Surface Waves

Why surface waves in Global Tomography?

- radial resolution
- data coverage in remote areas (e.g. no stations)

Typical Period Range

- 67 - 250 s (15 - 4 mHz),
  recently down to 35 s though off-great circle wave propagation becomes an issue

Applied Techniques

Waveform Modelling ↔ Dispersion Data

complete wave packets parametric data
### Surface Waves

**Applied Techniques**

<table>
<thead>
<tr>
<th>data type</th>
<th>complete wave packets</th>
<th>parametric data</th>
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<tbody>
<tr>
<td>modeling</td>
<td>direct inversion for structure</td>
<td>two-step procedure</td>
</tr>
<tr>
<td>overtones</td>
<td>included</td>
<td>complicated</td>
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<tr>
<td>scattering</td>
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<td>errorbars</td>
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<td>quick&amp;dirty</td>
<td>no</td>
<td>✓</td>
</tr>
<tr>
<td>size of problem</td>
<td>massive procedure</td>
<td>small, linear inverse problem</td>
</tr>
</tbody>
</table>

- e.g. Harvard for mantle waves; Berkeley
- e.g. Harvard for surface waves; Scripps Caltech
W&D Waveform Modelling

- Make a synthetic surface wave seismogram using mode representation (that is inherently insensitive to odd orders)
- odd order structure represented by epicentral distance shift

consider phase integral along great circle

\[
\delta \psi = \int_{0}^{T} \delta \omega(\theta, \phi) \, dl
\]  \hspace{1cm} (6.6)

odd wave orbits \( (R_{2n+1}, G_{2n+1}) \)

\[
\delta \psi = \int_{0}^{t_1} \delta \omega(\theta, \phi) \, dl + n \int_{0}^{T_c} \delta \omega(\theta, \phi) \, dl
\]  \hspace{1cm} (6.7)

even wave orbits \( (R_{2n}, G_{2n}) \)

\[
\delta \psi - \int_{0}^{t_1} \delta \omega(\theta, \phi) \, dt + n \int_{0}^{T_c} \delta \omega(\theta, \phi) \, dt
\]  \hspace{1cm} (6.8)

Define:

\[
\delta \tilde{\omega} = \frac{1}{T_c} \int_{0}^{T_c} \delta \omega(\theta, \phi) \, dt \quad \delta \tilde{\omega} = \frac{1}{t_1} \int_{0}^{t_1} \delta \omega(\theta, \phi) \, dl
\]  \hspace{1cm} (6.9)

To get:

\[
\begin{align*}
\delta \psi &= t_1 \delta \tilde{\omega} + nT_c \delta \tilde{\omega} \quad \text{(odd)} \\
\delta \psi &= T_c \delta \tilde{\omega} - t_1 \delta \tilde{\omega} + nT_c \delta \tilde{\omega} \quad \text{(even)}
\end{align*}
\]
W&D Waveform Modelling

\[ \delta \psi = t_1 \delta \omega + nT_c \delta \omega \quad \text{(odd)} \]
\[ \delta \psi = T_c \delta \omega - t_1 \delta \omega \mid nT_c \delta \omega \quad \text{(even)} \]

rearrange
\[ \delta \psi = t_1 \delta \omega + nT_c \delta \omega \quad \text{(odd)} \]
\[ \delta \psi = -t_1 \delta \omega + nT_c \delta \omega + T_c \delta \omega \quad \text{(even)} \]

add/subtract \( t_1 \delta \omega \)
\[ \delta \psi = -t_1 (\delta \omega - \delta \omega) + (nT_c + t_1) \delta \omega \]
\[ \delta \psi = \mid t_1 (\delta \omega - \delta \omega) \mid (nT_c \mid t_1) \delta \omega \]
same but opposite sign

So get:
\[ \delta \psi = -(l + \frac{1}{2}) \delta \Delta + (nT_c + l_1) \delta \omega \]
\[ \delta \psi = + (l + \frac{1}{2}) \delta \Delta + (nT_c + t_1) \delta \omega \]

Define: \( (6.9) \)
\[ \delta \Delta = \frac{t_1}{l} \left( \frac{1}{2} \delta \omega, \delta \omega \right) \]

\( \delta \omega; \delta \Delta \)
different for each path and each mode so they provide a convenient device to incorporate 3D structure
W&D Waveform Modelling

\[ \delta \omega; \delta \Delta \]

different for each path and each mode so they provide a convenient device to incorporate 3D structure

ficticious source shift is a consequence of along-branch coupling (e.g. \( 0S_{24}-0S_{25}-0S_{26} \))
W&D Waveform Modelling

\[ \delta \omega - \frac{1}{T_c} \int_0^{T_c} \delta \omega(\theta, \phi) \, dt \]

\[ \delta \psi = (l + \frac{1}{2}) \delta \Delta + (nT_c + l_1) \delta \omega \]

remember \[ s = \text{Re} \left\{ \sum_{j} A_j(\Delta) e^{i \vec{\omega}_j t} \right\} \]

\[ s = \text{Re} \left\{ \sum_{j} A_j(\Delta + \delta \Delta) e^{i(\vec{\omega}_j + \delta \omega) t} \right\} \quad (6.10) \]

\[ \omega \text{ relates to } \sum_{s,t} c_{s}^{t} Y_{s}^{t}(\theta, \phi) \quad (5.59) \]

Expand \( \delta m_{s}^{t}(r) \)

e.g. W&D used Legendre Polynomials; B-Splines are fine, too

\[ \delta m(r, \theta, \phi) = \sum_{s,t} \sum_{k} \delta m_{s}^{t} f_{k}(r) Y_{s}^{t}(\theta, \phi) \]

\[ c_{s}^{t} = \sum_{k} \delta m_{s}^{t} \int_{0}^{\alpha} K_{s}(r) f_{k}(r) r^2 \, dr \]

\[ D_{sk} \]
\[ \delta \dot{\omega} = \frac{1}{T_c} \int_0^{T_c} \delta \omega(\theta, \phi) \, dt \]

so we get

\[ \delta \dot{\omega} = \frac{1}{T_c} \int_0^{T_c} \sum c_s^t Y_s^t(\theta, \phi) \, dt \]

\[ = \sum P_s(0) \sum c_s^t Y_s^t(\Theta, \Phi) \]

\[ - \sum P_s(0) \sum_{l,k} \delta m_s^l \ell_s k Y_s^l(\Theta, \Phi) \]

and

\[ \delta \Delta = \frac{t_1}{l_1} (\delta \dot{\omega}) \]

\[ \delta \Delta = \frac{t_1}{l_1} \left[ \frac{1}{\Delta u} \int_0^\Delta \delta \omega(\theta, \phi) \, dx \right] \delta \dot{\omega} \]

which can also be cast in terms of

\[ \delta m_s^l(r) \]

So, calculate synthetic
determine derivative and
do iterative inversion:

\[ s_{\text{observed}} = s_0 + \sum_{k,s,t} \frac{\partial s_0}{\partial k \delta m_s^t} \delta (k \delta m_s^t) \]
So, calculate synthetic
determine derivative and
do iterative inversion:

\[ s_{\text{observed}} = s_0 + \sum_{k,s,t} \frac{\partial s_0}{\partial k \delta m_s^t} \delta(k \delta m_s^t) \]  (6.13)

- need good starting model
- need partial derivatives of seismogram
  with respect to model parameters
DATA:
- 53 events; depth mainly < 50km
- some 2000 seismograms
  recorded at GDSN + IDA (gravimeter)
  on 870 paths

MODEL:
- lateral: spherical harmonics up to l=8
- radial: “a cubic spline polynomial”
  (Legendre Polynomials) down to 670km (up to k=3)

“Mapping the Upper Mantle: Three-dimensional Modeling of Earth Structure by Inversion of Seismic Waveforms”
Woodhouse & Dziewonski, 1984

"path-by-path inversion"
(I.e. peak shift measurement)

Fig. 1a. Results of path by path inversion for the frequency shift of $0^S_{30}$ as a function of great circle pole position. The upper panel shows results for individual paths. The size of the symbols is proportional to the frequency shift according to the scale indicated. The lower panel shows a smoothed version, obtained by fitting spherical harmonics up to degree and order 8.
waveform fitting

Woodhouse & Dziewonski, 1984

Fig. 4. Same as Figure 3 but for a deep event.

3-component seismograms!
Woodhouse & Dziewonski, 1984

TABLE 2. Model M84A (No Crustal Correction): Spherical Harmonic Coefficients of the Perturbation in Squared Shear Velocity

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<thead>
<tr>
<th>m</th>
<th>k=0</th>
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<td>B R'</td>
<td>A RM</td>
<td>B R'</td>
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<td>0.157</td>
<td>0.69</td>
</tr>
</tbody>
</table>

the model
Woodhouse & Dziewonski, 1984

“model predictions reproduce much of what is known about dispersion of mantle waves”
- high phase velocity for shields
- low phase velocity at ridges
- strong degree 2 pattern for Rayleigh waves
- treatment of complete waveforms implicitly includes overtones

Features of the Model
- shields and ridges are major features at 25-250km depth
- EPR and larger shields down to 350km
- Southeast Indian Rise and Mid-Atl. Ridge underlain by high velocities at same depth
- 450-650km: broad regions of high velocities under S. America, S. Atl., Africa, West Pacific
- fits mode degree 2 pattern
L02.56 comes from Dziewonski, 1984, same issue of JGR. It was derived in and inversion of ISC p-travel time picks for P-velocity structure.
Plate 3. [Woodhouse and Dziewonski]. Four vertical sections through the upper mantle model M84C and the lower mantle model L02.56 of Dziewonski [this issue]. The maps show the complete great circles along which the sections are taken. The upper mantle (depths 25-670 km; vertical exaggeration 8:1) and lower mantle (depths 670-2891 km; vertical exaggeration 4:1) models are shown in separate panels. For the upper mantle, colors indicate relative shear velocity perturbations and, for the lower mantle, relative P velocity perturbations. Note the differing scales of variation for the upper (±2%) and lower mantle (±0.5%) models. Anomalies exceeding the scale range are clipped at the maximum absolute value.
A Slight Complication

Surface Waves are Sensitive to
more than shear velocity

\[
\frac{\delta c}{c} = \int_0^a r^2 dr (\tilde{A} \cdot \delta \alpha + \tilde{B} \cdot \delta \beta + \tilde{R} \cdot \delta \rho)
\]

\[
\frac{1}{1.7} \tilde{B} \cdot \delta \beta \quad \frac{1}{2.5} \tilde{B} \cdot \delta \beta
\]
Phase Velocity Sensitivity Kernels

Rayleigh waves (ρ)  Rayleigh waves (VP)  Rayleigh waves (VS)

Love waves (ρ)  Love waves (VP)  Love waves (VS)

Depth [km]

4 - 9 mHz
10 - 15 mHz
16 - 20 mHz