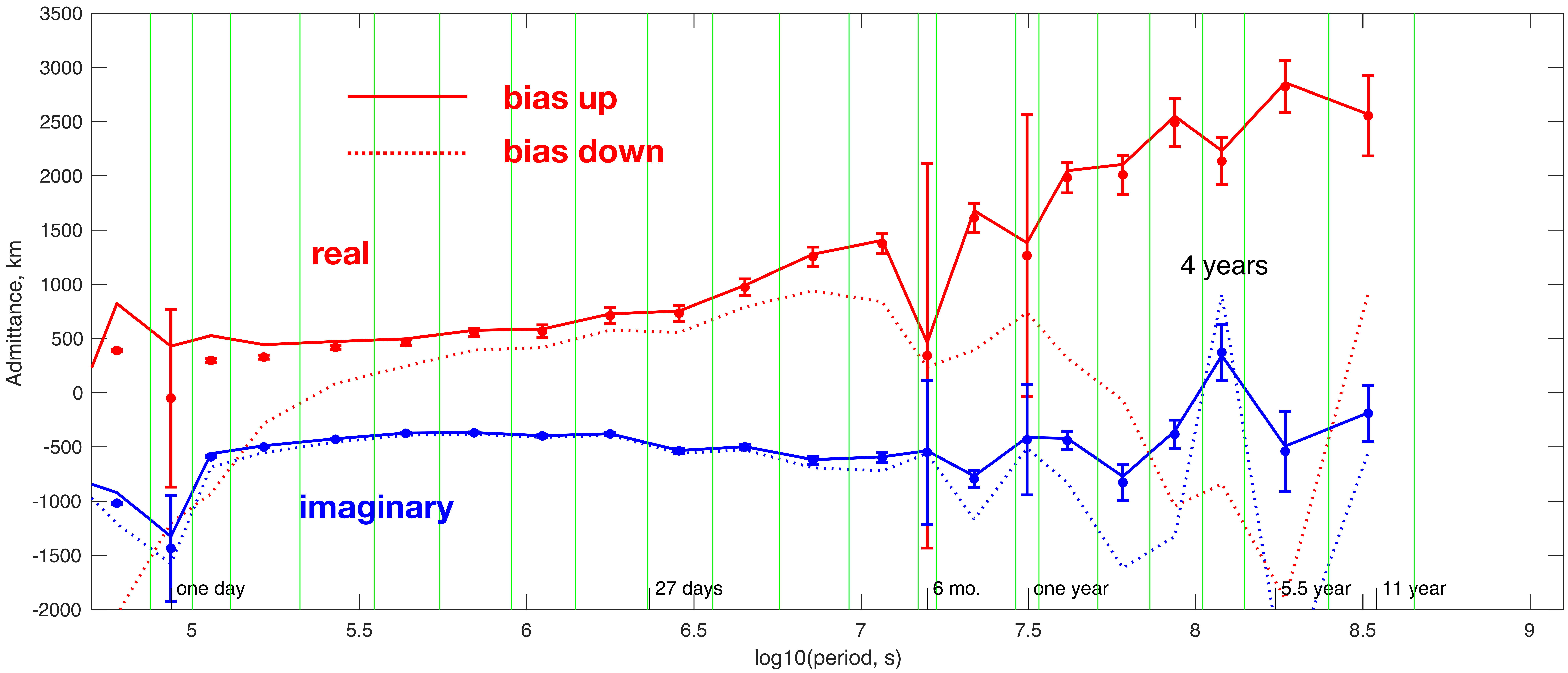
$$c = \frac{r}{2}\frac{(1-2Q)}{(1+Q)}$$

Here we assume the absolute noise is the same in the internal and external field spectra. Finally, for P_1^0 source field geometry the inductive scale length is

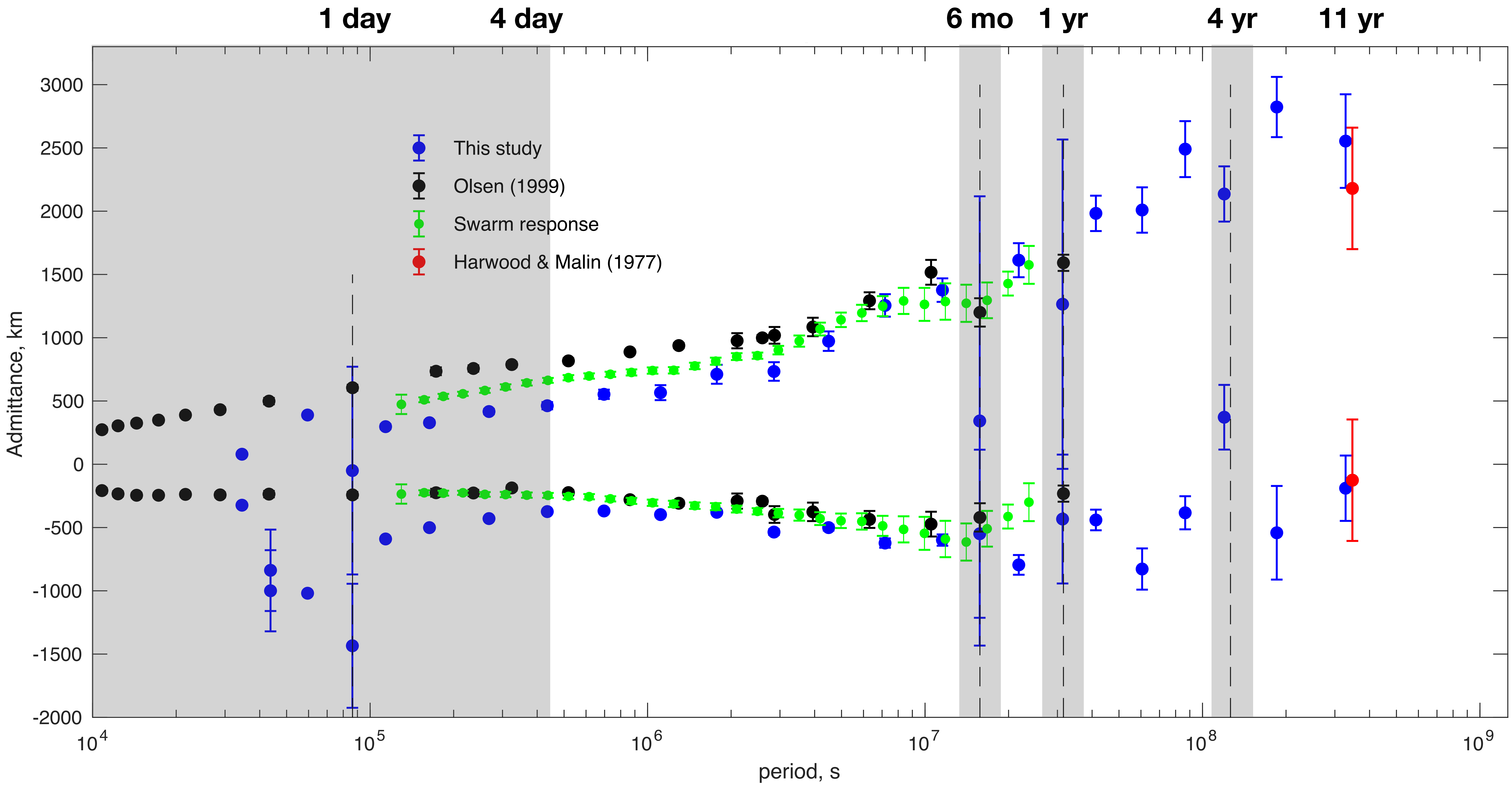
$$c = \frac{r}{2}\frac{(1-2Q)}{(1+Q)}$$

Errors. Errors are important! While errors can be expected to be equal in the real and imaginary components of the cross spectrum, the relationship to c is non-linear. We estimate errors in the spectrum from the variance in the band averages by running 1,000 simulations sampling these errors and projecting them into c.

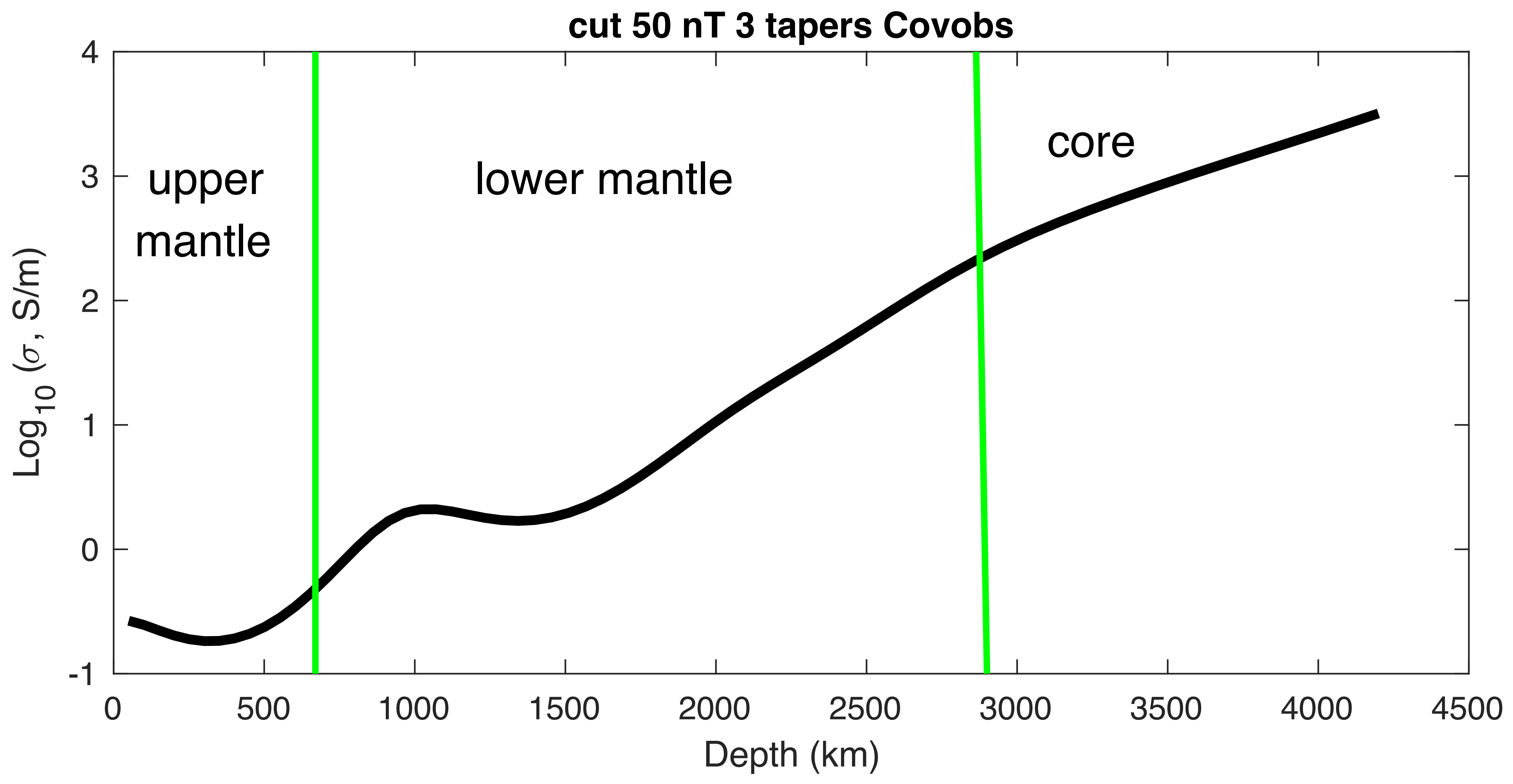
Finally we get our estimates of c, corrected for bias and with errors generated from the variance in the mean of the band averages. Periods of 1 day, 6 months, and 1 year are known not to be P_1^0, but ~4 years appears so too.

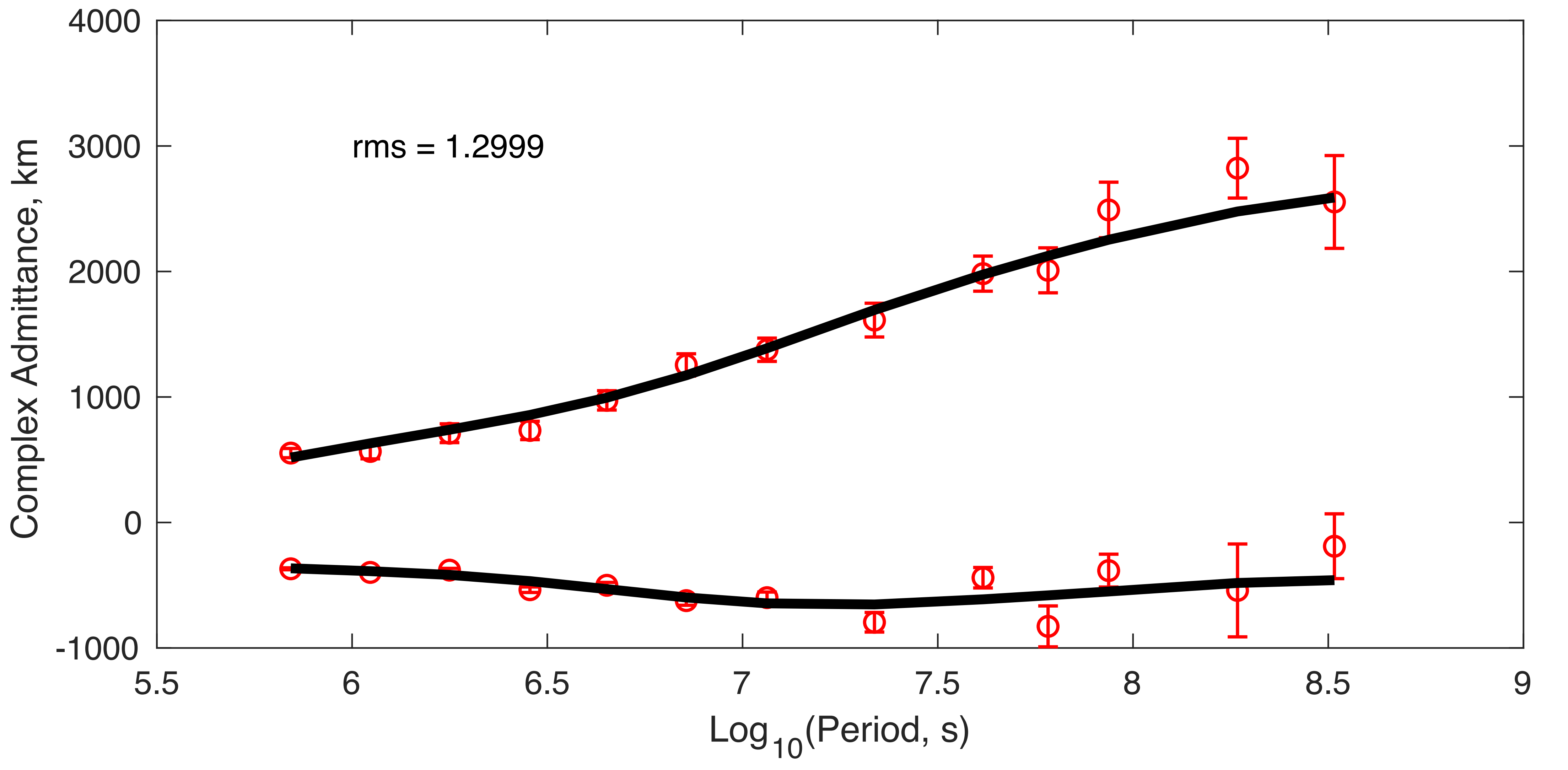


We have good agreement between our results and previous work but have filled in the 1—11 year gap. For inversion we invert periods > 4 days, excluding 1/2, 1, and 4 years.

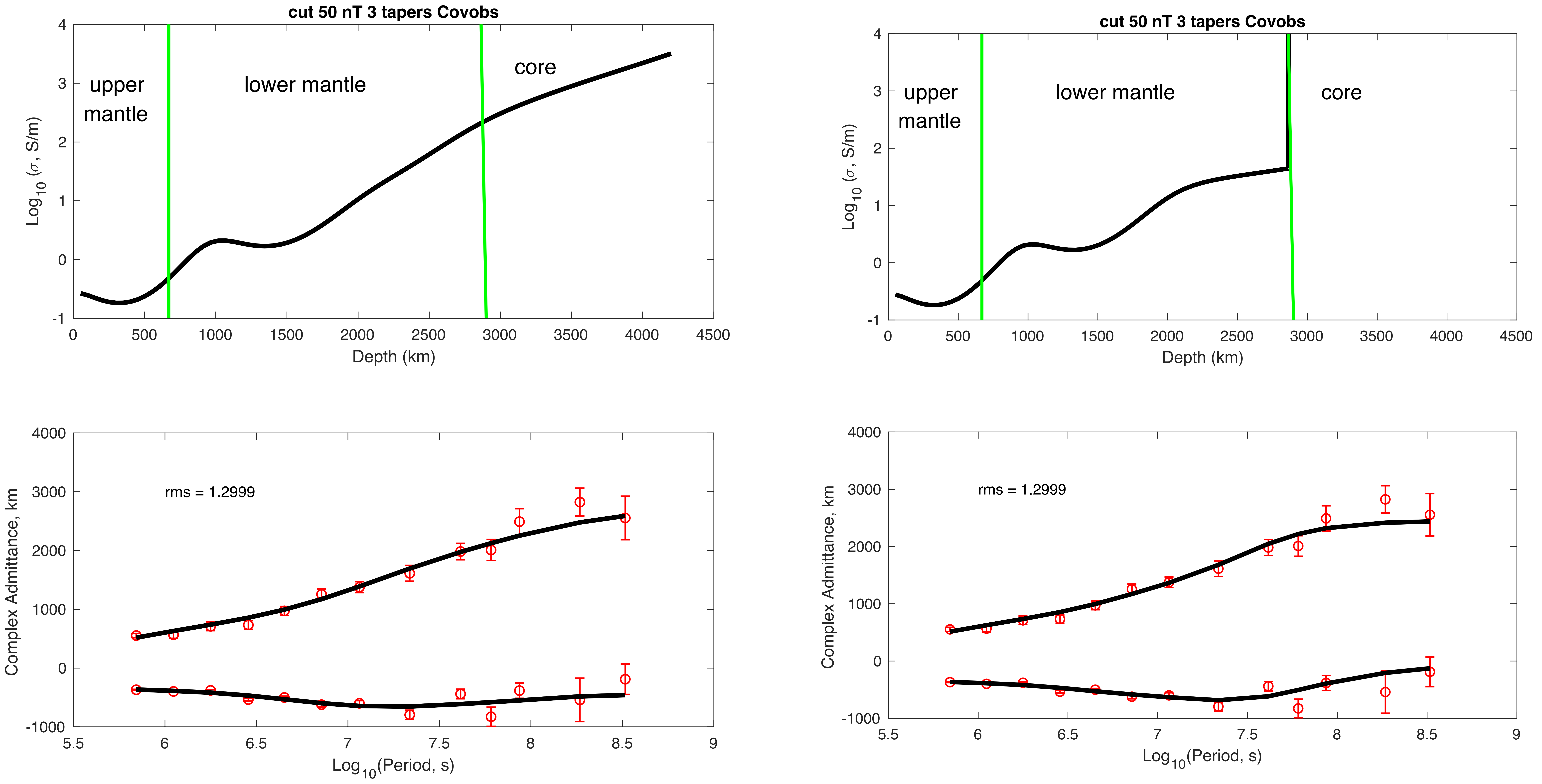


A regularized ("Occam") inversion that is maximally smooth in a 1st derivative sense. Perhaps we see the core?

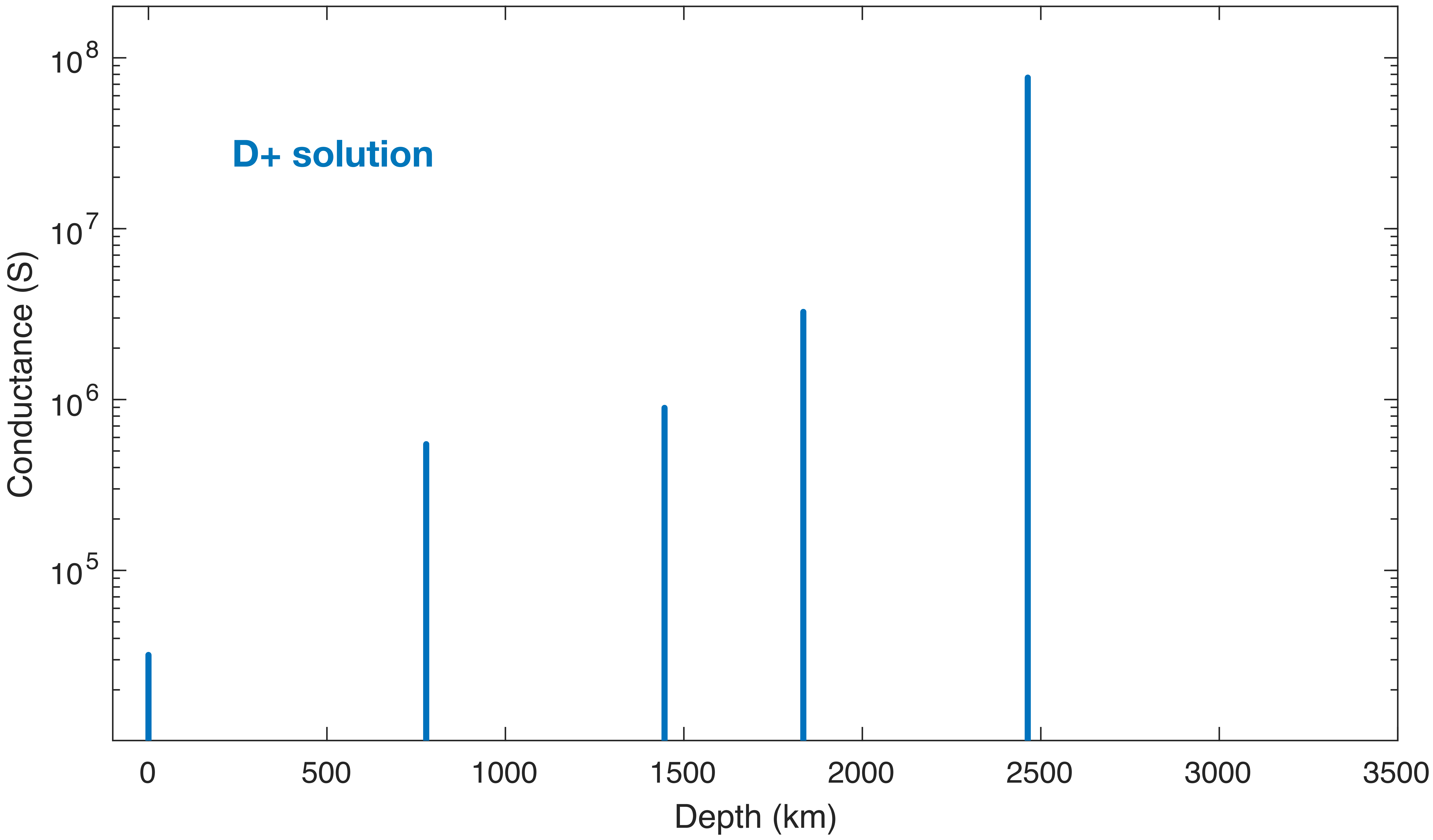




Indeed, fixing the core at its known conductivity and depth does not alter the ability to fit the data. But do we need it? And is the jump in conductivity at ~1800 km depth real? We need Uncertainty Quantification (UQ)!



Bring on the Bayesians? One thing we know about 1D-MT is that an acceptable model, indeed the **best** model in a LS sense, is Parker's D+ model, which has conductivities that are either zero or infinite. So we can't hope to bound conductivity (or resistivity) at any given depth.   We need some smoothing, either by regularization or layering.



The Bayesian methods seek to quantify uncertainty by randomly sampling models and seeing which ones fit the data, but this is slow and needs careful tuning. An alternative: **Randomize Then Optimize.** We can work with Bayes' theorem by randomizing the data rather than the model. We draw randomly and repeatedly from the data, guided by the error structure, and use a standard optimization to compute the associated models, either by regularized inversion or Marquardt fitting.

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Uncertainty quantification for regularized inversion of electromagnetic geophysical data—Part I: motivation and theory

Uncertainty quantification for regularized inversion of electromagnetic geophysical data – Part II: application in 1-D and 2-D problems

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RTO: Randomize Then Optimize. This is very much more efficient because we rarely have to reject models, since our optimization machinery is good at finding models that fit the data. And because we are not doing a random walk through model space, each optimization is independent and can be parallelized.

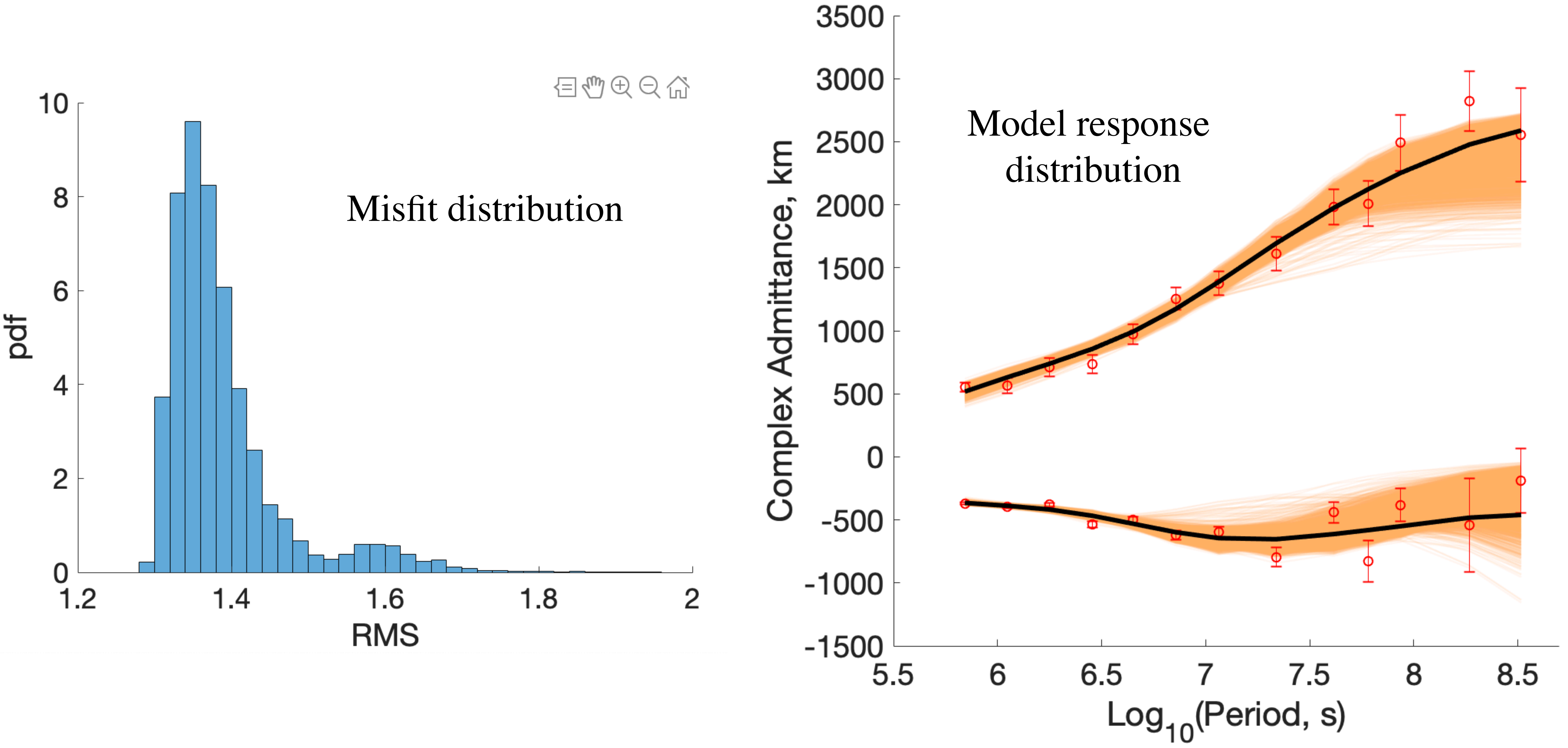
randomize this → $P(data|model)$

instead of this → $P(model)$

$$P(model|data) = \frac{P(data|model)P(model)}{P(data)}$$

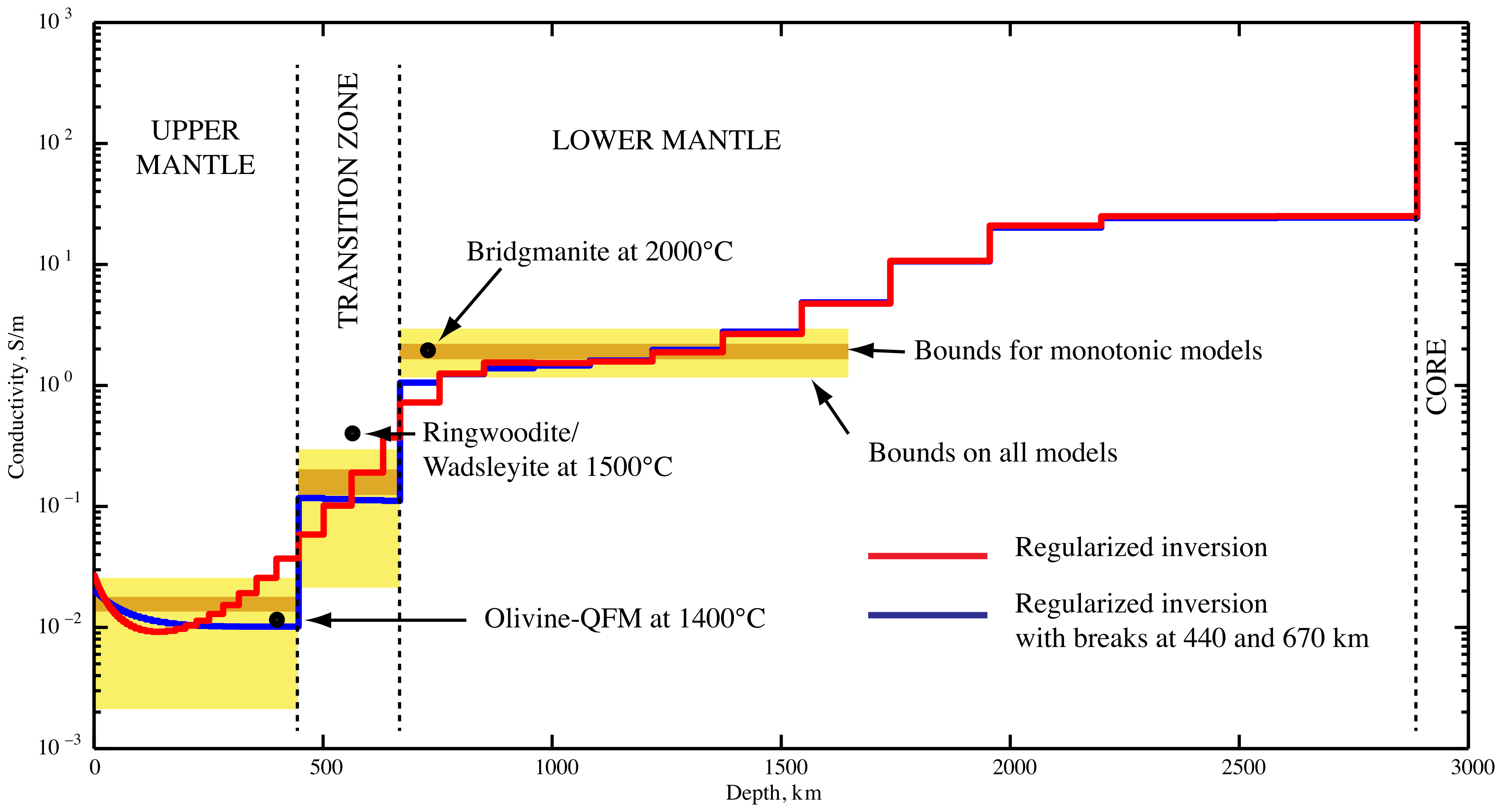
(Bayes' Theorem)

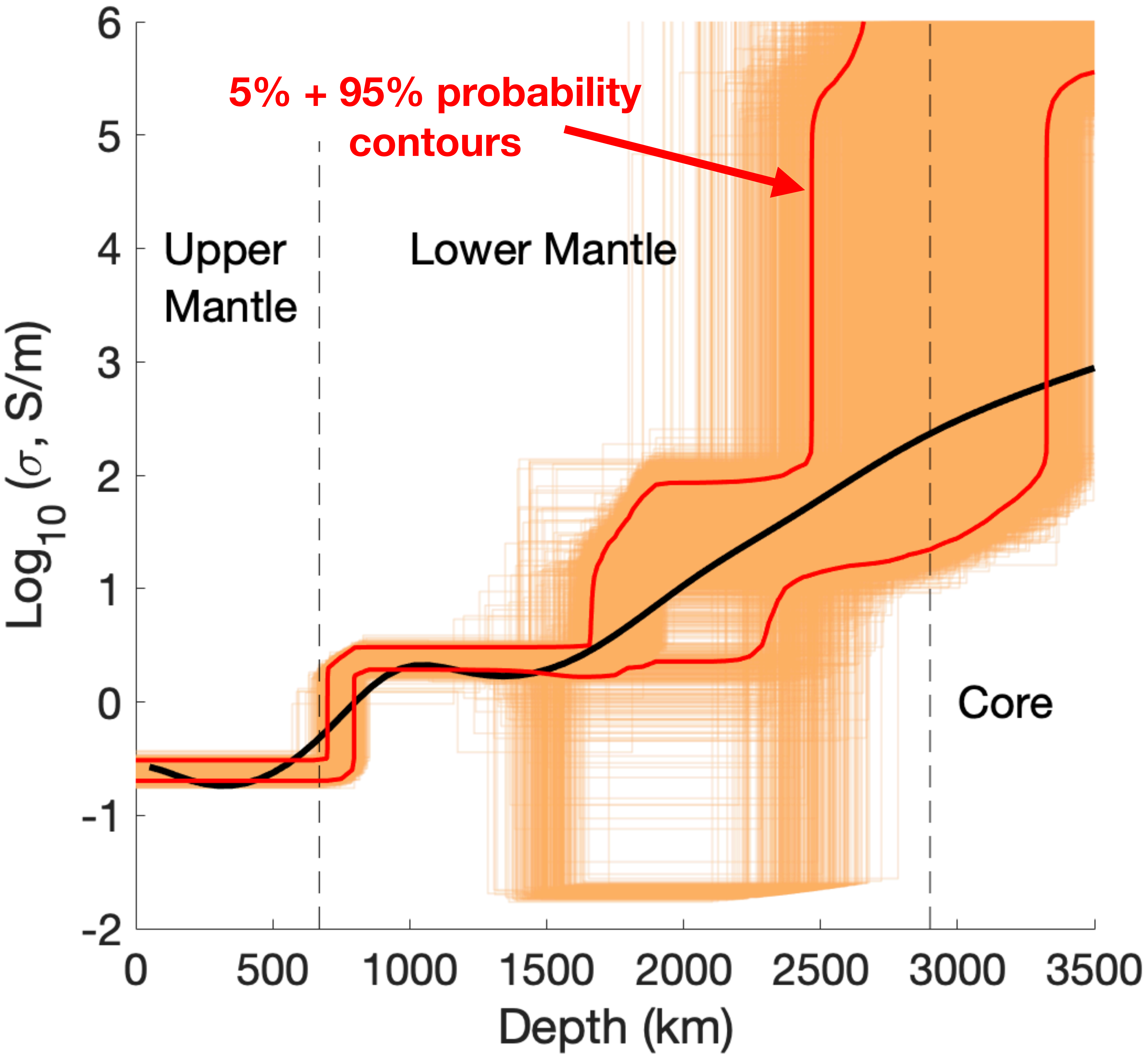
Here we have fit 10,000 4-layer models using Marquardt inversion, using random data sets drawn from our observed data distributions (i.e. data + errors). The Occam model response (rms 1.3) is shown for reference (black lines).



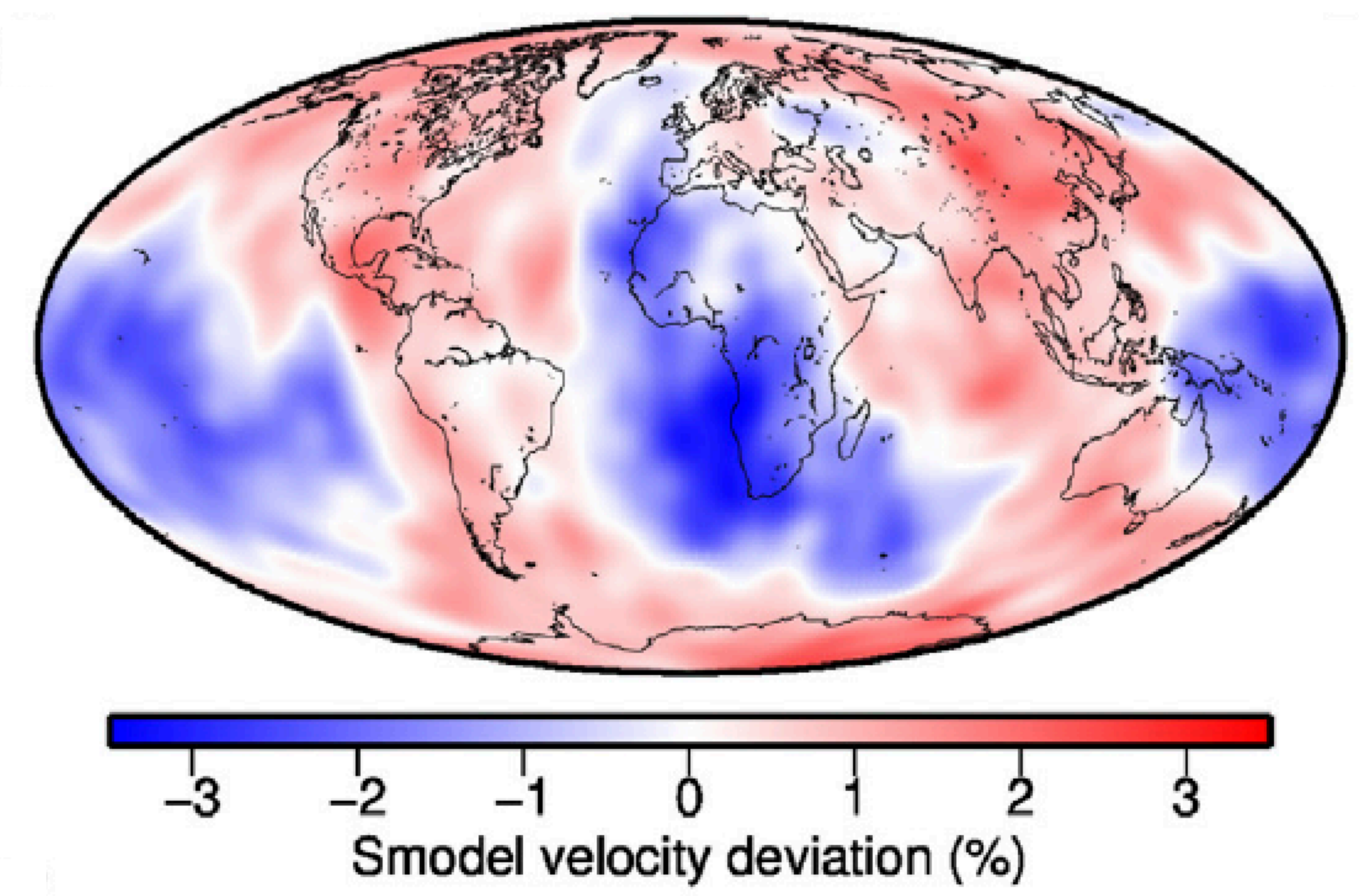
Here are the models (orange), with the Occam models shown again in black. They cover a lot of model space, but 90% of conductivities fall with in the contours. Jumps at 670 km and 2900 km are known to exist, so perhaps the jump at 1800 km is real too.

What causes the 1800 km jump?

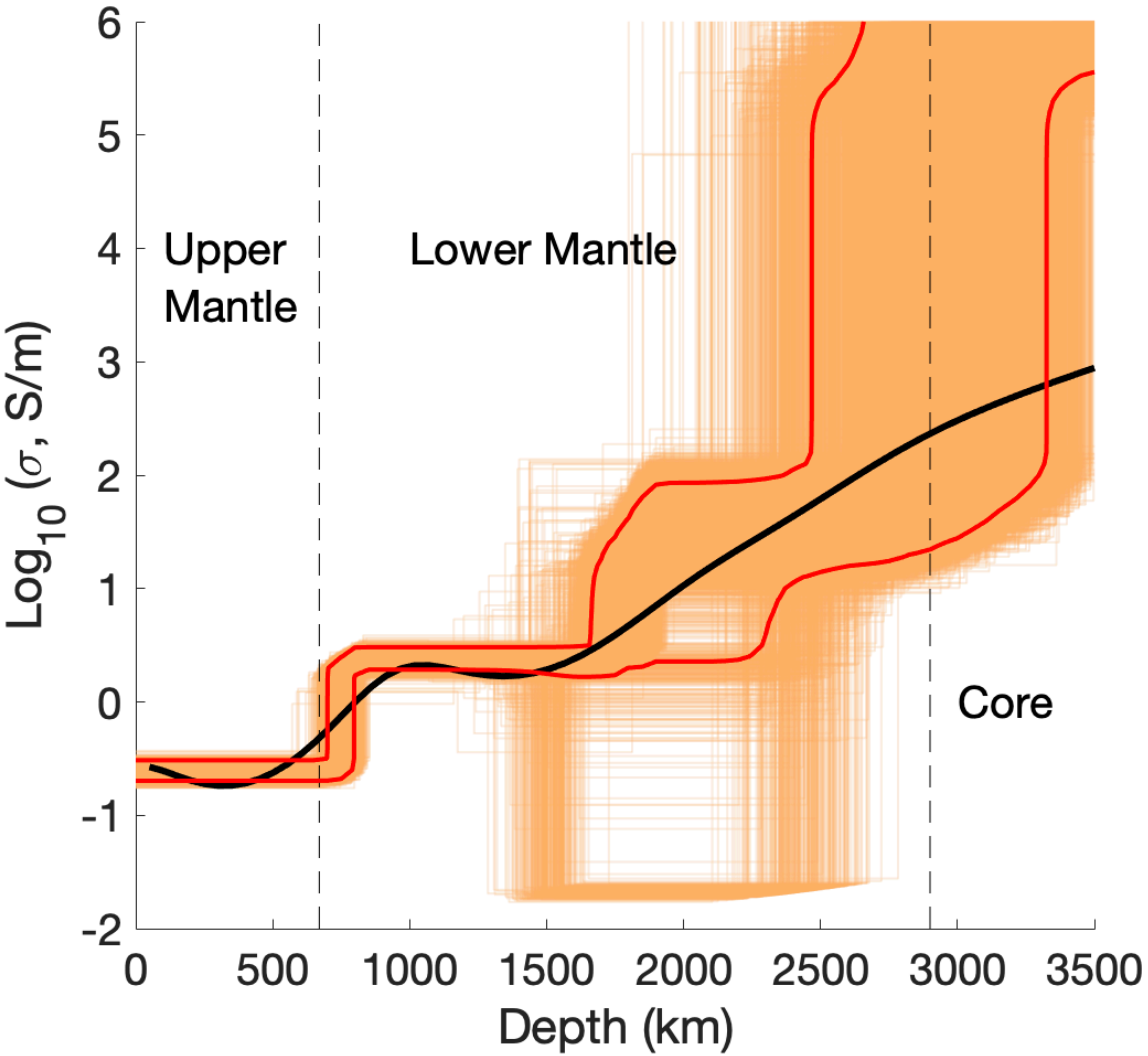




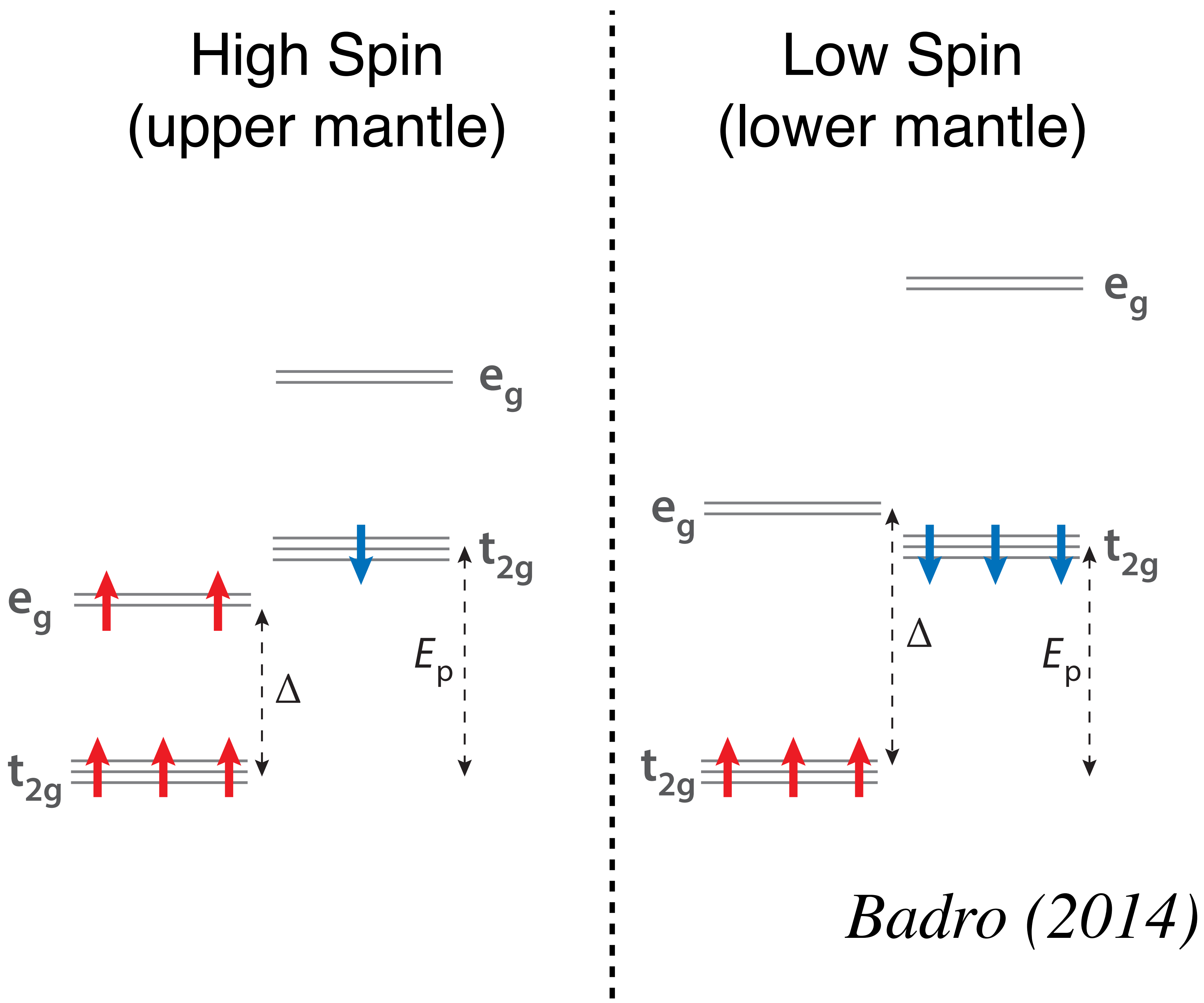
Maybe we are seeing a response to the Large Low Seismic Velocity Provinces (LLSVPs). However, if we re-run our analysis using observatories on and off the LLSVPs, we get the same result.



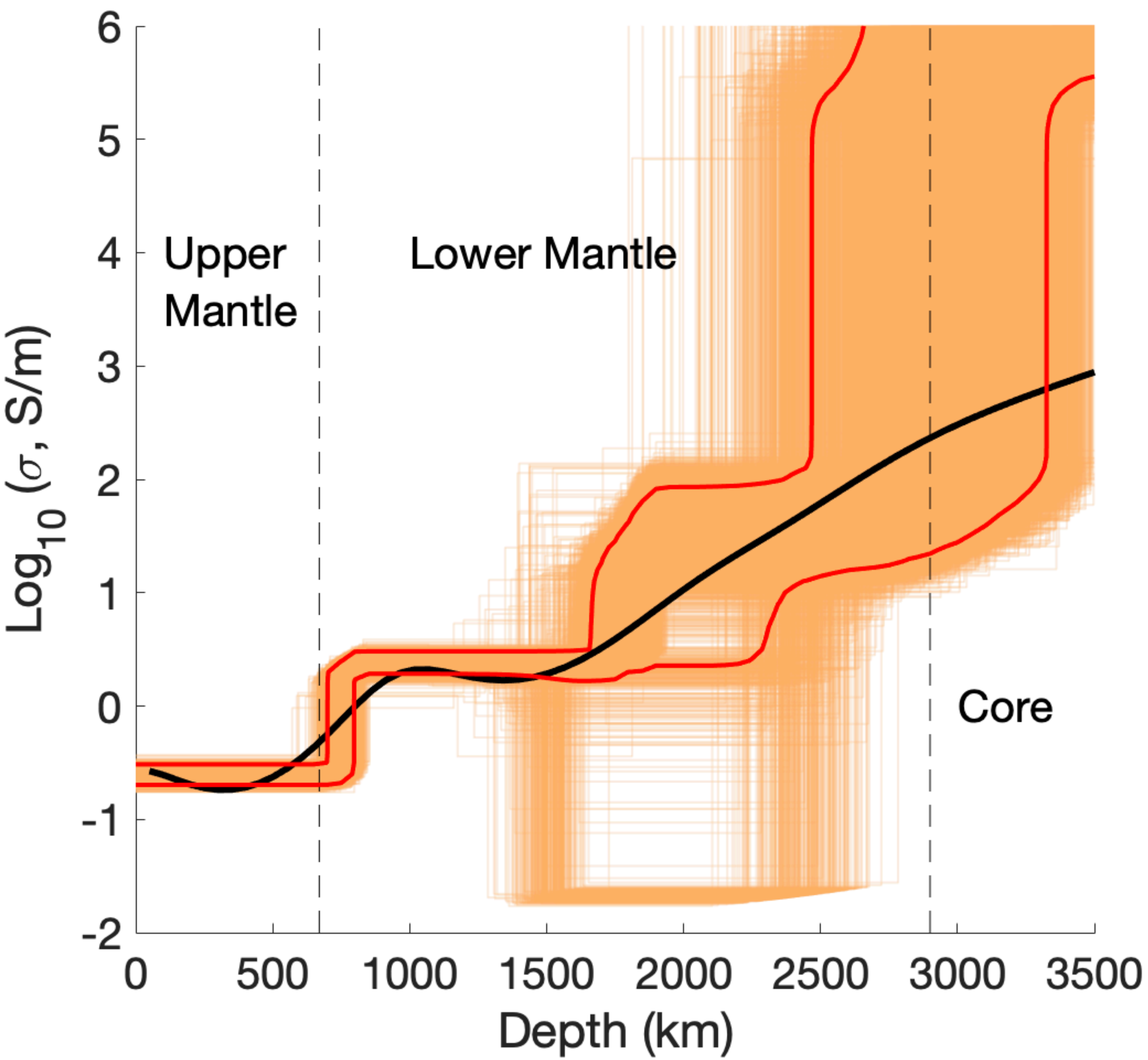
Korte et al. (2022)



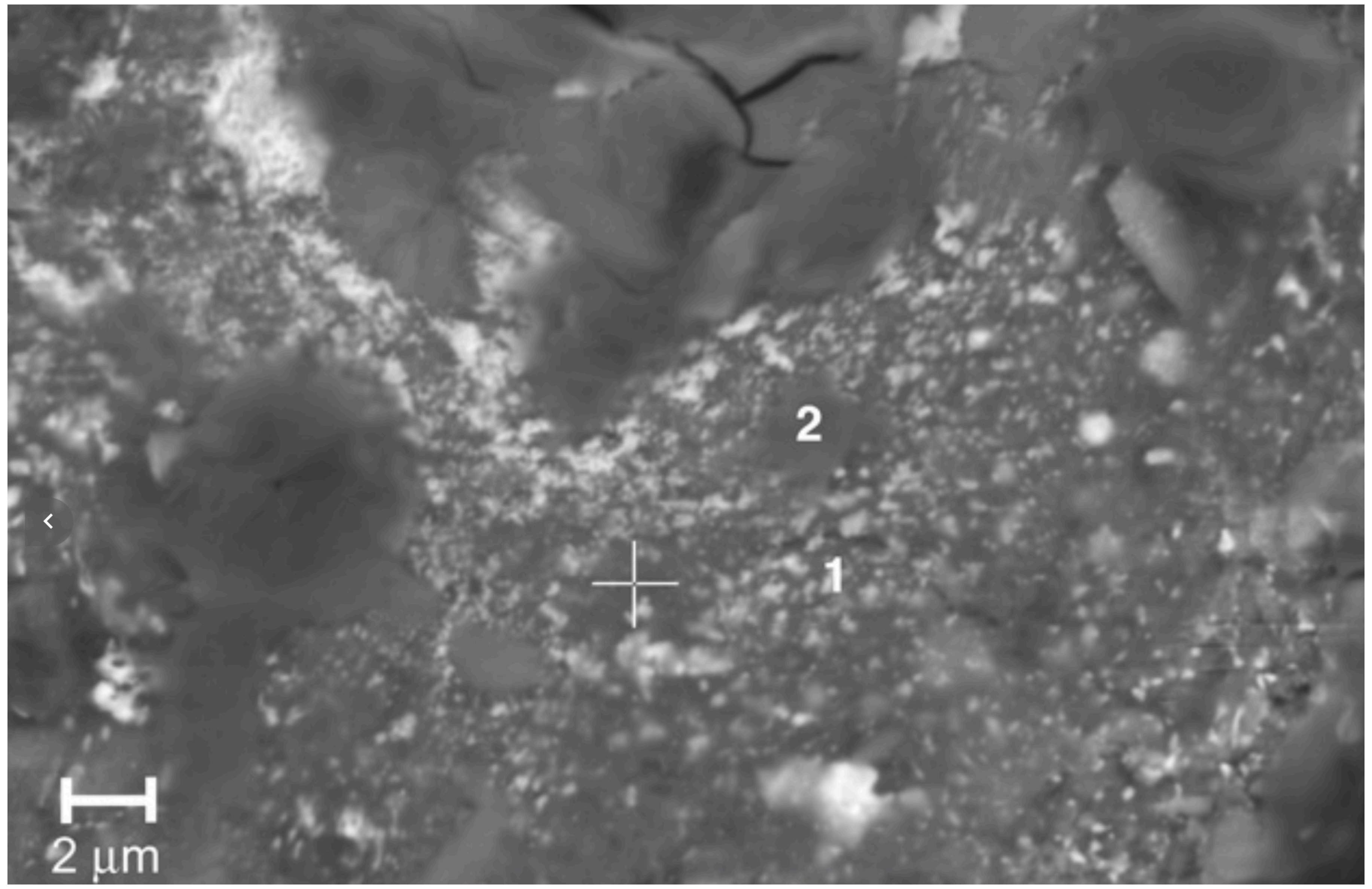
The depth is about where we expect the spin transition in iron. However, laboratory measurements suggest that the spin transition reduces conductivity .



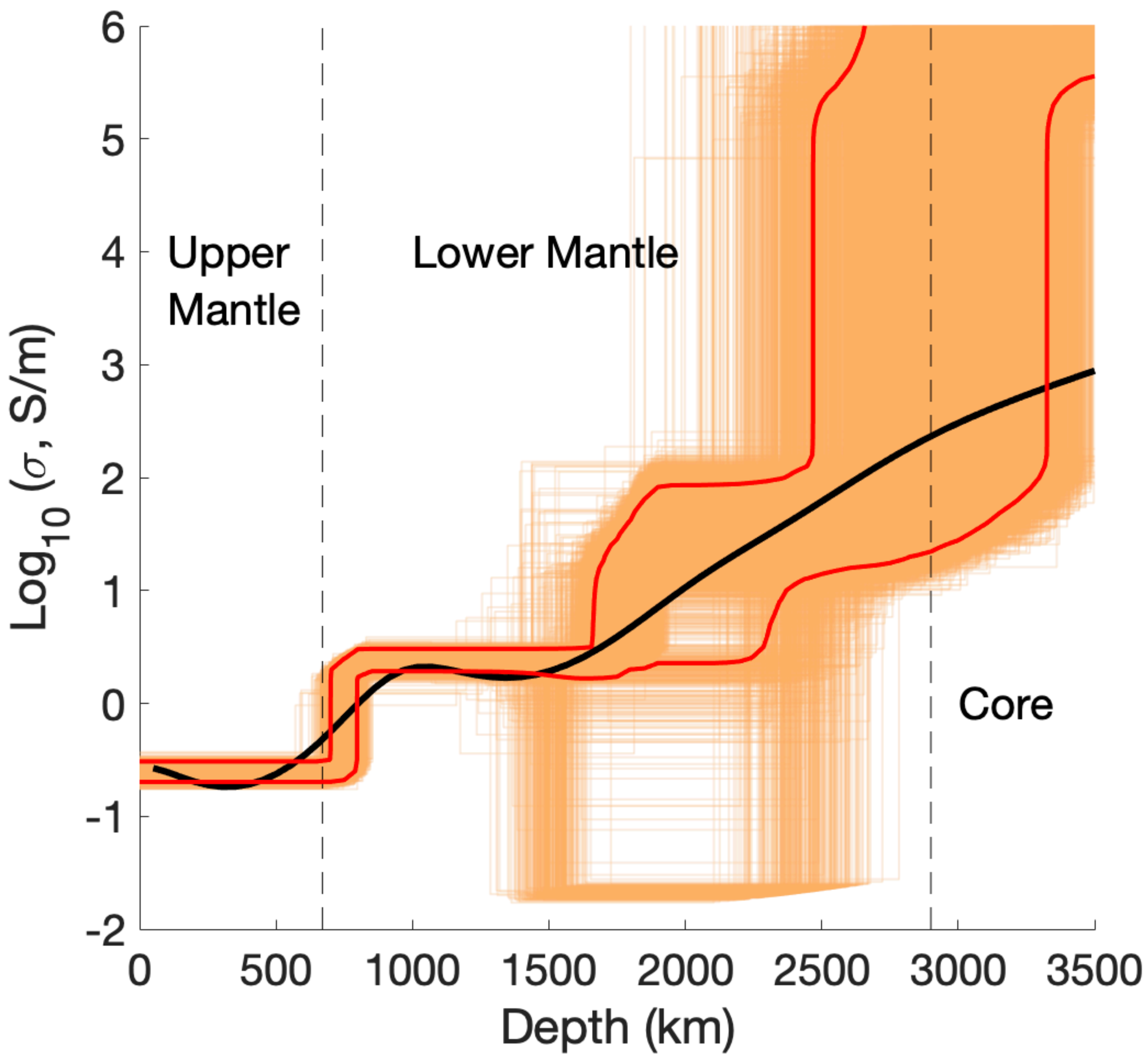
Badro (2014)



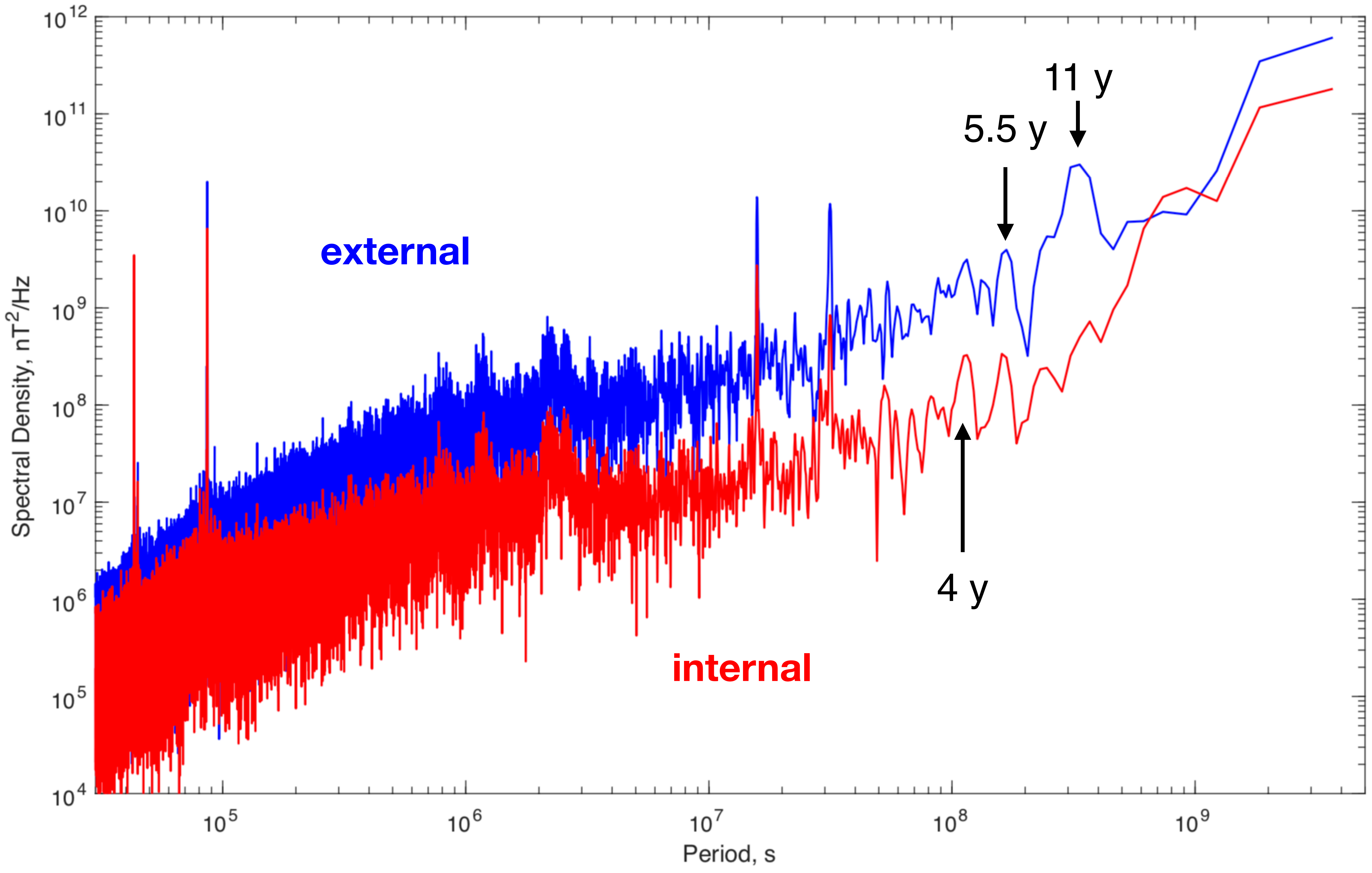
Since upper mantle conductivity is consistent with bridgmanite, but ferropericlase is much more conductive, we could be seeing a change in connectivity between these minerals driven by temperature and pressure.



Chandler et al. (2021)



A provocation: Finally, the link that has been suggested between a 6-year periodicity in the length of day (LOD) and internal magnetic field may instead be the 5.5 year harmonic of the sun spot cycle, since we see a peak in the external field as well as the internal. On the other hand, the 4-year peak in the spectrum, strongest in the internal field, may be the signature of a torsional oscillation in the core.



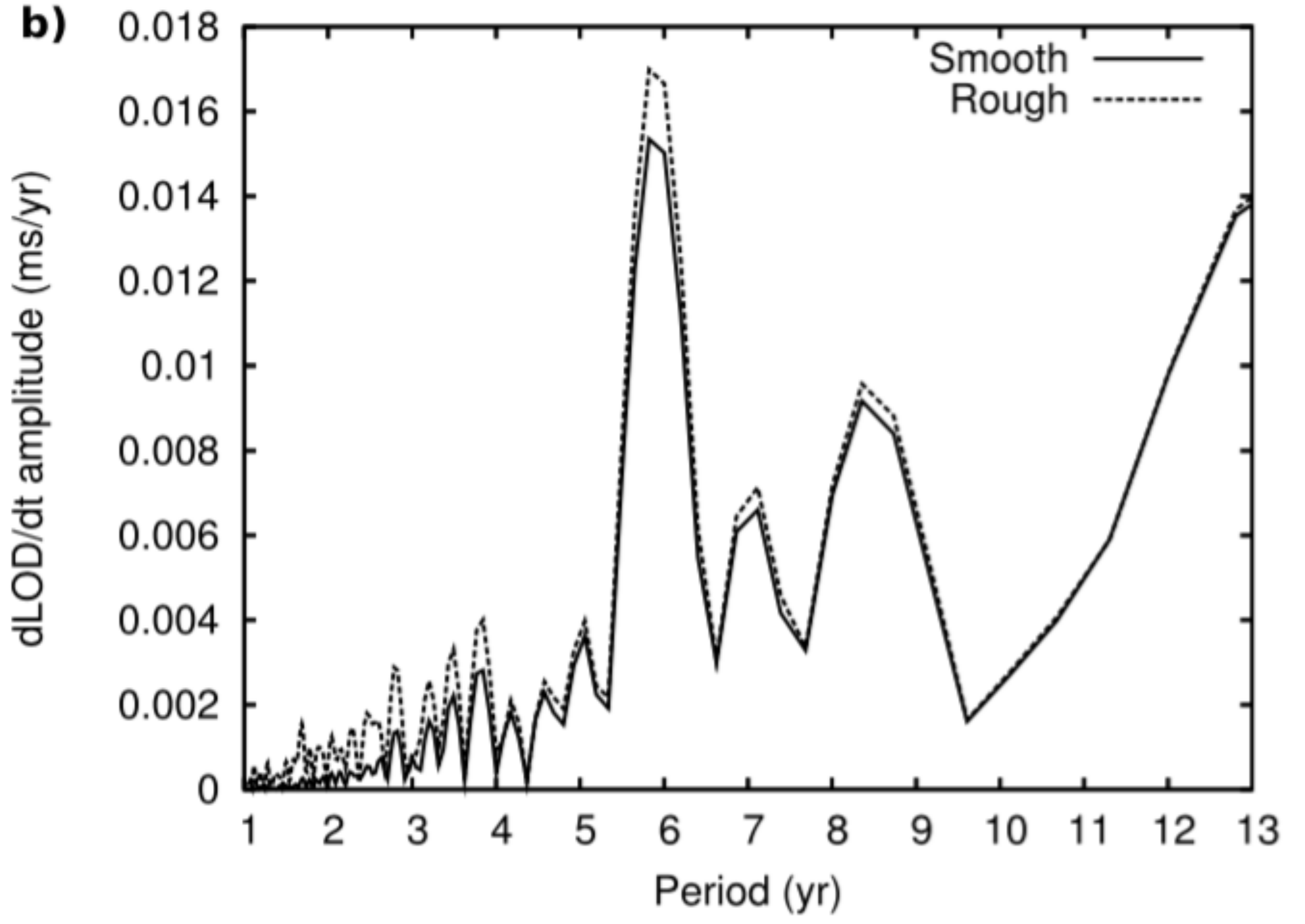


Figure 1. Filtered time series of dLOD/dt from *Holme* [2010]: (a) original signal and (b) frequency spectrum.

Silva et al. (2012)

Summary

- 117 years of hourly data from 180 observatories provides a time series of internal and external fields.
- Multi-taper cross-spectrum analysis shows peaks at 11 and 5.5 years with high coherency.
- An EM response function is estimated, correcting for bias from noise.
- Error bars are estimated by a parametric bootstrap based on variance in spectral band averages.
- Results are consistent with previous work, but fill in periods between 1 and 11 years.
- Data can be fit to RMS 1.25 using a radially symmetric Earth.
- An RTO algorithm provides uncertainty on conductivity, showing jumps at 670 km and the CMB.
- A jump of ~1 OM at ~1800 km suggests a transition in mantle properties.
- It is likely that an inferred 6 year geomagnetic LOD oscillation is a harmonic of the sunspot cycle.

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