SIO 227B – Practical 3

This practical requires you to find the source mechanisms of some earthquakes. The problem is a linear inverse problem which is over determined (i.e. you have more data than unknowns). The equation you will be solving is

$$a_j(\omega) = \sum_{i=1}^{6} B_{ij}(\omega)\psi_i(\omega)$$

which is equation 4.50 in the notes. a_j is the spectrum of the recording at the j'th station, B_{ij} is a matrix which we compute and the 6-vector, ψ, is the seismic moment-rate tensor. The many problems encountered in solving for the moment-rate tensor are discussed in the notes (page 84). Here, we consider some of the practicalities involved.

Low-frequency seismology requires fairly large earthquakes to get a usable signal-to-nose ratio (SNR). We typically use events with M_S greater than about 6.5 and there are usually about 20 such events per year. At very low frequencies (less than about 2mHz), ground noise rapidly increases as frequency decreases and there is only usable signal for extremely large earthquakes. Routine analysis of source mechanisms therefore uses frequencies above 2mHz. As frequency increases, the fundamental modes (which dominate the seismogram) sample more and more of the near-surface structure which also happens to be the most heterogeneous. Above about 6mHz, the error incurred by assuming the Earth is spherically symmetric becomes extremely large and the source is not accurately retrieved. We therefore fit the above equation in a 4mHz frequency band centered about 4mHz. The optimum record length for estimating the amplitude of a mode is about 0.5 Q cycles which is about 10 hours for a 250 second surface-wave equivalent mode.

When we construct Green's functions, we include all modes with frequencies less than 8mHz. This means that we can apply a low-pass filter with a corner at about 6mHz and get clean looking seismograms without the ringing asociated with a sharp spectral cut-off. The data must be filtered in exactly the same way so that a meaningful time-domain comparison can be made. The program that you use to filter the data is called NDEC. This program applies a zero phase shift convolution filter to your data. The Greens functions are computed using program GREEN and are then filtered with the same filter as you applied to the data in program FILGRN. Note that GREEN also applies the individual instrument responses to the Green's functions.

The linear fitting for the moment tensor elements is done in program MTTC. This program asks you several questions before using a singular value decomposition (SVD) to construct a generalised inverse of the matrix $\mathbf{B}$. One question is whether or not you want to normalise the data. This means that the average amplitude at each station is made to appear the same using row weights. This is a good idea because the fact that there is not much power at a station tells you as much about the source as when there is a lot of power. Least-square solutions tend to fit the large signals so that stations with little power will tend to be ignored in the fitting procedure unless the weighting is applied. Another question that is asked is whether or not you wish to apply a time-domain taper before FFTing. Tapering sharpens up the spectrum but this is one case where you don't necessarily want high resolution. The data, and the columns of the Greens functions are sums of decaying sinusoids so their spectra are spiky. Accurate source retrieval requires that the spikes be alligned in frequency. We have seen that 3D structure tends to move peaks around and anything which accentuates the misallignment of the Green's functions with the data is a bad thing – this includes tapering.

Now we get to the real fitting. The program will ask for a time constant (which is the baseline length of a triangle function – a reasonable shape for the moment-rate tensor elements) and solves the above equation. You can apply the deviatoric constraint so that no explosive or implosive component to the source is allowed and, if you have information about the orientation of a possible fault plane, you can apply a plane constraint. The fitting is done using a SVD which is described in several texts. Briefly, $\mathbf{B}$ is decomposed into left- and right-hand eigenvectors and a diagonal matrix of singular values, Λ, as

$$\mathbf{B} = \mathbf{U}\Lambda\mathbf{V}^T$$

where $\mathbf{U}^T\mathbf{U} = \mathbf{I}$ and $\mathbf{V}^T\mathbf{V} = \mathbf{I}$ where superscript T stands for transpose and $\mathbf{I}$ is the unit matrix. The generalized inverse is given by

$$\hat{\psi} = \mathbf{V}\Lambda^{-1}\mathbf{U}^T\mathbf{a}$$

which reduces to the standard least-squares solution when all singular values are taken. If there are some small singular values, the system of equations is not well-conditioned and we do not have enough information to completely constrain the solution. A reasonable response is to ignore all small singular values when constructing the solution. The fit to the data will probably not be degraded by much and the result is a solution with the smallest Euclidian norm for this fit to the data. The program asks for a "qmin-qmax" ratio which is the largest range of singular values you are prepared to accept. A value of 0.01 to 0.1 is appropriate and you can try experimenting with this parameter.

The program prints out some interesting numbers at each stage of the fitting, including fits of various recordings and an overall variance reduction. You should vary the time constant of the event until the overall variance reduction is optimised. Events with M_S below about 7 will probably have time constants of less than 30 seconds. The largest events can have time constants in excess of 100 seconds.

Once you have settled on a best fit, you can use program SYNDAT to compute synthetic seismograms. This program accepts double couples or moment tensors as input. You can view the fit of individual synthetics to the data using program RECP. This is where you may find some problems with the data. You may need to fine tune some editing or you may find that a time series has a timing error or an incorrect instrument reponse. The variance reductions printed out by MTTC for individual stations are useful for diagnosing data problems. If a variance ratio is less than about 0.4, you will find that time-domain fits are excellent. A variance ratio between 0.4 and 0.6 is acceptable. 0.6 to 0.8 is barely acceptable and suggests a problem with the data. Above 0.8 means that there is almost certainly something wrong with the data or, if many recordings are badly fit, the event may have had a complex rupture history and cannot be modeled by a point source.

One last program for you to play with is DCSEARCH. This program was written to try and remove some of the bias associted with the linear fitting procedure. It takes a guess of a double couple model for the source (strike, dip and slip) and adjusts this until the power at each station is correctly predicted. This program calcualates power in the time domain so you should use a relatively short time series including about two Rayleigh waves and you should not use time series which have a lot of low frequency noise (unless you are going to high pass everything).

When writing this assignment up, please discuss how well-determined your solution is and how reasonable it is from a tectonic standpoint. If your data is incapable of completely constraining the solution, try looking through the literature to get a better guess of source orientation. Discuss how well your solution fits the data and describe any adjustments you made to your dataset during the fitting procedure.