SIOG 231
GEOMAGNETISM AND ELECTROMAGNETISM

Lecture 7

Geomagnetic Power Spectra and Secular Variation

1/30/2024

Where we are in the Syllabus

1. Introduction, motivation, history of the physics and geophysics, overview of the applications.
2. Vector calculus basics, Maxwell's equations, constitutive relationships, scalar and vector potentials.
3. Lorentz force, diffusion equations, skin depth.
4. Instruments, observatory networks, satellite observations.
5. Gauss' theory and the main field.
6. Spherical harmonic representation, internal/external separation, upward and downward continuation.
7. Geomagnetic power spectra, secular variation.
8. Earth's external geomagnetic and electromagnetic environment.
9. Geomagnetic depth sounding + Fitting data: Least squares, errors, parameter estimation.
10. Main field modeling, regularization.
11. Lithospheric fields, magnetic remanence, Runcorn's theorem.
12. Core processes and the internal field.
13. Geodynamos, toroidal and poloidal fields, frozen flux approximation.
14. The magnetotelluric method.
15. Electrical conductivity of rocks, minerals, and melts. The crust, mantle, and core.
16. Modeling induction in one dimension.
17. Forward modeling using finite differences and finite elements.
18. Inverse modeling MT and GDS data.
19. TBD
20. TBD

Today’s Class

- Starting to move on from static fields
- Geomagnetic Secular Variation - 2 ways to think about time variations
    - ✦ parametrized variations
    - ✦ statistical variability
- Using the Lowes spectrum to estimate the core radius

Spherical Harmonic Representation

$$\vec{B}(r,\theta,\phi,t) = -\nabla\Psi(r,\theta,\phi,t)$$

- adding in time variations

$$\Psi = a\sum_{l=1}^{\infty}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+1}(g_l^m(t)\cos m\phi + h_l^m(t)\sin m\phi)P_l^m(\cos\theta).$$

P_l^m are partially normalized Schmidt functions, radius r, θ co-latitude, ϕ longitude, t time. g_l^m, h_l^m are the Gauss coefficients representing the field model.

Spatial Power Spectrum for a Field Model is $R_l(t)$

$$R_l(t) =< \vec{B}_l \cdot \vec{B}_l >_{r_a} = (l+1)\sum_{m=0}^{l}[(g_l^m(t))^2 + (h_l^m(t))^2]$$

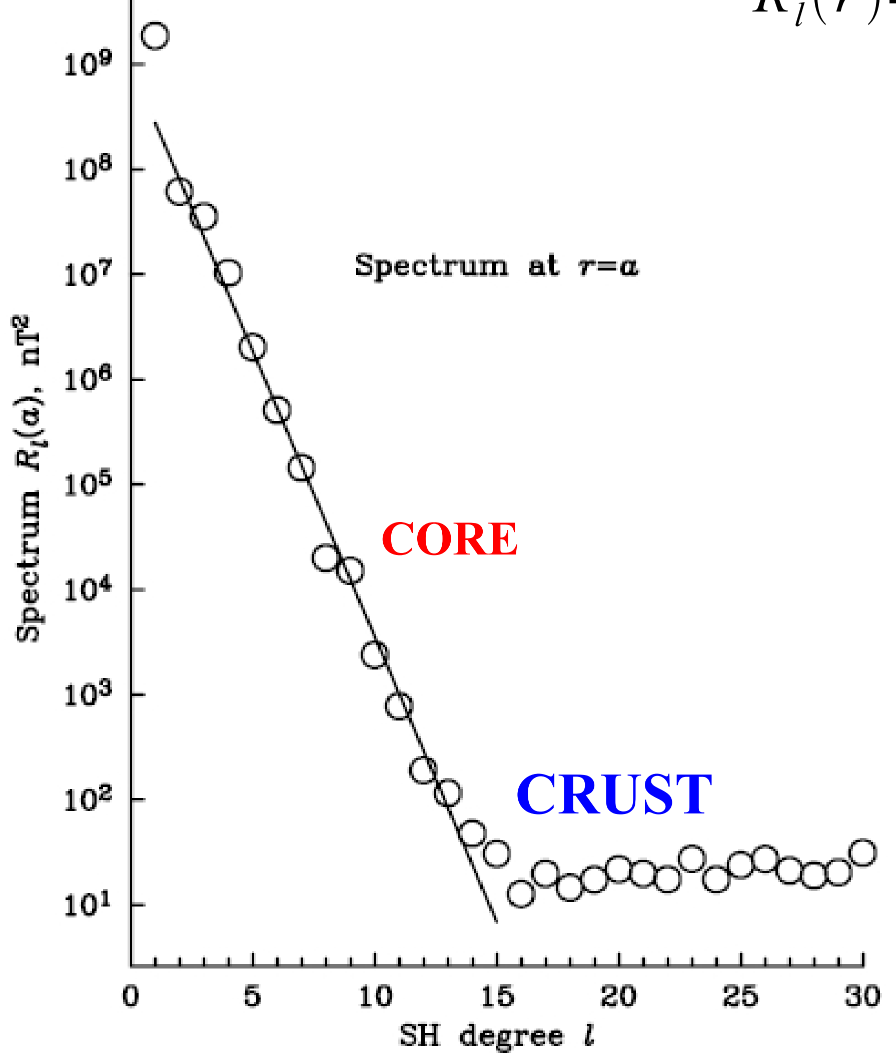
Spatial Power Spectrum for a Secular Variation Model is $S_l(t)$

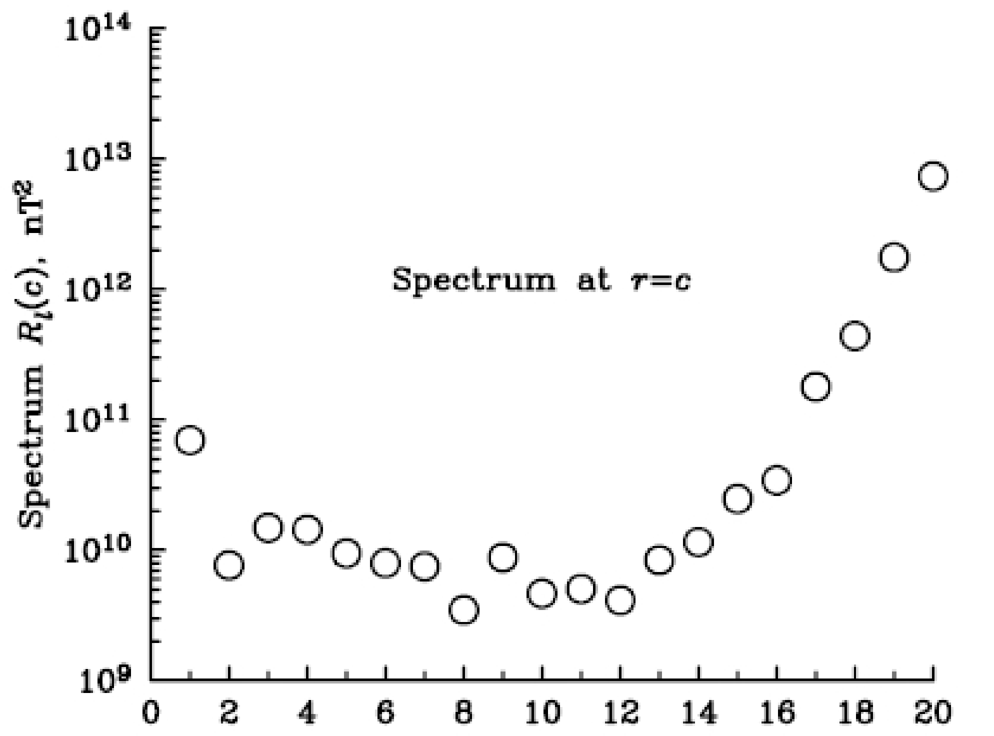
$$S_l(t) =< \frac{d\vec{B}_l}{dt} \cdot \frac{d\vec{B}_l}{dt} >_{r_a} = (l+1)\sum_{m=0}^{l}[(\dot{g}_l^m(t))^2 + (\dot{h}_l^m(t))^2]$$

The Geomagnetic Spectrum

$$R_l = \frac{(2l+1)(l+1)}{4\pi}\sum_{m=-l}^{l}|b_l^m|^2 = (l+1)\sum_{m=0}^{l}\left[(g_l^m)^2 + (h_l^m)^2\right]$$

$$R_l(r) = \left(\frac{a}{r}\right)^{2l+4} R_l(a)$$





Figs. 3.4.6.1 and 3.4.6.2 notes

- assume no time variations in crustal field

The International Geomagnetic Reference Field and World Magnetic Models

The IGRF is the International Geomagnetic Reference Field. See https://www.ngdc.noaa.gov/IAGA/vmod/igrf.html for downloads along with information about how to use it. It is updated every 5 years by members of the IAGA working Group V-MOD, and was last revised (to 13th generation) in 2019. It represents the Schmidt normalized geomagnetic internal potential to SH degree and order $L = 13$.

$$\Psi(r,\theta,\phi) = a\sum_{l=1}^{13}\sum_{m=0}^{l}\left[\left(\frac{a}{r}\right)^{l+1}\left(g_l^m\cos(m\phi) + h_l^m\sin(m\phi)\right)\tilde{P_l^m}(\cos\theta)\right]$$

A close relative of the IGRF is the World Magnetic Model (https://www.ngdc.noaa.gov/geomag/WMM/DoDWMM.shtml), a joint product of the United States' National Geospatial-Intelligence Agency (NGA) and the United Kingdom's Defence Geographic Centre (DGC), and is used by the U.S. Department of Defense, the U.K. Ministry of Defence, the North Atlantic Treaty Organization (NATO) and the International Hydrographic Organization (IHO), for navigation, attitude and heading referencing systems using the geomagnetic field.

Magnetic Field Calculators

Magnetic Field Estimated Values

Magnetic field is calculated using the most recent World Magnetic Model (WMM) or the International Geomagnetic Reference Field (IGRF) model. For 1590 to 1900 the calculator is based on the gufm1 model. A smooth transition from gufm1 to IGRF was imposed from 1890 to 1900. The Enhanced Magnetic Model (EMM) is a research model compiled from satellite, marine, aeromagnetic and ground magnetic surveys which attempts to include crustal variations in the magnetic field too fine to appear in the World Magnetic Model. The calculator provides an easy way for you to get results in HTML, XML, CSV, or JSON programmatically (API). **Registration is required to access this service.** Please register using the API registration link on the top right.

Calculate Magnetic Field

Latitude: S N

Longitude: W E

Elevation: GPS **Mean sea level**

0 Kilometers

Model: **WMM (2019-2024)** **IGRF (1590-2024)** **EMM (2000-2019)**

Start Date: Year 2024 Month 1 Day 29

End Date: Year 2024 Month 1 Day 29

Step size: 1.0

Result format: HTML XML CSV JSON

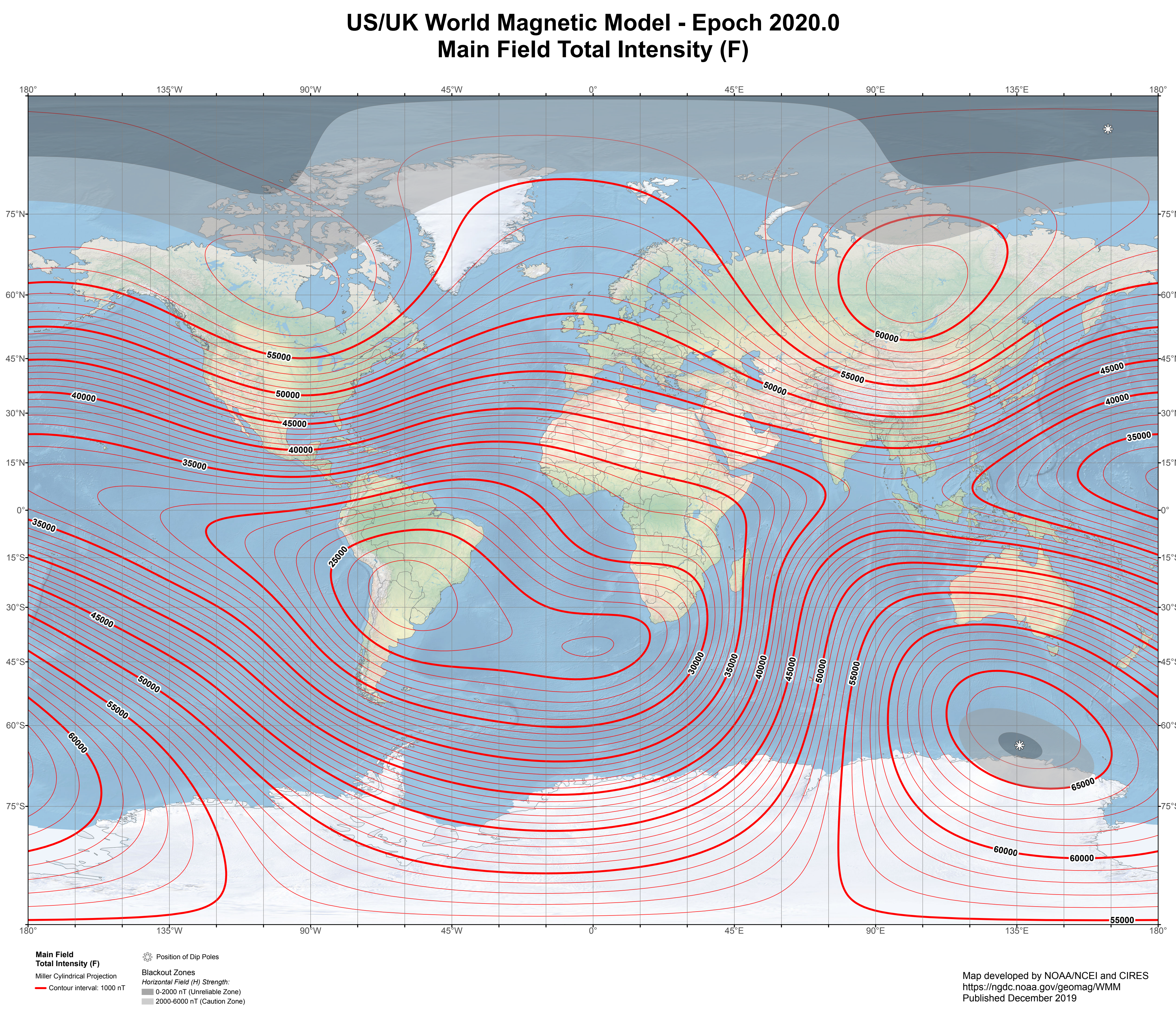
Calculate

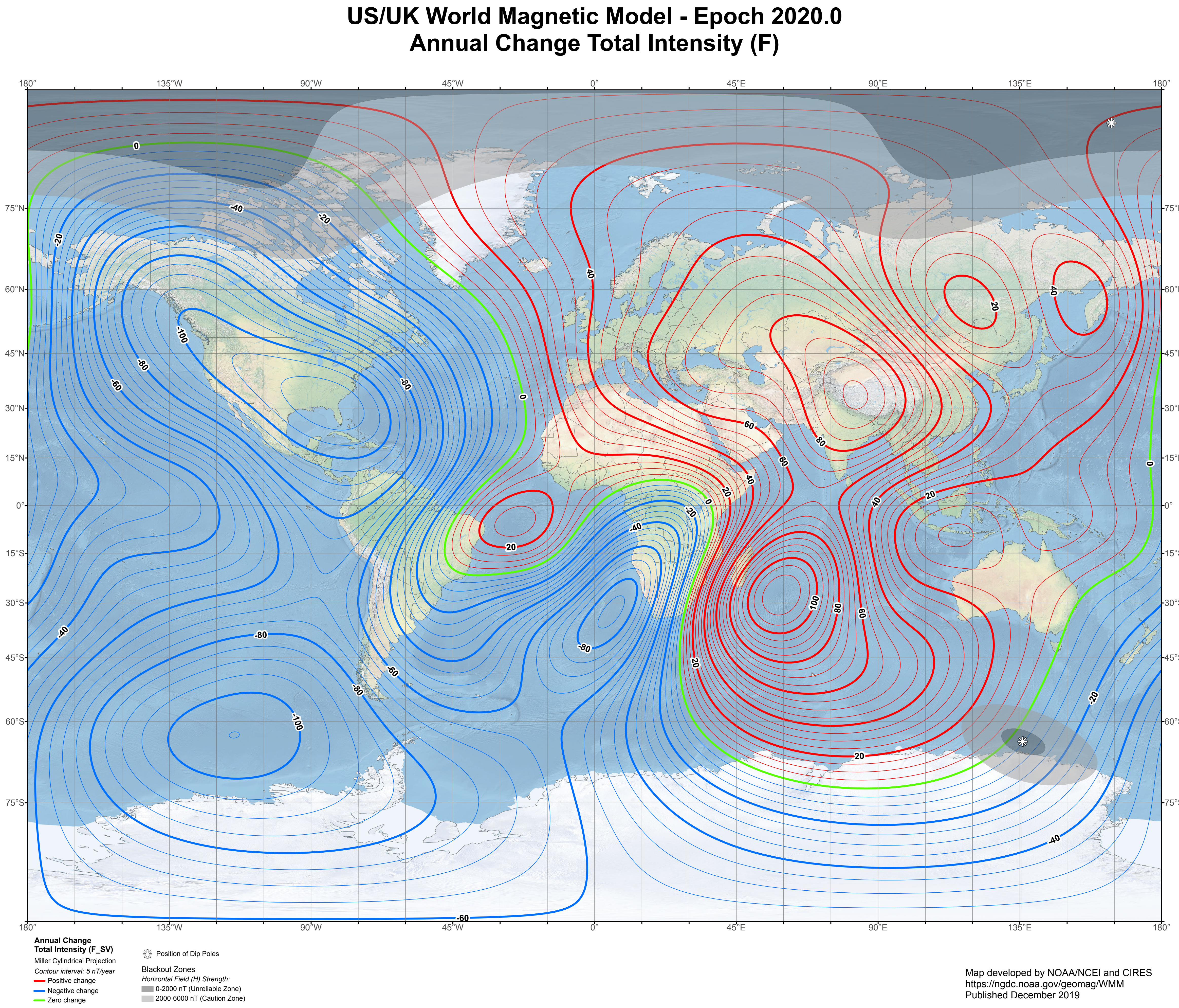
Lookup Latitude / Longitude

Enter a street address, street name, or street intersection. For best results, include as much location information as possible with the street address in your search, such as city, state, zip code.

Location:

Get & Add Lat / Lon

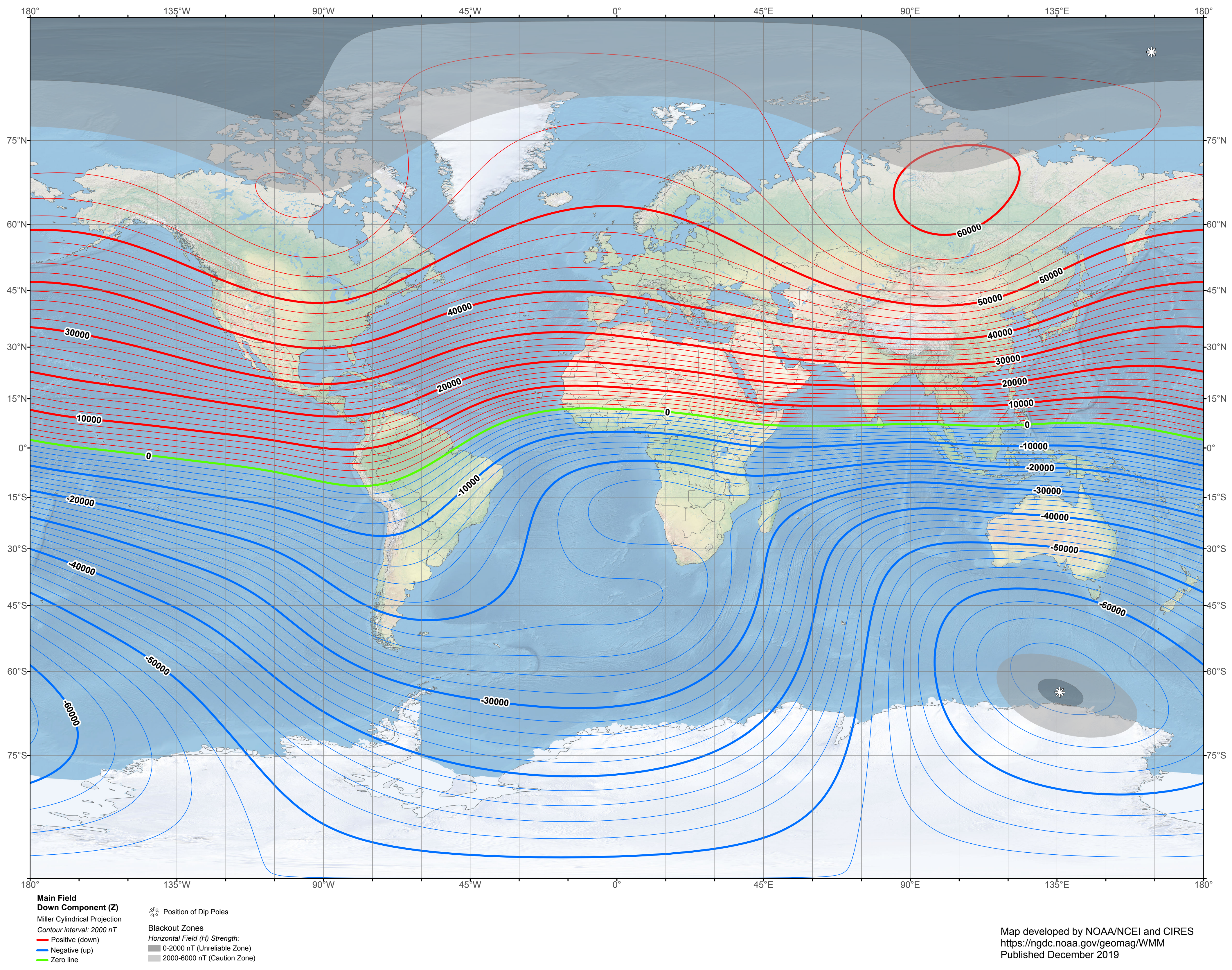




Any of the magnetic elements D, I, H, X, Y, Z, F can be predicted from WMM Models. IGRF and WMM Models also come with a prediction of linear rates of change in time for the Gauss coefficients for the next five years. These can be used to predict rates of change in the various magnetic elements.

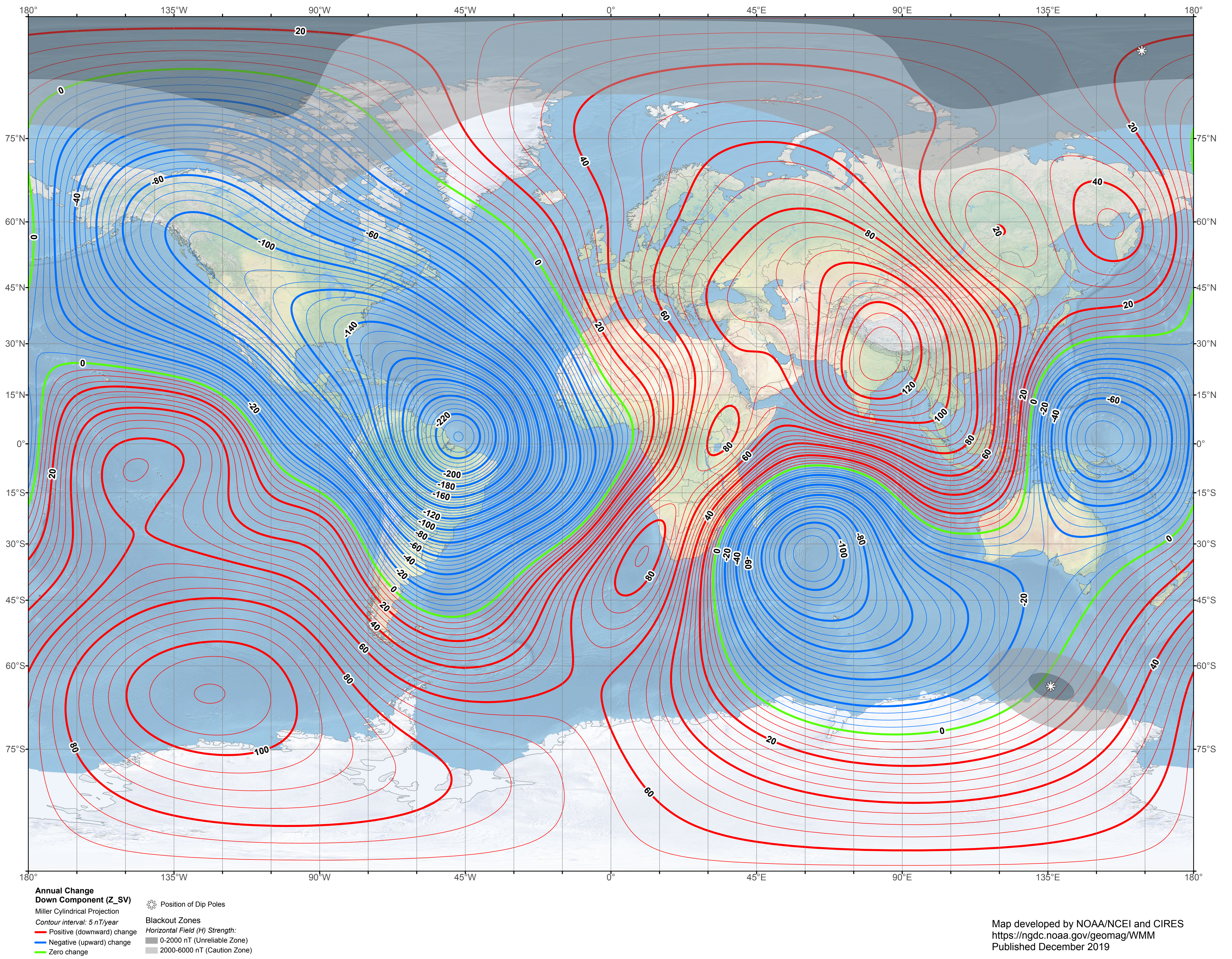
https://www.ngdc.noaa.gov/geomag/WMM/image.shtml

US/UK World Magnetic Model - Epoch 2020.0
Main Field Down Component (Z)



Any of the magnetic elements D, I, H, X, Y, Z, F can be predicted from WMM Models. IGRF and WMM Models also come with a prediction of linear rates of change in time for the Gauss coefficients for the next five years. These can be used to predict rates of change in the various magnetic elements.

US/UK World Magnetic Model - Epoch 2020.0
Annual Change Down Component (Z)



For more than just a couple of years linear time variations aren't good enough.

For longer term field modeling we need some more comprehensive basis to expand the Gauss coefficients in time. Cubic or higher order B-splines are often adopted for this purpose.
We write

$$g_l^m(t) = \sum_{k=1}^{N_{spl}} g_l^{mk} S_k(t)$$

Here N_{spl} is the number of B-splines need to represent the time interval covered. There is a similar equation for each h_l^m. The time-dependent Gauss coefficients are thus linear combinations of the spline coefficients and the piecewise polynomial functions $S_k(t)$, e.g., a polynomial of degree 3 (order 4).

Splines are piecewise degree *j* polynomials in time used to make a continuous function and up to degree (*j-1*) derivatives at knot points where they join

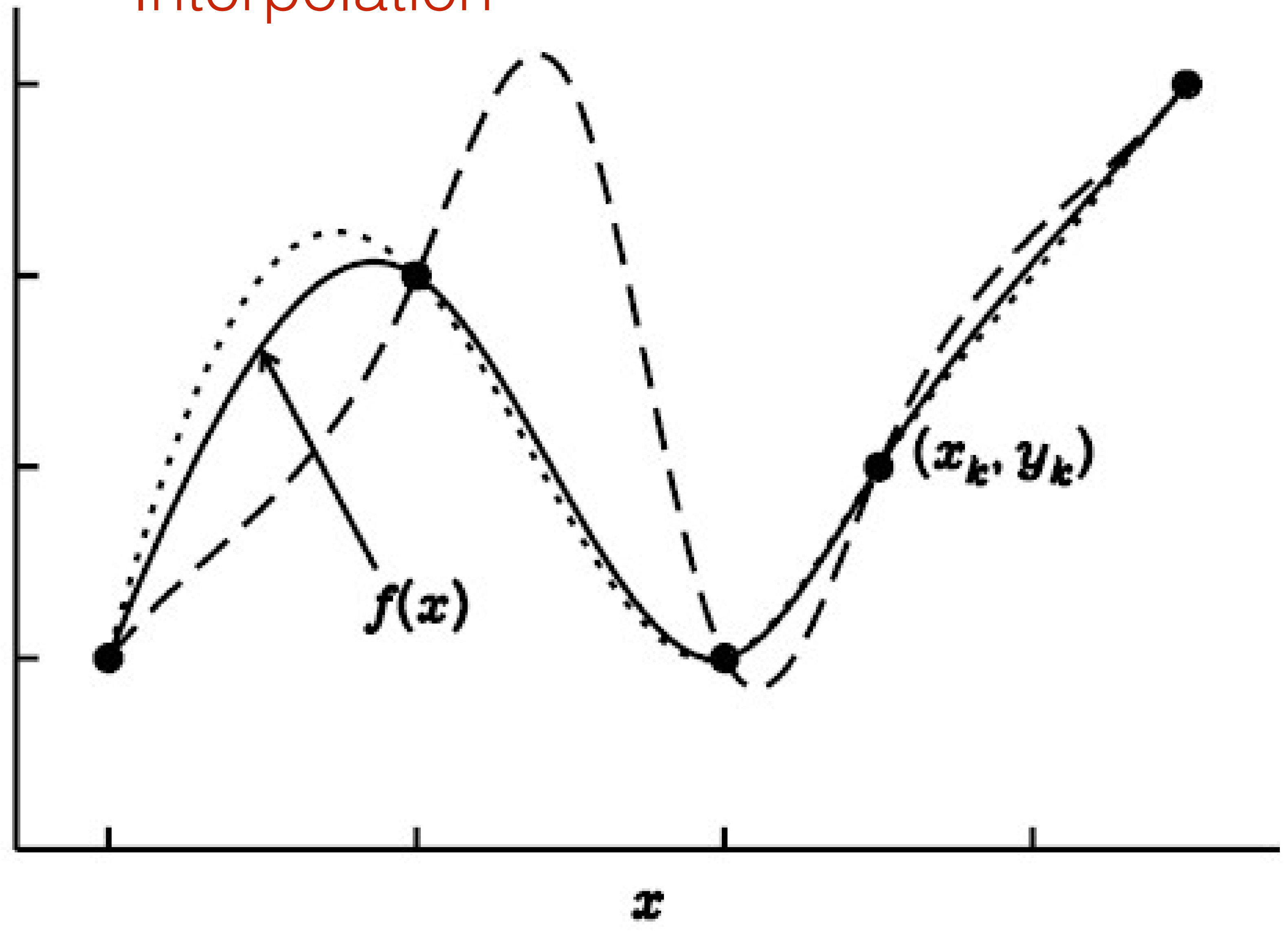
Example: cubic spline interpolation

$$U = \sum_{j=1}^{N} \frac{\left(f(x_j) - y_j\right)^2}{\sigma_j^2} + \lambda \int_{x_1}^{x_N} \left[\frac{d^2 f}{dx^2}\right]^2 dx$$

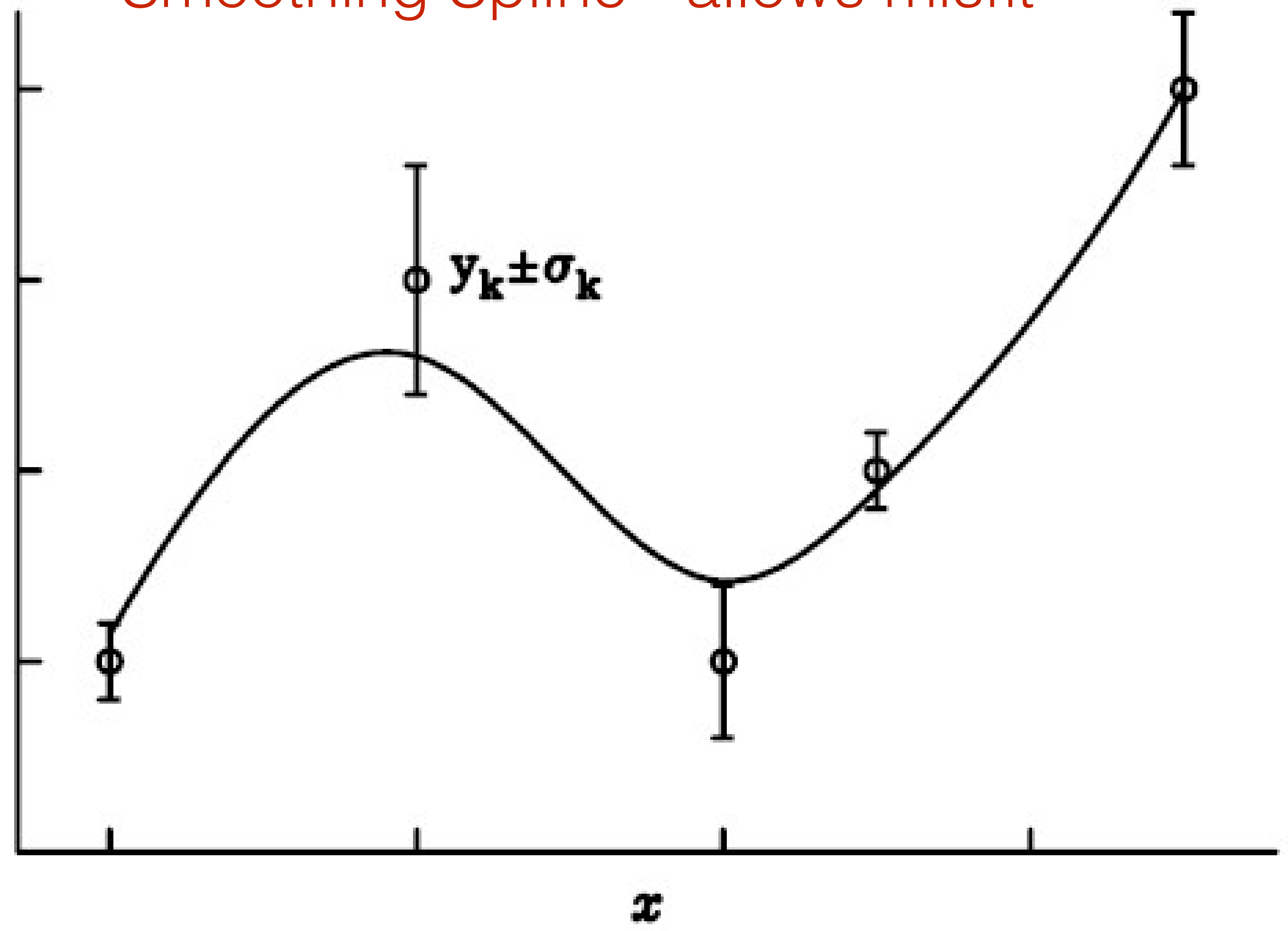
smoothest curve connecting the points (RMS second derivative)

subject to $$\sum_{j=1}^{N} \frac{\left(f(x_j) - y_j\right)^2}{\sigma_j^2} = T$$

Interpolation

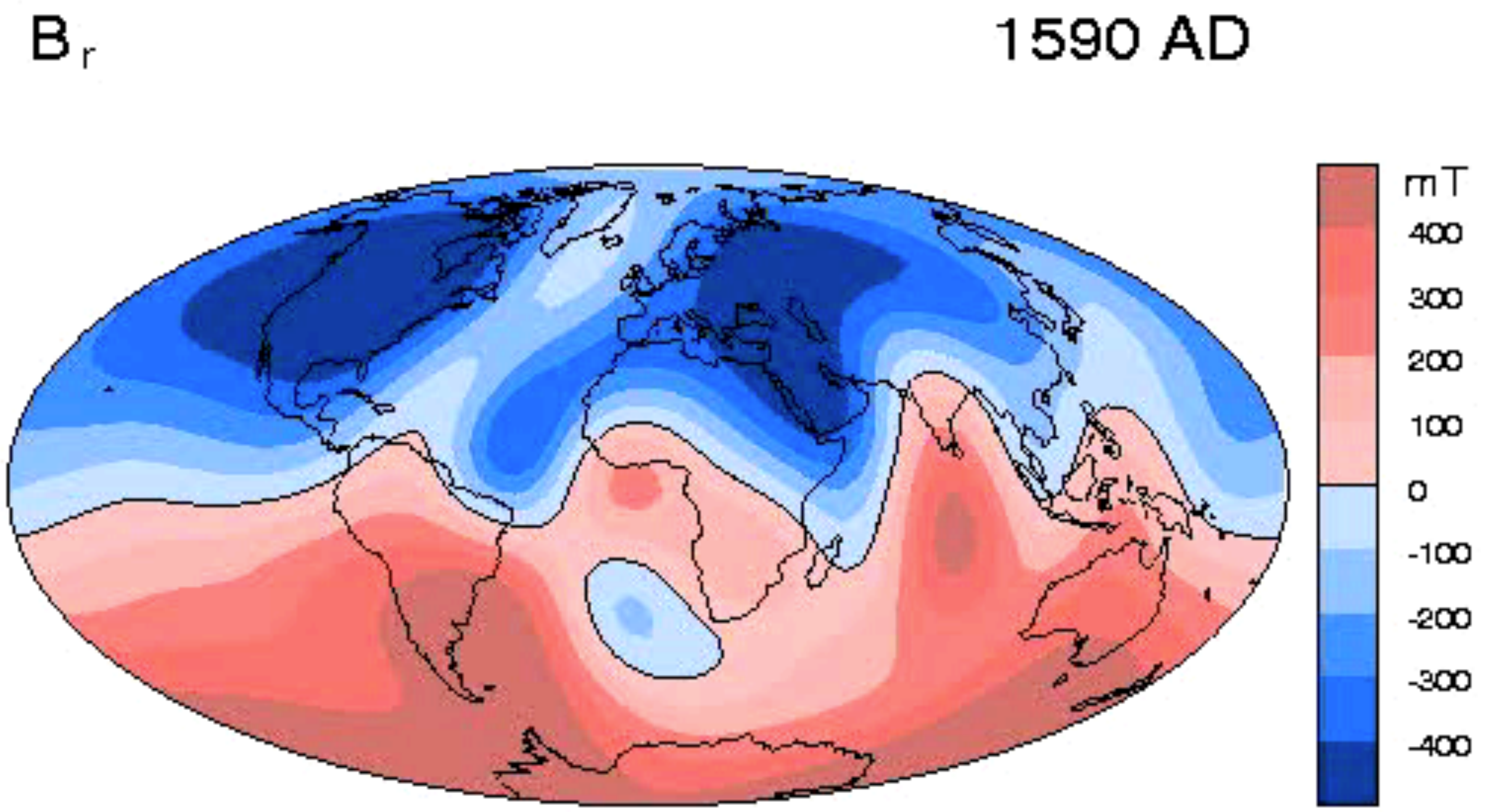


Smoothing Spline - allows misfit



Figs. 3.4.4.1 and 3.4.4.2 notes

Cubic splines and regularized inversion were used to make this now classic field model called *gufm1,* spanning 1590-1990 CE. It is based on direct observations of the field.



SPHERICAL HARMONIC MODEL

Outside its source region the core field is represented by Gauss coefficients describing a scalar potential. The field B is given by the gradient of this scalar potential.

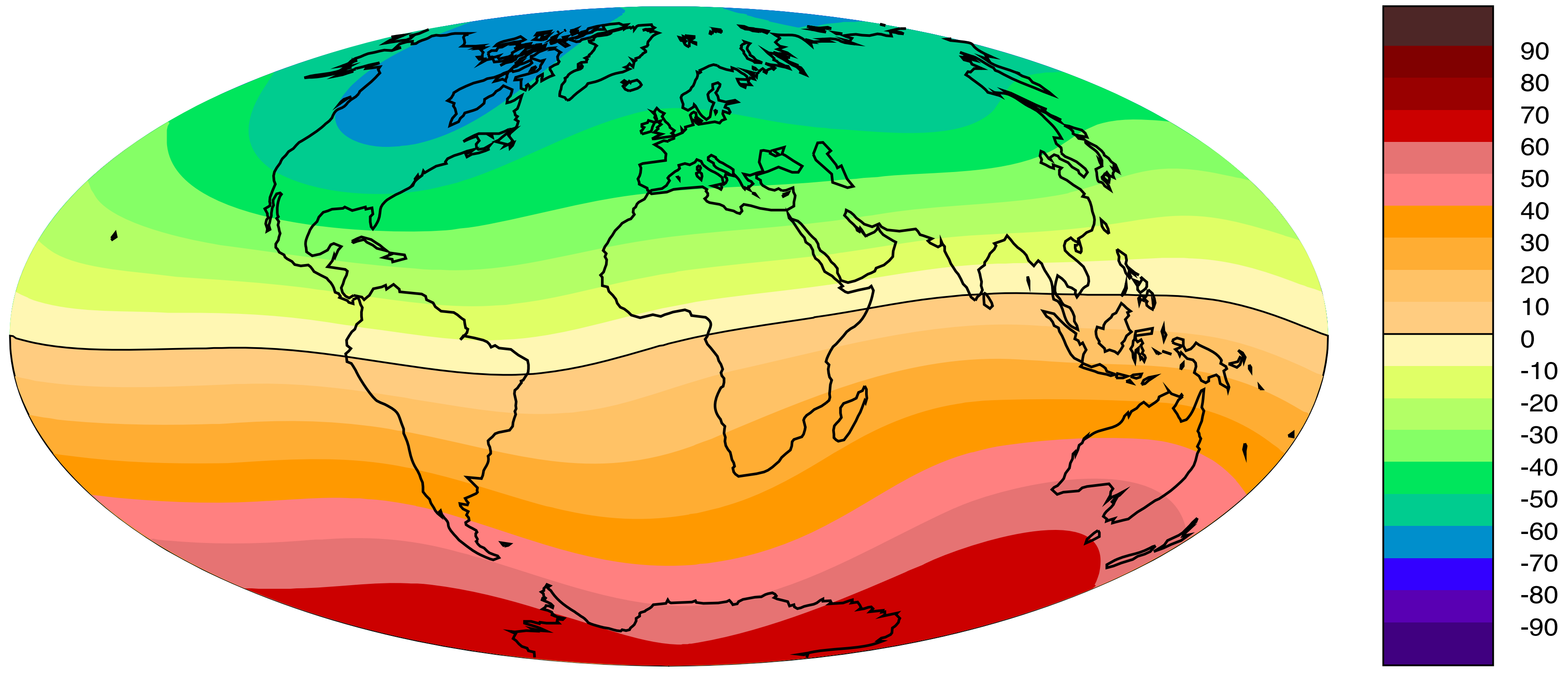
$$\vec{B}(r,\theta,\phi,t) = -\nabla\Psi(r,\theta,\phi,t)$$

$$\Psi = a\sum_{l=1}^{\infty}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+1}\left(g_l^m(t)\cos m\phi + h_l^m(t)\sin m\phi\right)P_l^m(\cos\theta).$$

P_l^m are partially normalized Schmidt functions, radius r, θ co-latitude, ϕ longitude, t time. g_l^m, h_l^m are the Gauss coefficients representing the field model.

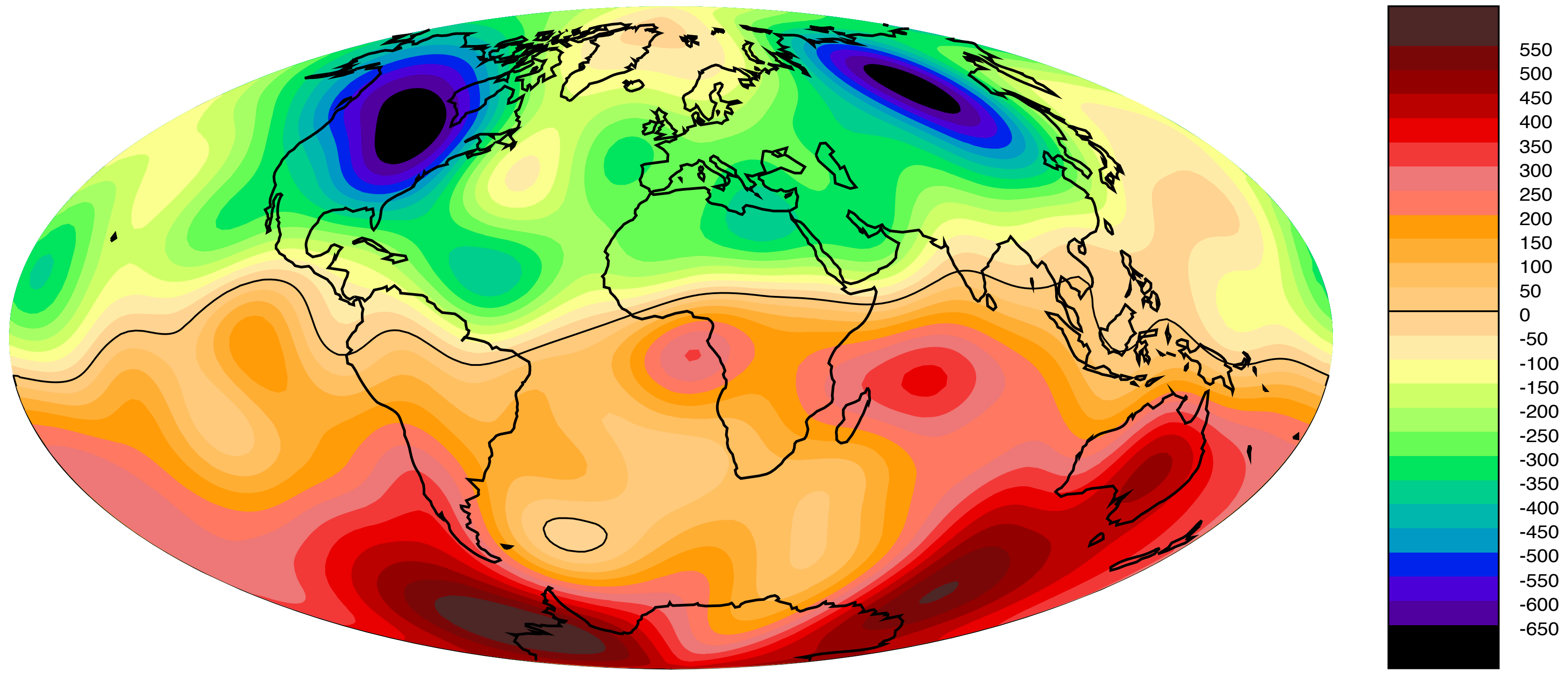
GUFM1, 1590-1990 AD

Earth Surface



Average radial field at r=a

Core-Mantle Boundary

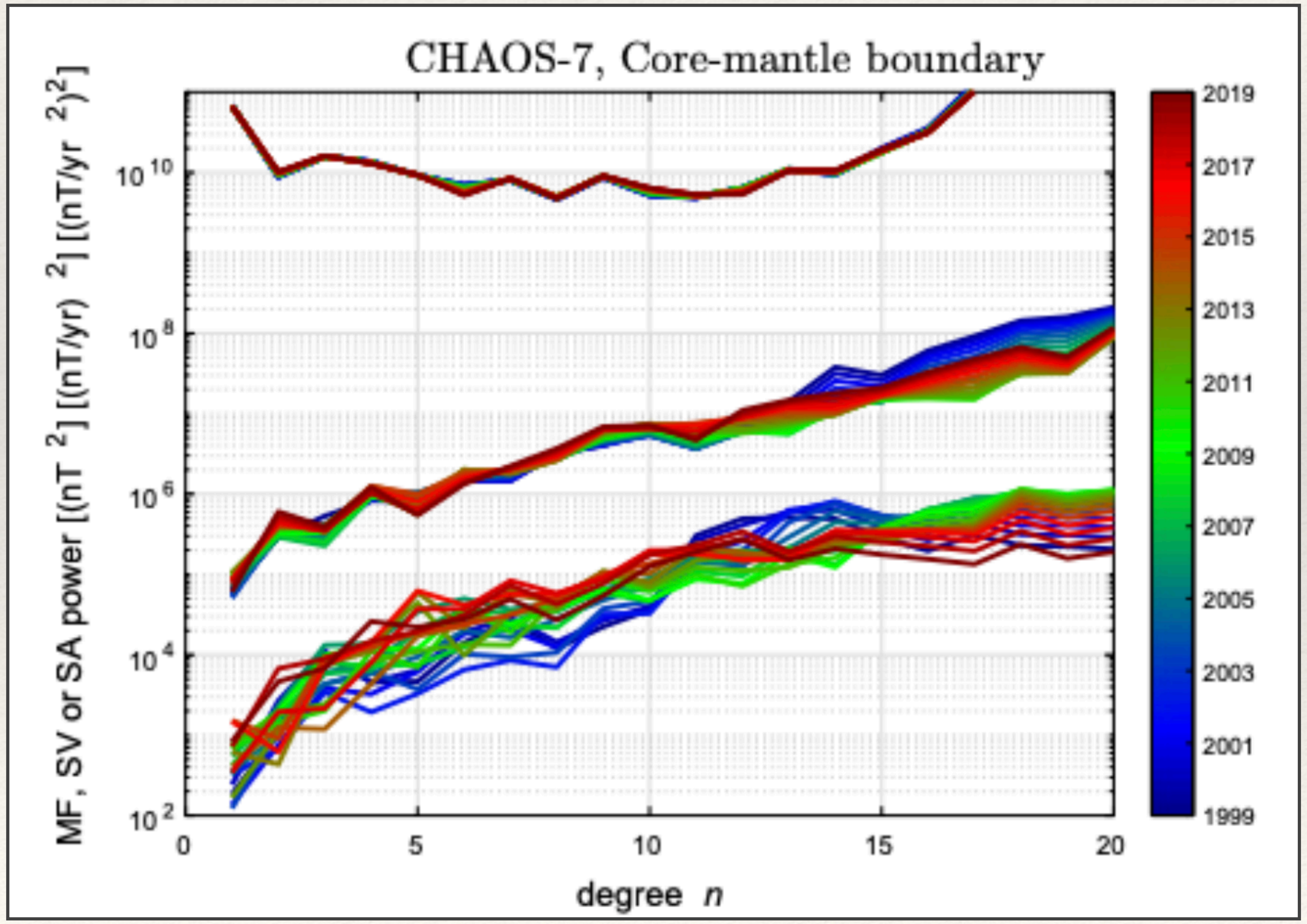


Average radial field at r=c

SH degree *l* represents spatial wavelength

Spatial Power Spectrum for a Field Model is $R_l(t)$

$$R_l(t) = < \vec{B}_l \cdot \vec{B}_l >_{r_a} = (l+1) \sum_{m=0}^{l} [(g_l^m(t))^2 + (h_l^m(t))^2]$$



High quality modern field models (1999-on) are based on comprehensive satellite and observatory data and use order 6 splines for better temporal resolution. These are used to calculate secular acceleration and its spatial power spectrum.

Finlay et al. , Earth, Planets and Space (2020)

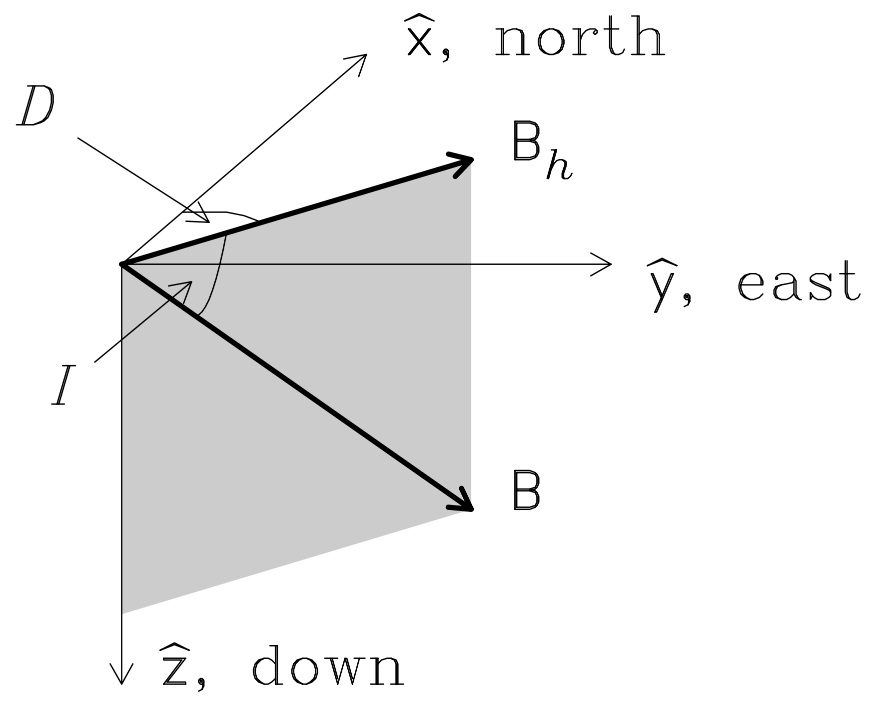
If we want go further back in time we have to use paleomagnetic observations

Globally distributed oriented rock samples serve as proxy magnetometers

We need to know when they acquired their magnetization by some independent chronological method, such as radiometric dating (radiocarbon or Ar/Ar) or correlation to oxygen isotope records.

MEASURING THE PAST FIELD

- Paleomagnetism and Archeomagnetism



Depositional Remanence

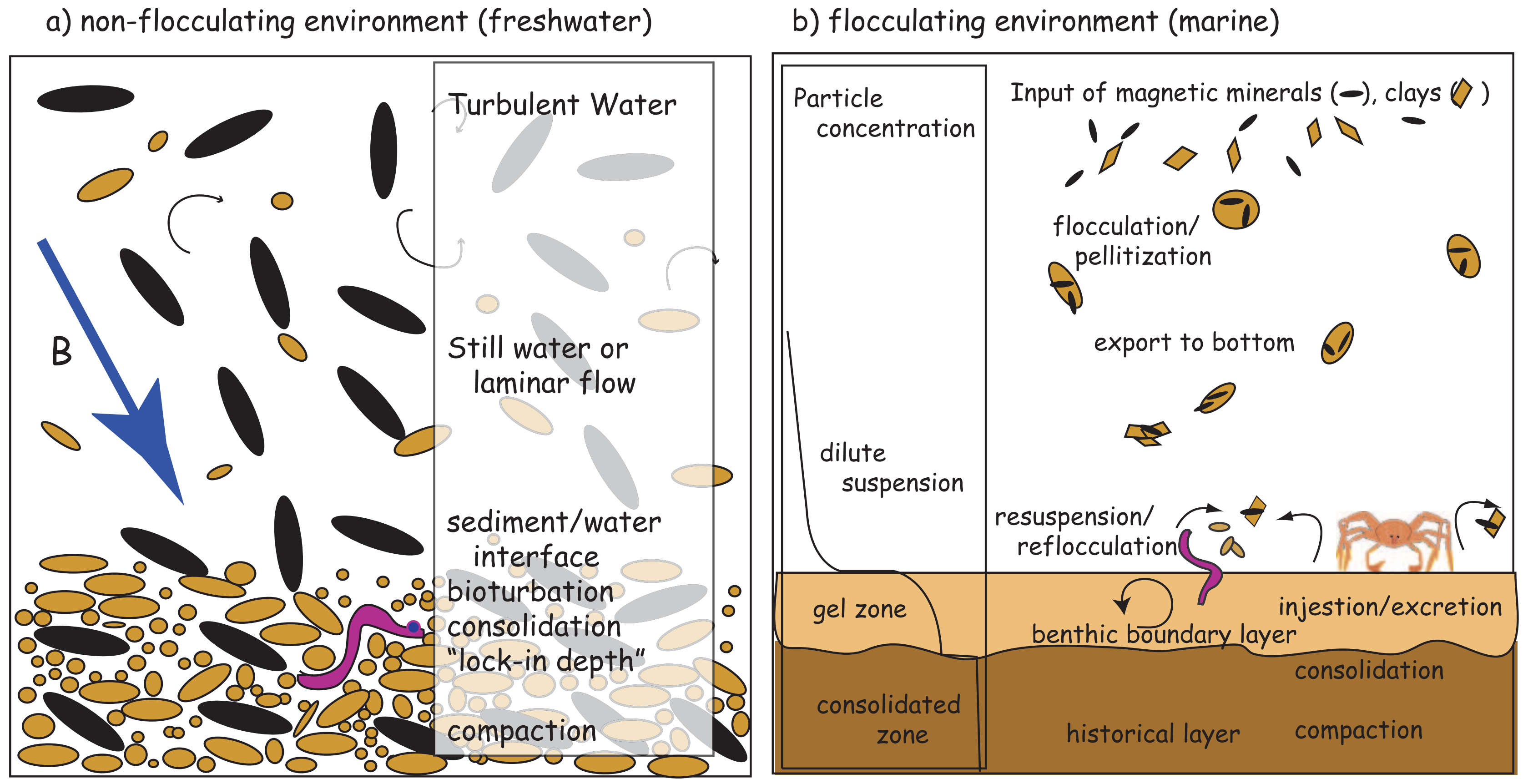
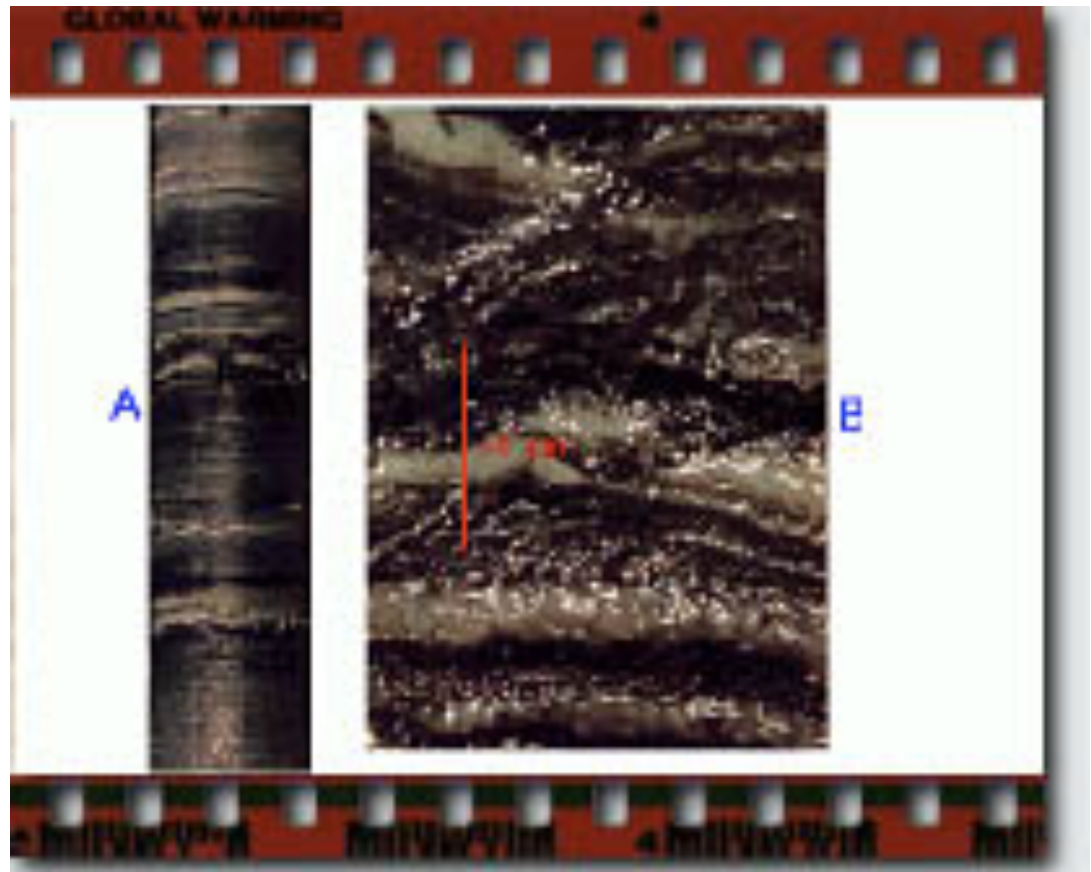
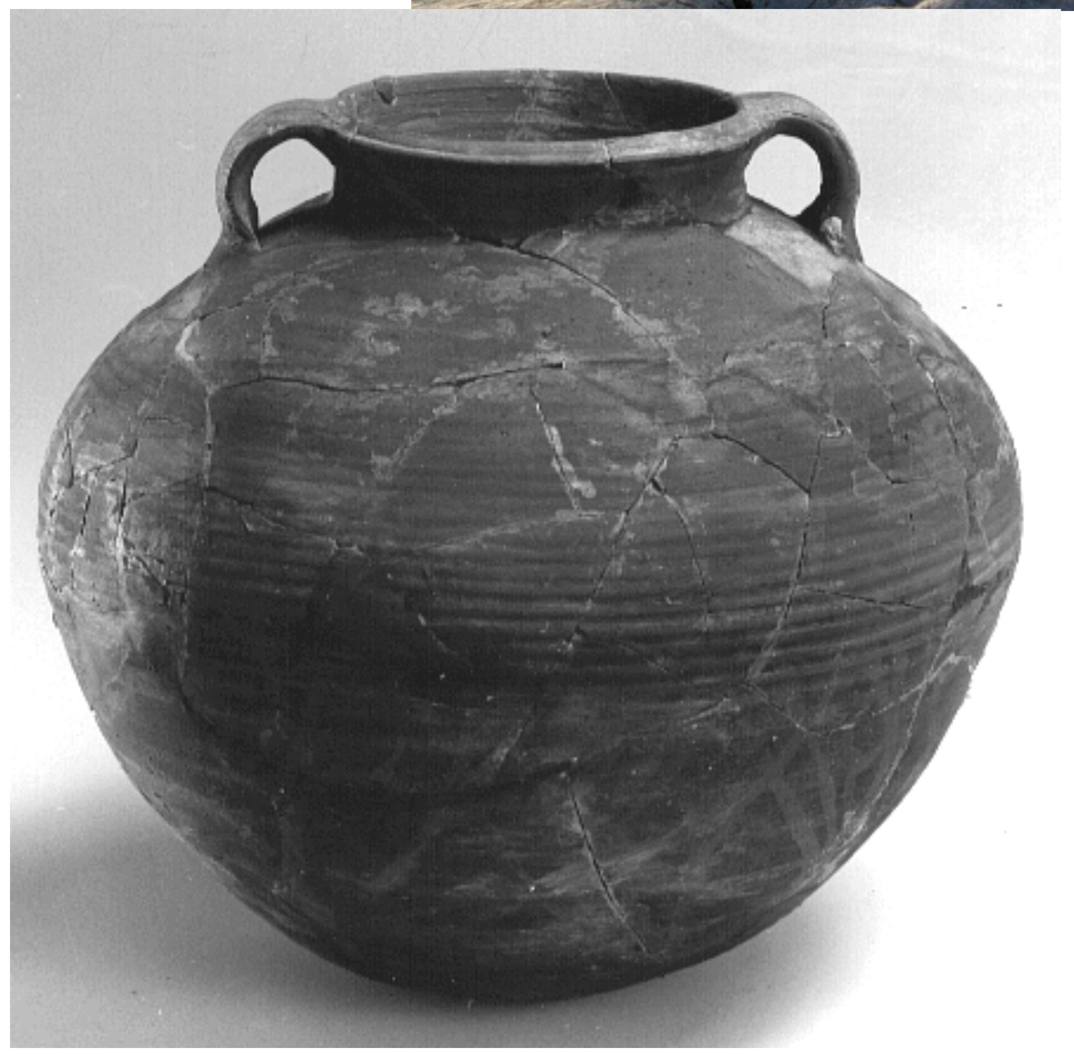


Figure 13.12: a) Schematic drawing of traditional view of the journey of magnetic particles from the water column to burial in a non-flocculating (freshwater) environment. Magnetic particles are black. (Redrawn from Tauxe, 1993.) b) View of depositional remanence in a flocculating (marine) environment. (Redrawn from Tauxe et al. 2006.)



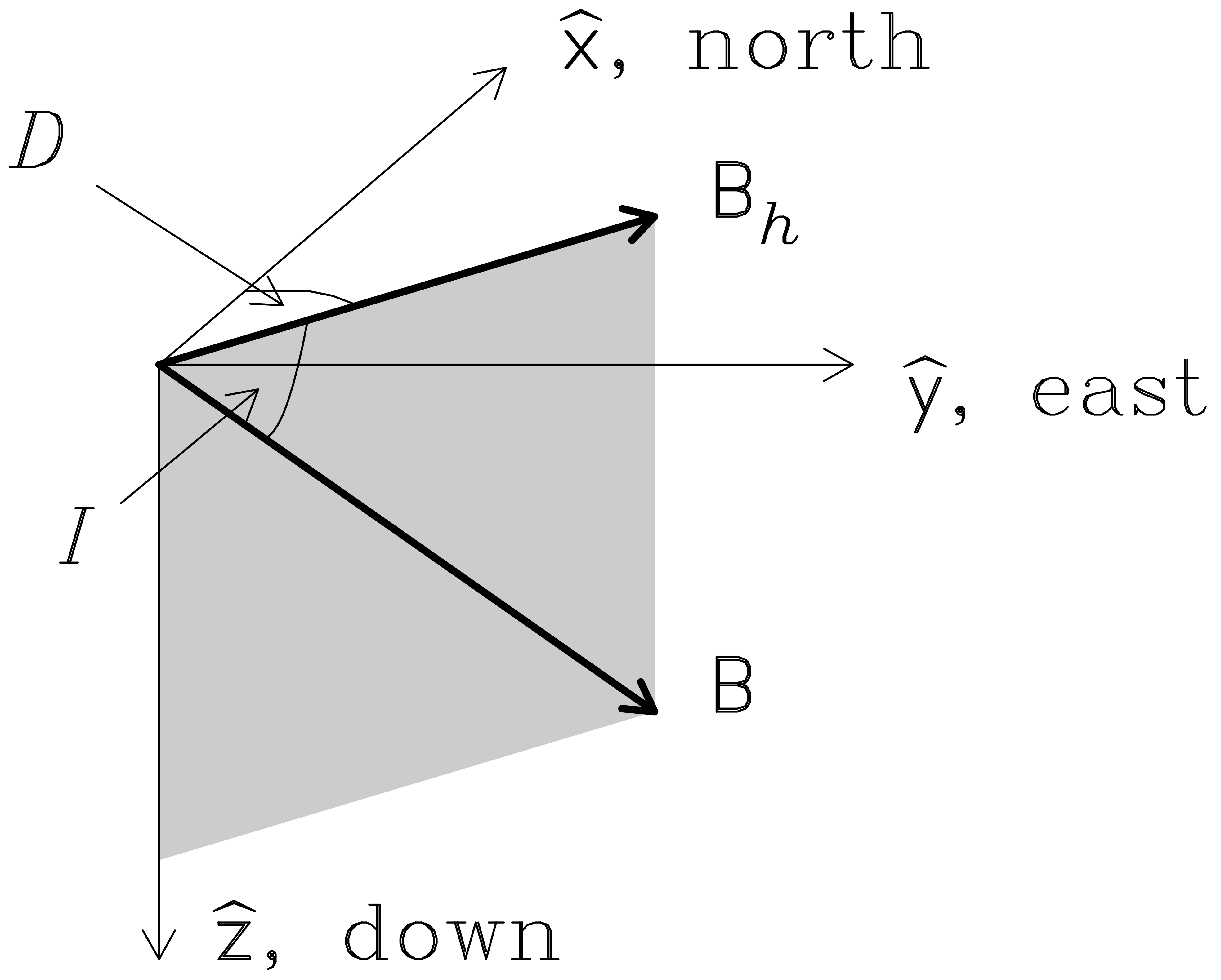
Thermal Remanence



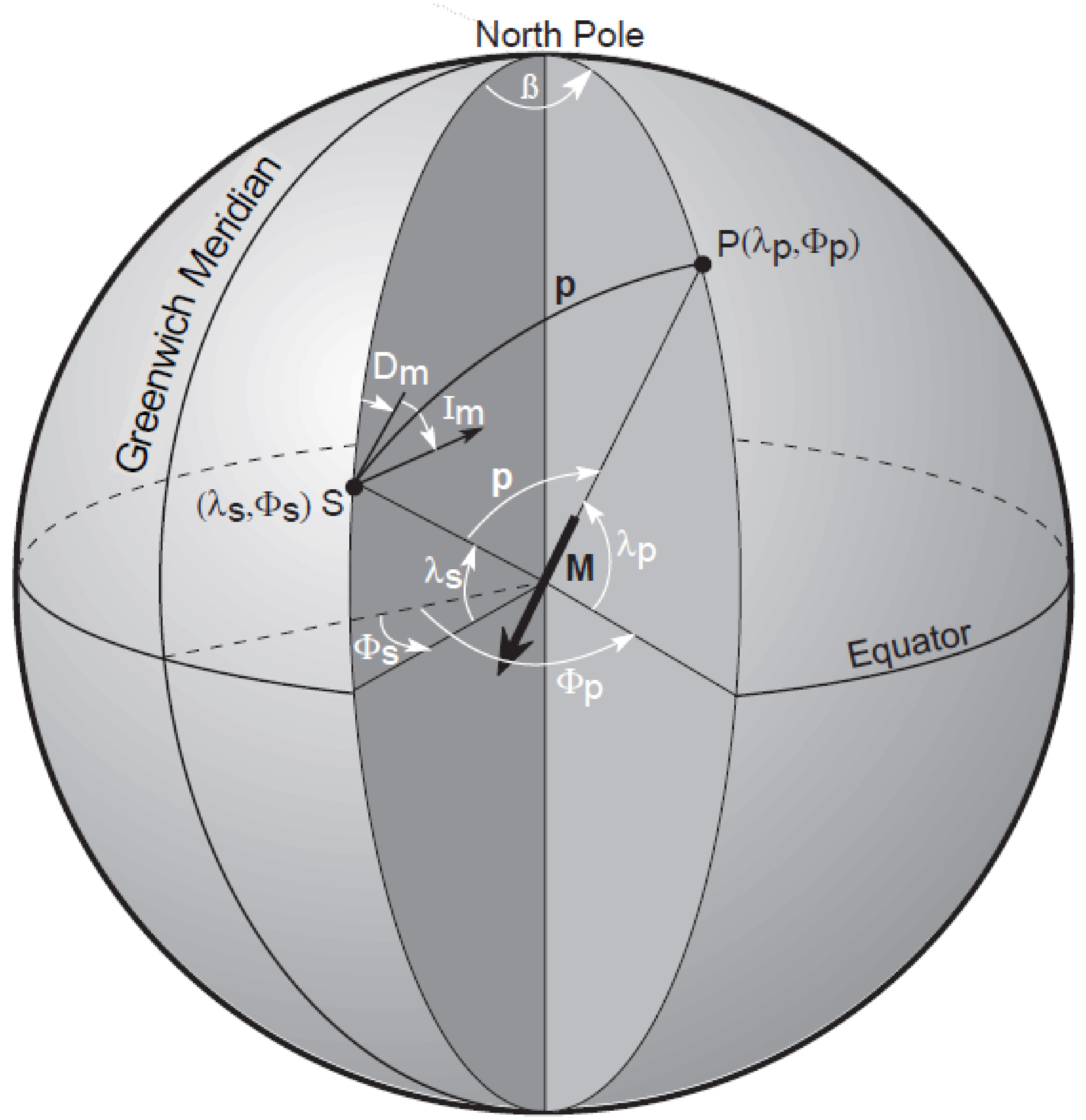
MAGNETIC FIELD IS A VECTOR QUANTITY

A Single Local Observation can be represented by

- B - magnetic field vector
- D - Declination
- I - Inclination

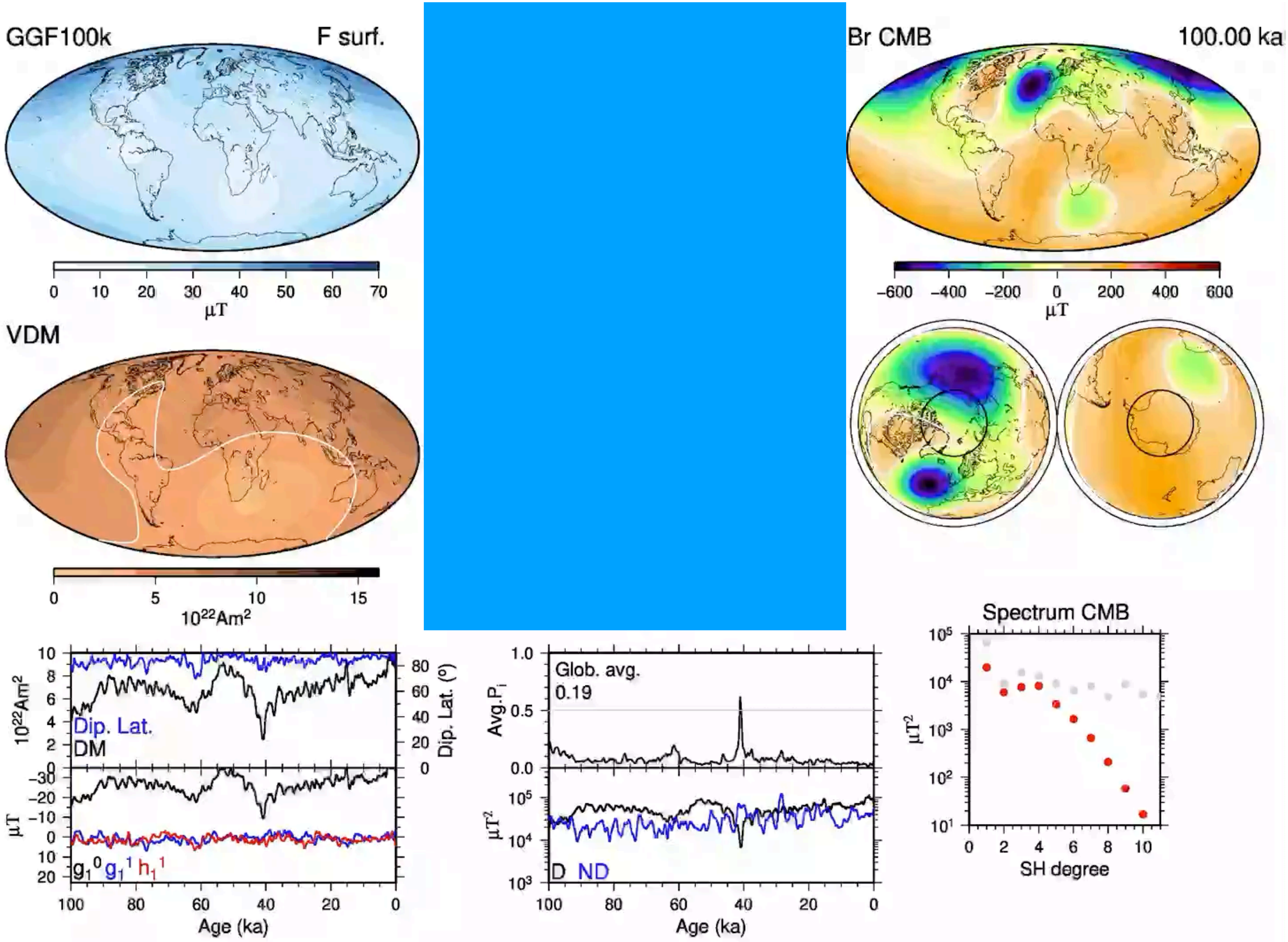


or equivalently by Virtual Geomagnetic pole (VGP) and Virtual Dipole Moment (VDM)

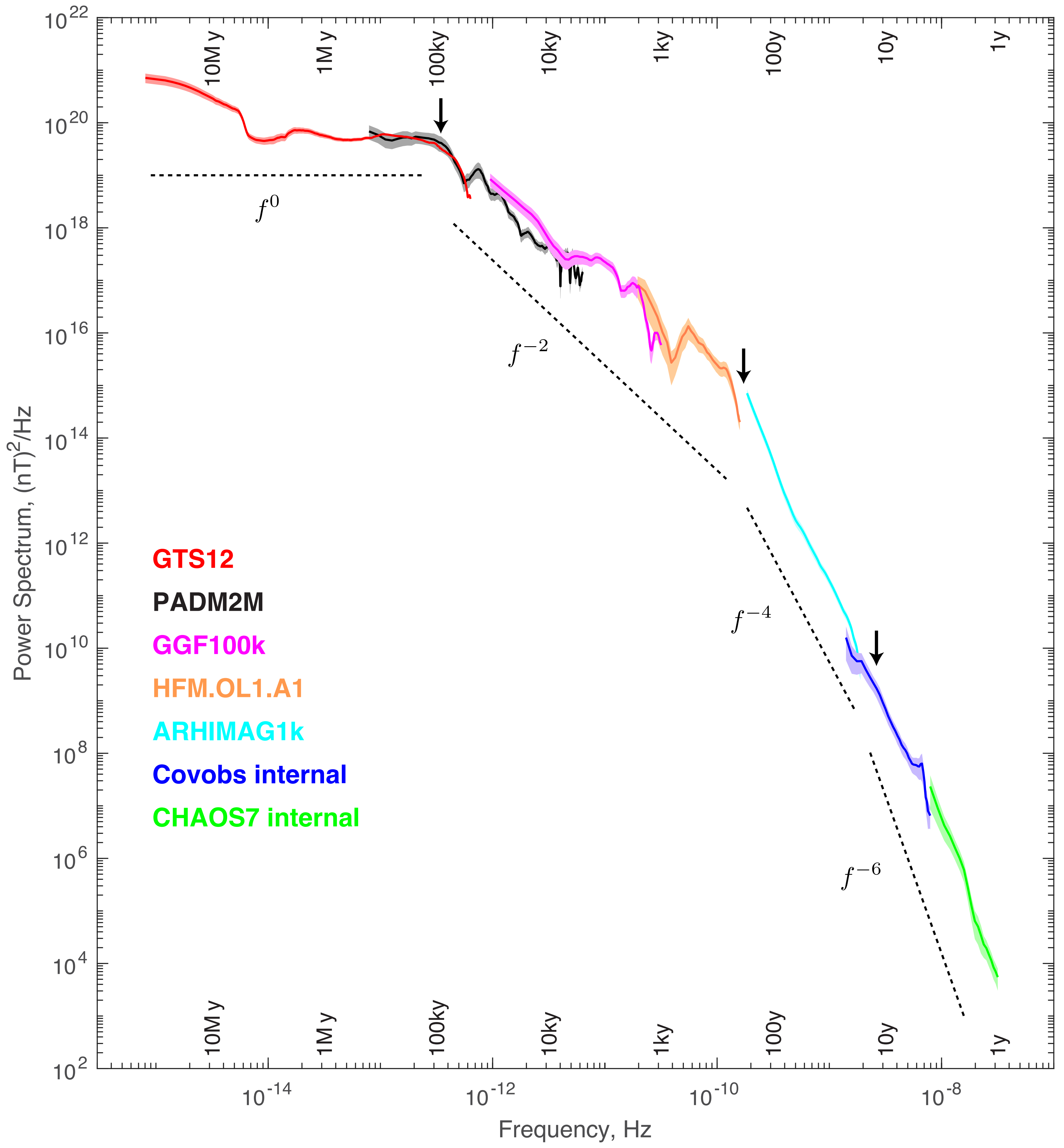


With lots of observations and age constraints to date them we can make time varying paleofield models.

100 Thousand Years of Geomagnetic Field Variations



Time series of g_1^0 can be used to look at the power spectrum in frequency for Earth's axial dipole moment



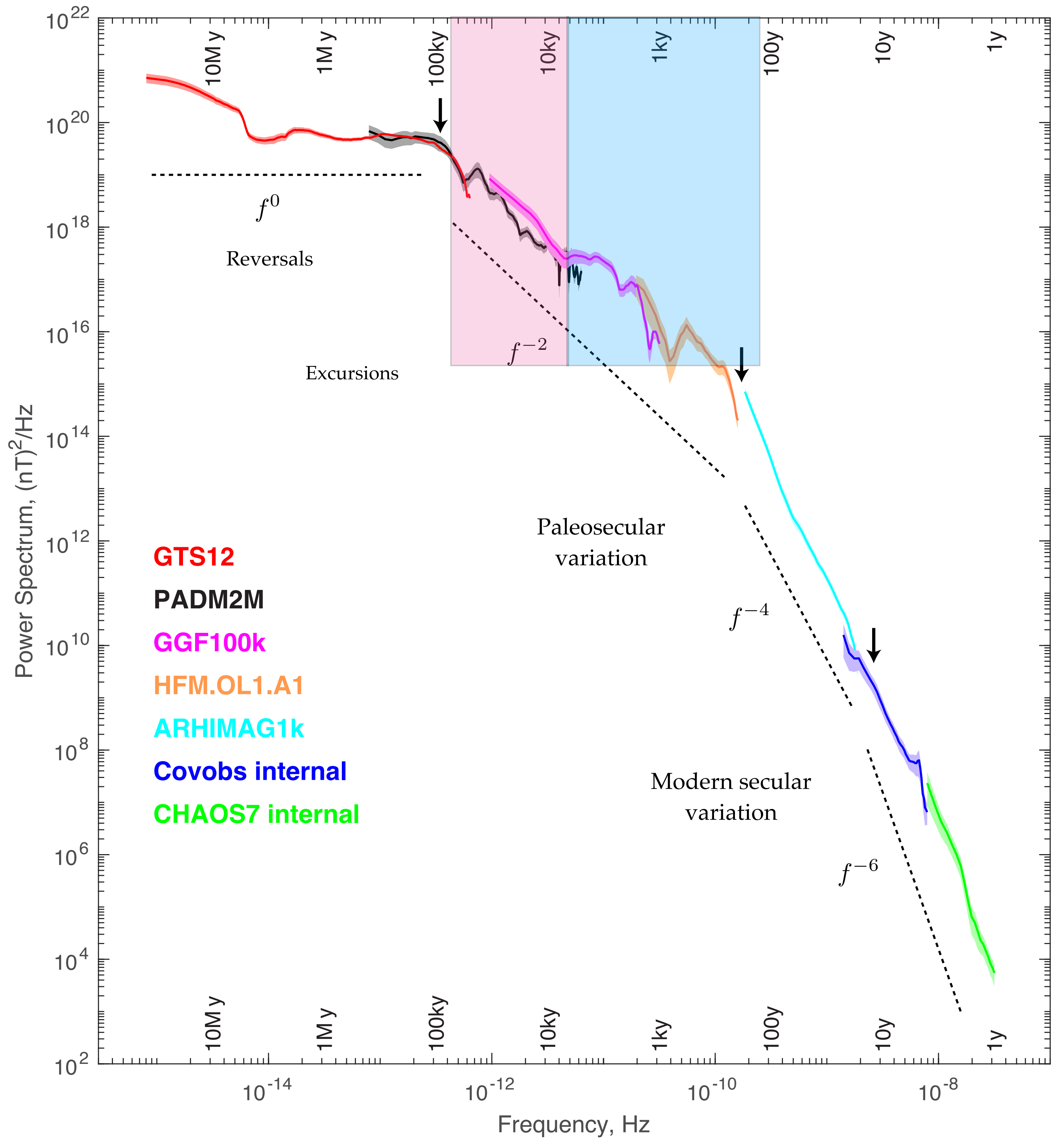
Axial Dipole Moment

$$p(t) = \frac{4\pi a^3}{\mu_0} |g_1^0(t)|$$

Power Spectrum of Axial Dipole Moment

$$S_p(f) = \mathscr{E}(|P(f)|^2)$$

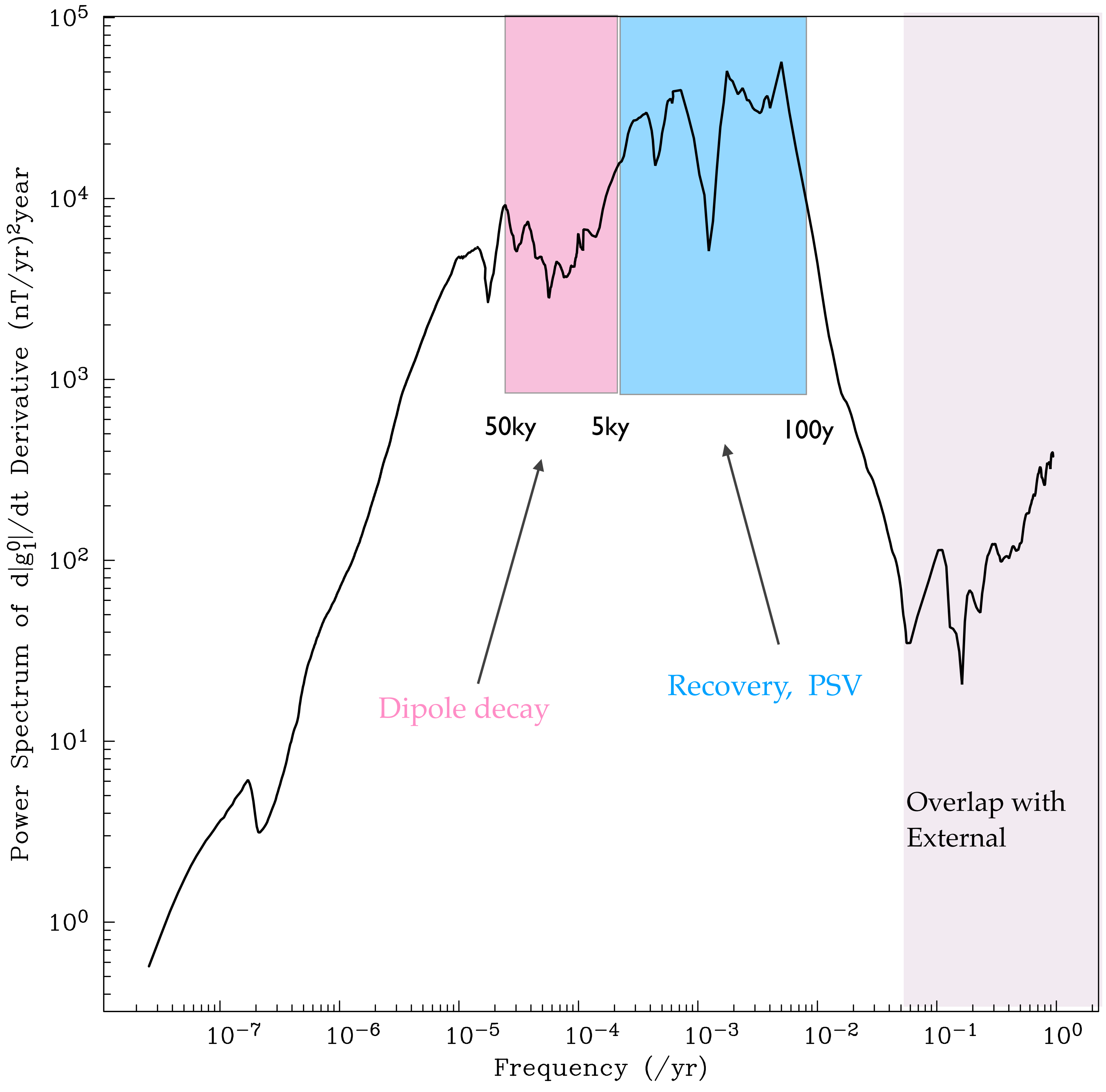
Knowing the power spectrum of g_1^0 we can directly estimate the power spectrum of its secular variation or rate of change with time dg_1^0/dt



Power Spectrum of g_1^0 is

$$S_{g_1^0}(f) = \mathscr{E}(|\tilde{g}_1^0(f)|^2)$$

$\tilde{g}_1^0(f)$ is the Fourier Transform of $g_1^0(t)$.

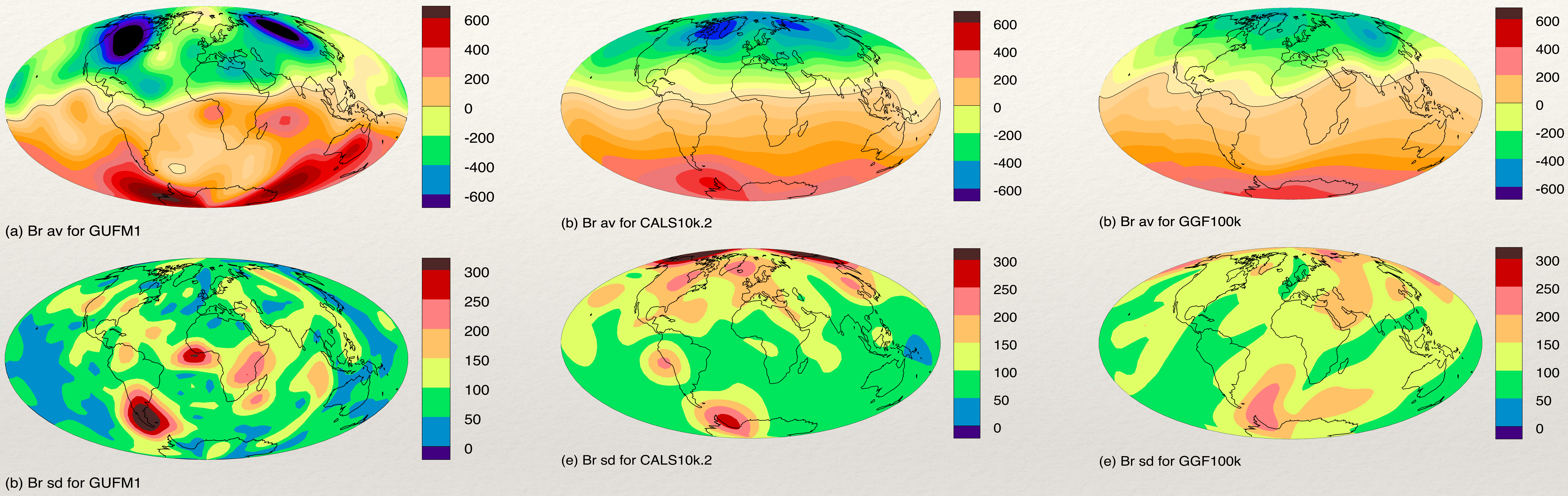


Power Spectrum of $dg_1^0/dt = \dot{g}_1^0(t)$

$$S_{\dot{g}_1^0}(f) = (4\pi^2 f^2) S_{g_1^0}(f)$$

Power spectrum for $\dot{g}_1^0(t)$ peaks in 50 ky - 100 year range.

Average B_r and Standard Deviation over Time



400 years — 10,000 years — 100,000 years

- B_r in μT at CMB - average across time-varying models
- Features are attenuated in time average
- N/S hemispheric asymmetry and longitudinal structure in field strength and variability

For Long term Global View from Paleomagnetic Measurements without good chronological constraints -

Spherical Harmonic Paleosecular Variation Models
A Statistical Approach

$$\vec{\mathcal{B}}(r,\theta,\phi) = -\nabla\mathcal{V}(r,\theta,\phi)$$

$$\mathcal{V} = a\sum_{l=1}^{\infty}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l+1}(\mathcal{G}_l^m \cos m\phi + \mathcal{H}_l^m \sin m\phi)P_l^m(\cos\theta).$$

[redacted]

- Gauss coefficients $\mathcal{G}_l^m$ and $\mathcal{H}_l^m$ are statistical samples uniformly distributed in time and

$$\mathcal{G}_l^m \sim N(\mu_{gl}^m, \sigma_{gl}^m)^2$$
$$\mathcal{H}_l^m \sim N(\mu_{hl}^m, \sigma_{hl}^m)^2$$

- Under this model, magnetic field directions at a location (r,θ,ϕ) follow a 3-dimensional Gaussian distribution

$$\vec{\mathcal{B}}(r,\theta,\phi) \sim N(\vec{\mu}_B, \Sigma)$$

with

$$\Sigma = \begin{pmatrix} \sigma_\theta^2 & \sigma_{\theta\phi} & \sigma_{\theta r} \\ \sigma_{\theta\phi} & \sigma_\phi^2 & \sigma_{\phi r} \\ \sigma_{\theta r} & \sigma_{\phi r} & \sigma_r^2 \end{pmatrix}$$

If $\sigma_{gl}^m = \sigma_{hl}^m = \sigma_l$ then Σ will be diagonal with

$$\Sigma = \begin{pmatrix} \sigma_\theta^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_r^2 \end{pmatrix}$$

A Giant Gaussian Process (GGP)

- A complete description of the field at any point in time requires that we know the Gauss coefficients, $g_l^m(t)$, h_l^m(t).
- But often we don't have good enough age constraints to make a time varying model - especially for lava flow data
- Resort to a statistical description, assuming that paleomagnetic observations have random temporal sampling.
- That allows us to calculate mean field properties and variability.
- Temporal correlations can be included, but this has not yet been done effectively.

Giant Gaussian Process (GGP) models

Constable & Parker, 1988, doi:10.1029/JB093iB10p11569

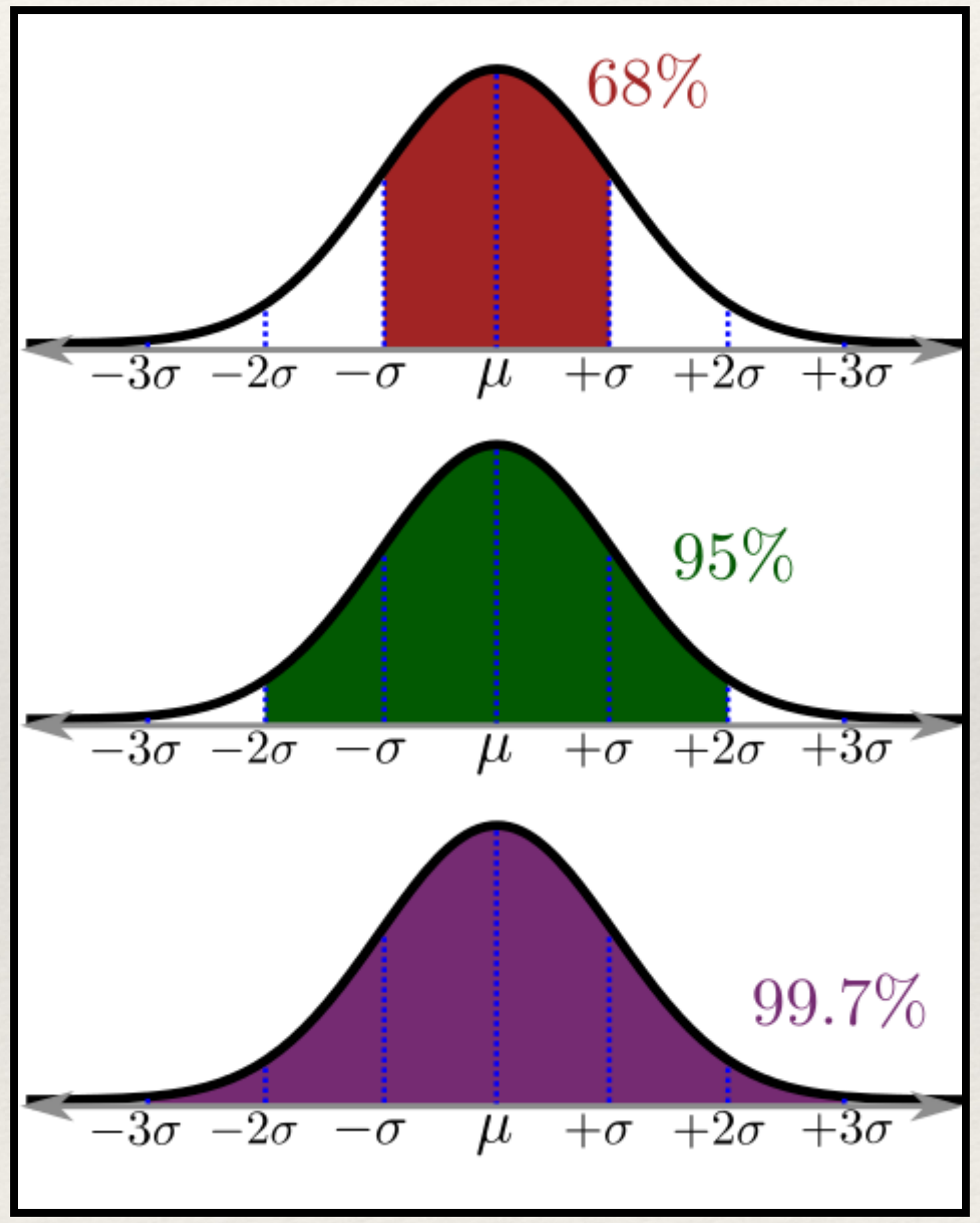
- At any point in time each Gauss coefficient in a spherical harmonic representation of the field looks like a random sample from a normal (Gaussian) distribution with specified mean and standard deviation.

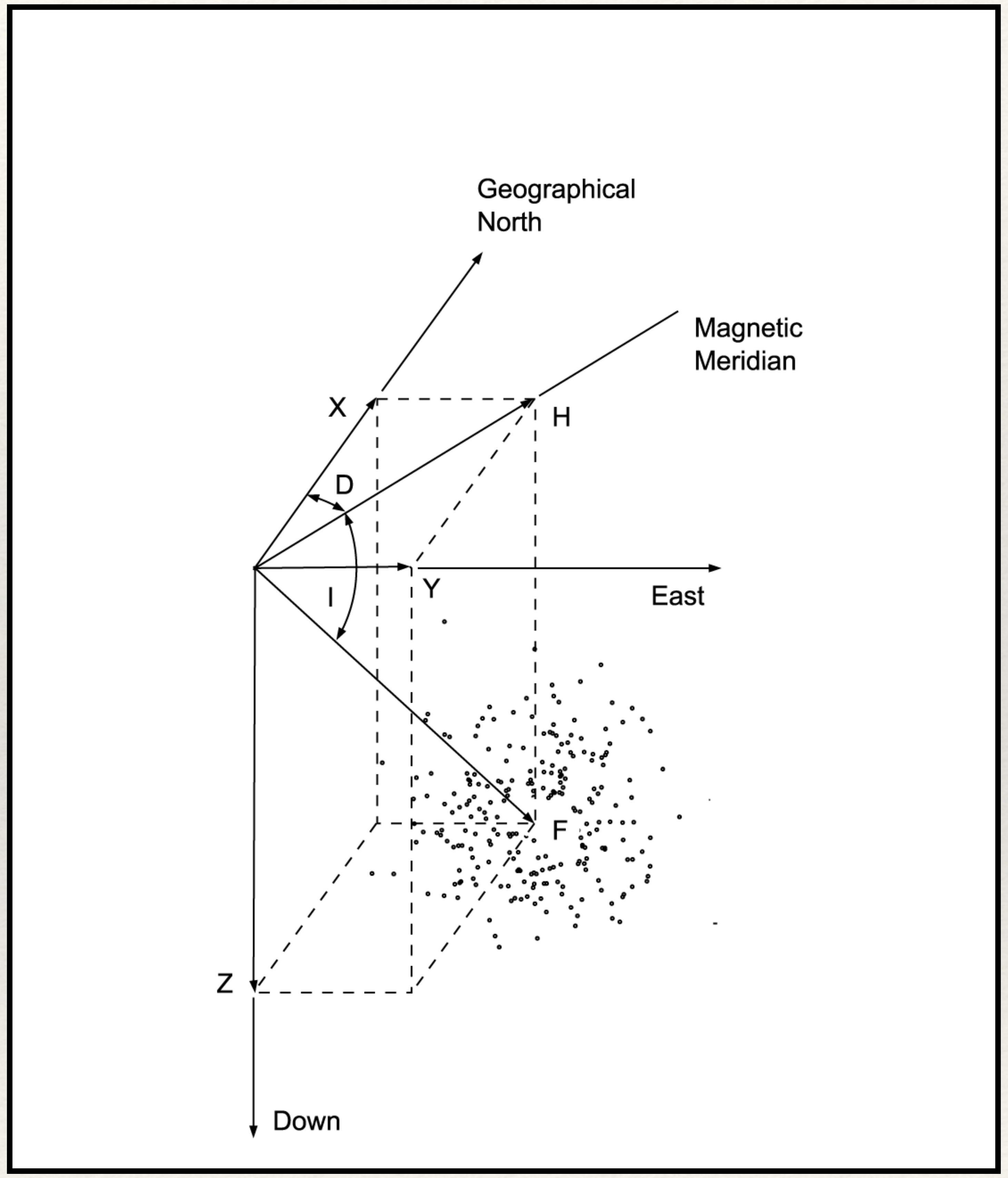
- Simplest GGP models have zero mean for the time averaged field except for the geocentric axial dipole.

- Local field components (X,Y, Z) will have a 3D-Gaussian distribution, representing PSV.

- Expected directional and intensity statistics can be calculated for any location, sometimes analytically, more often by random sampling of specified distributions.

- Local directional and intensity distributions are neither Fisherian nor Gaussian, respectively.





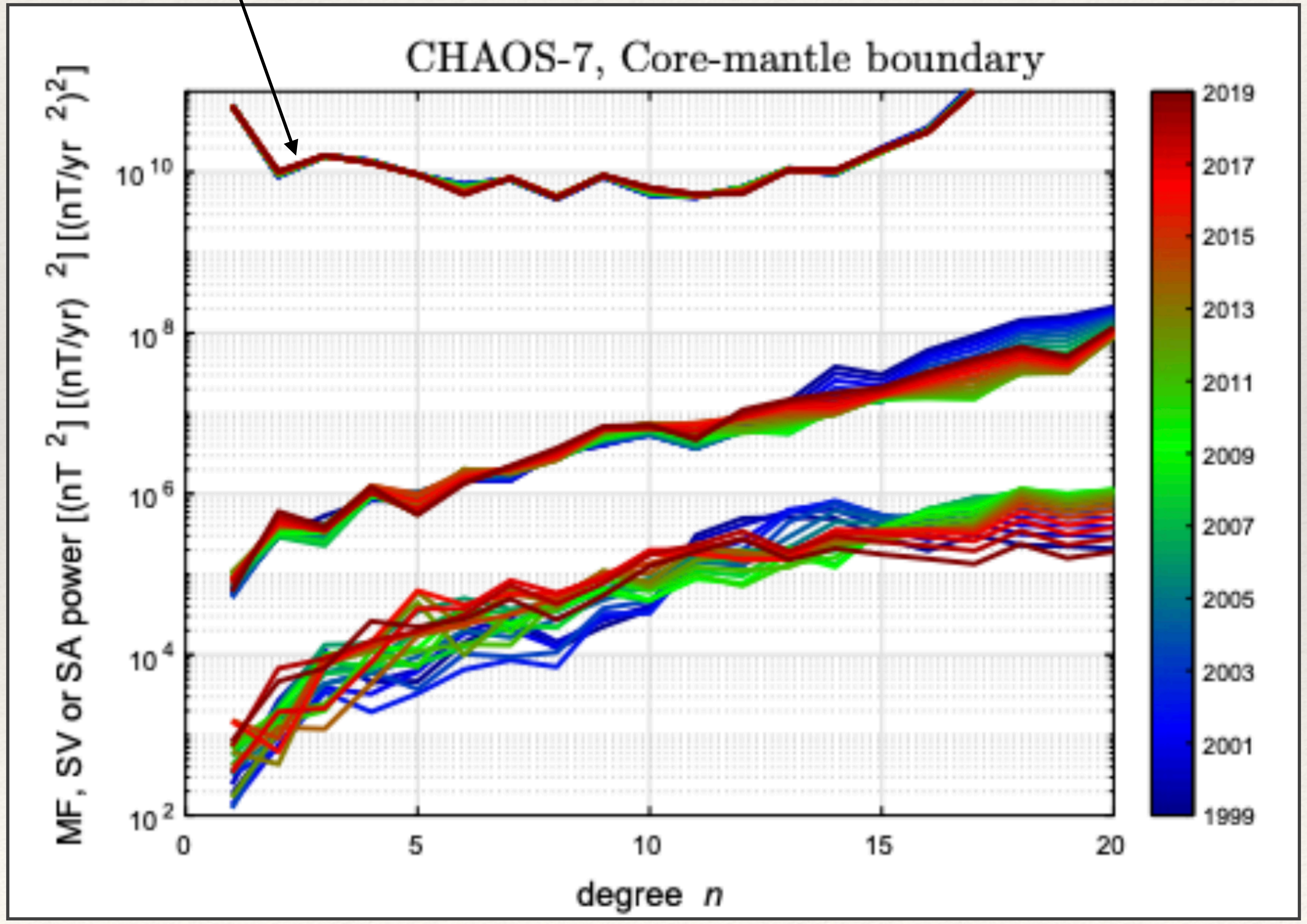
isotropic 3-D Gaussian distribution

SH degree *l* represents spatial wavelength

Spatial Power Spectrum for a Field Model is $R_l(t)$

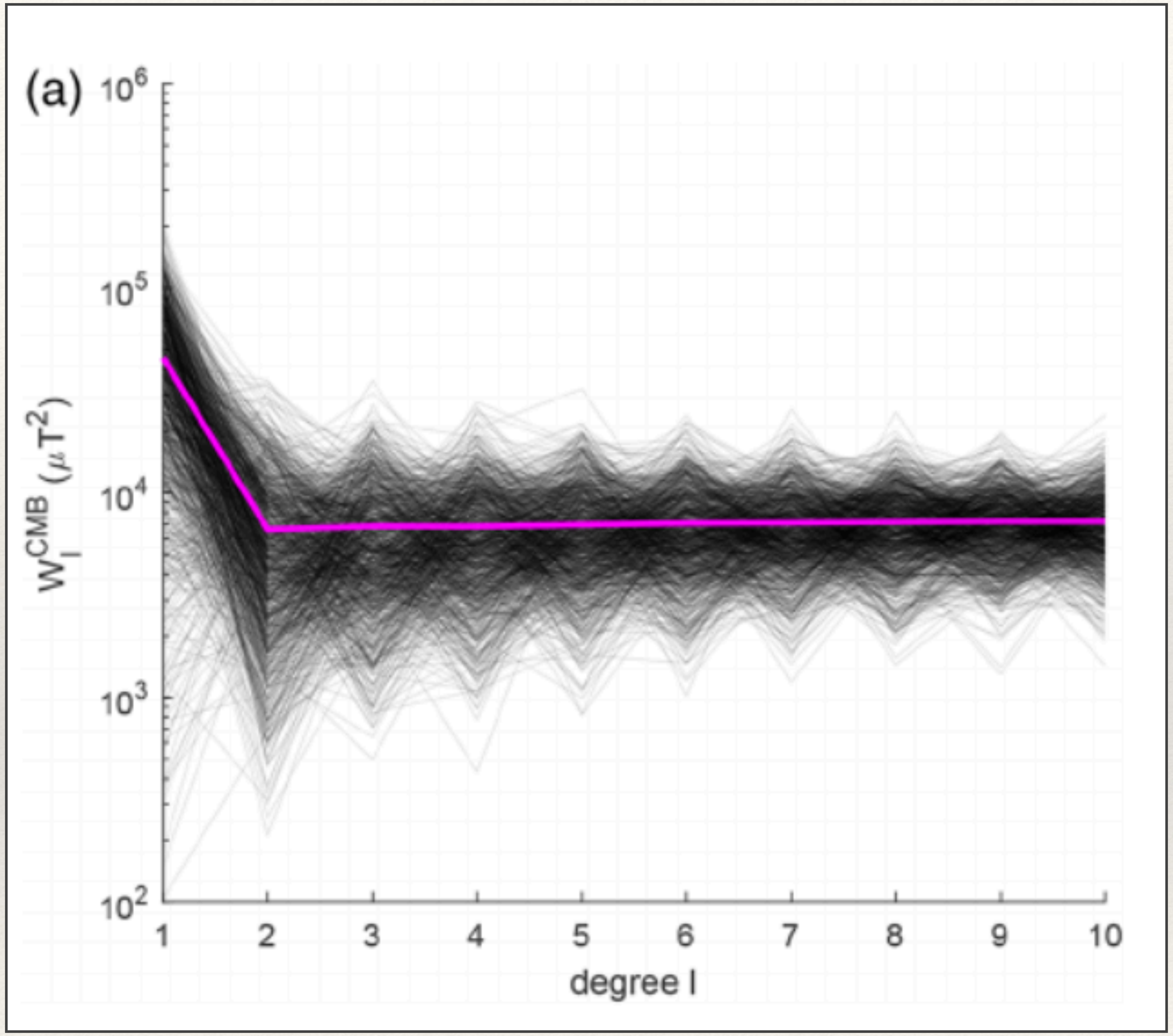
$$R_l(t) = <\vec{B}_l \cdot \vec{B}_l>_{r_a} = (l+1)\sum_{m=0}^{l}[(g_l^m(t))^2 + (h_l^m(t))^2]$$

R_l is approximately white at the CMB



Finlay et al. , Earth, Planets and Space (2020)

Spatial power spectra at the core-mantle boundary of 1000 realizations from GGP model BB18.Z3 for 0-5 Ma



Bono et al., G-cubed (2020)

$$R_l(r) = \left(\tfrac{a}{r}\right)^{2l+4}(l+1)\sum_{m=0}^{l}\left[(g_l^m)^2 + (h_l^m)^2\right] \tag{4}$$

is the Lowes spectrum, or sometimes the Lowes–Mauersberger spectrum (Mauersberger, 1956). It follows that

$$R_l(r) = \left(\tfrac{a}{r}\right)^{2l+4} R_l(a). \tag{5}$$

The downward continuation relation (5) gives the Lowes spectrum $R_l(r)$ at some depth r in terms of $R_l(a)$ at the surface. It relies crucially on $\boldsymbol{B}$ being purely potential.

To estimate the depth of the dynamo region, we need one further assumption. It has been argued that the large-scale part of $R_l(a)$ mainly originates from the Earth's outer core and turbulence there results in a uniform distribution of magnetic energy over different scales l. In particular, at some depth r_{lowes} near the core-mantle boundary, $R_l(r_{\text{lowes}})$ is independent of l. This 'white source hypothesis' (Backus et al., 1996), together with (5) implies the linear relation

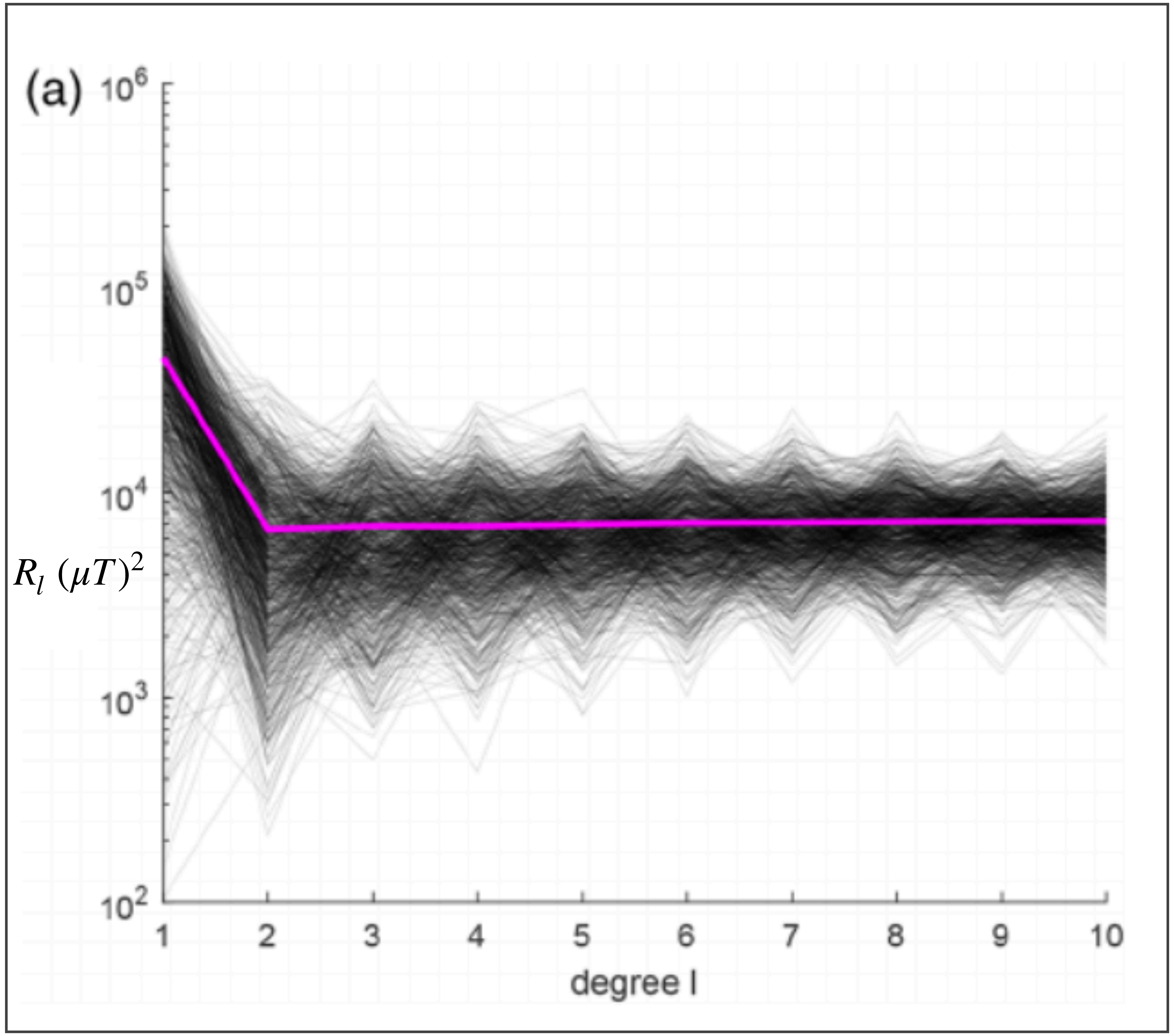
$$\log_{10} R_l(a) \sim -\beta(a)l \tag{6}$$

for the large scales with $\beta(a)$ satisfying

$$r_{\text{lowes}} = 10^{-\beta(a)/2} \cdot a. \tag{7}$$

Thus, (7) gives the Lowes radius r_{lowes} in terms of the spectral slope β which can be determined solely from magnetic measurement at the surface. The Lowes radius provides an estimate to the location of the Earth's core-mantle boundary that agrees reasonably with seismic measurement. Langlais et al. (2014) found $r_{\text{lowes}} = 3294.5$ km compared to the seismically determined 3481.7 km. Langlais et al. (2014) also found that omitting the $m = 0$ axisymmetric components in (4), so that only the non-zonal components are used, greatly improves the fit to the seismic core radius.

Spatial power spectra at the core-mantle boundary of 1000 realizations from GGP model BB18.Z3 for 0-5 Ma



Bono et al., G-cubed (2020)

Spherical Harmonic Paleosecular Variation Models
A Statistical Approach

$$\vec{\mathcal{B}}(r,\theta,\phi) = -\nabla \mathcal{V}(r,\theta,\phi)$$

$$\mathcal{V} = a \sum_{l=1}^{\infty} \sum_{m=0}^{l} (\frac{a}{r})^{l+1} (\mathcal{G}_l^m \cos m\phi + \mathcal{H}_l^m \sin m\phi) P_l^m(\cos\theta).$$

- no explicit time variation (although temporal correlations can be included – see Hulot & le Mouël,1994)

- Gauss coefficients $\mathcal{G}_l^m$ and $\mathcal{H}_l^m$ are statistical samples uniformly distributed in time and

$$\mathcal{G}_l^m \sim N(\mu_{gl}^m, \sigma_{gl}^m)^2$$
$$\mathcal{H}_l^m \sim N(\mu_{hl}^m, \sigma_{hl}^m)^2$$

- Under this model, magnetic field directions at a location (r,θ,ϕ) follow a 3-dimensional Gaussian distribution

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If $\sigma_{gl}^m = \sigma_{hl}^m = \sigma_l$ then Σ will be diagonal with

$$\Sigma = \begin{pmatrix} \sigma_\theta^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_r^2 \end{pmatrix}$$

Note the local field distributions are not isotropic and depend on location

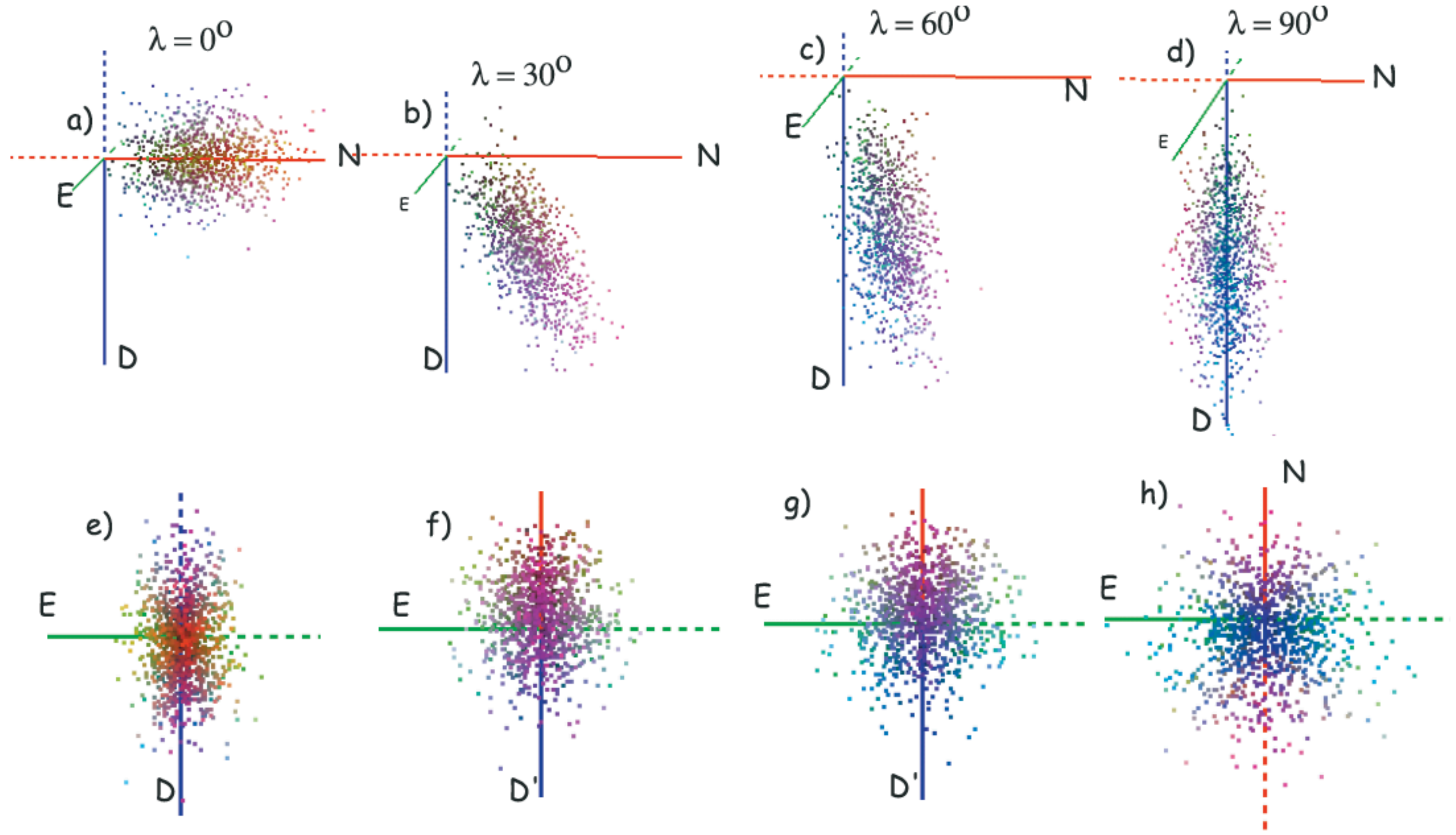


Plate 1. 1000 realizations of TK03.GAD projected as North (red), East (green) and Down (blue) components. Each dot is assigned the RGB color corresponding to the contributions from each component. a–d) All North axes are 40 μT long. (South, East and Up axis are the dashed lines. a) Equator, b) 30°N, c) 60°N, d) 90°N. e–h) Same data as a–d) but projected along the principal axis for each data cloud. All East axes are 20 μT. Axes labelled D' are projections in the N–S plane looking along the expected direction at that latitude. e) Equator, f) 30°N, g) 60°N, h) 90°N.

from Tauxe & Kent, 2004

So what is a GGP model?

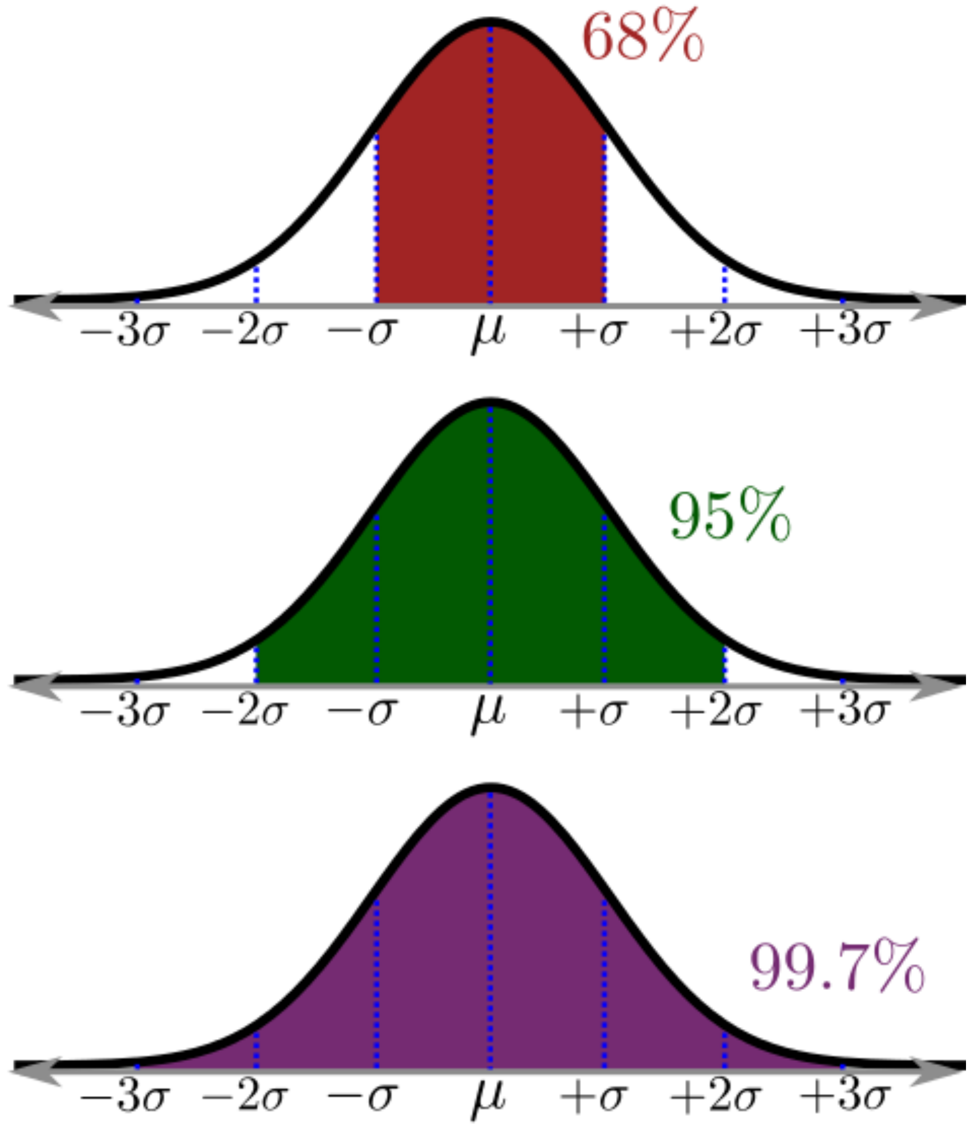
- At any point in time each Gauss coefficient in a spherical harmonic representation of the field looks like a random sample from a normal (Gaussian) distribution with specified mean and standard deviation.

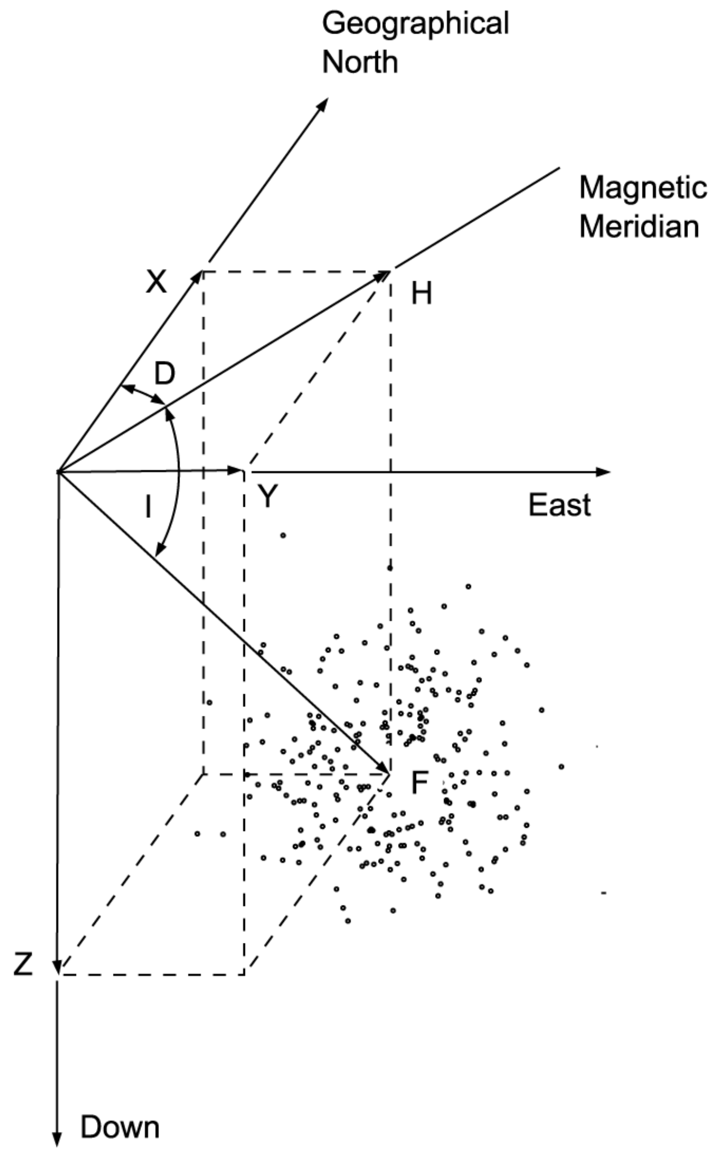
- Simplest GGP models have zero mean for the time averaged field except for the geocentric axial dipole.

- Local field directions (X,Y, Z) will have a 3D Gaussian distribution, representing PSV.

- Expected directional and intensity statistics can be calculated for any location, sometimes analytically, more often by random sampling of specified distributions.

- Directional and intensity distributions are not Fisherian or Gaussian, respectively.





isotropic 3-D Gaussian distribution

GGP model of Tauxe & Kent (2004) - TK03.GAD

- TK03 model has a non zero mean only for the Geocentric Axial Dipole
- Variances for Gauss coefficients of degree *l* and order *m* depend on symmetry

α and β:

$$\sigma_l^{m^2} = \frac{(c/a)^{2l}\alpha^2}{(l+1)(2l+1)} \quad \text{for} \quad l - m = \text{even},$$

symmetric terms

$$\sigma_l^{m^2} = \frac{(c/a)^{2l}\alpha^2\beta^2}{(l+1)(2l+1)} \quad \text{for} \quad l - m = \text{odd},$$

antisymmetric terms

c/a is the ratio of the core radius to that of Earth (0.547)

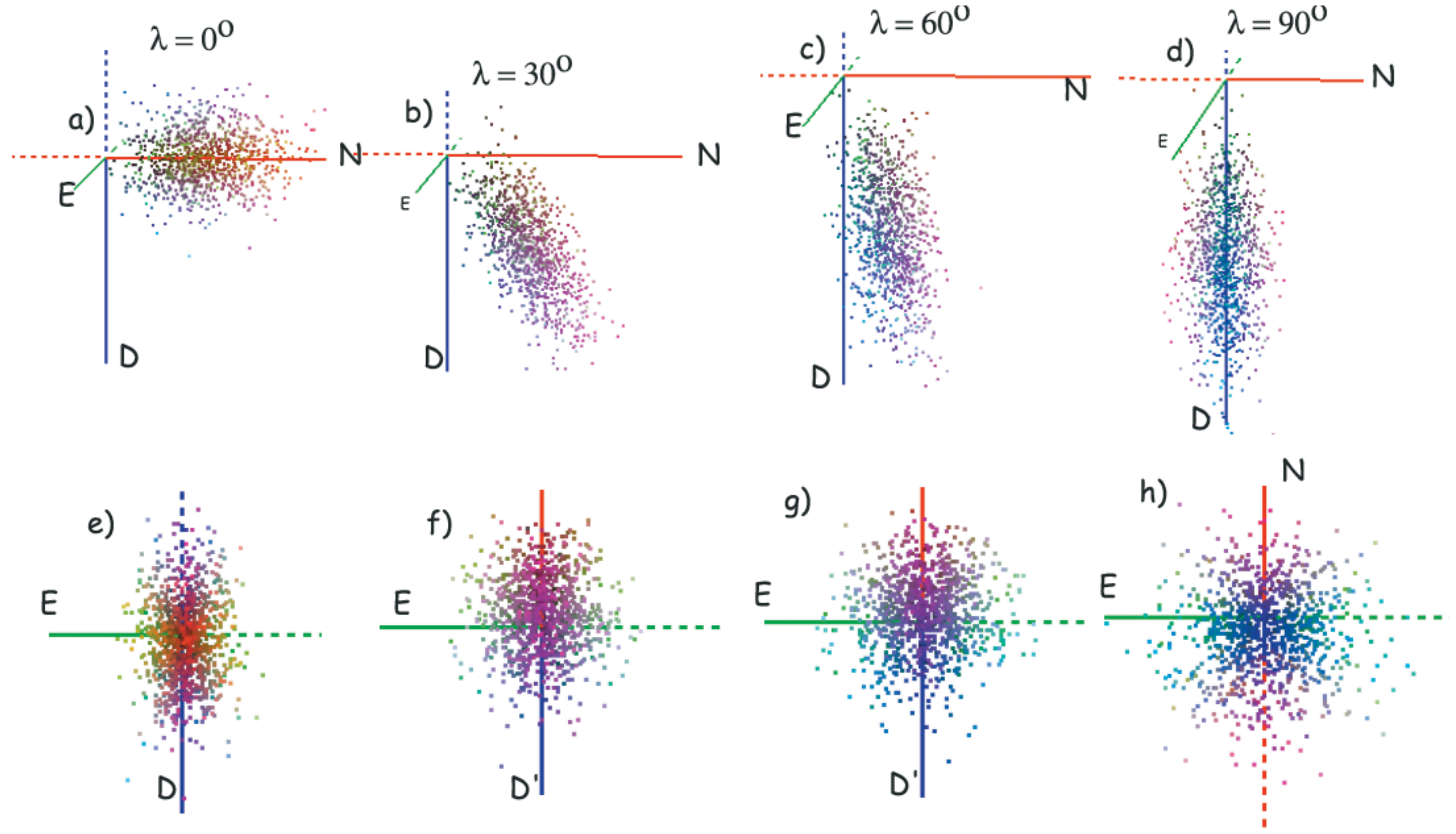


Plate 1. 1000 realizations of TK03.GAD projected as North (red), East (green) and Down (blue) components. Each dot is assigned the RGB color corresponding to the contributions from each component. a–d) All North axes are 40 μT long. (South, East and Up axis are the dashed lines. a) Equator, b) 30°N, c) 60°N, d) 90°N. e–h) Same data as a–d) but projected along the principal axis for each data cloud. All East axes are 20 μT. Axes labelled *D´* are projections in the N–S plane looking along the expected direction at that latitude. e) Equator, f) 30°N, g) 60°N, h) 90°N.

Instead of functional time variations like splines we can use a statistical approach

A complete description of the field at any point in time requires that we know the Gauss coefficients.

But often we don't have good enough age constraints to make a time varying model.

Then we resort to a statistical description, assuming that paleomagnetic observations have random temporal sampling.

That allows us to calculate mean field properties and variability.

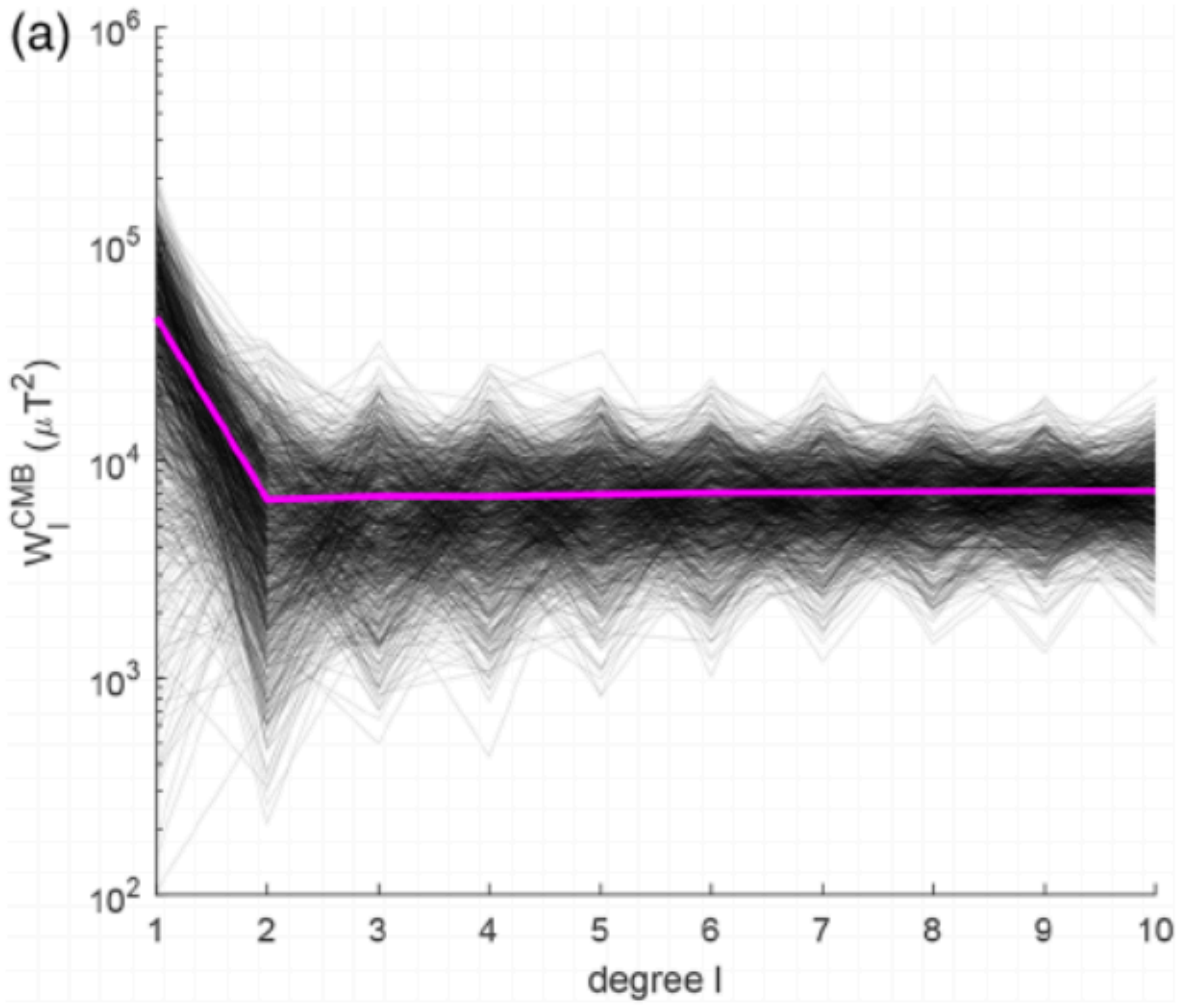


Figure 3. Power spectra at the core-mantle boundary (Lowes, 1974) of 1,000 realizations of *BB18.Z3* (black lines). Magenta line shows the mean power spectrum for *BB18.Z3*.

What happens to the Lowes' spectrum when the field is downward continued to the CMB?

For Earth the spectrum is approximately white at the CMB from *l=2* up to about degree 14.
The depth at which the large scale spatial power spectrum is white has been proposed to estimate the depth to the dynamo surface in other planets.

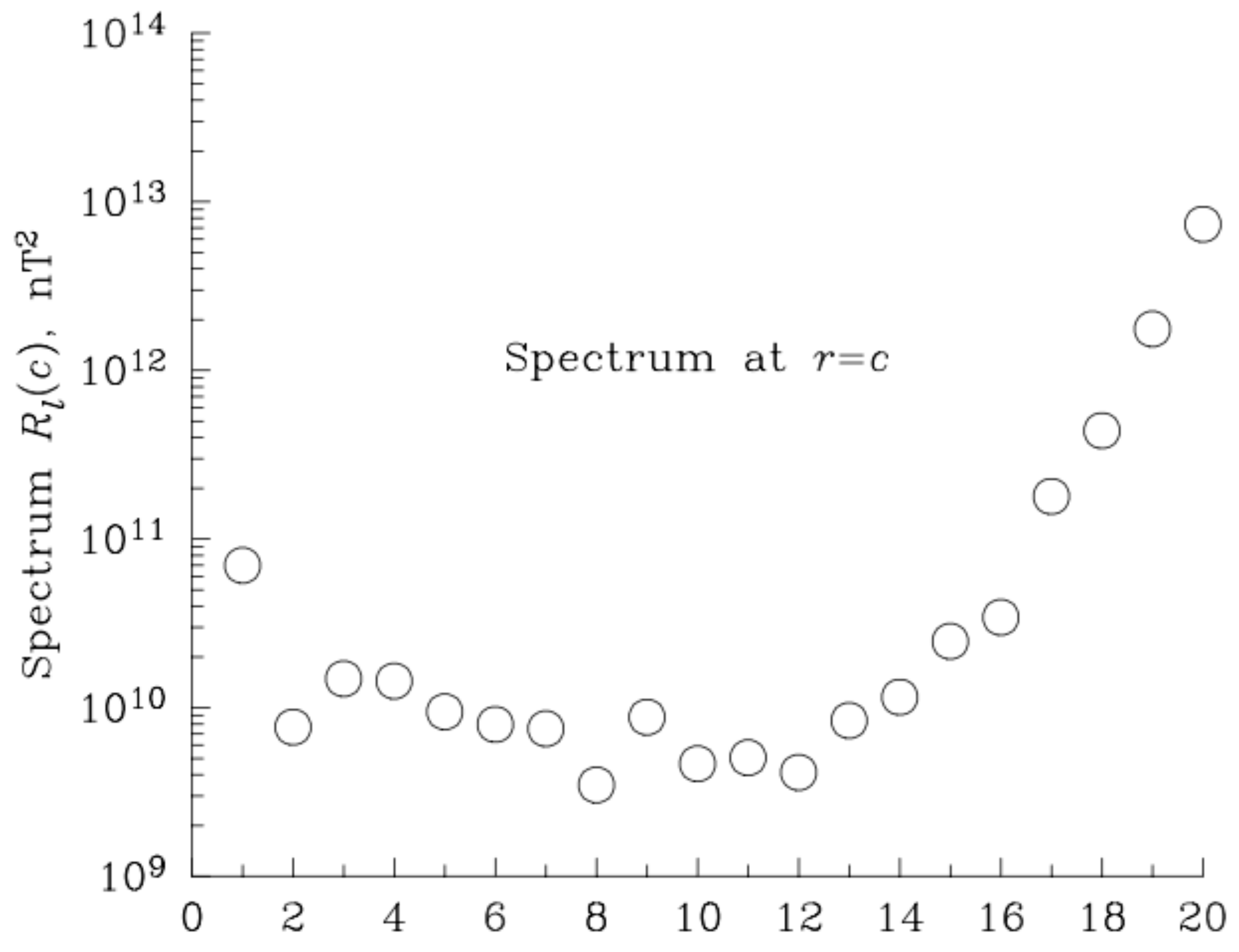


Figure 3.6.4.2 Lowes spectrum evaluated at the core-mantle boundary.

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Characterising Jupiter's dynamo radius using its magnetic energy spectrum

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Highlights

- The dynamo radius (top of the dynamo region) is a key property of Jupiter's dynamo.
- The shape of the magnetic energy spectrum becomes invariant inside the dynamo region.
- Transition in the magnetic spectral slope defines a dynamo radius for Jupiter.
- The traditional Lowes spectrum gives a lower bound to the dynamo radius.
- The Lowes spectrum derived from the Juno data is significantly steeper than expected.

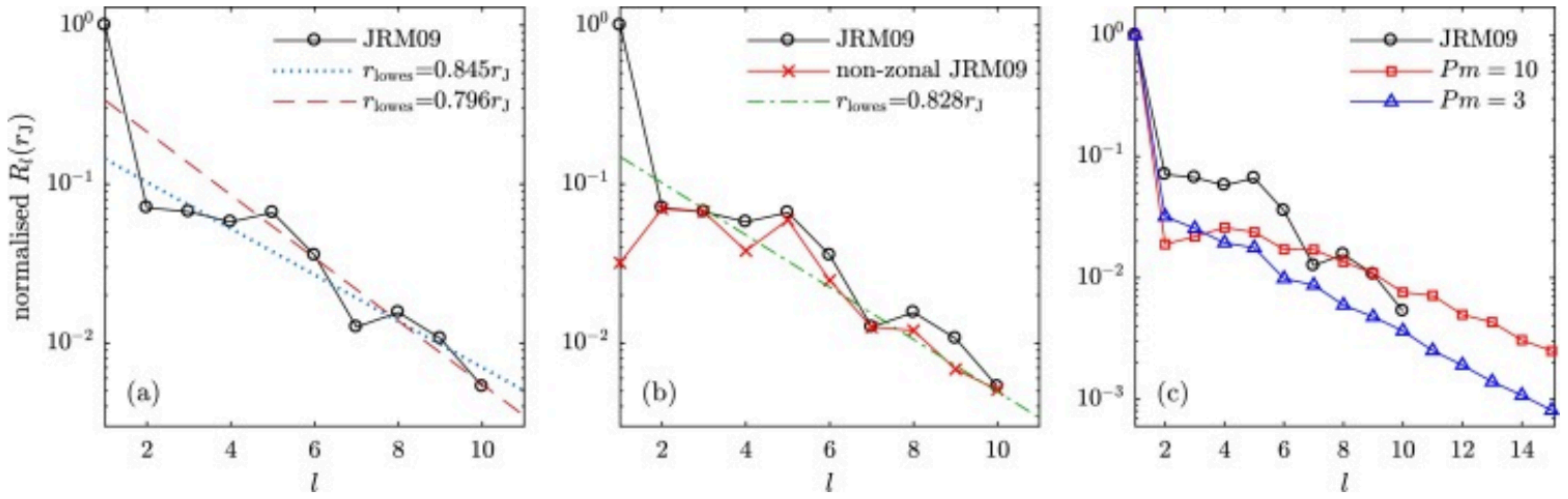

Fig. 1. (a) Normalised Lowes spectrum R_l/R_1 at $r = r_J$ calculated from the Gauss coefficients in the JRM09 model of Connerney et al. (2018). A linear fit to $\log_{10} R_l(r_J)$ for $2 \leq l \leq 10$ gives the Lowes radius $r_{lowes} = 0.845r_J$. Changing the fitting range to $5 \leq l \leq 10$ results in $r_{lowes} = 0.796r_J$. Connerney et al. (2018) suggest the data is compatible with $r_{lowes} = 0.87r_J$. Here, $r_J = 6.9894 \times 10^7$ m. (b) The non-zonal part of the JRM09 data is compared with the full spectrum. Both spectra are normalised by the value of R_1 of the full spectrum. Note that the non-zonal data gives a much closer fit to a straight line in the range $6 \leq l \leq 10$ than the full data and a linear fit in this range gives $r_{lowes} = 0.828r_J$. (c) Comparison of normalised Lowes spectrum R_l/R_1 at $r = r_J$ from the Juno data JRM09 and our Jupiter dynamo model at $Pm = 10$ and $Pm = 3$.