1) Derive a second-order accurate finite difference equation for a first derivative $du/dx$ that uses a “one-sided” stencil - that is, values $u_{i-2}$, $u_{i-1}$ $u_i$ (or $u_i$, $u_{i+1}$, $u_{i+2}$) to evaluate the derivative of $u$ at point $x_i$.

2) Determine the order of accuracy of the following FD equation to the PDE

$$u'_t + vu'_x = 0,$$

$$u_{i,j+1} = u_{i,j-1} - \frac{v\Delta t}{\Delta x}(u_{i+1,j} - u_{i-1,j})$$

where $v = \text{const}$, and indexes $i$ and $j$ correspond to discrete values of $u(x_i, t_j)$.

3) Implement a finite difference solution to the flux-conservative initial value problem

$$u'_t + vu'_x = 0,$$

on an interval $x=[0 1]$, subject to the initial condition $u(x, 0) = \sin(4\pi x)$ and boundary condition $u(0, t) = 0$. Assume $v = 1$. Use the first-order FD approximation for the time derivative $u'_t$, and the following FD approximations for $u'_x$: (i) first-order upwind scheme; (ii) second-order centered scheme; (iii) second-order centered scheme with the Lax substitution ($u_{i,j} \rightarrow \frac{1}{2}(u_{i+1,j} + u_{i-1,j})$). Choose $\Delta x$ to ensure an adequate spatial resolution of your solution (a simple visual inspection will suffice for now), and explore how your FD solutions depend on a time step $\Delta t$. 