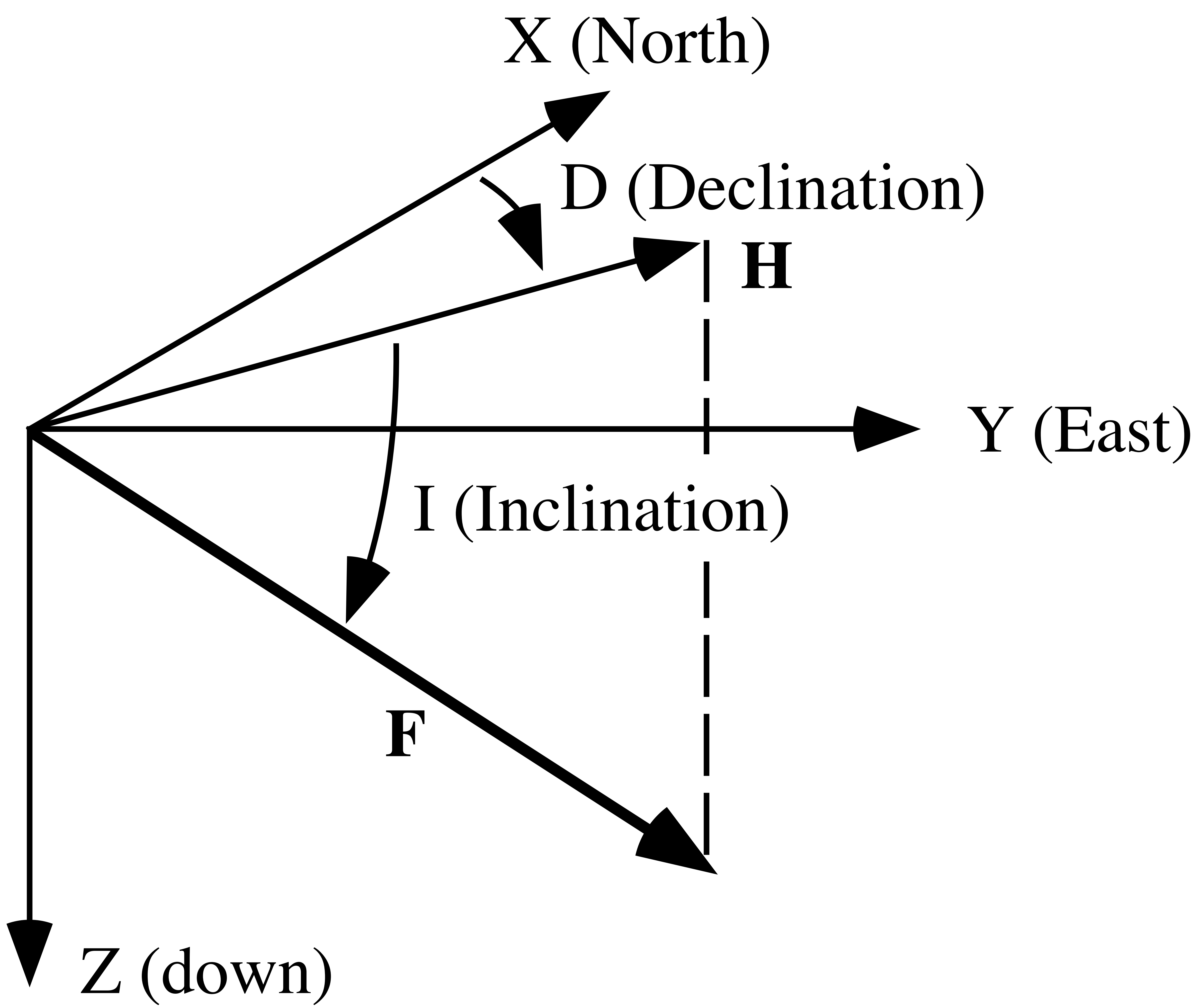
SIOG 231
GEOMAGNETISM AND ELECTROMAGNETISM

Lecture 4
Instrumentation
1/18/2024

The magnetic coordinate system:



Various types of magnetic measurements:

Can measure directions, D and I

Can measure the total field, |**F**|

Can measure the components, (X, Y, Z)

Sensors are either total field or vector.

Can measure only the high frequency variations (variometers).

Magnetometers:

Torsion fiber magnetometers: Early observatory and low-power field instruments.

Nuclear precession (PPM, Alkali Vapor, Overhauser): Total field, intrinsic calibration.

Fluxgate: Vector, low frequency.

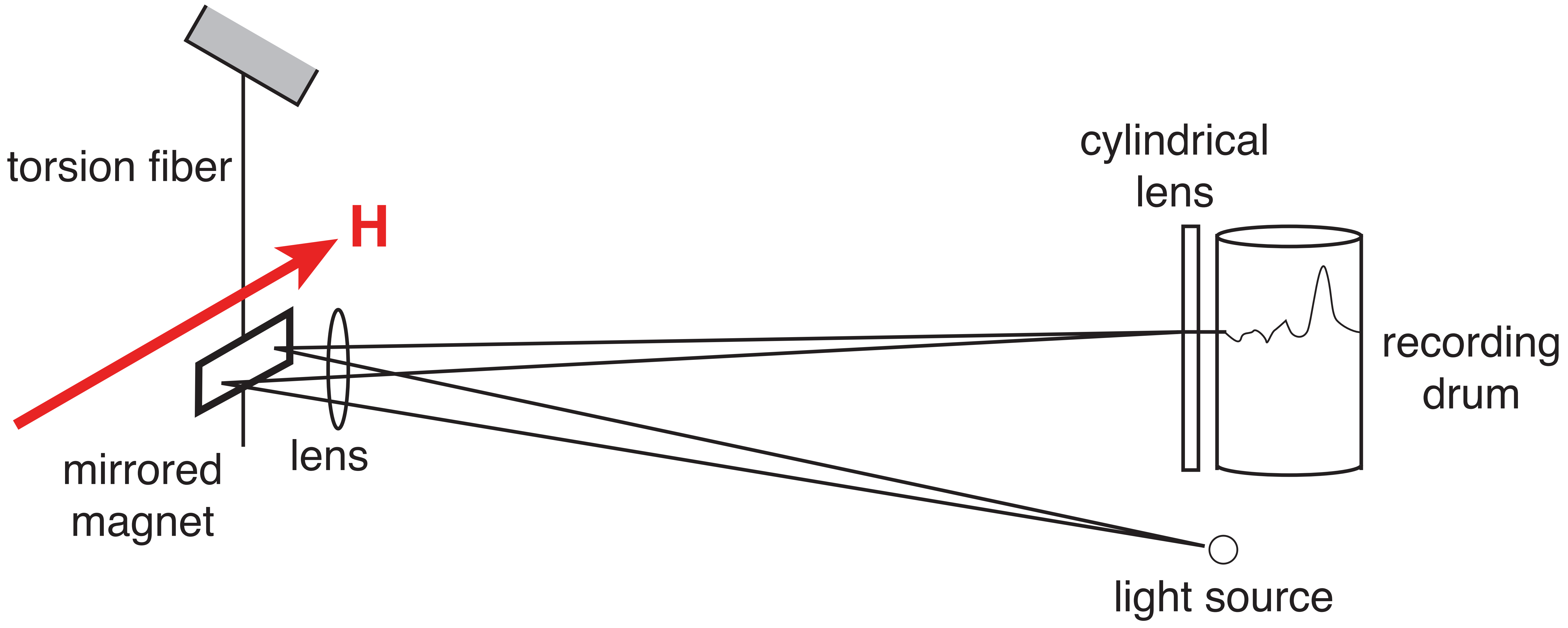
Induction coil: Vector, high frequency.

Theodolite/fluxgate: High precision observatory measurements of D and I.

Torsion fiber magnetometers:

A magnet with moment $\mathbf{m}$ will experience a torque $\mathbf{L}$ in a magnetic field $\mathbf{B}$ given by

$$\mathbf{L} = \mathbf{m} \times \mathbf{B}$$



angular change in the declination (in minutes of arc) which produces a deflection of one millimeter on the photographic recording drum is given by

$$S_D = \frac{G}{M}\frac{3438}{2R}$$

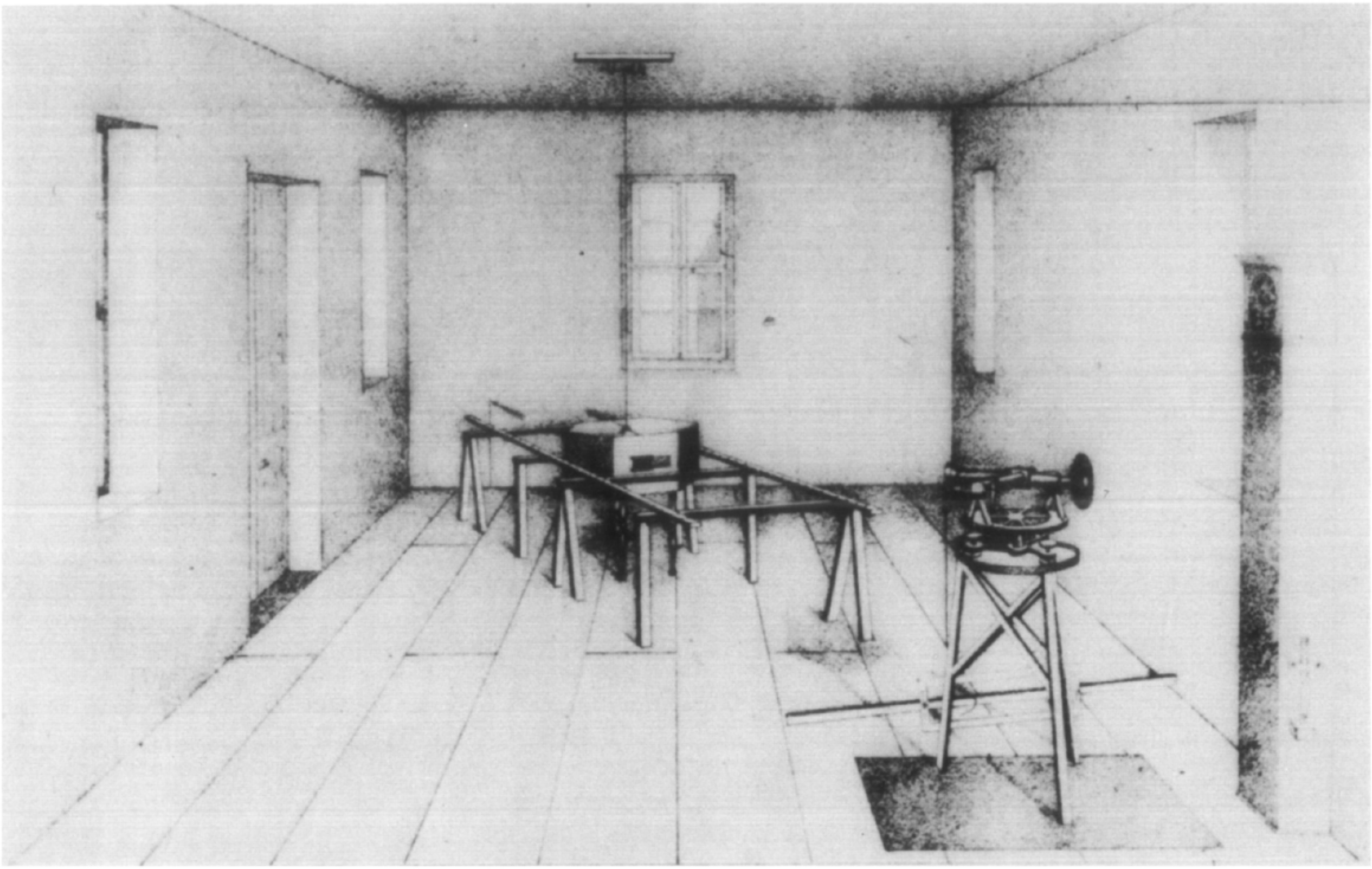
G is the torsion coefficient, M is the magnetic moment (3438 = number of arc-minutes in a radian)

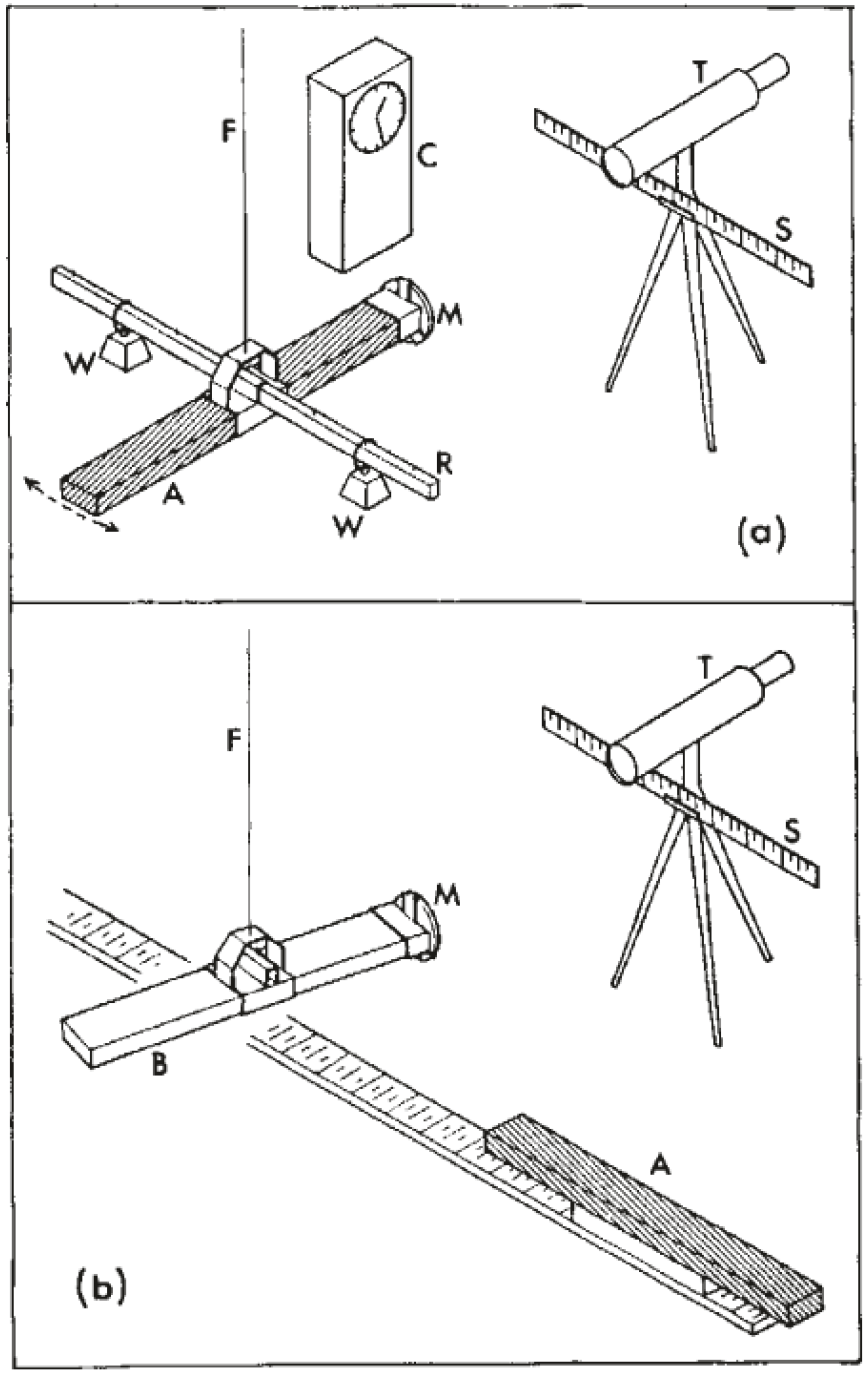
Torsion fiber magnetometers:

$$\mathbf{L} = \mathbf{m} \times \mathbf{B}$$

$$T = 2\pi\sqrt{\frac{I}{mB}}$$

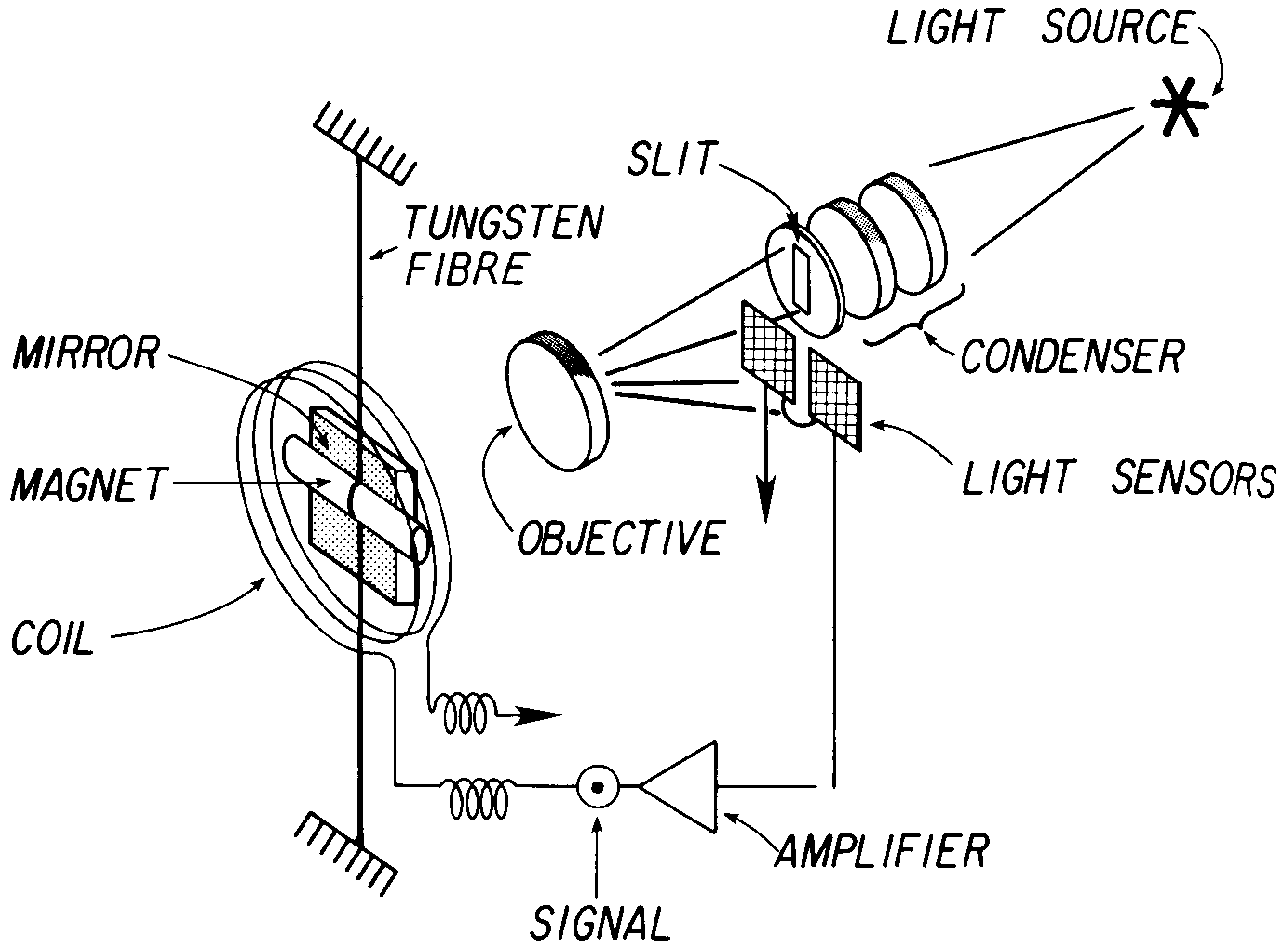
Period of oscillation T given by the moment of inertia I of the magnet divided by the torque.





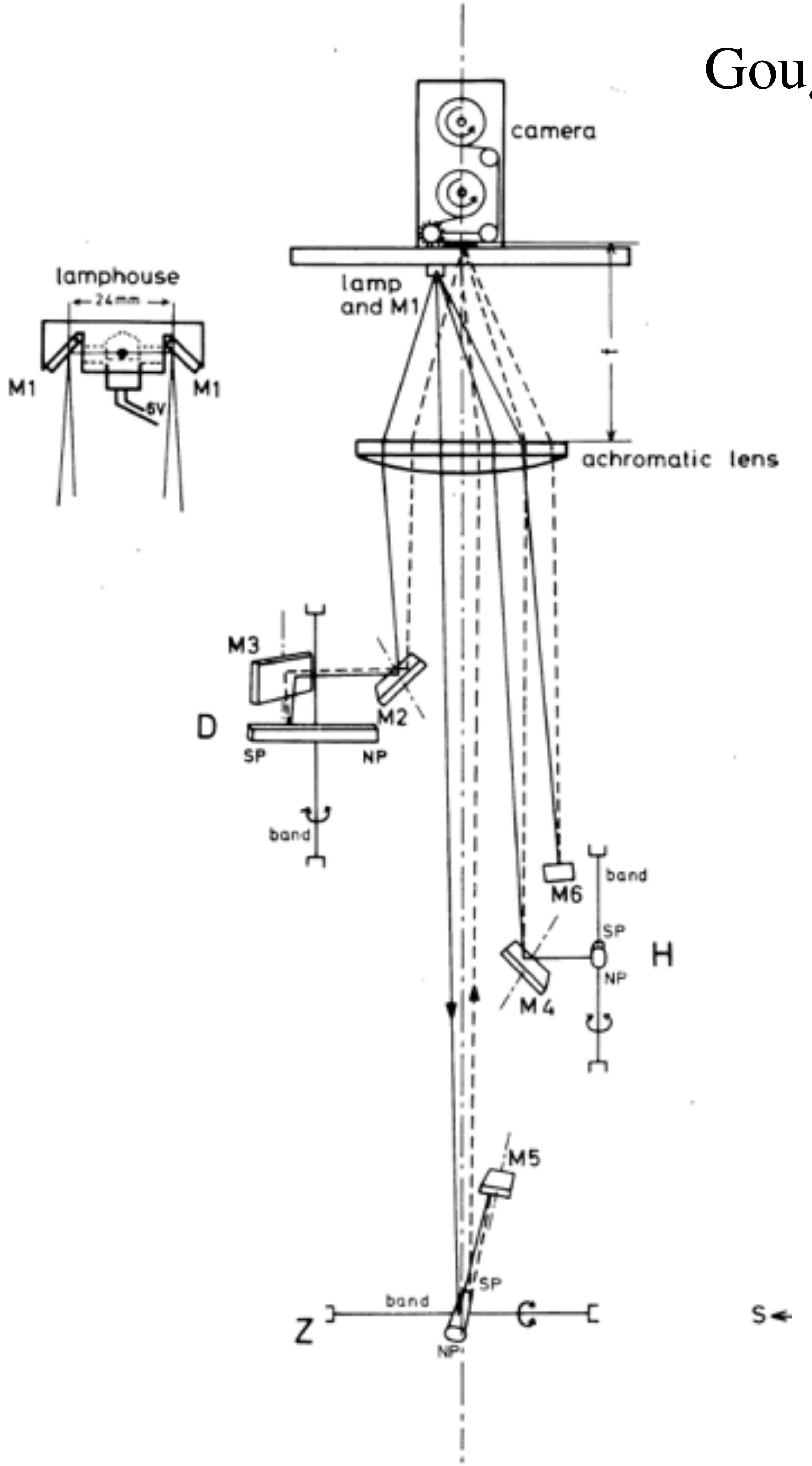
Torsion fiber magnetometers:

Were used in the field until early 1980's - advantage of low power.



Filloux seafloor D-component

Gough-Reitzel



Nuclear precession magnetometers:

Proton precession magnetometer (PPM)

Lamor precession frequency f is proportional to the magnitude of the magnetic field B

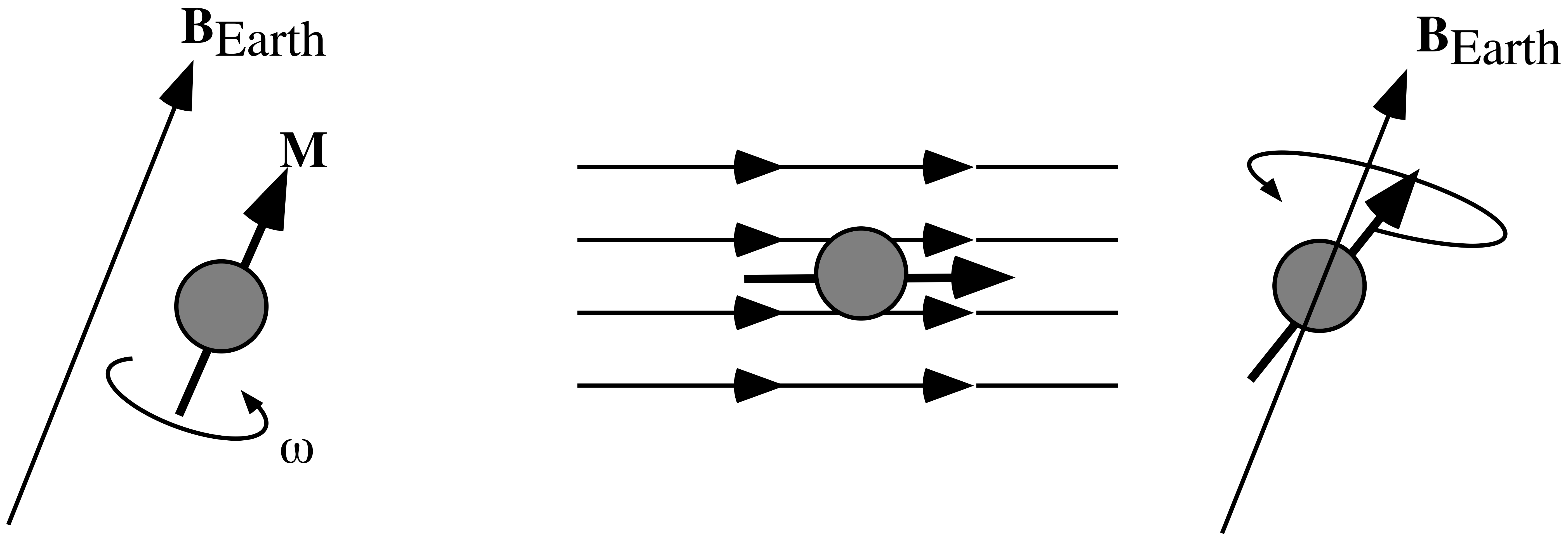
$$\omega = \gamma_p B$$

$$f = \frac{\gamma_p}{2\pi} B$$

A proton has a magnetic moment as well as spin angular momentum

A strong polarizing magnetic field aligns the proton at a high angle to Earth's field

Upon removal of the polarizing field, the proton precesses about Earth's field



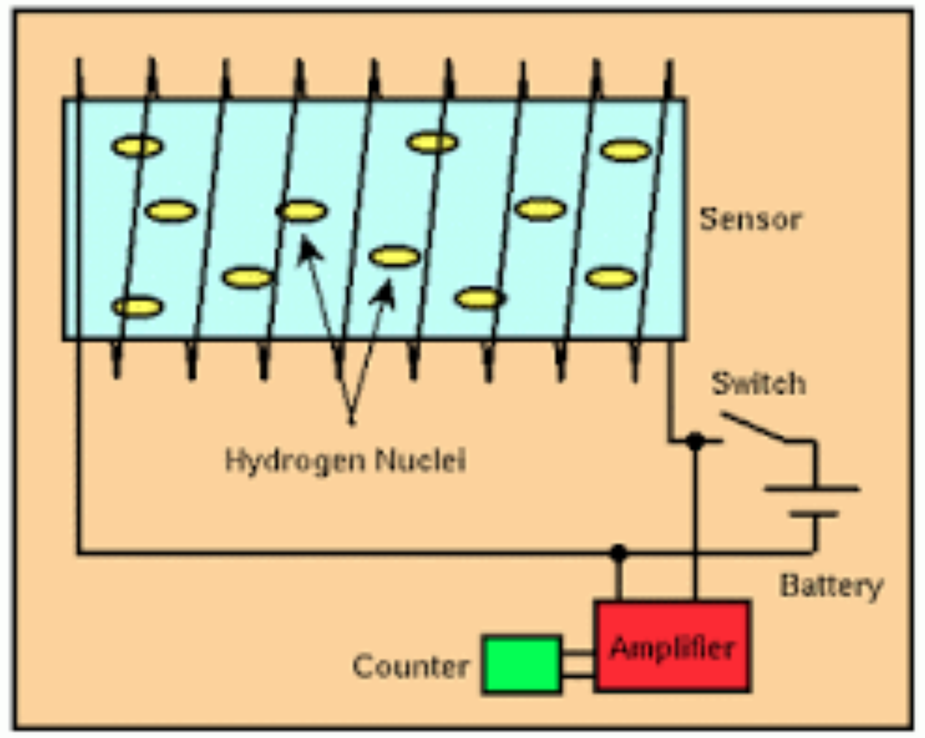
The gyromagnetic ratio of the proton is the ratio of magnetic moment to spin angular momentum:

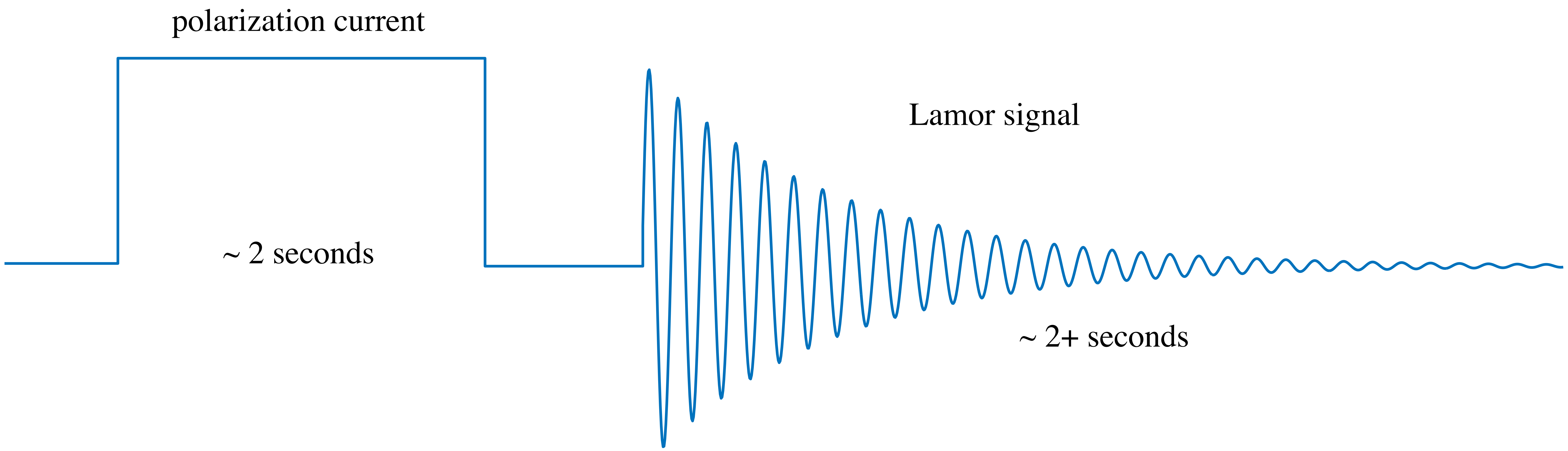
$$\gamma_p = 2.675 \times 10^8 \text{ radians T}^{-1} \text{ s}^{-1}$$

$$B_{\mathrm{Earth}} = 2\pi f / \gamma_p \quad (23.4859 \text{ nT/Hz})$$

Nuclear precession magnetometers - PPM:

The PPM is simple and gives an intrinsically calibrated measurement of total field. However, it is power hungry, it takes about several seconds to integrate a signal, and has a resolution of around 1 nT.





Nuclear precession magnetometers:

We can estimate the gyromagnetic ratio by considering the charge on the spinning proton or electron as being distributed in a circular circuit:

The current in this circuit is then charge times velocity divided by circumference:

$$I = \frac{qv}{2\pi r}$$

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The magnetic dipole of a single loop is current times area:

$$D_M = IA = \frac{qv}{2\pi r}\pi r^2$$

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Cancelling *pi-r* and multiplying by *m/m* where *m* is the mass of the proton or electron:

$$D_M = \frac{q}{2m}mvr$$

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The second term *mvr* is just angular momentum, so the ratio of dipole moment to angular momentum is

$$\frac{D_M}{mvr} = \gamma = \frac{q}{2m}$$

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This gets you close - the estimate of off by the *g-factor,* 5.58 for the proton and 2.002 for the electron. These are quantum mechanical effects that are sorta predictable.

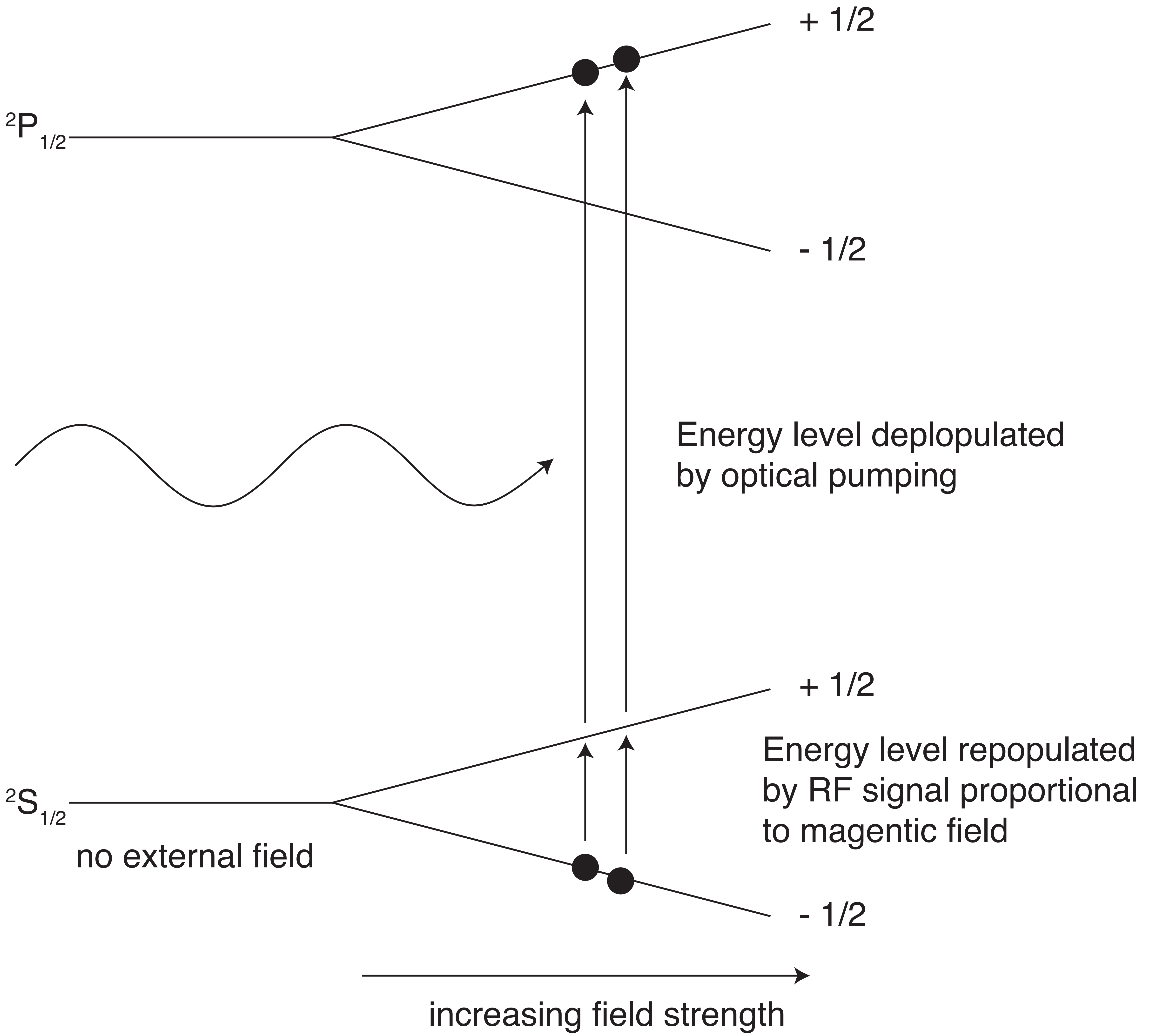
Nuclear precession magnetometers:

$$\gamma_e = 1.761 \times 10^{11} \text{ radians } \mathrm{T}^{-1}\mathrm{s}^{-1}$$

Alkali vapor magnetometers use Zeeman splitting.

Cesium

Rubidium

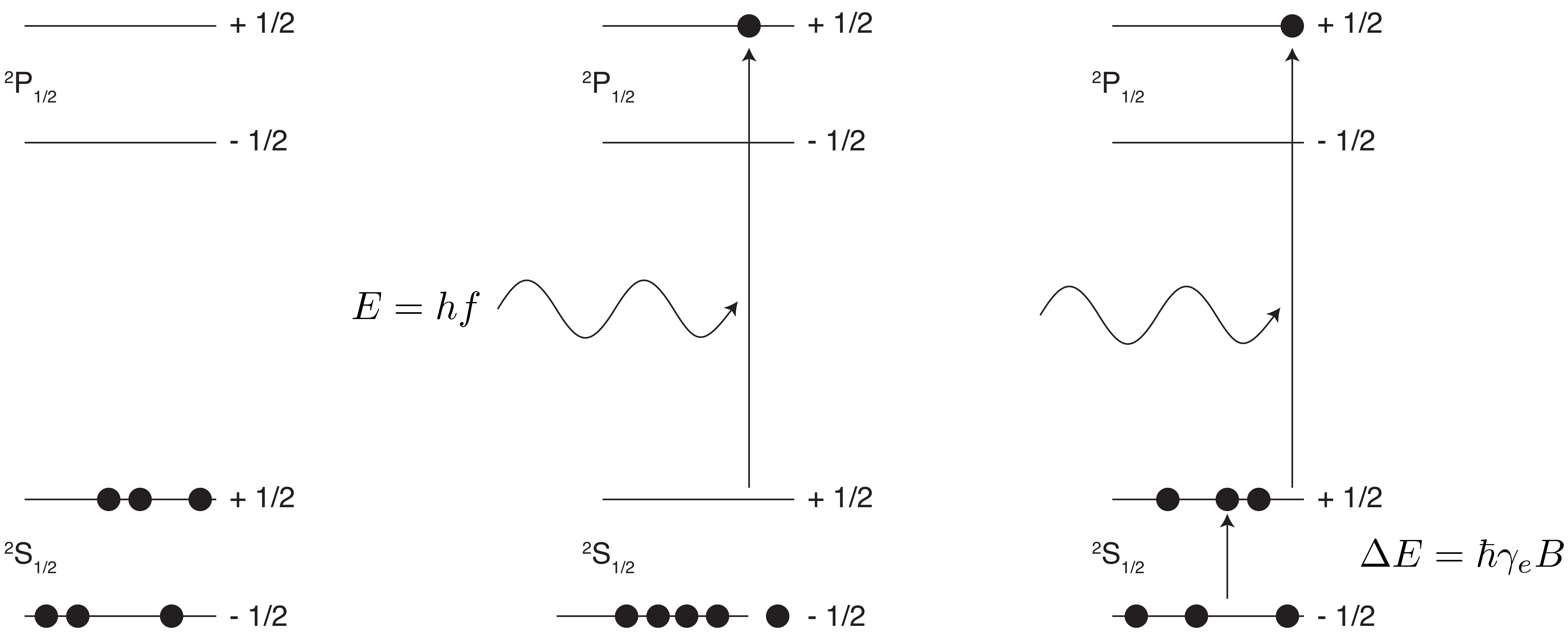


$$E = hf$$

$$\Delta E = \hbar \gamma_e B$$

$$f = \gamma / 2\pi B$$

Nuclear precession magnetometers:



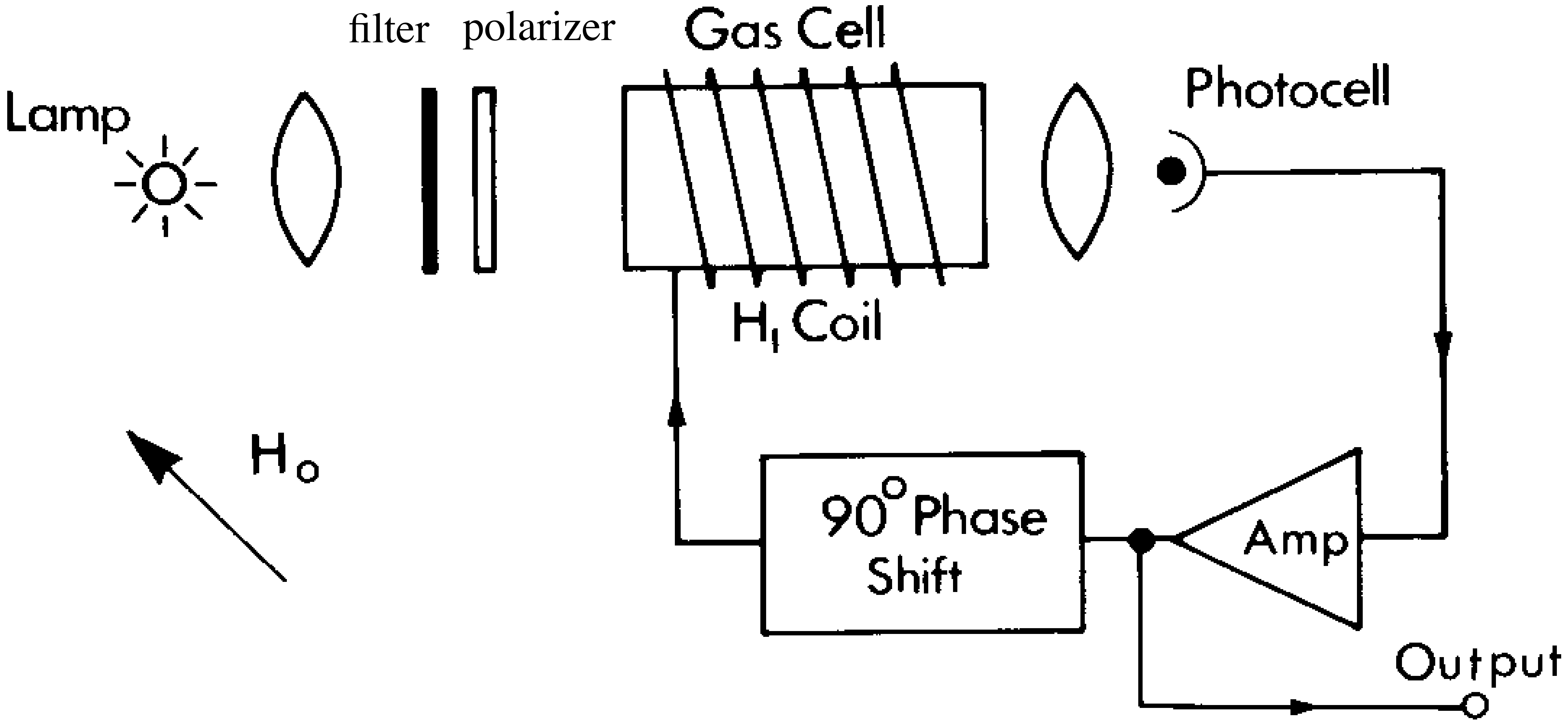
Electrons divided equally in the ground states.

+1/2 energy level deplopulated by optical pumping. Polarized light only couples to +1/2. Cell goes transparent.

+1/2 level repopulated by RF signal proportional to magentic field. Cell starts absorbing light and goes dark.

Nuclear precession magnetometers:

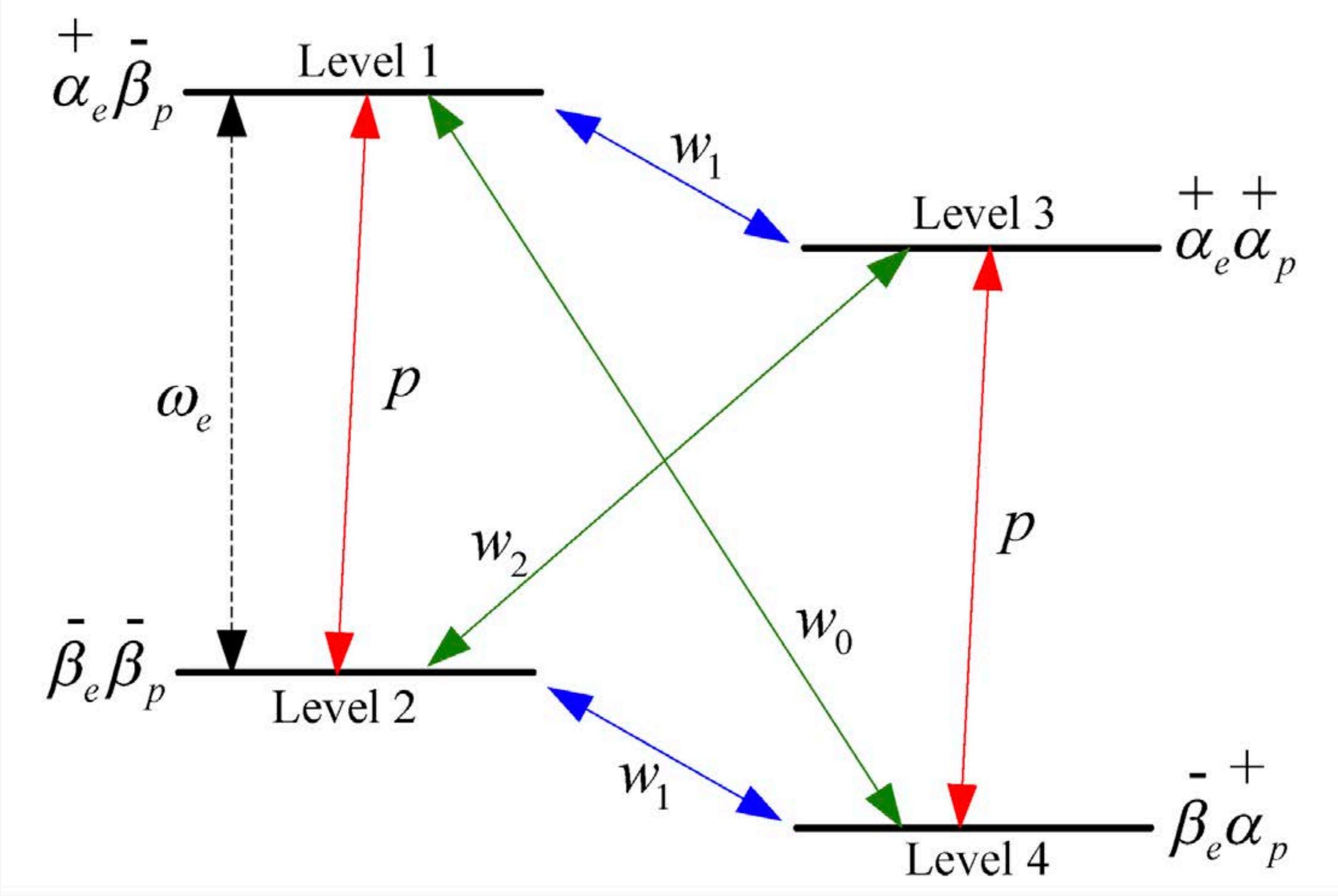
Rather than sweep through the radio frequencies, cesium magnetometers are run as a feed-back circuit.



Nuclear precession magnetometers - Overhauser:

Ge, J., Dong, H., Liu, H., "Overhauser Geomagnetic Sensor Based on the Dynamic Nuclear Polarization Effect for Magnetic Prospecting,," Sensors, 16 (6), 806 (2016). https://doi.org/10.3390/s16060806

Figure 1. Four-level energy diagram for an electron spin S = 1/2 coupled to a ^{1}H proton spin I = 1/2.



Uses Overhauser coupling between proton and electron.

Chemical free radicals supply electrons to a PPM fluid.

Electrons are polarized using an RF signal, and couple to the protons, enhancing magnetic moment.

Figure 2. Scheme of the Overhauser geomagnetic sensor.

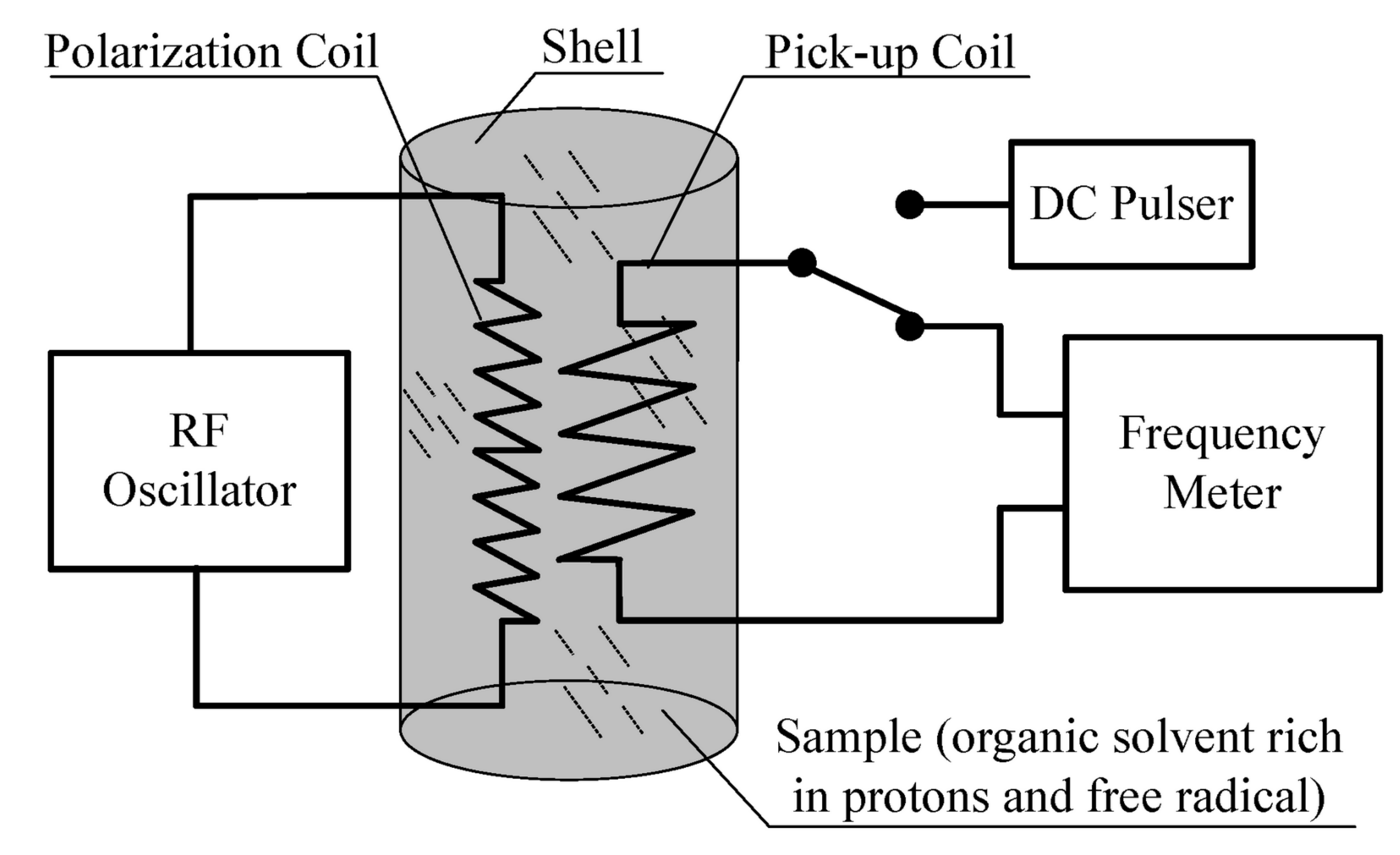
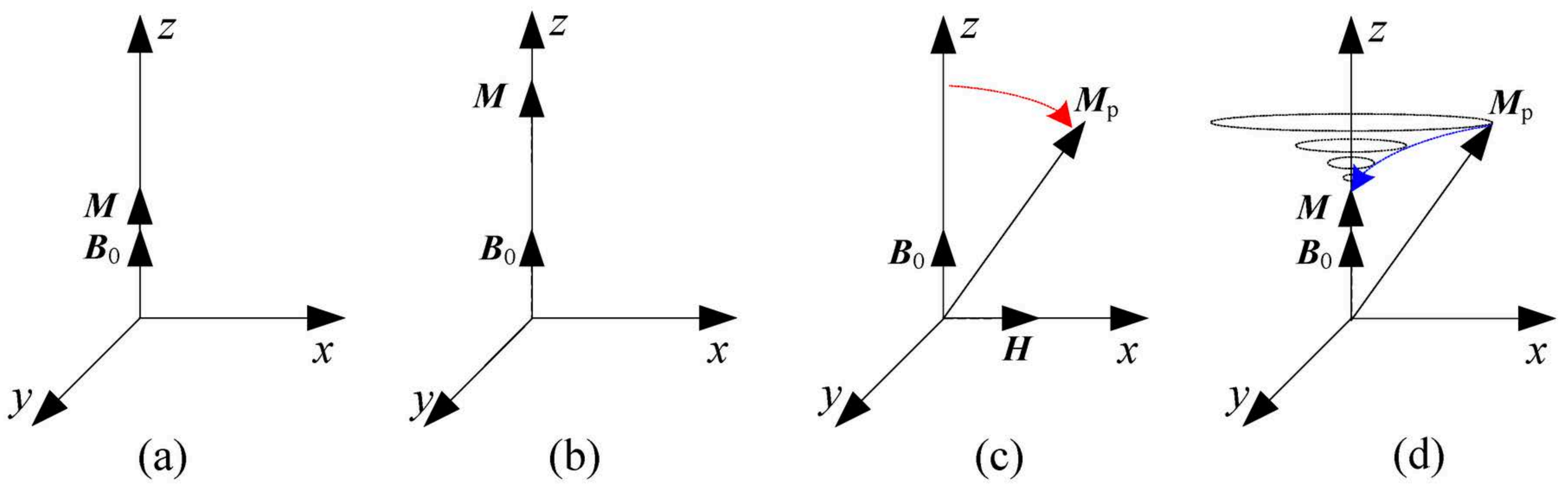
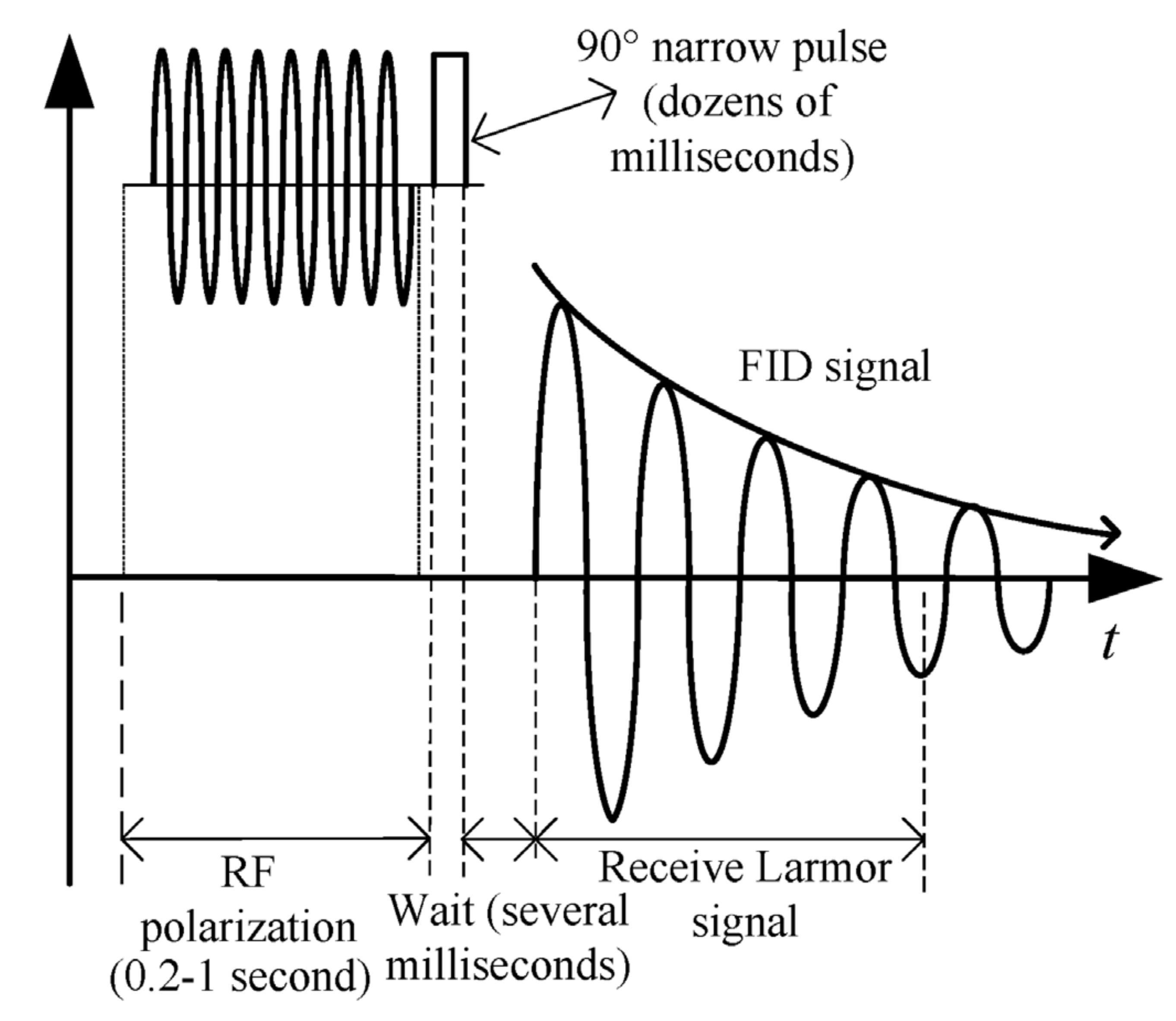


Figure 3. Enhancement and deflection of proton magnetization. (**a**) Initial state; (**b**) enhancement; (**c**) deflection; (**d**) rotating.

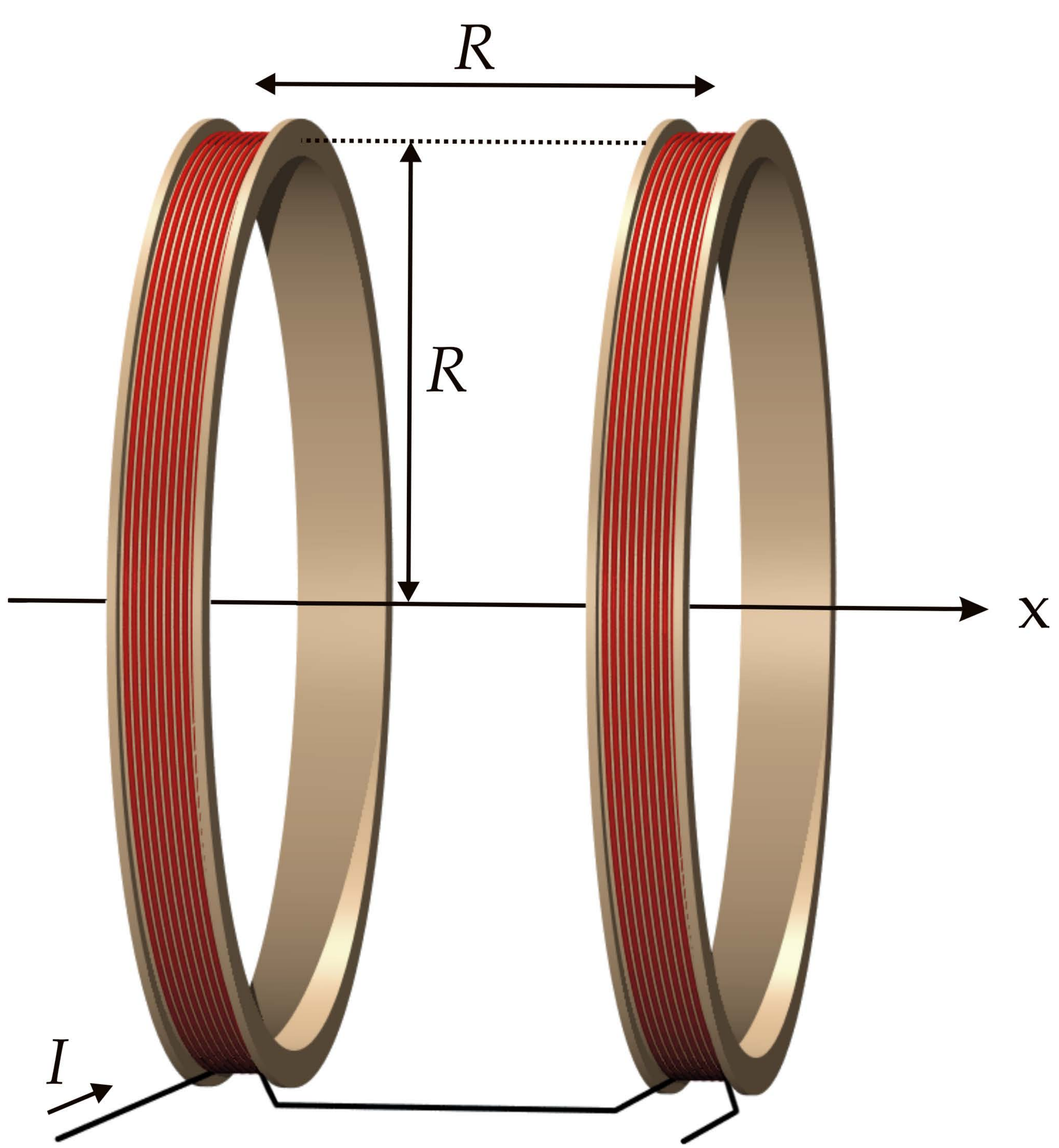


Measurement is then made as for a PPM, but using lower power and higher accuracy.

Figure 4. Workflow of the Overhauser sensor.



Helmholtz coils:



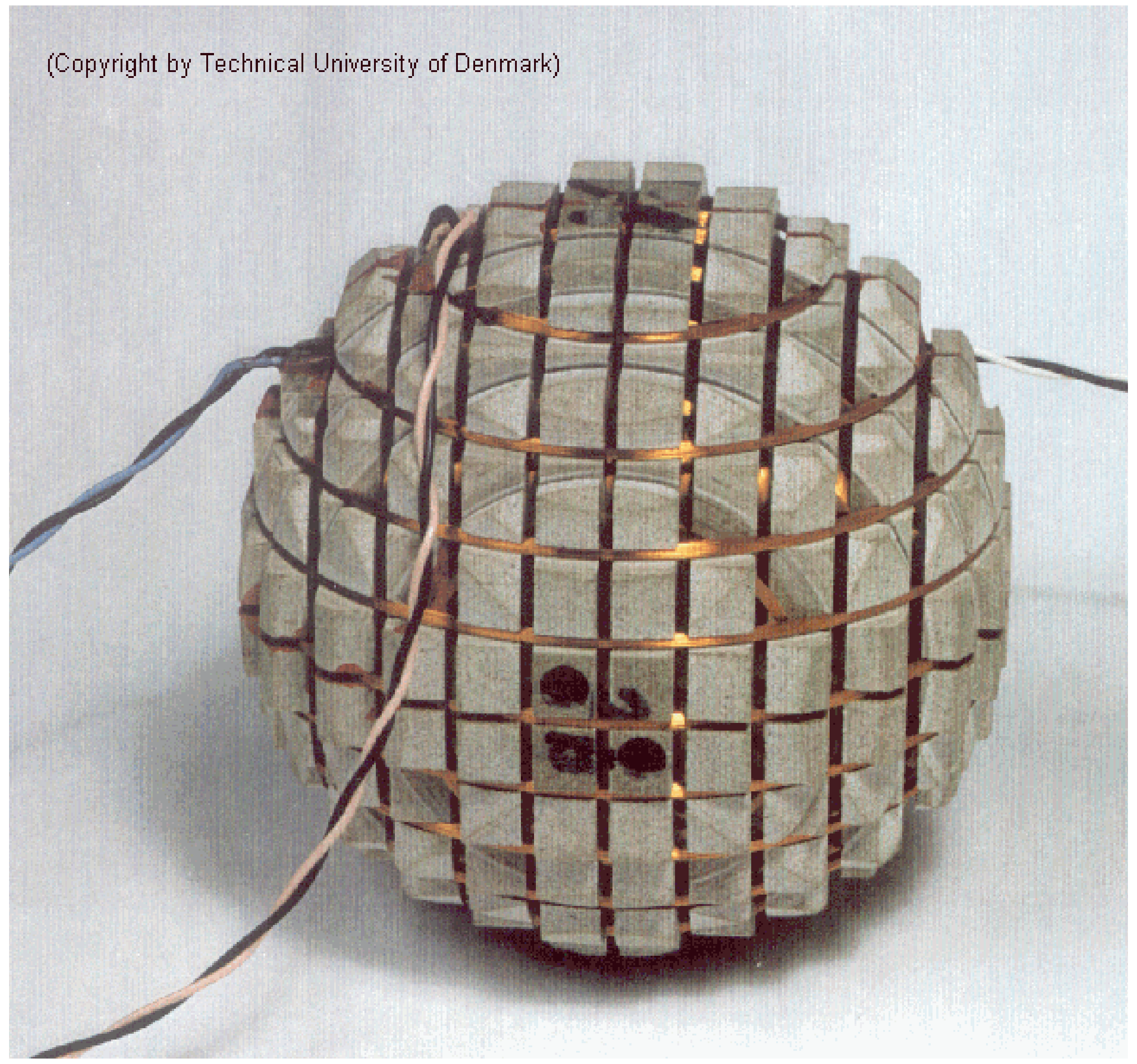
The field at the center is

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_o n I}{R}$$

and only varies by about 1% in the central half of the volume.

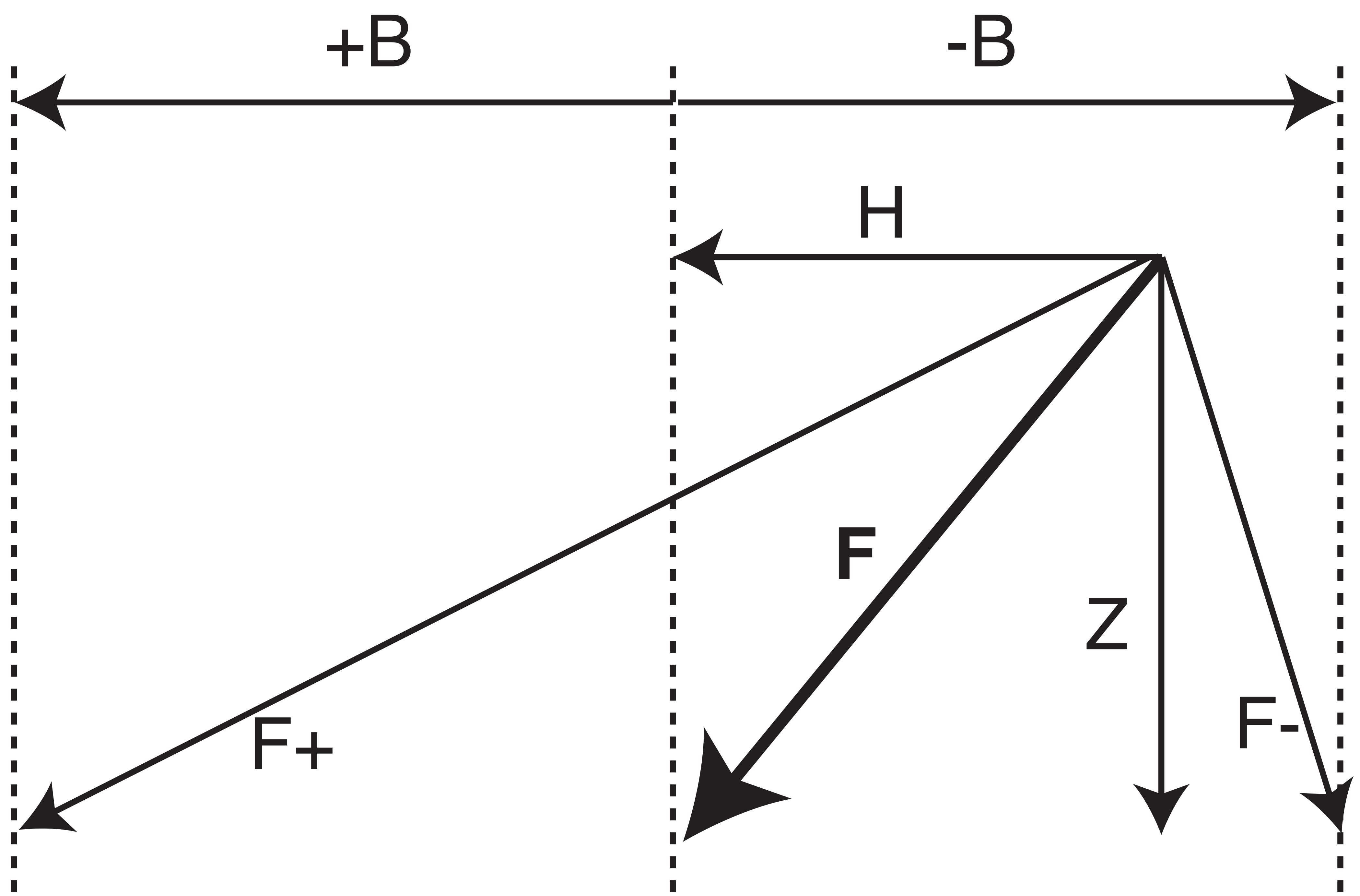
Adding a third coil in the center improves uniformity and is called a Maxwell coil.

Winding an infinite number of turns on a sphere exactly produces a uniform field inside. Sometimes called a Backus coil.



Nuclear precession magnetometers:

Observatories turn proton precession instruments into vector instruments by using a reversed pair of bias fields +B and -B using a pair of Helmholtz coils. Here we show just H, but three sets of coils allows one to measure all 3 components.



$$F_+^2 = (H+B)^2 + Z^2 = F^2 + B^2 + 2HB$$

$$F_-^2 = (H-B)^2 + Z^2 = F^2 + B^2 - 2HB$$

Subtracting:

$$H = \frac{F_+^2 - F_-^2}{4B} \quad .$$

Adding the same two equations:

$$F_+^2 + F_-^2 = 2(F^2 + B^2)$$

giving

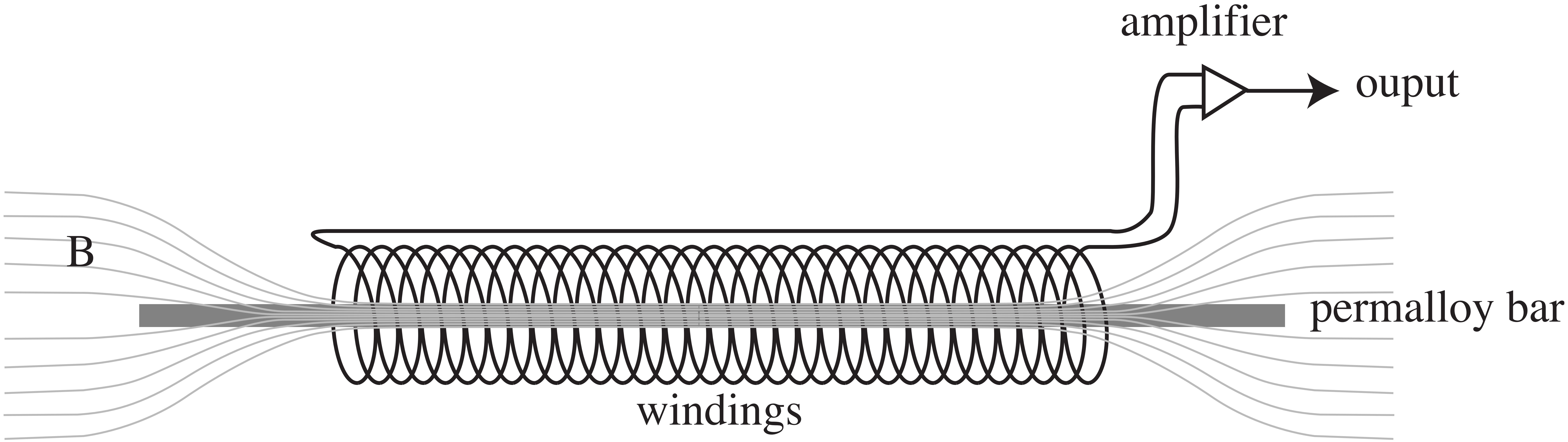
$$B = \sqrt{(F_+^2 + F_-^2 - 2F^2)/2}$$

so

$$H = \frac{F_+^2 - F_-^2}{2\sqrt{2(F_+^2 + F_-^2 - 2F^2)}}$$

Induction coil magnetometers:

Induction coil magnetometers. Vector instruments, low power, highly sensitive at higher frequencies (above 0.1 Hz). But large and heavy.



Induction coil magnetometers:

An application of Faraday's Law:

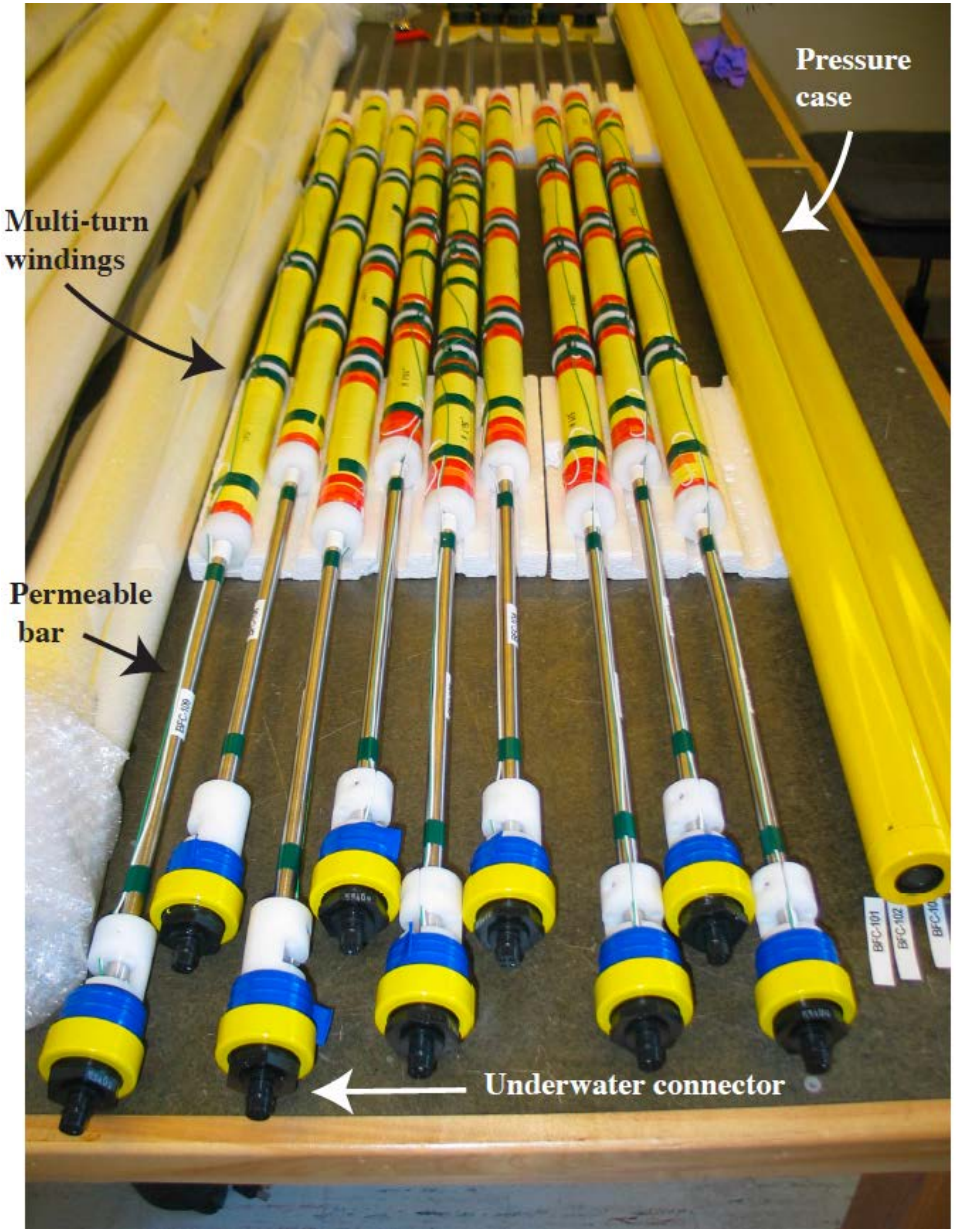
$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

A loop of wire of area A will generate a voltage

$$V = -A\frac{dB}{dt}$$

but it is not very big. We make it bigger by winding N turns (tens of thousands) and adding a permeable bar which multiplies the flux:

$$V = -NA\frac{dB}{dt} = -NA\mu\frac{dH}{dt}$$

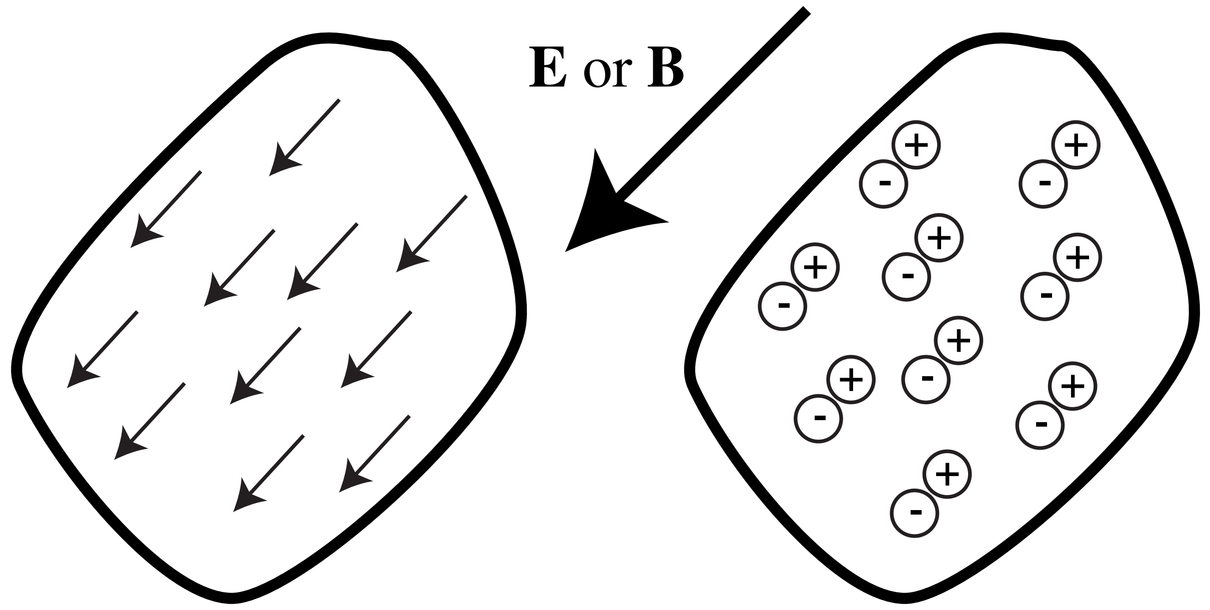


Induction coil magnetometers:

Magnetic susceptibility:

Introducing the magnetizing field **H** the internal magnetization in a material is

$$\mathbf{M} = \chi \mathbf{H}$$



But the external magnetic field **B** can be written as

$$\mathbf{B} = \mu_{\mathbf{o}} \mathbf{H}$$

so

$$\mathbf{B} = \mu_{\mathbf{o}}(\mathbf{H} + \mathbf{M}) \quad = \mu_o(\mathbf{H} + \chi \mathbf{H}) \quad = \mu \mathbf{H}$$

where the material permeability is

$$\mu = \mu_o(1 + \chi)$$

which can also be written as a relative permeability

$$\mu_r = \mu / \mu_o$$

Mu-metal and permalloy can have relative permeabilities of ~100,000

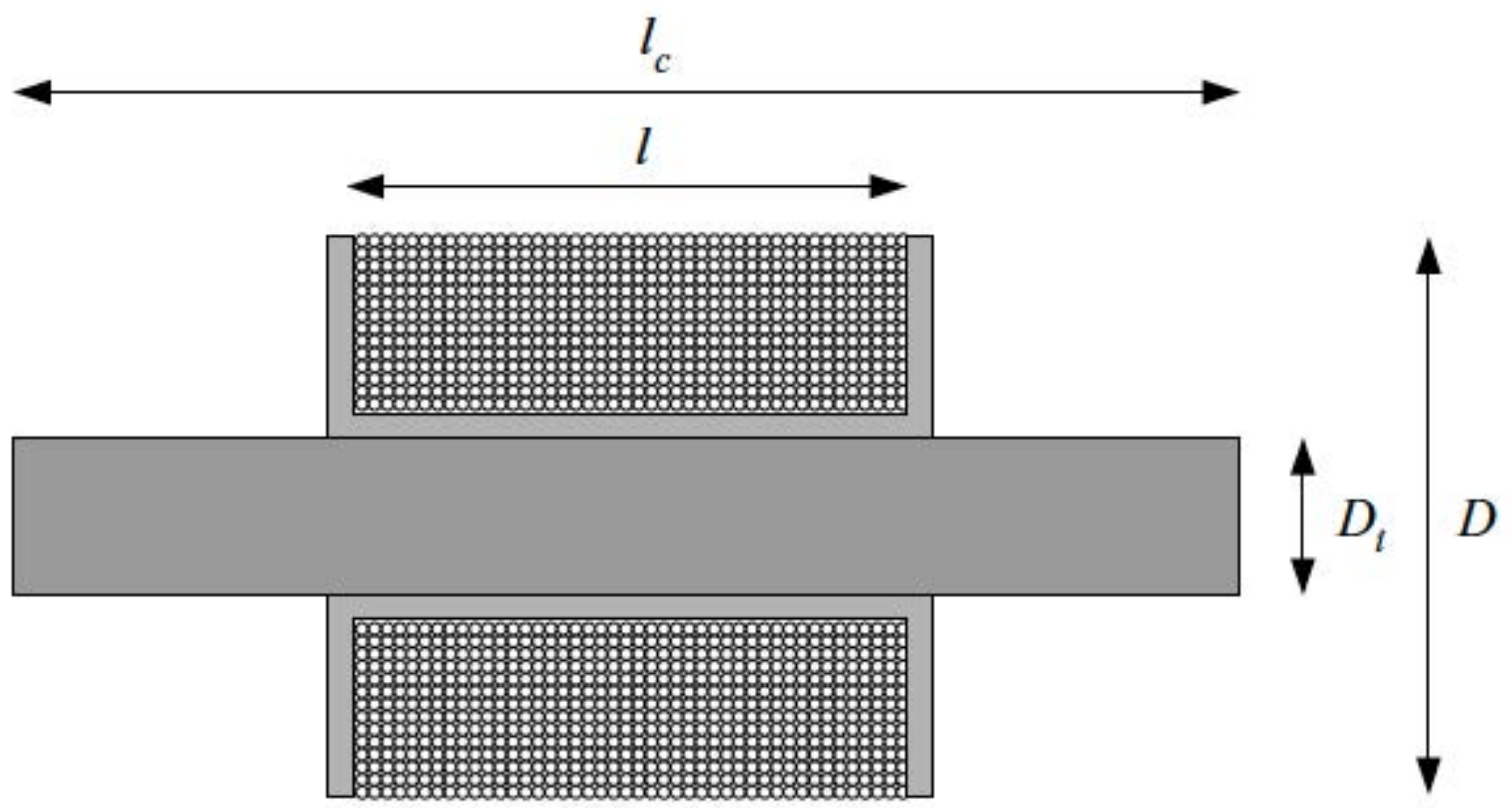
Metglass ~1,000,000

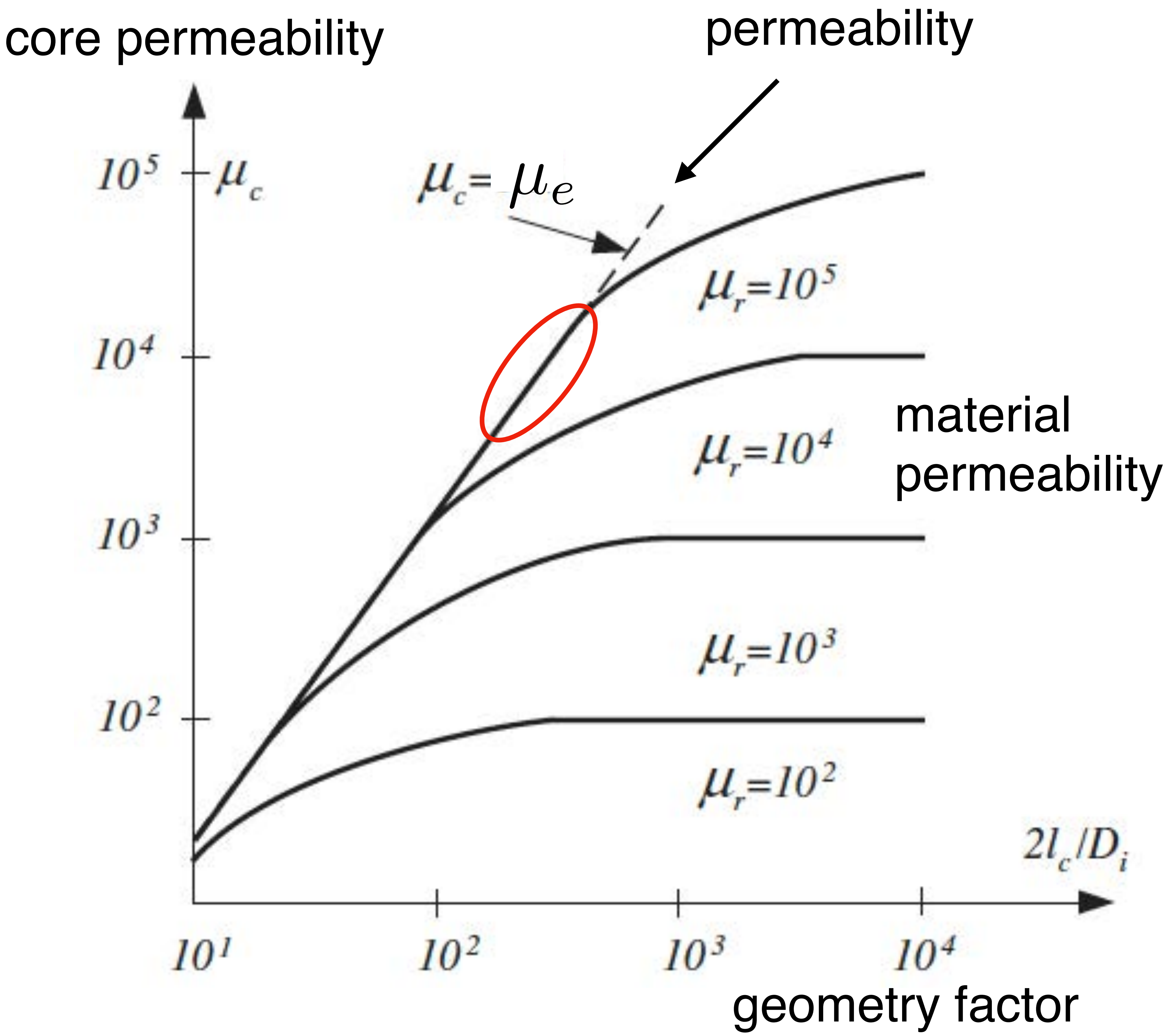
Induction coil magnetometers:

$$V^2 = 4kTR\Delta f$$

Induction coils are made long and thin. You want to increase the number of windings, N, but increasing the diameter increases resistance noise in the windings, so winding along the core is better.

$$V = -\mu_c N A \frac{dB}{dt}$$





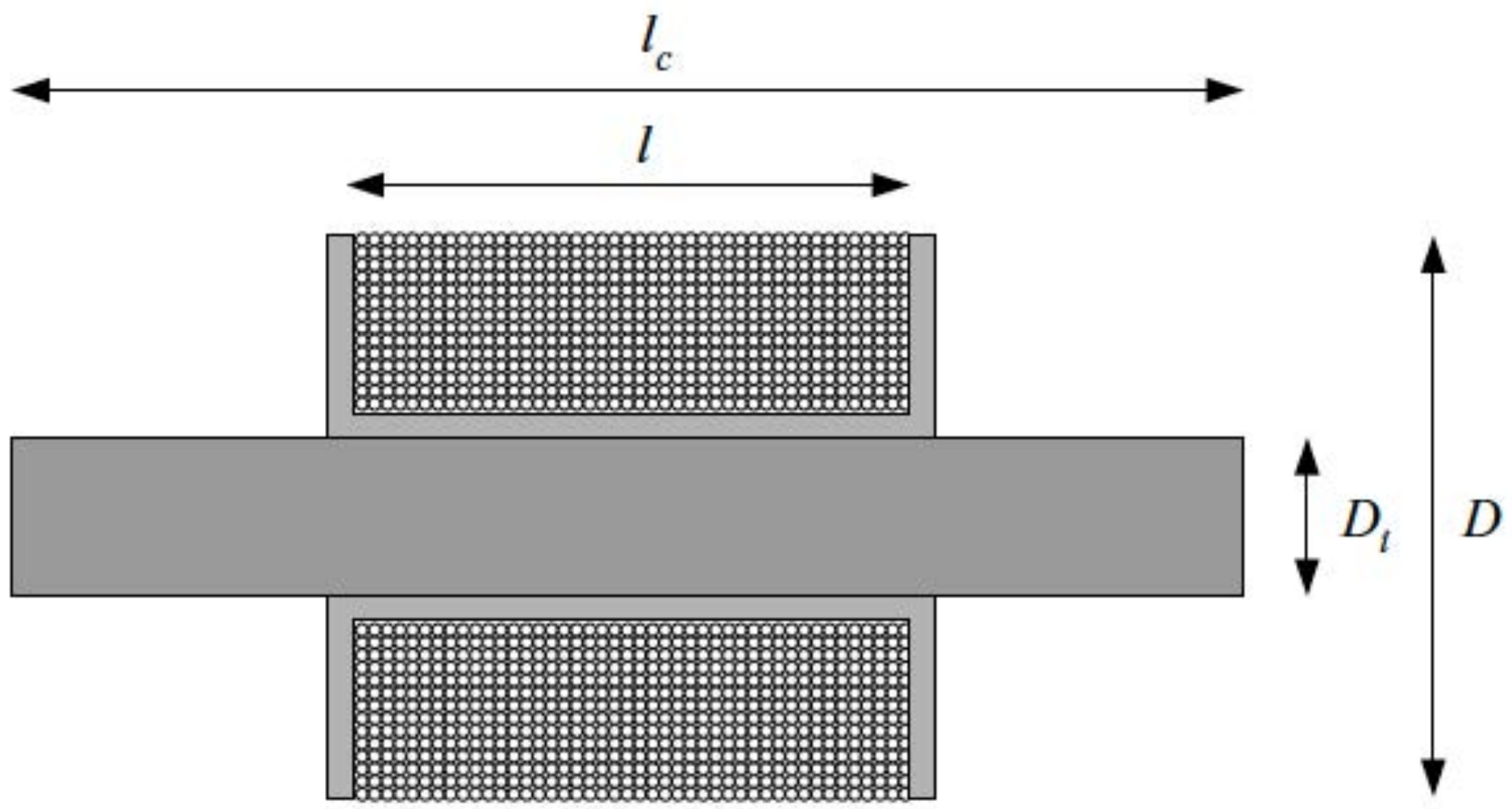
The effective flux magnification depends on the coil geometry, and increases for long coils:

$$\mu_e = \frac{a^2}{\ln(2a) - 1} \quad \text{where} \quad a = \frac{l_c}{D_i}$$

Increasing length increases the core permeability, but only up to the point where the relative permeability of core takes over. By staying on the effective permeability curve, you are not sensitive to temperature and age induced changes in material properties.

(Tumananski, 2007)

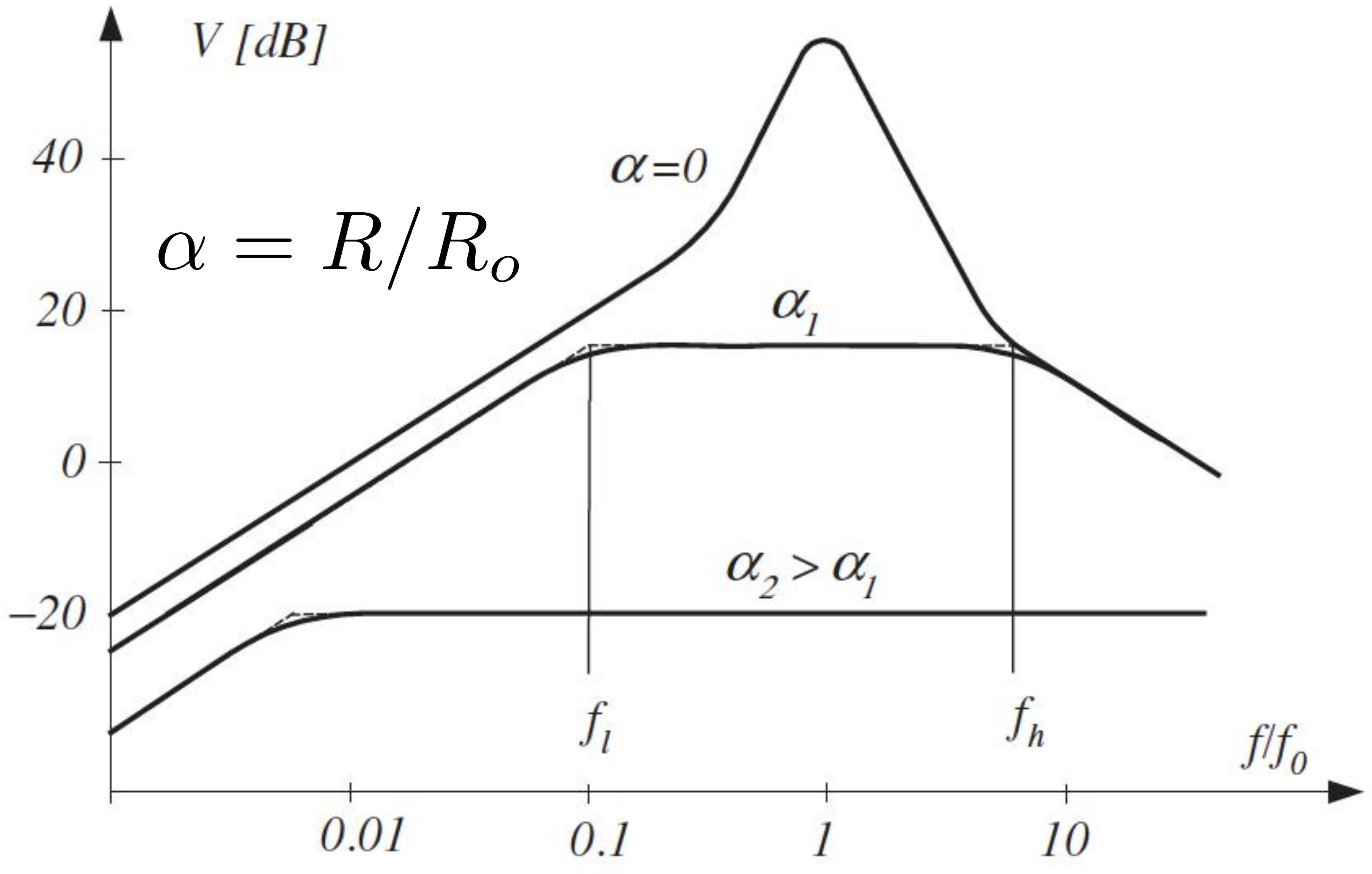
Induction coil magnetometers:



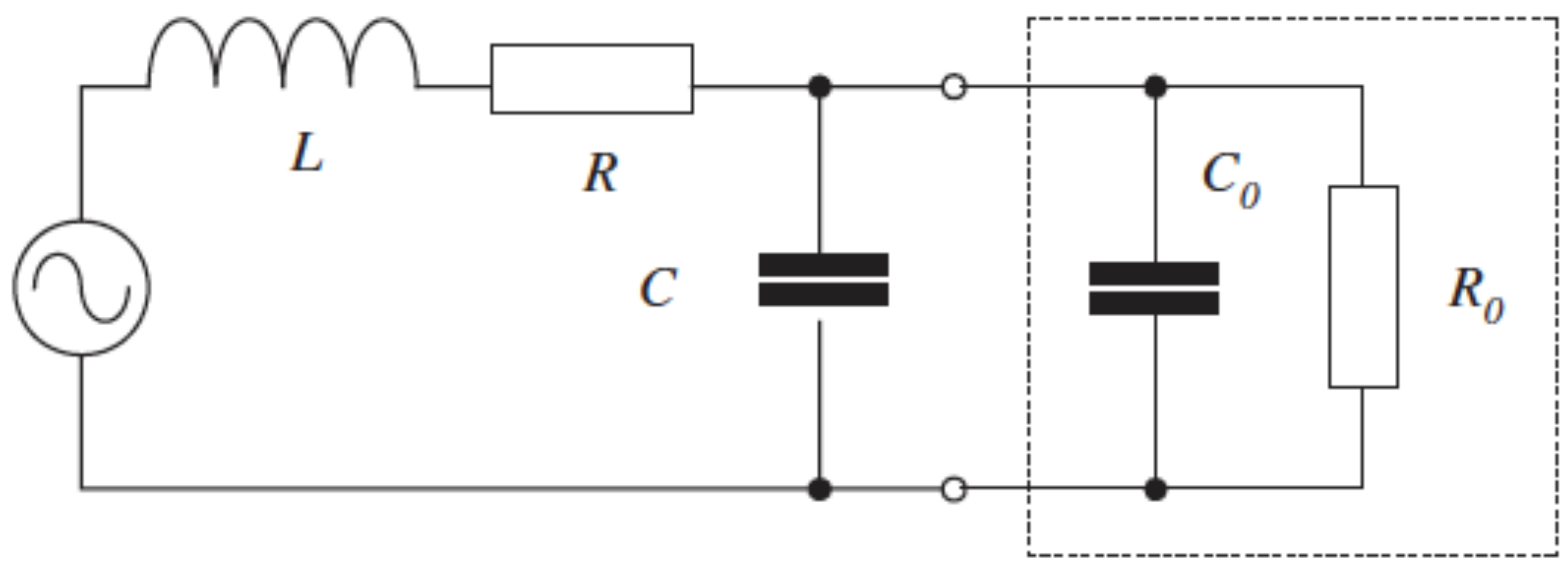
The output and SNR are both increased by making the core long.

The output is increased by making the wire diameter small (this maximizing the number of turns), but the noise is determined by the Johnson noise of the wire resistance, which goes up just as fast as the output does, so is SNR is independent of wire diameter. Effectively, SNR is proportional to weight of copper.

At high frequencies, coil makers laminate the core to reduce eddy currents, and worry about capacitance in the windings.



The capacitance of the coil creates a resonant circuit. This is usually loaded with a resistance to flatten the response curve.



(Tumananski, 2007)

Copper, by the way, is awfully hard to beat as a conductor. Only silver is better. Aluminum is pretty good and a lot lighter if weight is an issue.

Periodic Table of the Elements
Electrical Conductivity (MS/m)

	Group																	
Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	H																	He
2	Li 11.7	Be 25											B 5e-11	C 0.07	N	O	F	Ne
3	Na 20.1	Mg 22.4											Al 37.7	Si 4e-4	P 1e-17	S 5e-17	Cl	Ar
4	K 16.4	Ca 31.3	Sc 1.5	Ti 2.6	V 4	Cr 7.9	Mn 0.5	Fe 11.2	Co 17.9	Ni 14.6	Cu 60.7	Zn 16.9	Ga 1.8	Ge 3e-6	As 3.8	Se 8	Br 1e-16	Kr
5	Rb 47.8	Sr 5	Y 1.8	Zr 2.3	Nb 6.6	Mo 17.3	Tc 0.001	Ru 14.9	Rh 23	Pd 10	Ag 62.9	Cd 14.7	In 3.4	Sn 8.7	Sb 2.6	Te 2e-4	I 1e-11	Xe
6	Cs 5.3	Ba 2.8	La 1.9	Hf 3.4	Ta 8.1	W 18.2	Re 5.8	Os 12.3	Ir 21.3	Pt 9.4	Au 48.8	Hg 1	Tl 5.6	Pb 4.8	Bi 0.9	Po 0.7	At	Rn
7	Fr	Ra 1	Ac	Rf	Db	Sg	Bh	Hs	Mt	Uun	Uuu	Uub						
	Lanthanides				Ce 1.4	Pr 1.5	Nd 1.6	Pm 2	Sm 1.1	Eu 1.1	Gd 0.8	Tb 0.9	Dy 1.1	Ho 1.1	Er 1.2	Tm 1.3	Yb 3.7	Lu 1.5
	Actinides				Th 7.1	Pa 5.6	U 3.6	Np 0.8	Pu 0.7	Am 0.7	Cm	Bk	Cf	Es	Fm	Md	No	Lr

Induction coil magnetometers:

Induction coil noise sources:

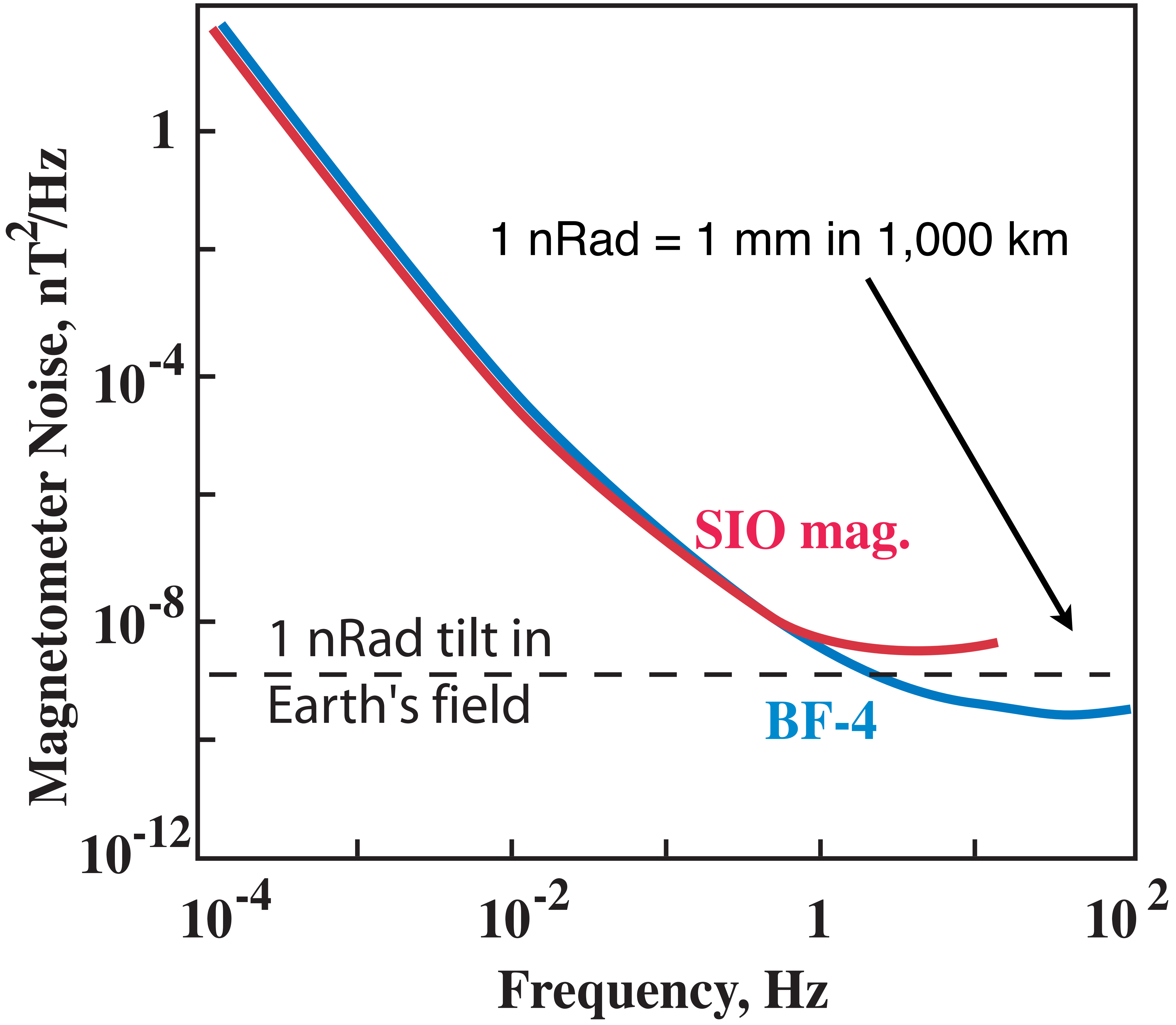
Noise floors ~ 0.01 pT

Environmental noise (power, trains, cathodic protection)

Sensors - thermal resistance noise in windings

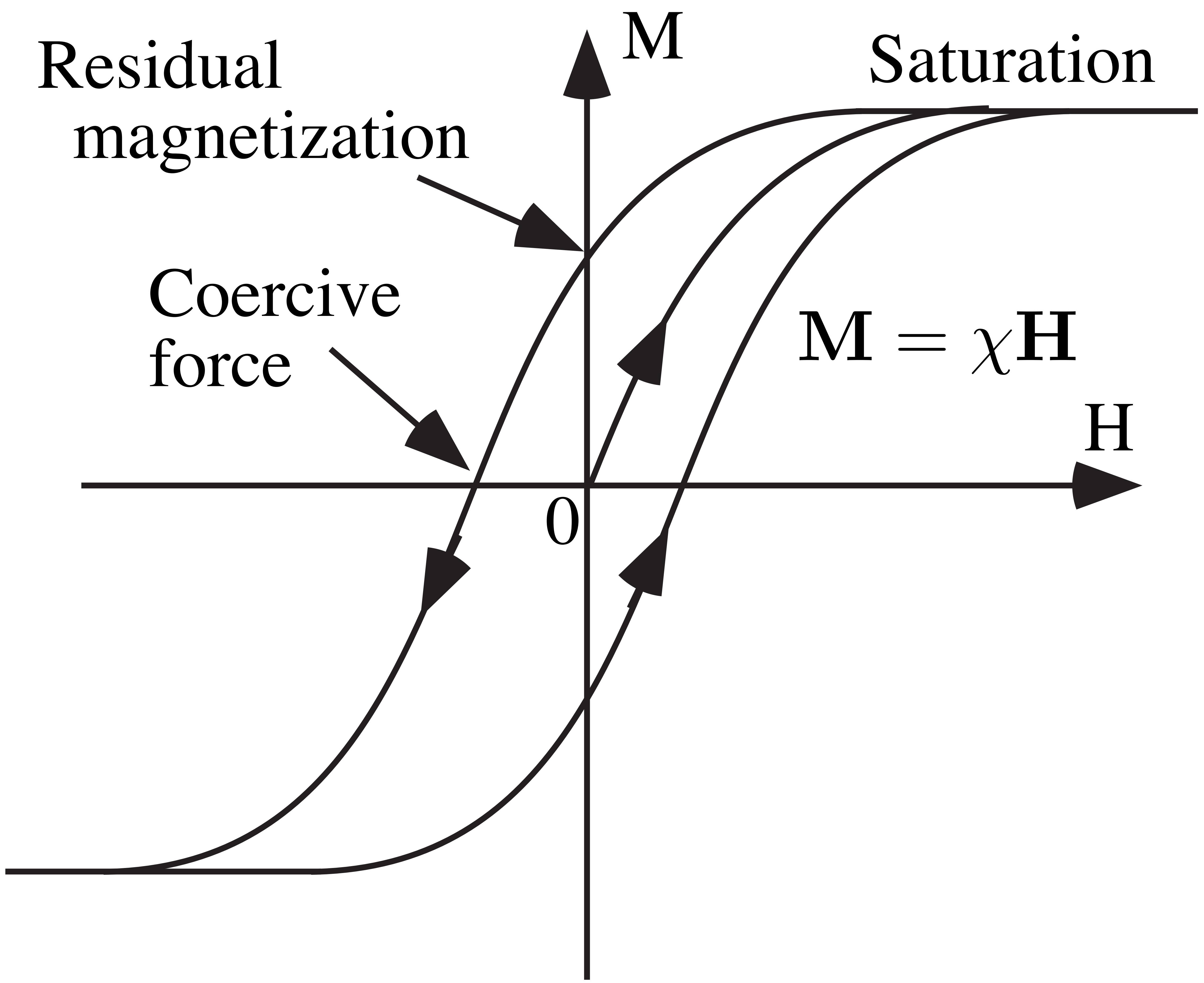
Instrument - Amplifiers (1/*f*), ADC, noise from other circuits

Sensor motion



Fluxgate magnetometers:

Fluxgates rely on the hysteresis curve of soft magnetic materials.



Fluxgate magnetometers:

Fluxgates generate a time varying flux B by creating a time varying μ_e

Recall from our induction coil: $V = -NA\mu_e \dfrac{dB}{dt}$

We could write this in terms of the magnetizing field H

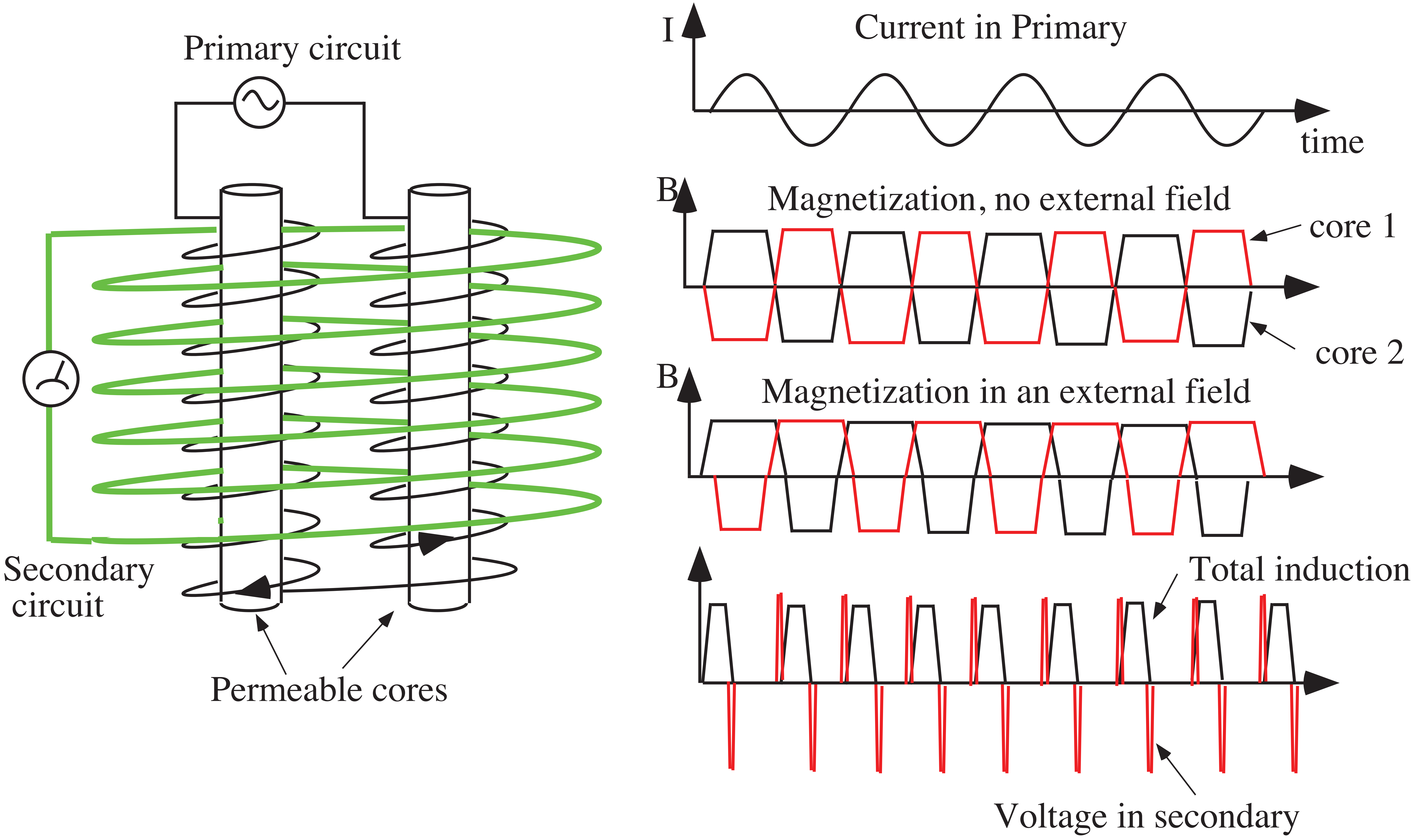
$$V = -NA\mu_e \frac{dB}{dt} = -NA\mu_e\mu_o \frac{dH}{dt} \qquad \text{where} \qquad B = \mu_o H$$

Imagine a time varying permeability instead of a time varying field:

$$V = -NA\mu_e \frac{dB}{dt} = -NA\mu_o \frac{d\mu_e}{dt} H = -NA\frac{d\mu_e}{dt} B$$

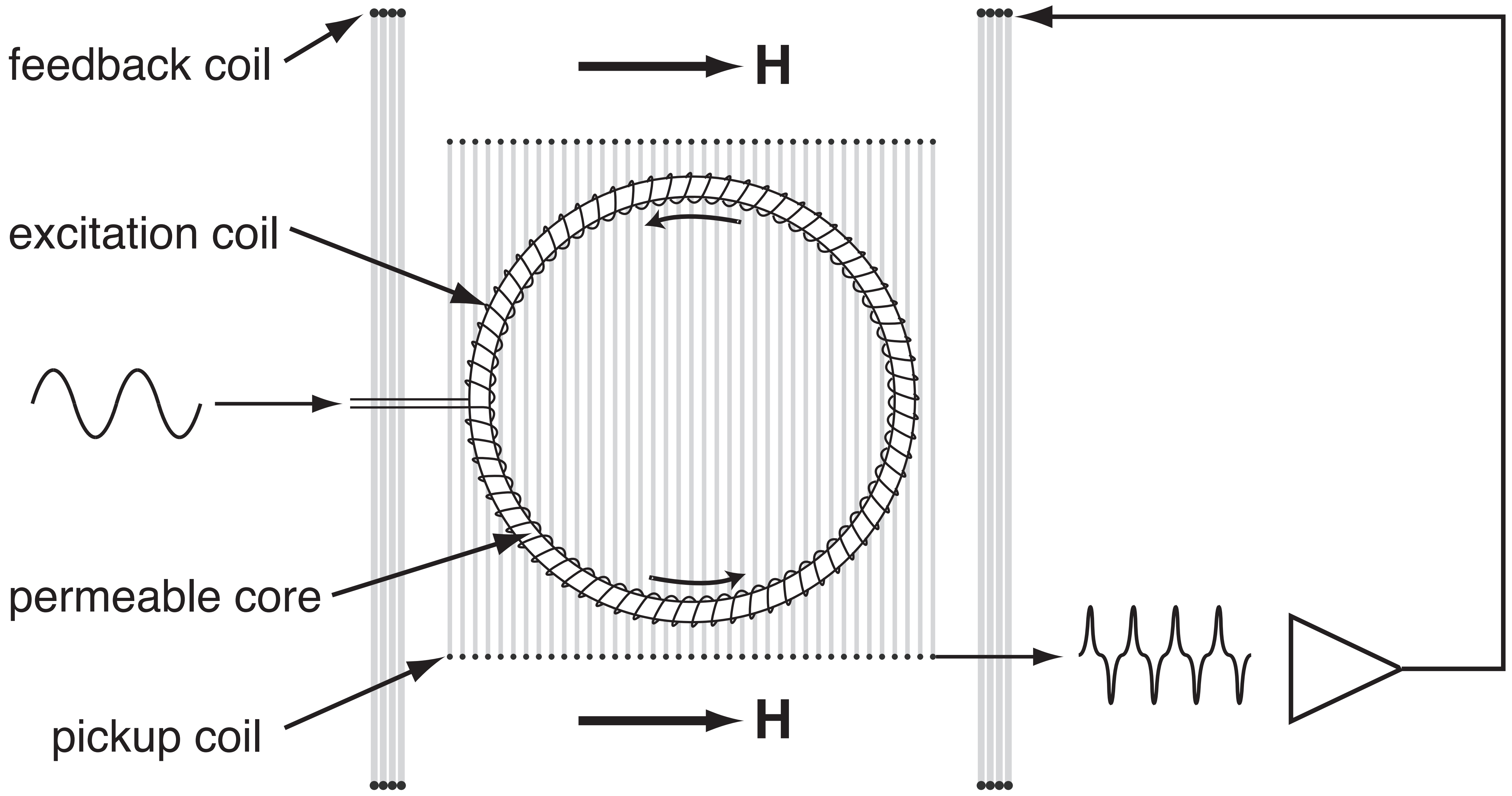
Fluxgate magnetometers:

This can done by saturating the core with an excitation coil, since a saturated core has zero effective permeability. Excitation frequencies ~ kHz



Fluxgate magnetometers:

Most good fluxgates are ring-cores operated at a null using feed-back.



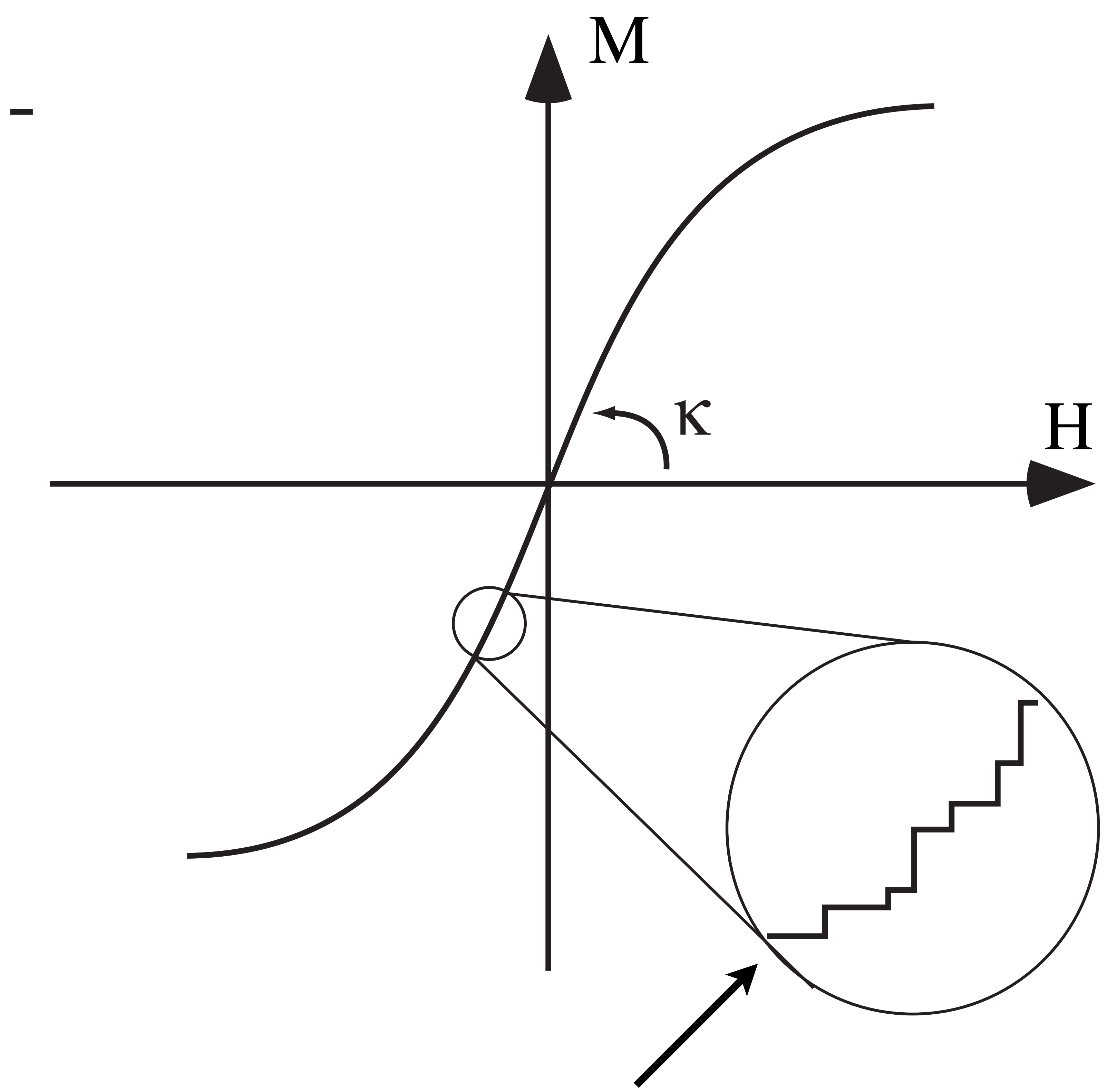
The effective permeability of the core is approximately diameter divided by thickness

$$\mu_e = \frac{L}{d}$$

Fluxgates are good between DC and a few Hertz, but are power hungry and the excitation signal can create noise in electric field measurements.

Fluxgate magnetometers:

The limiting source of noise for a fluxgate is individual magnetic domains being magnetized.

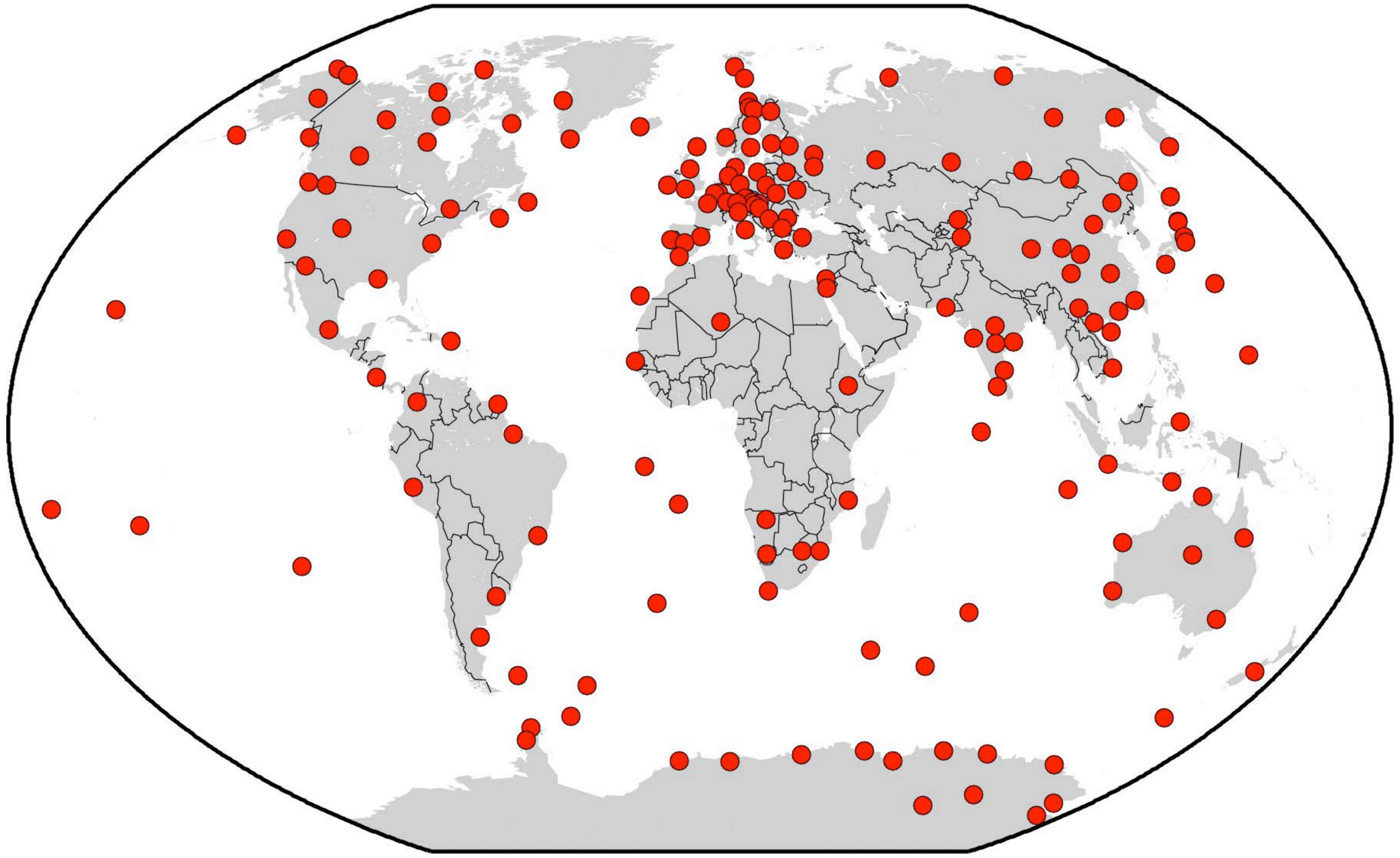


Barkhausen noise

Fluxgates are, by design, DC measurements, but can be operated at several hundred hertz

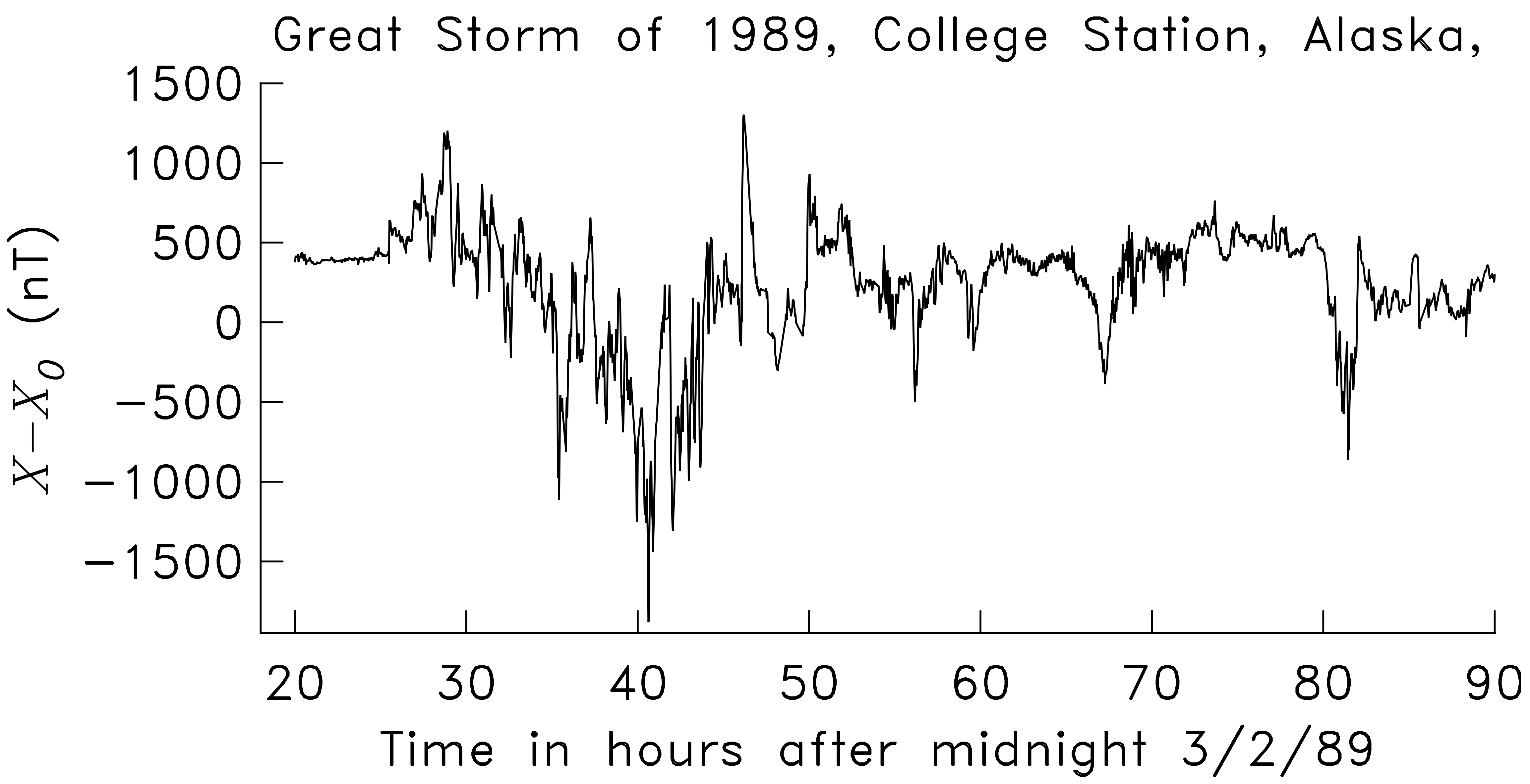
Noise floors ~ 0.01 nT

Magnetic observatories:

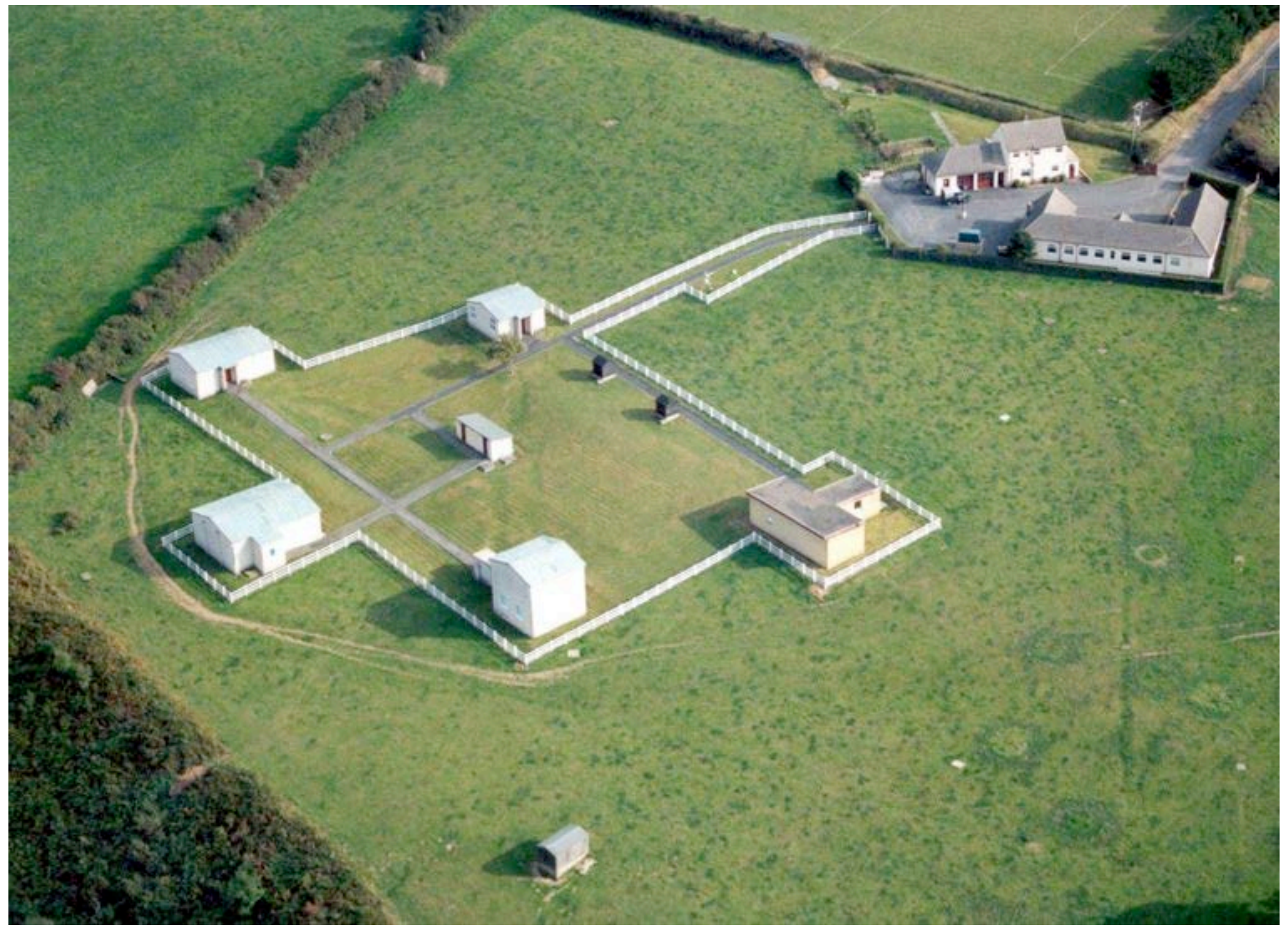


Locations of currently operating geomagnetic observatories

http://www.geomag.bgs.ac.uk/education/earthmag.html

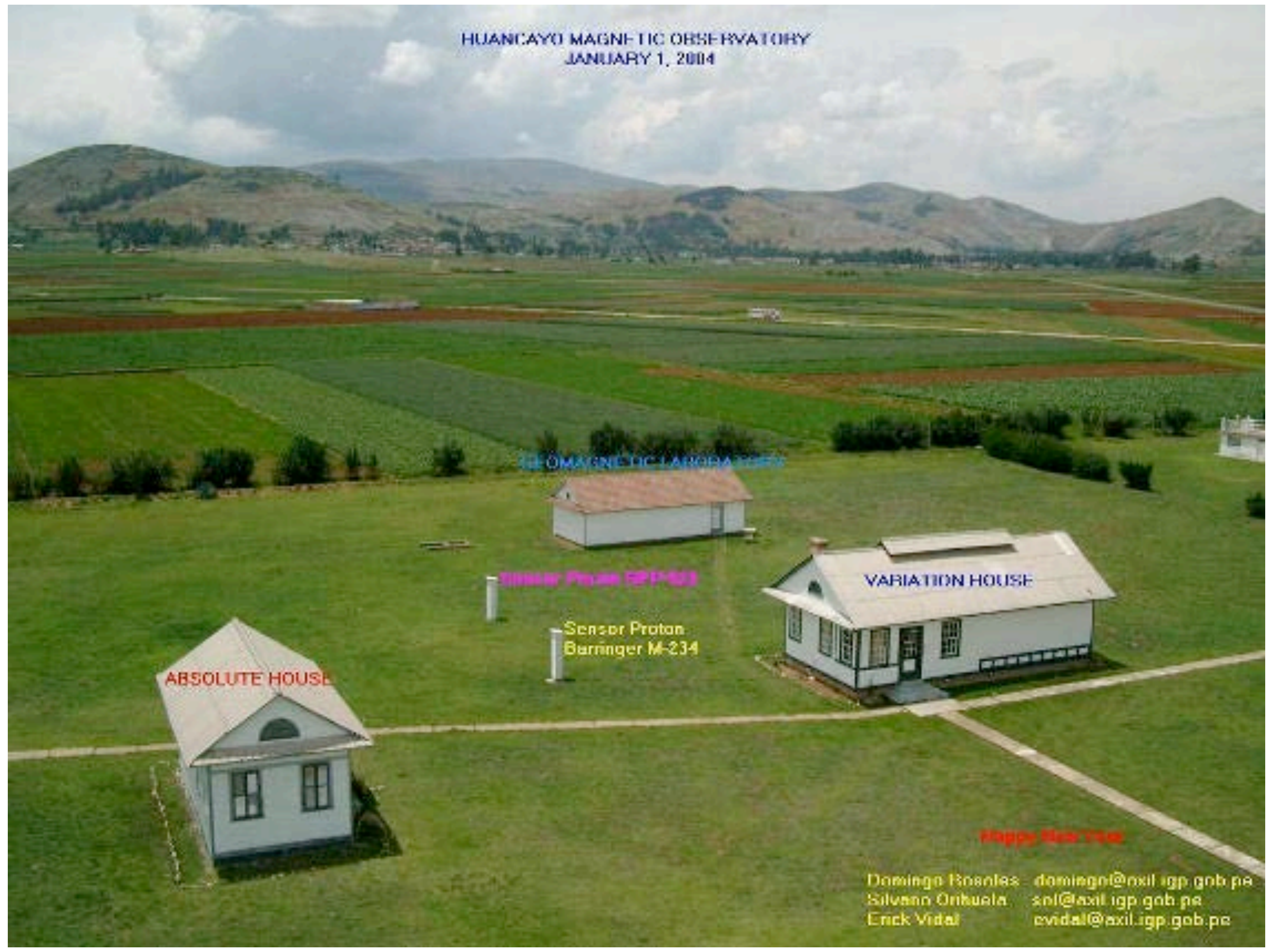


Magnetic observatories:



Hartland, UK

http://www.geomag.bgs.ac.uk/operations/hartland.html



Huancayo, Peru

https://www.intermagnet.org/images/photos/hua.jpg

Boulder, USA

Azimuth mark, Absolutes building and Coil building at Boulder magnetic observatory.

Theodolite, Boulder, USA

Zeiss Jena 010B Theodolite for making absolute measurements at Boulder magnetic observatory.(Public domain.)

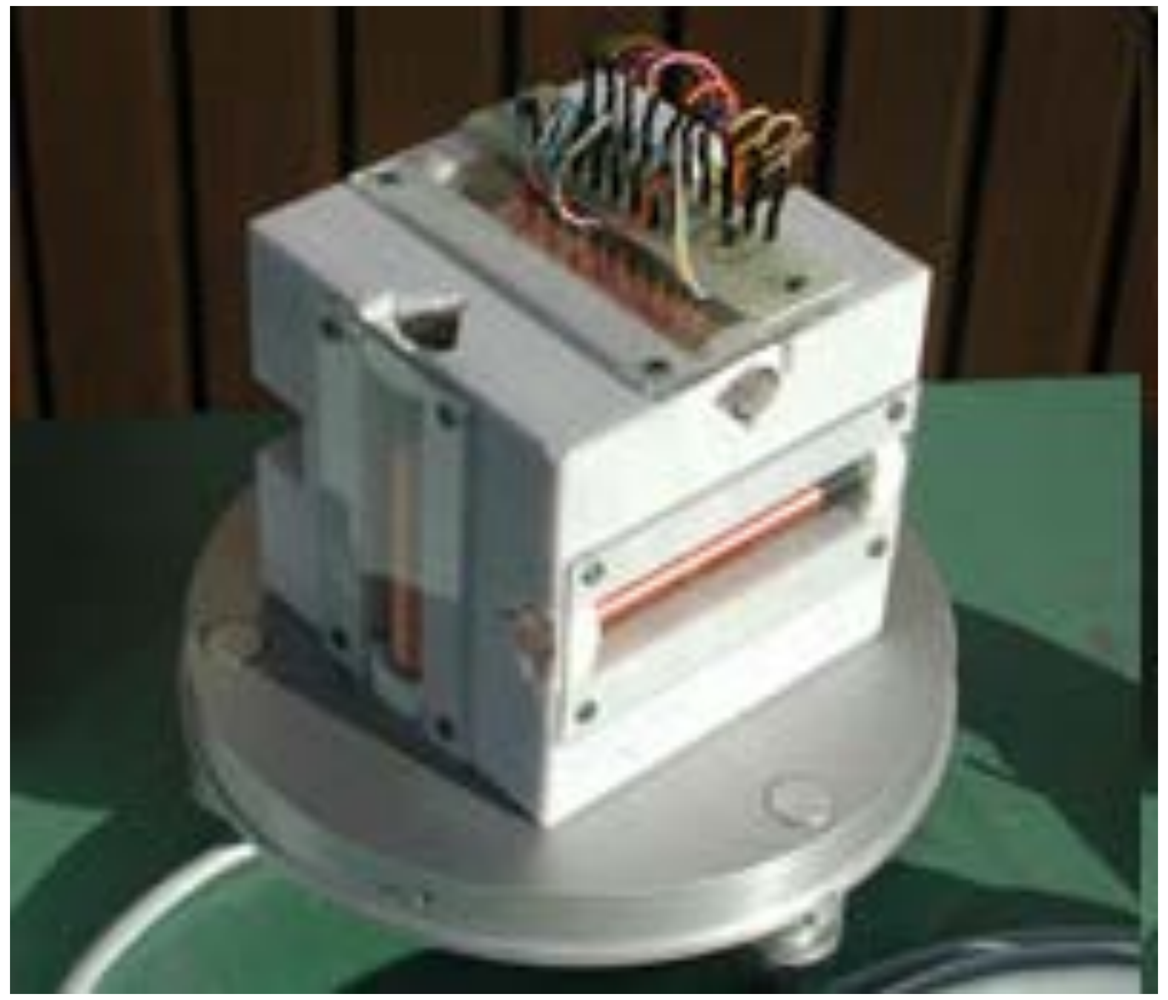
Magnetic observatories:

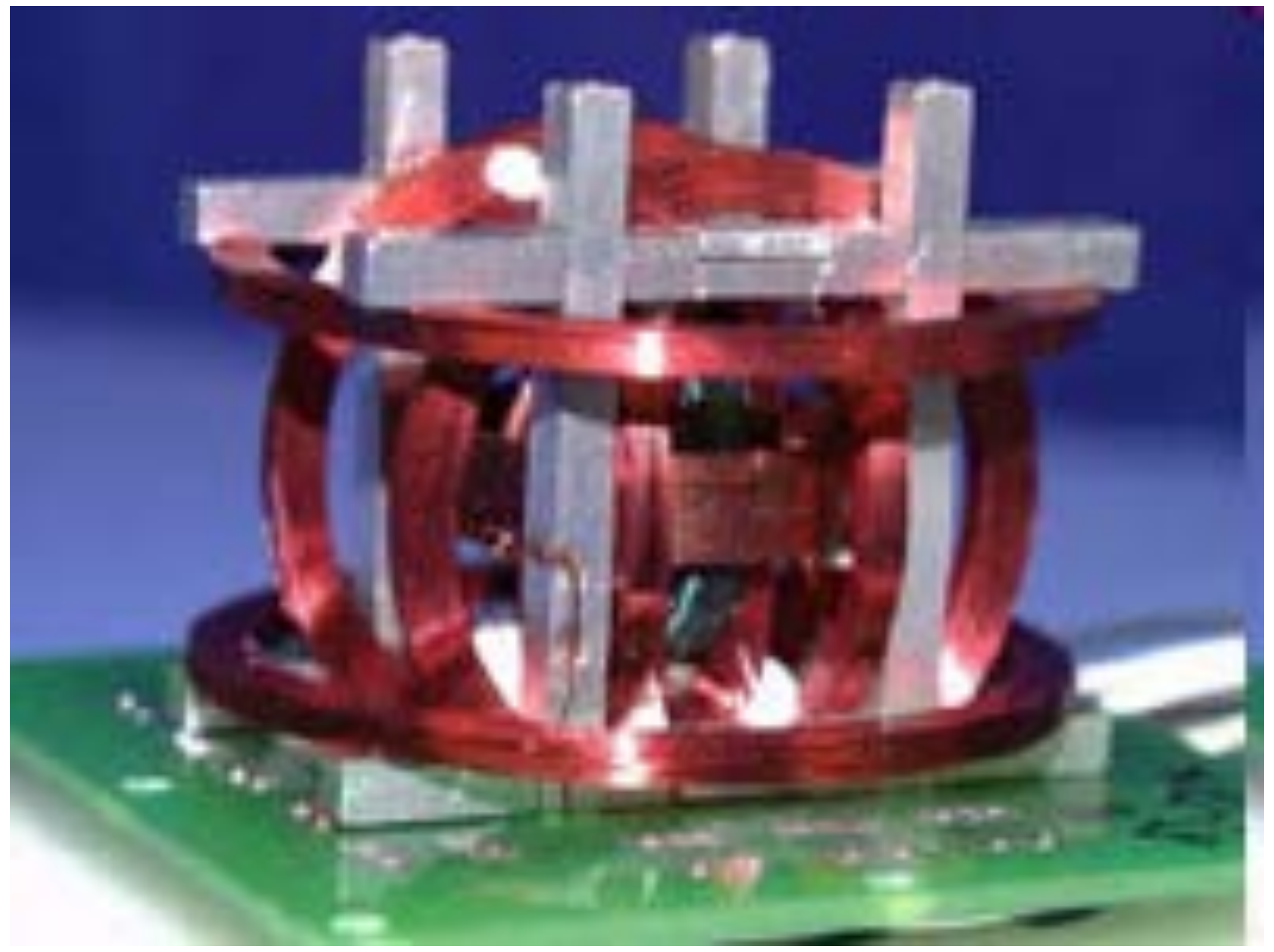
Started with torsion magnetometers. Now use PPM (absolute calibration), fluxgates, and direction measurements

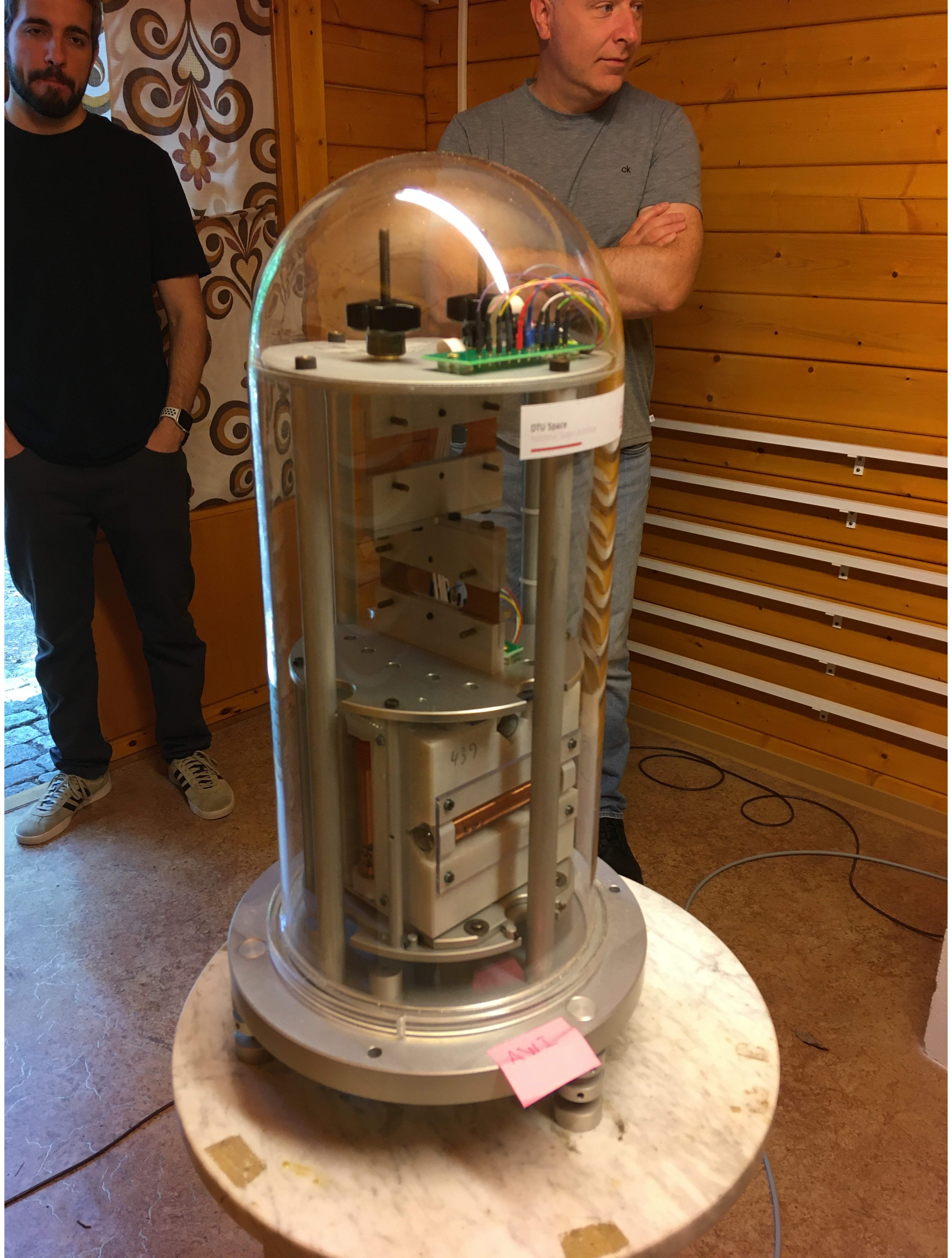
Magnetic observatories:

Fluxgates are temperature sensitive, need mounts that won’t move with soil, and need careful calibration.

They are often operated as feed-back sensors.







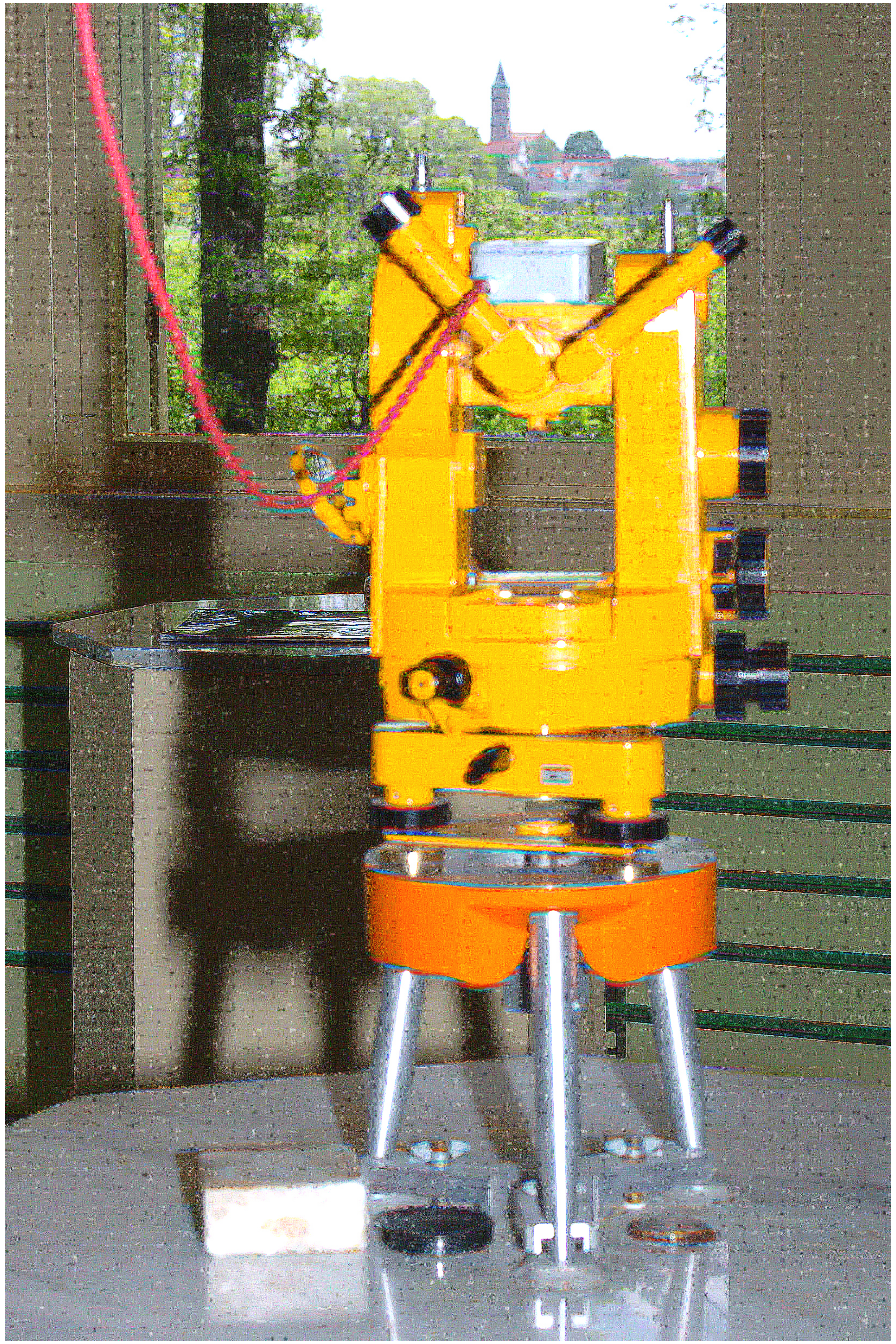

DTU Space

Magnetic observatories:

Declination and inclination are measured using a 1-axis fluxgate mounted on a non-magnetic theodolite.

The theodolite is leveled (*I*) or pointed to an azimuthal reference.

It is then rotated vertically and horizontally until the fluxgate reads zero. It is then perpendicular to that field component. Precision is measured in arc-seconds.



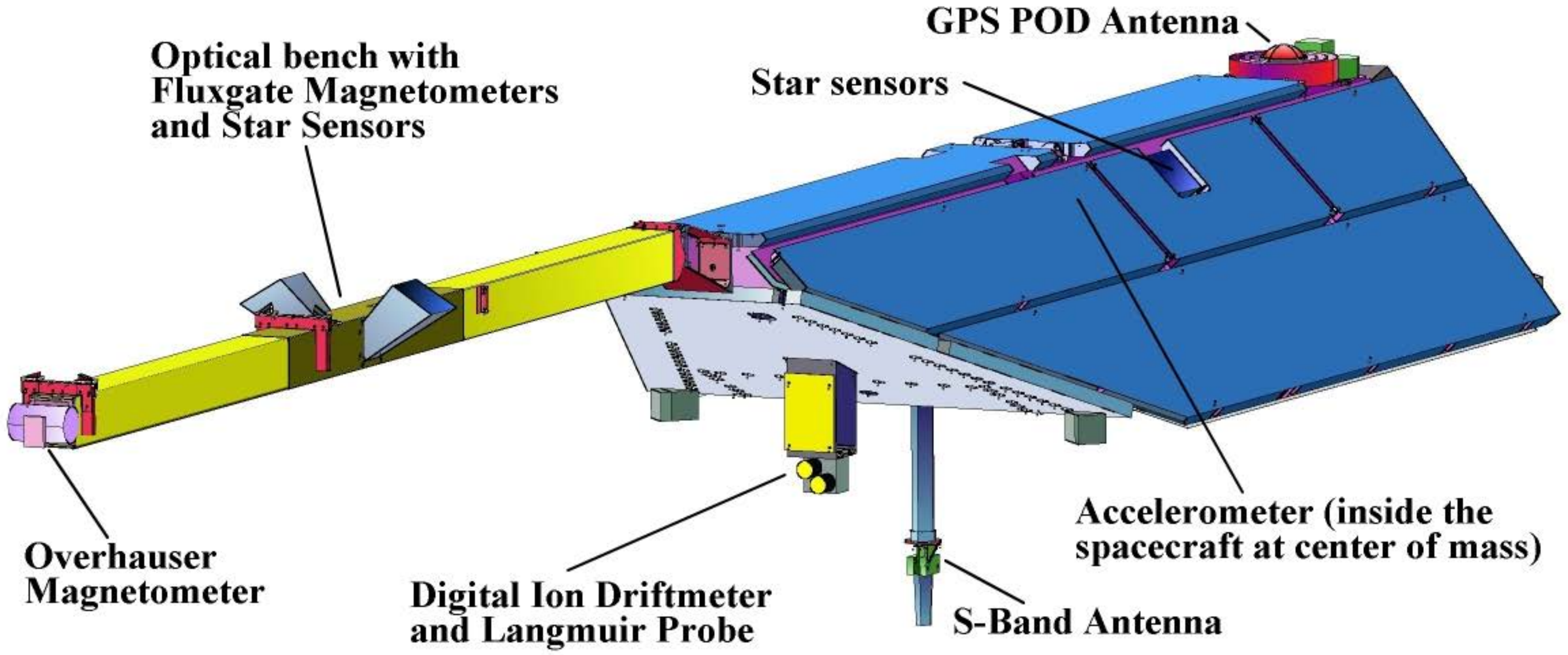
Magnetic satellites:

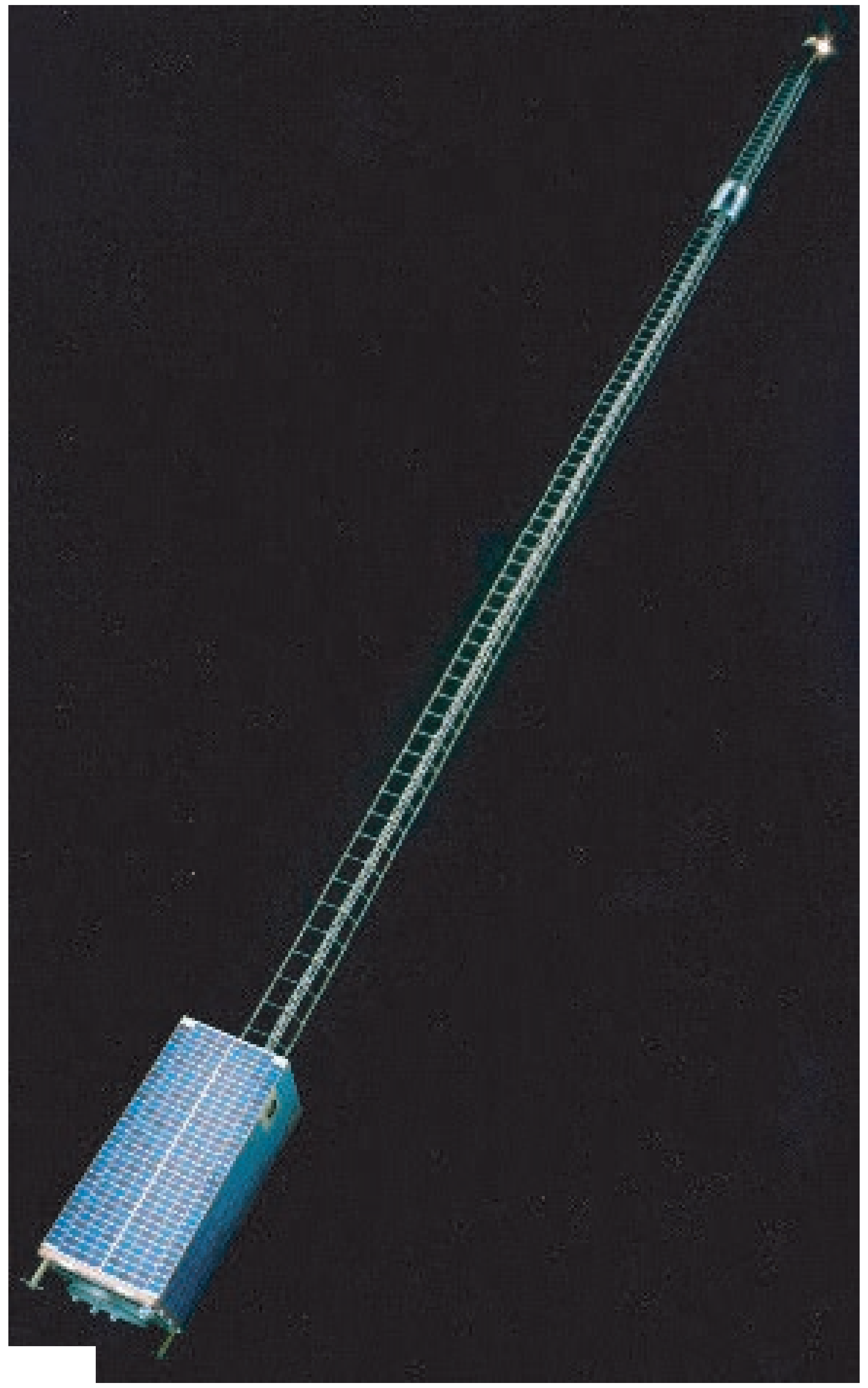
Cover the whole globe.

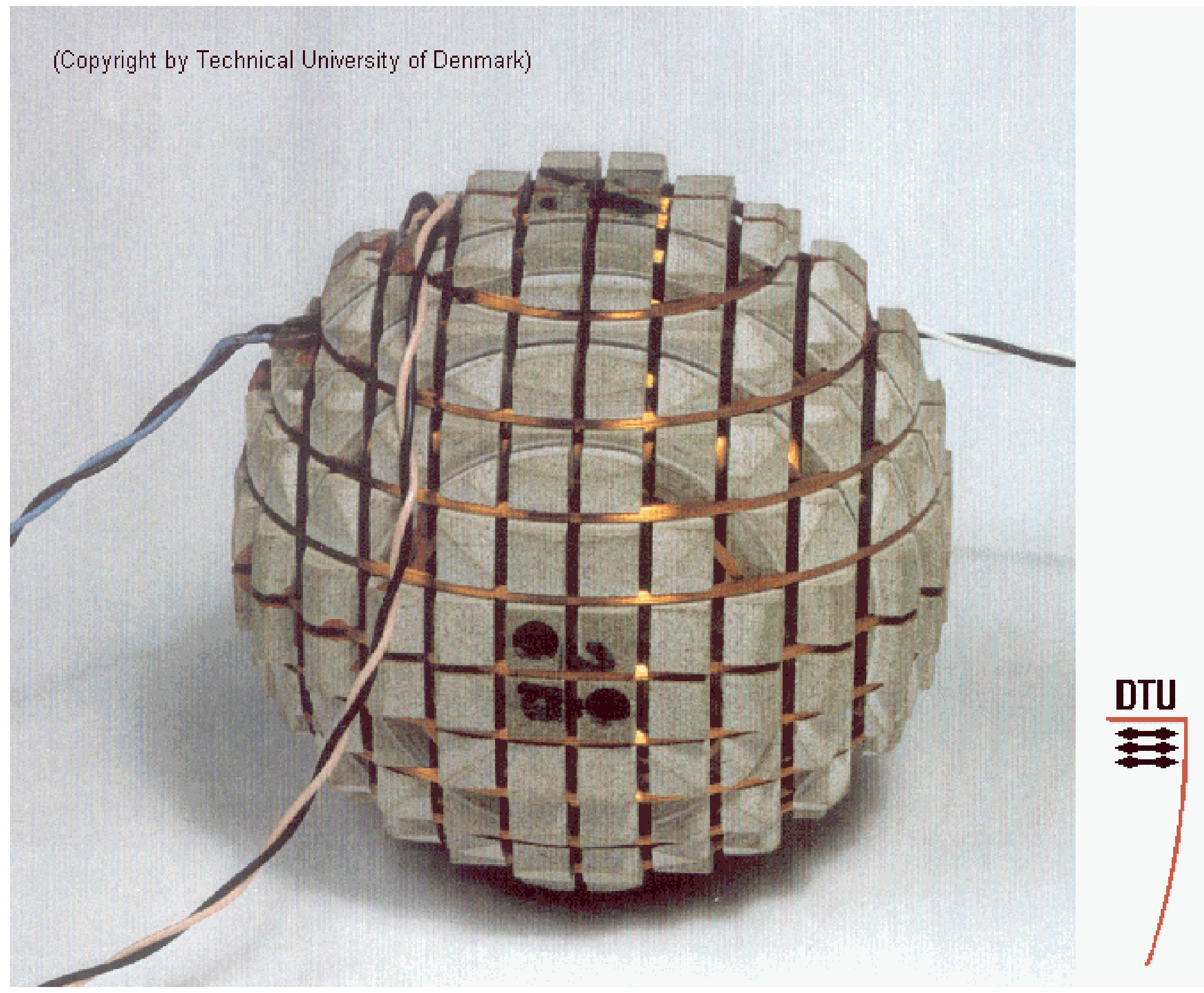
Use both fluxgates, oriented by star cameras, and total field instruments.

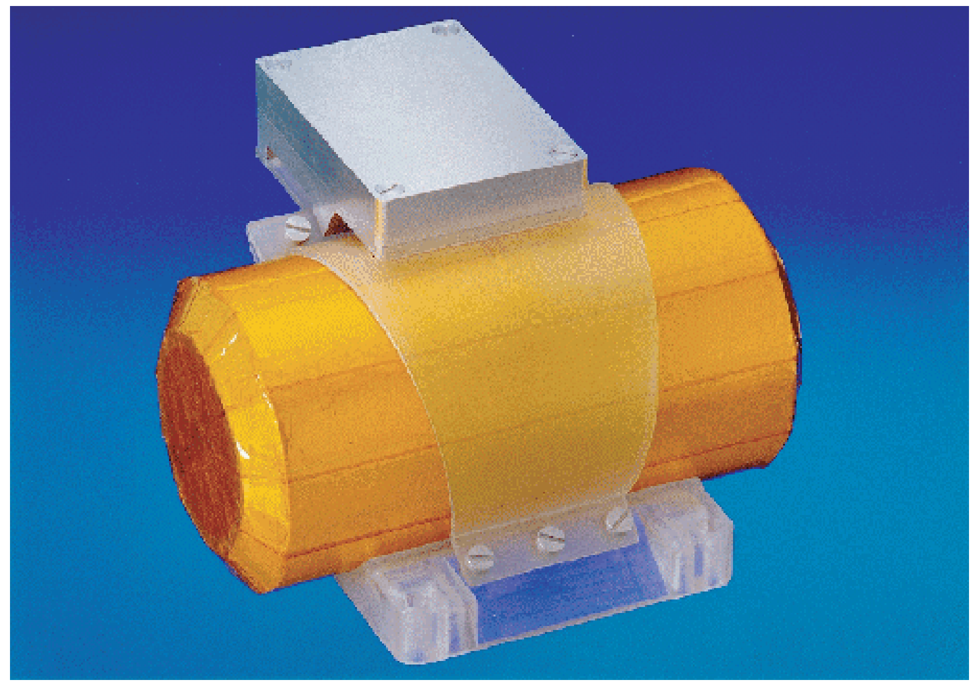
Need to distance the sensors from the satellite.

Mix space and time.









Overhauser Magnetometer

leti

Magnetic satellites:

Magsat

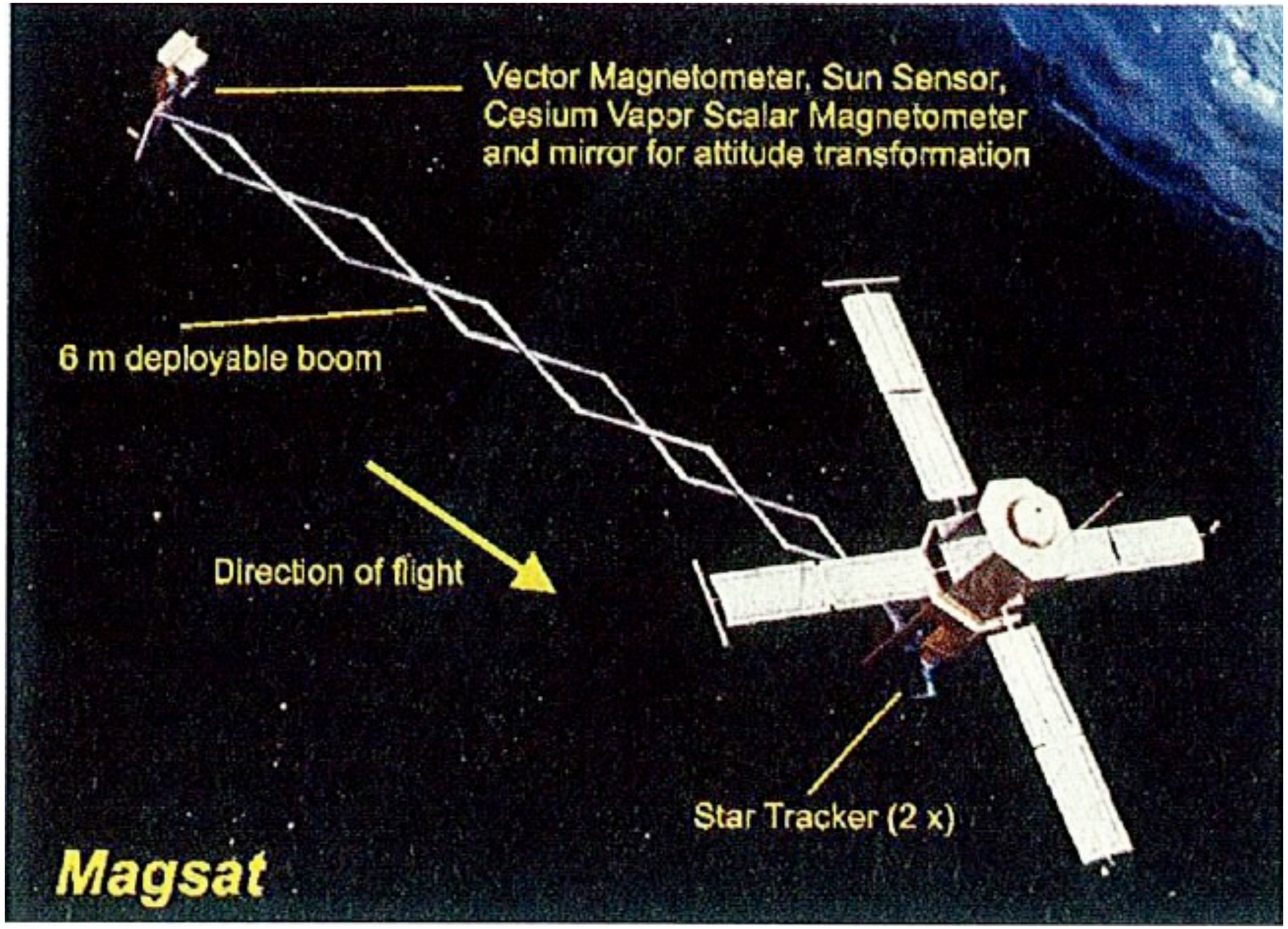
Nov 1979 — June 1980

Altitude 450 — 350 km

Period 90 min

Inclination 97°

Fixed local time (dawn/dusk)



Magnetic satellites:

Ørsted

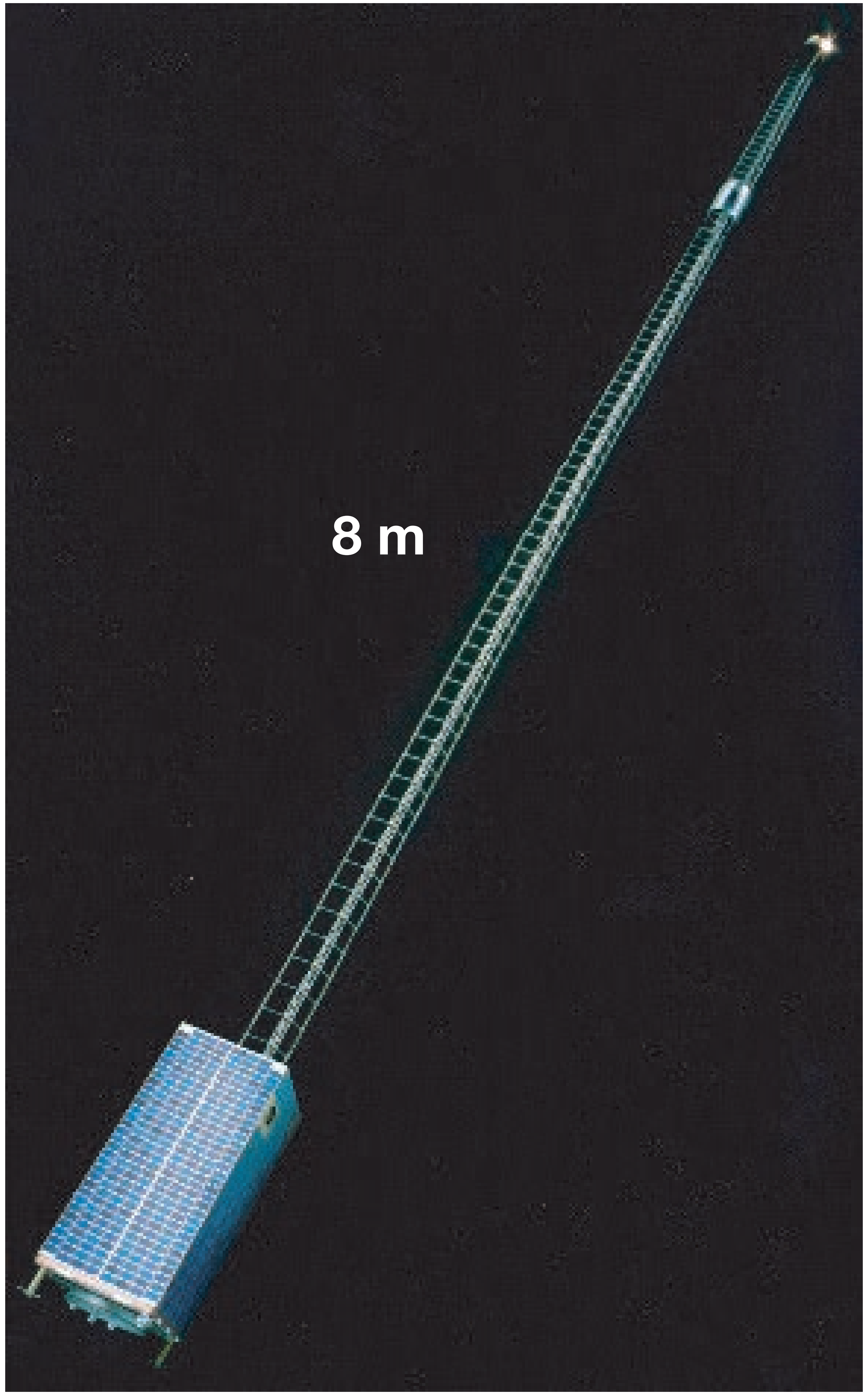
Feb 1999 — 2005

Altitude 649 — 865 km

Period 100 min

Inclination 96.48°

Time drift 0.91 min/day



Magnetic satellites:

CHAMP

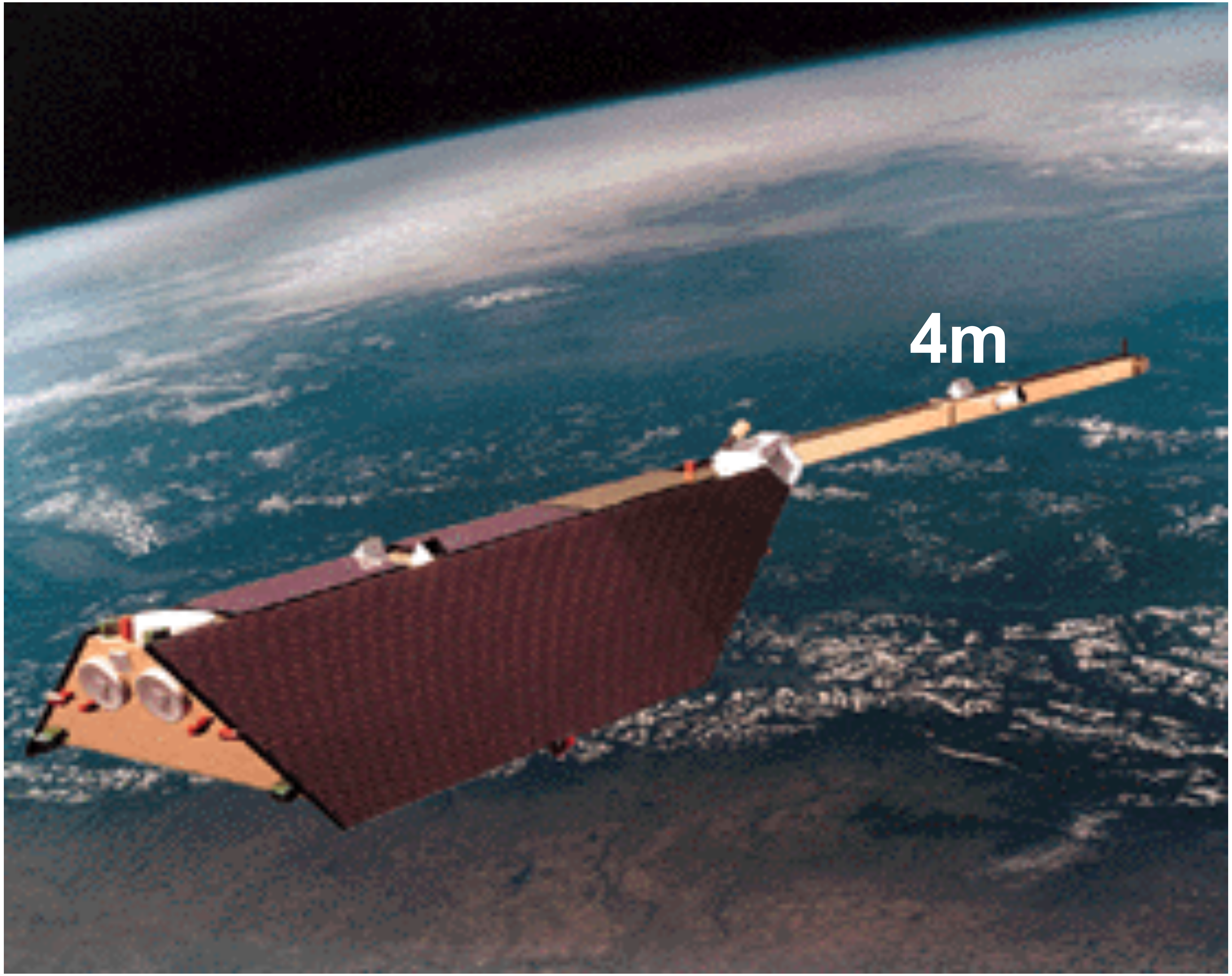
July 2000 — Sept 2010

Altitude 454 — 280 km

Period 93 min

Inclination 87.3°

Time drift 5.45 min/day



Magnetic satellites:

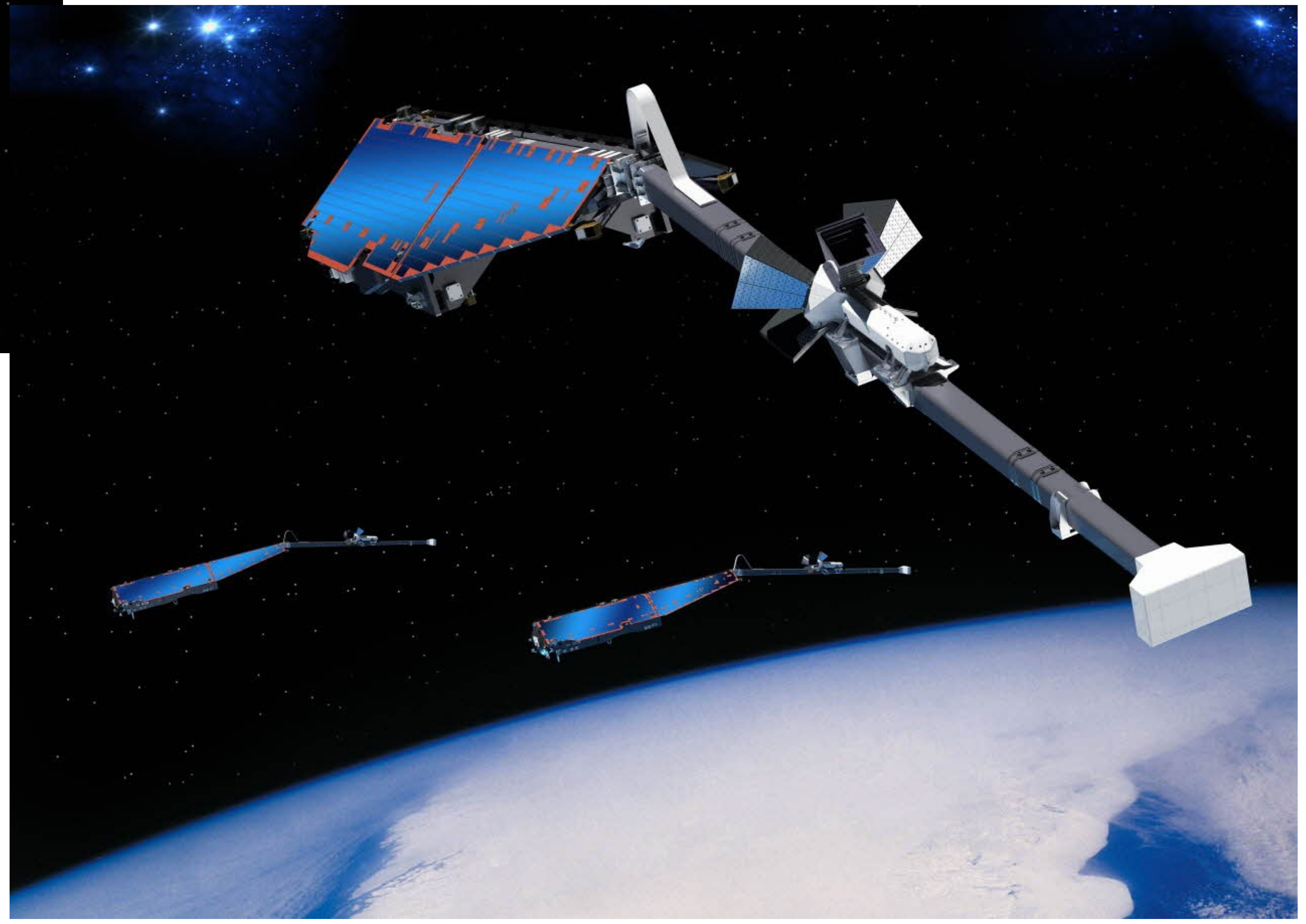
SWARM

Nov 2013 — today

Altitude 1 @ 530 km
Altitude 2 @ 450 km

Inclination 87.3°
Inclination 88°

Variable time drift

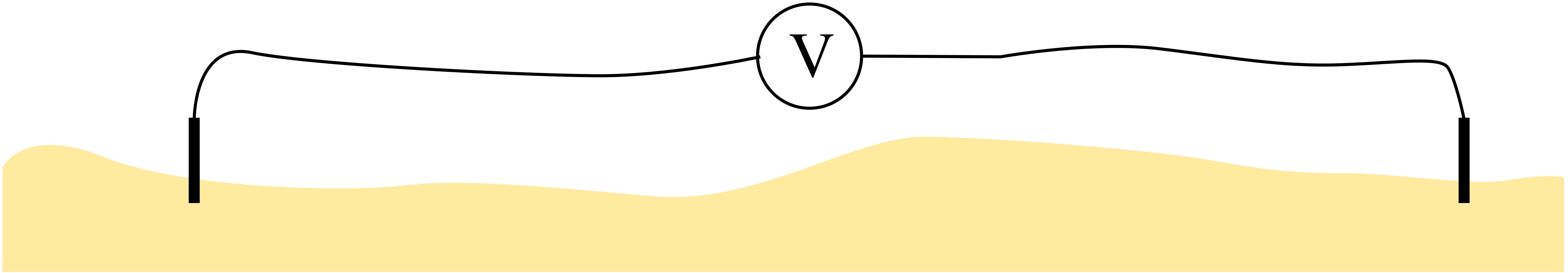


(credit: ESA)

https://www.esa.int/Applications/Observing_the_Earth/Swarm

Electric field:

Electric field measurements. Simple in theory...



$$E = \frac{\Delta V}{l}$$

For high frequencies, metal electrodes can be used, but they suffer from polarization and corrosion noise.

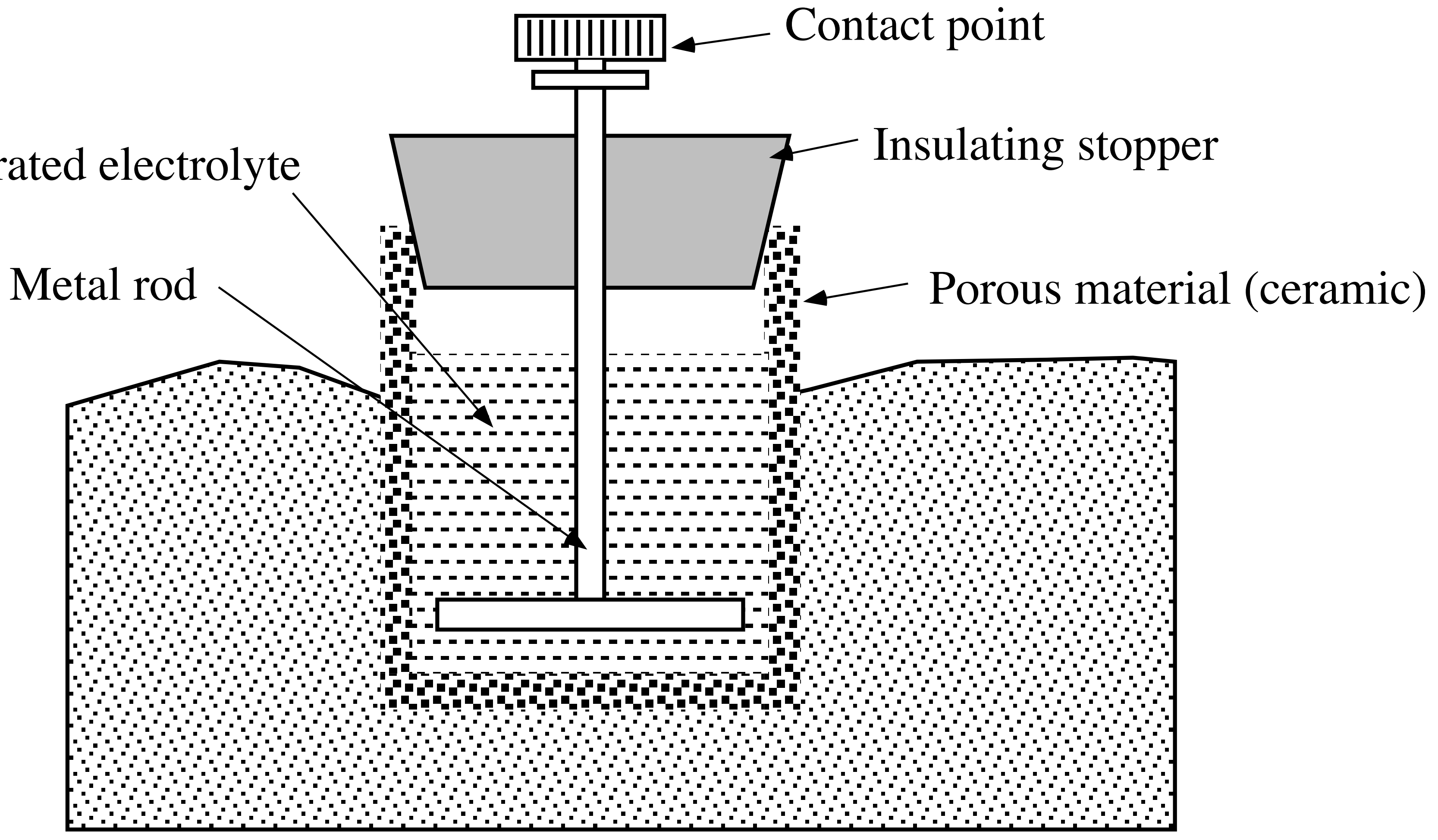
Electric field:

Non-polarizing electrodes:

Electrodes of the first kind (metal in electrolyte):

$$\mathrm{M} \rightleftharpoons \mathrm{M}^{+} + e^{-}$$

e.g. copper - copper sulphate

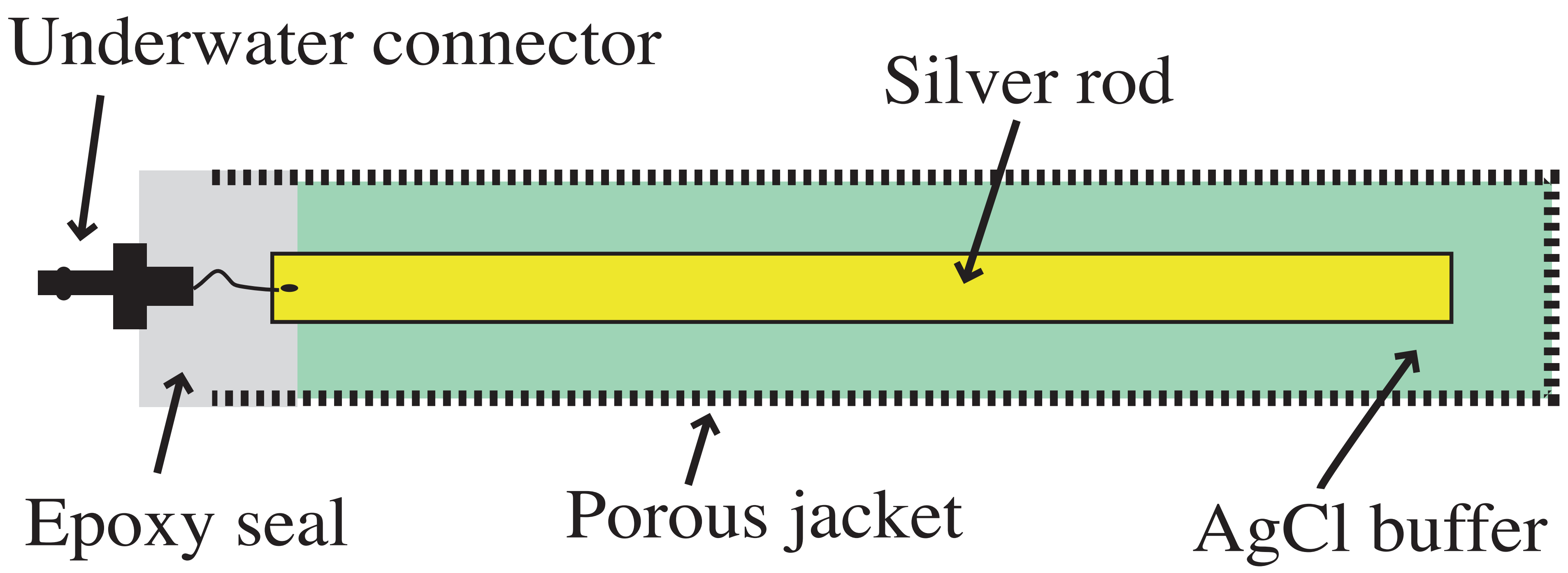


$$\mathrm{Cu} - \mathrm{CuSO_4}$$

Electrodes of the second kind (metal coated with insoluble metal salt):

$$\mathrm{M} + \mathrm{A}^{-} \rightleftharpoons \mathrm{MA} + e^{-}$$

e.g. silver - silver chloride, lead - lead chloride



$$\mathrm{Ag}^{0} + \mathrm{Cl}^{-} \rightleftharpoons \mathrm{AgCl} + e^{-}$$

Electric field: **Amplifiers**

Johnson, or thermal noise, found in all resistors, thermal motion of *e*[-] $V^2 = 4kTR\Delta f$

Shot noise is caused by quantization of charge during current flow. Poisson variance: $\sigma^2 = 2qI\Delta f$

For a current flowing though a resistor $V^2 = 2qIR\Delta f$

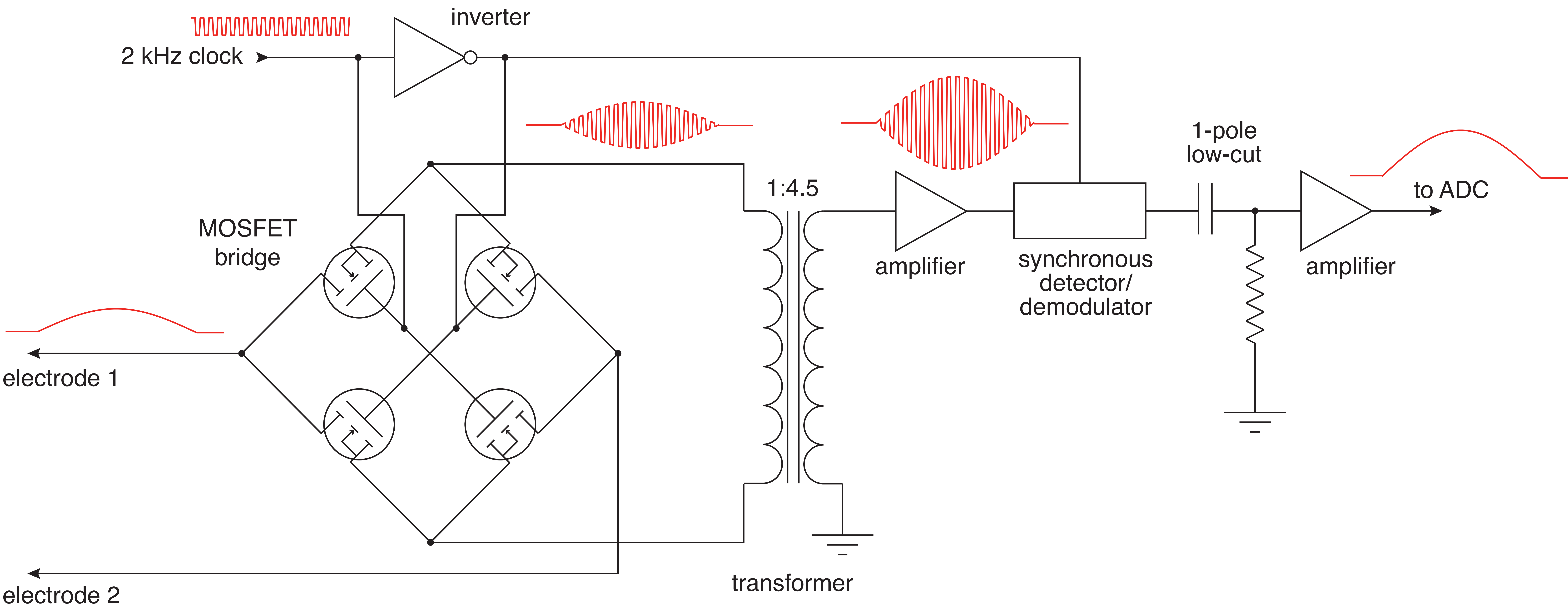
1/*f* noise is a ubiquitous and poorly understood property of semiconductors: $V^2 = \frac{\alpha}{f} R^2 \Delta f$

Alpha is about 10-4 and depends on semiconductor quality.

The low frequency limit of 1/f has not been found, but must exist.

Electric field: Amplifiers

Chopper amplifiers move low frequencies to a higher part of the $1/f$ noise spectrum:



Electric field:

E-field noise sources:

Cultural noise (cathodic pipeline protection, power systems etc.)

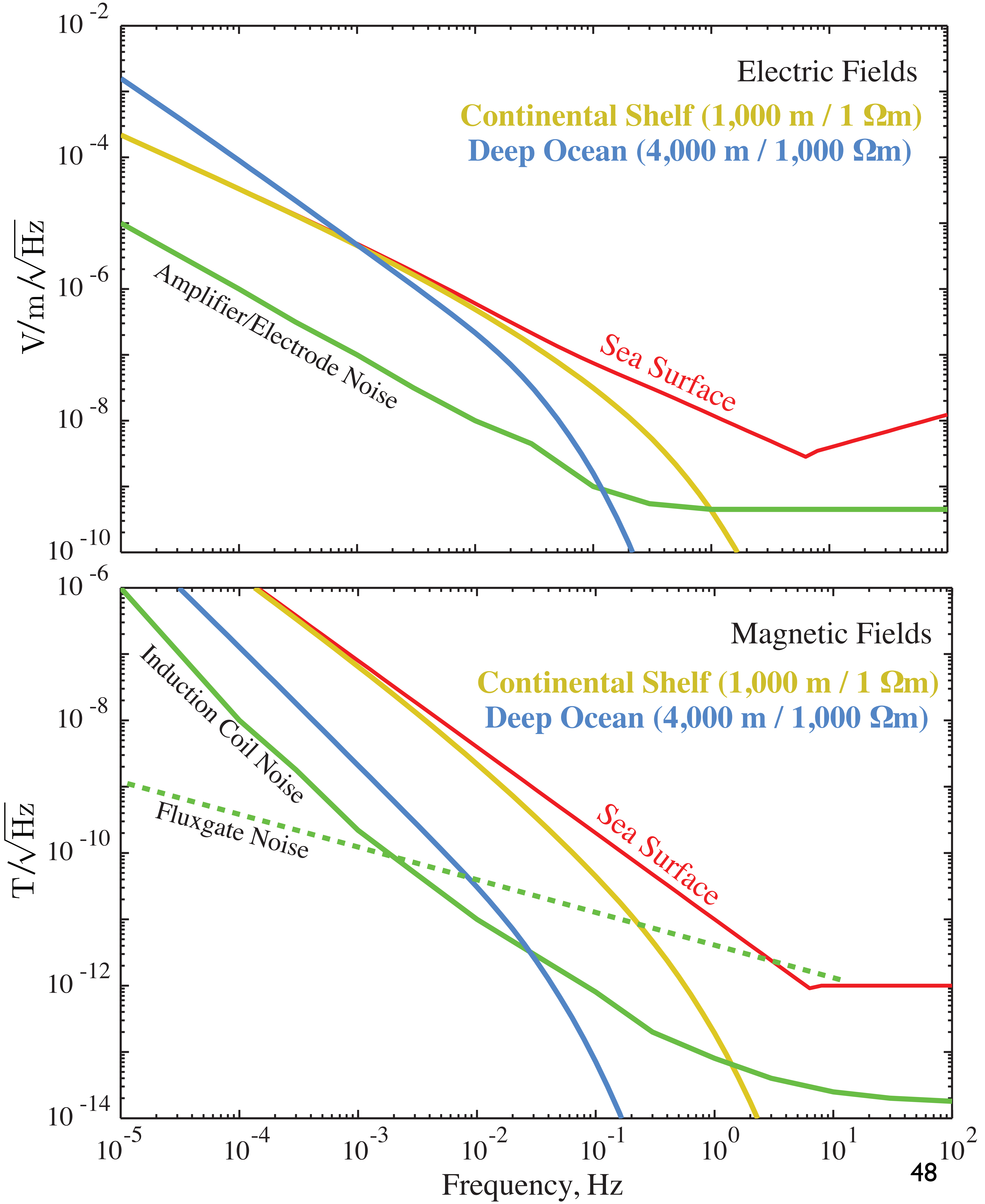
Electrodes: $1/f^2$ (?)

Instrument: Amplifiers ($1/f$), ADC, noise from other circuits

Streaming potentials, instrument motion, wave action, currents

$\mathbf{E} = \mathbf{v} \times \mathbf{B}$ where **B** is magnetic field, **v** is water velocity

Johnson noise limit



These plots show MT signals compared to sensor noise.

Fluxgate/induction coil noise crosses at about 500 s.

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