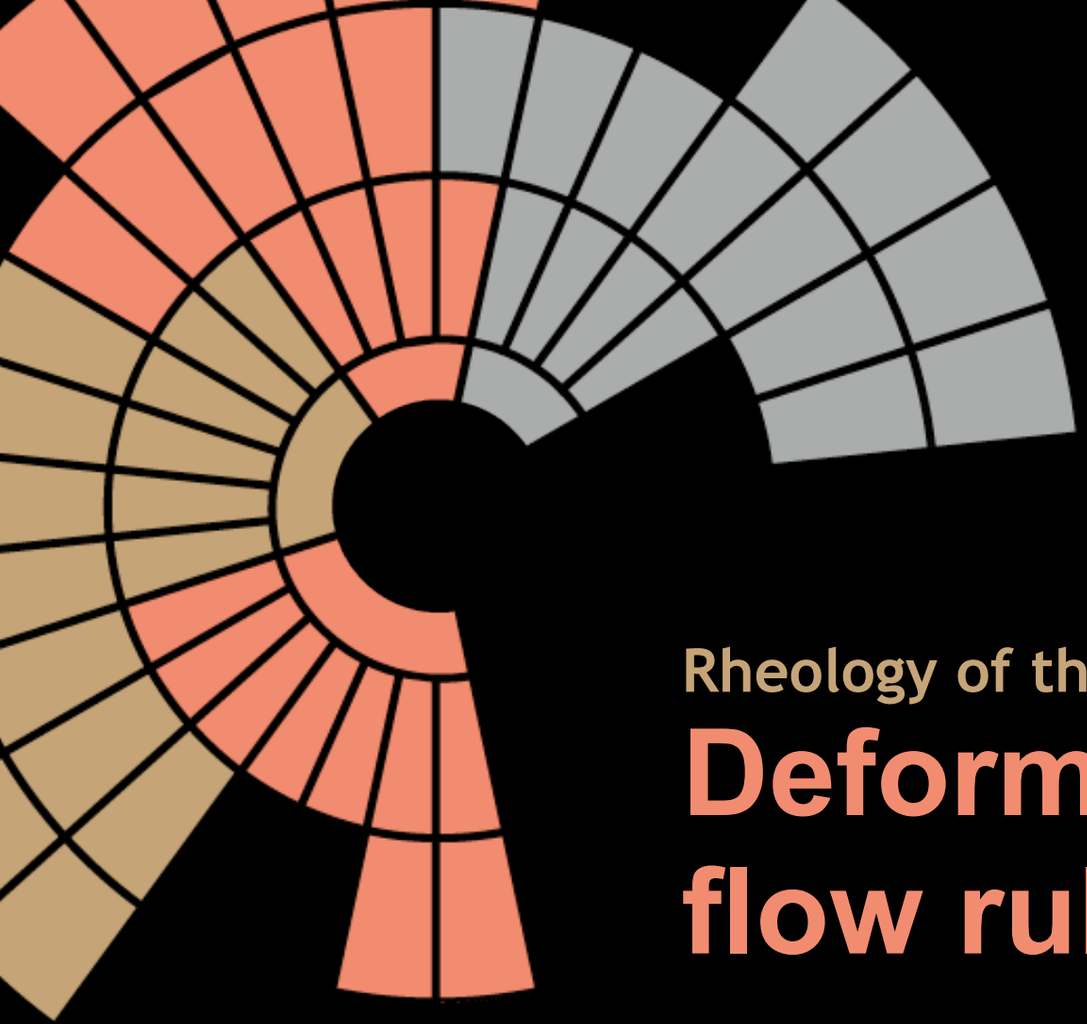


Rheology of the Mantle and Plates (part 1):

Deformation mechanisms and flow rules of mantle minerals

What is rheology?

- Rheology is the physical property that characterizes deformation behavior of a material (solid, fluid, etc)

- Rheology of Earth materials includes elasticity, viscosity, plasticity, etc

$$\sigma = E\varepsilon$$ solid mechanics

$$\sigma = 2\eta\dot{\varepsilon}$$ fluid mechanics

- For the deep Earth: mantle is fluid on geological timescales so we focus on its viscosity

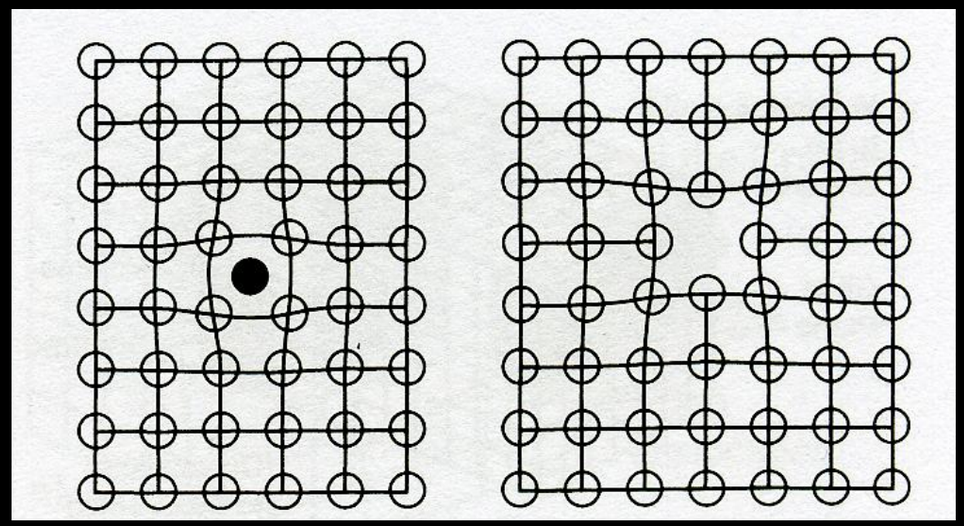
- For tectonic plates: still viscous on geological timescales, but the effective viscosity is a subject of debate

What is viscosity?

- constitutive relation between stress and strain-rate (deformation rate)
- in the continuum description, it is the analog of the elastic moduli which relate stress and strain
- measure of a fluid's ability to flow
- diffusivity of momentum

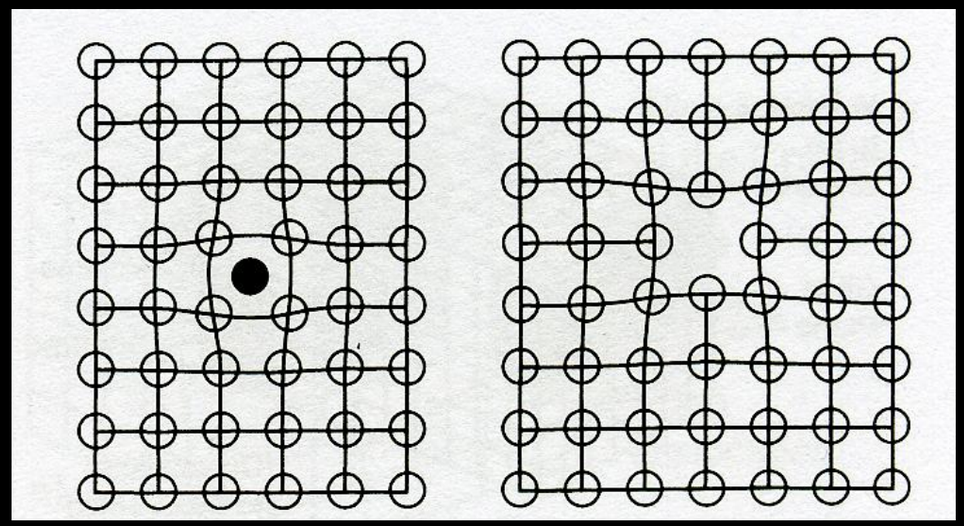
Definition of creep

- movement of crystal defects
  - point defects - extra atoms or vacancies



Definition of creep

- movement of crystal defects
  - point defects - extra atoms or vacancies
  - line defects - dislocations which represent a rearrangement of atomic bonds



Definition of creep

- movement of crystal defects
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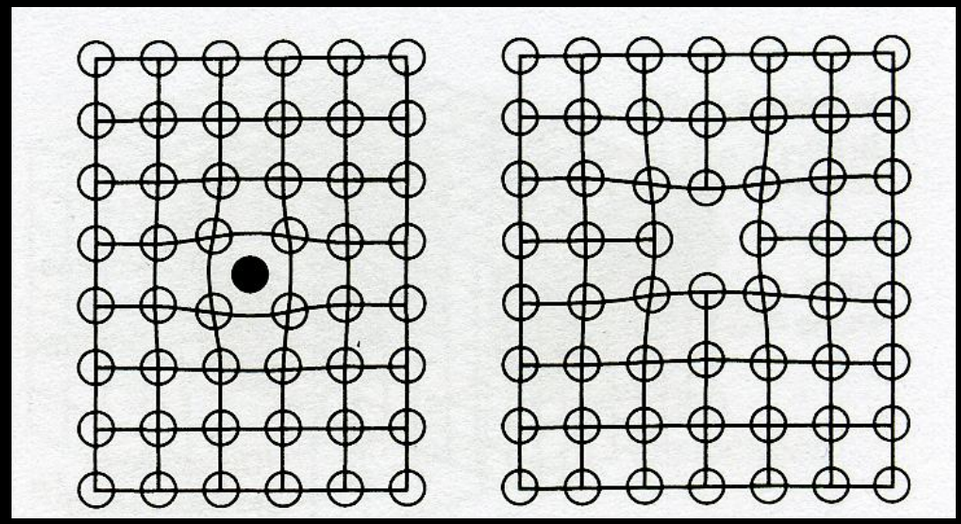
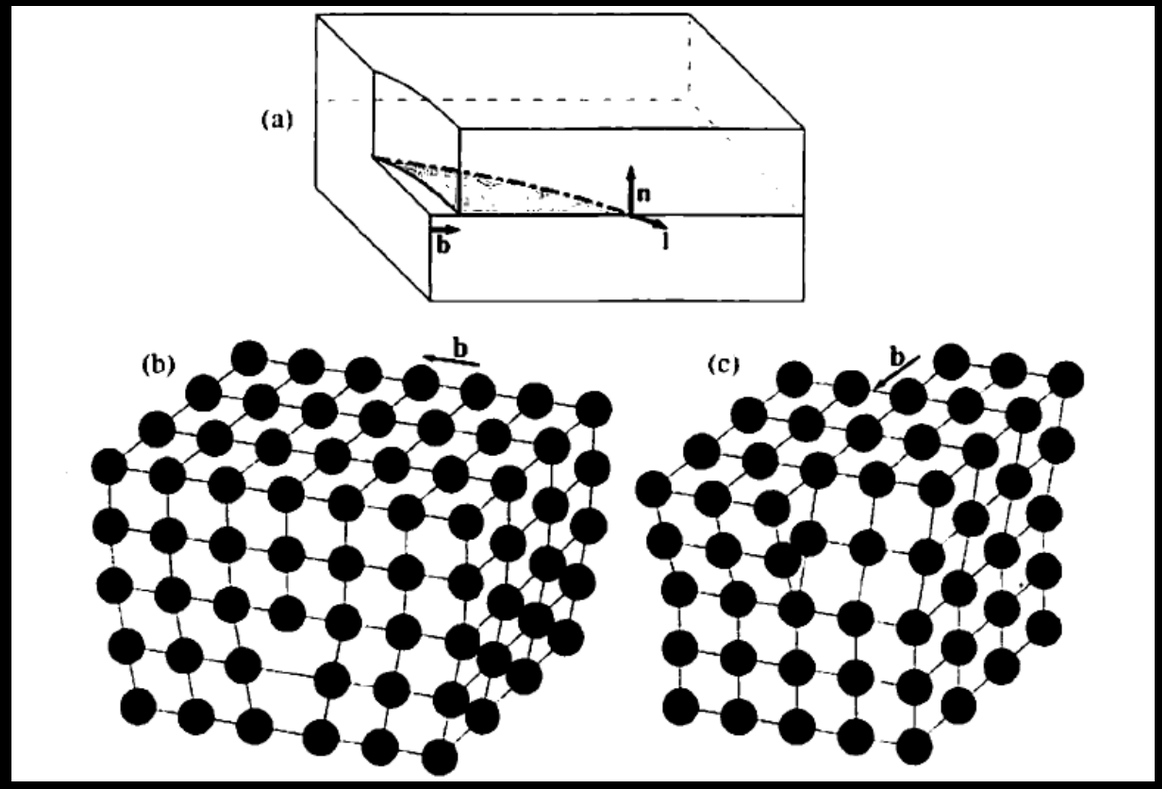
  - line defects - dislocations which represent a rearrangement of atomic bonds
    - two types of dislocations: “edge” and “screw”
    - each described by parallel or normal Burgers vector (b*)





Definition of creep

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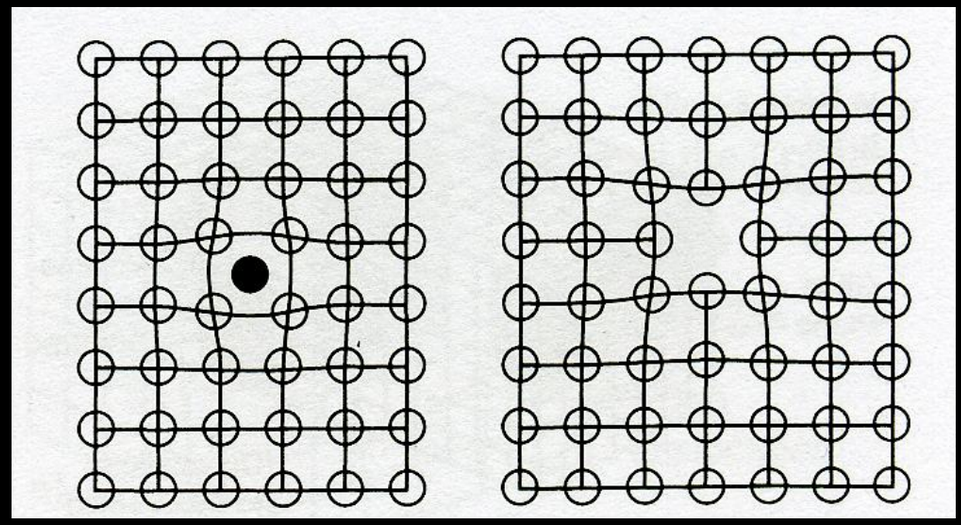
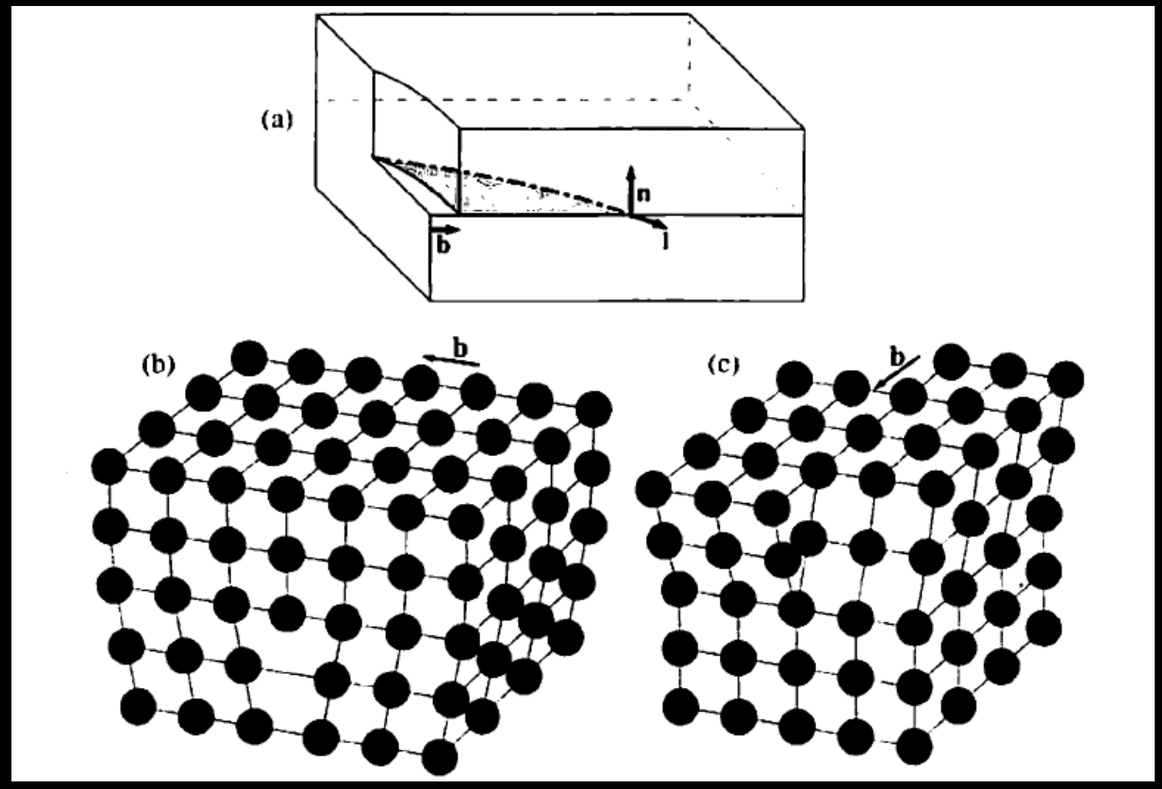
  - point defects - extra atoms or vacancies
  - line defects - dislocations which represent a rearrangement of atomic bonds
    - two types of dislocations: “edge” and “screw”
    - each described by parallel or normal Burgers vector (b*)
- creep will occur through whichever mechanism requires least amount of energy

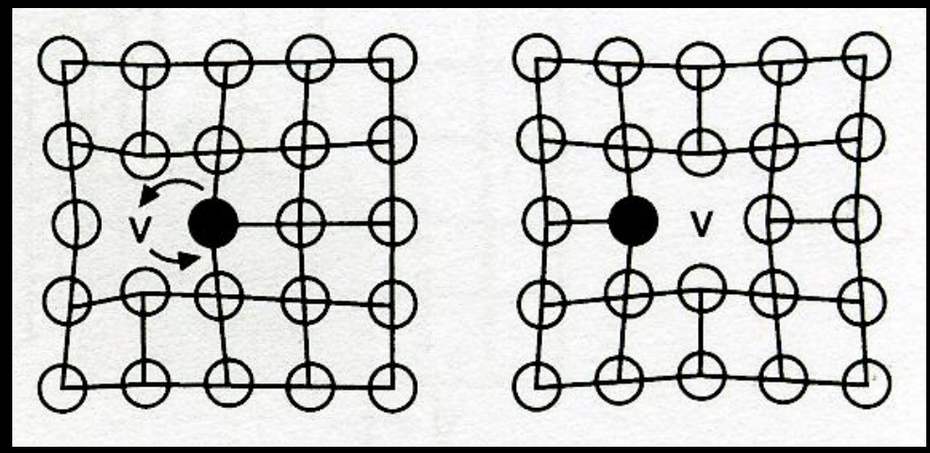




Diffusion creep

point defects move by *diffusion*

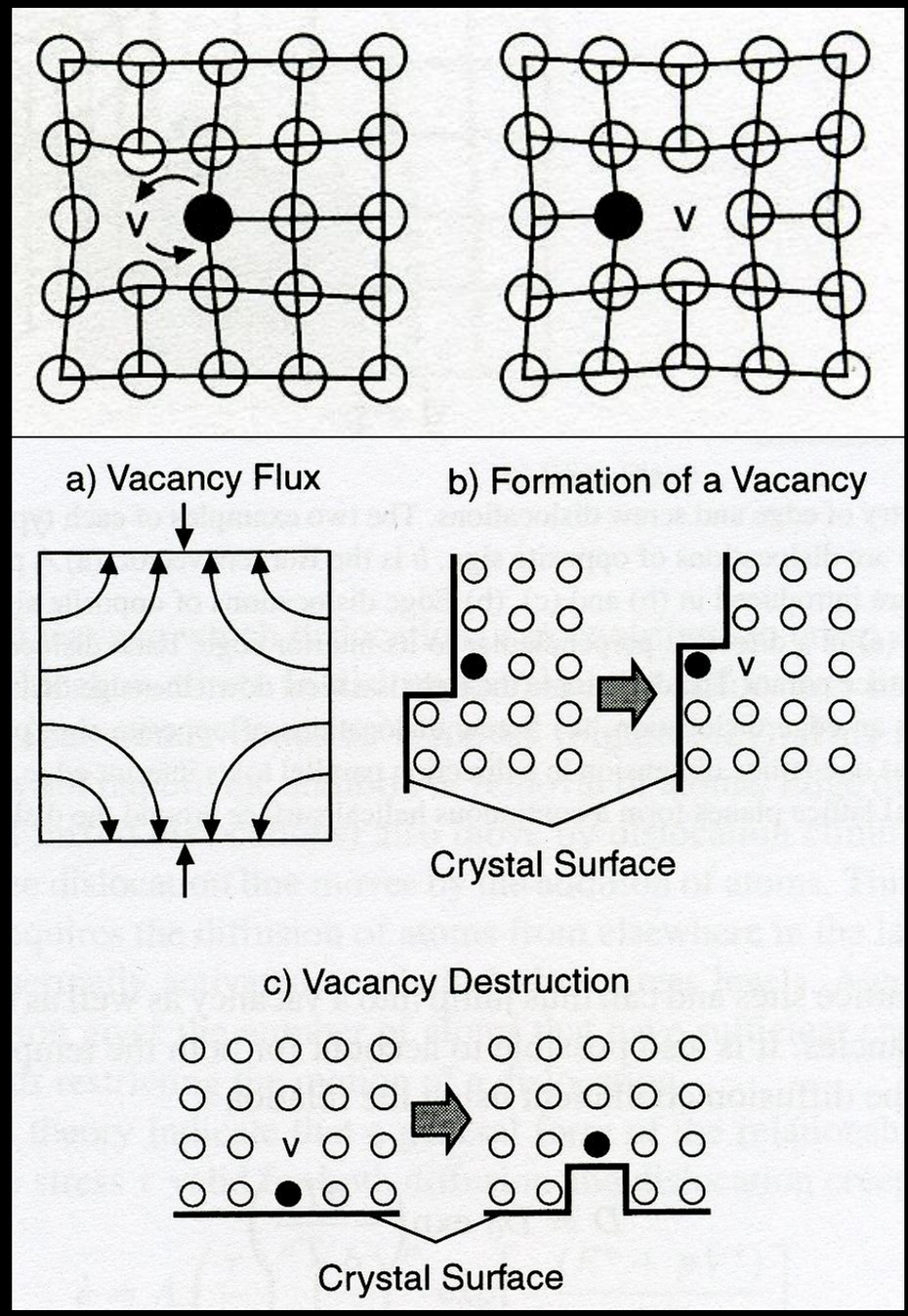
- **through the crystal matrix (Nabarro-Herring)**



Diffusion creep

point defects move by *diffusion*

- **through the crystal matrix (Nabarro-Herring)**
- **along the grain boundaries (Coble)**

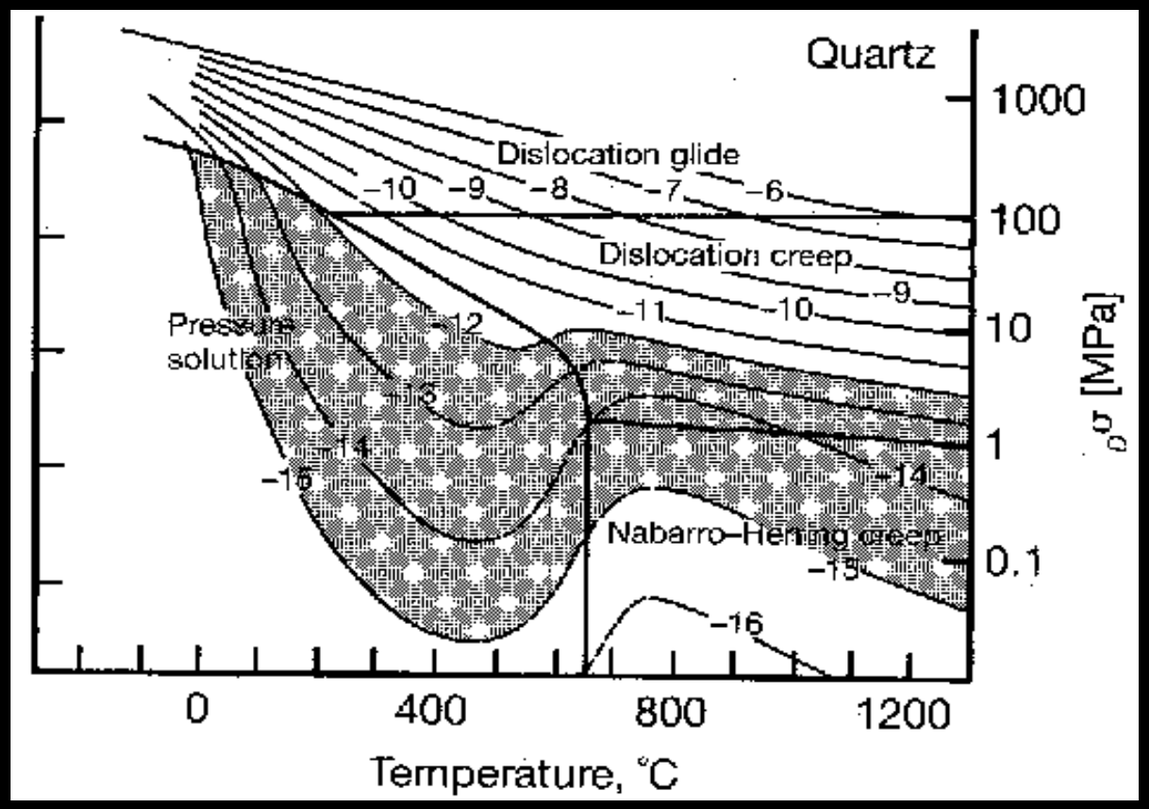


Deformation maps

- for any given stress + temperature, one mechanism will be weaker (and preferred) over all others

$$\dot{\varepsilon} = \dot{\varepsilon}_{\text{disl}} + \dot{\varepsilon}_{\text{diff}}$$

- map assumes constant grain size



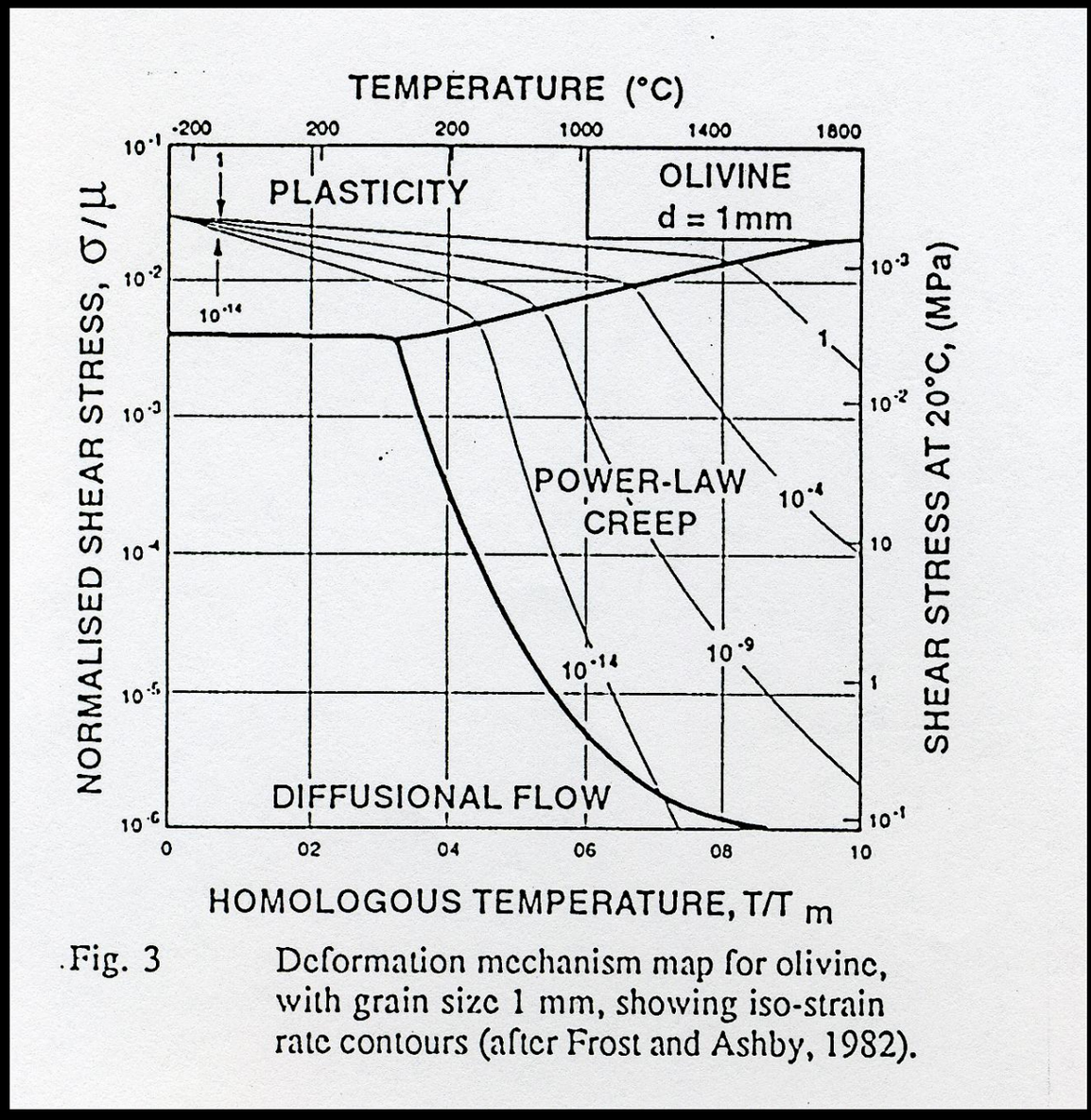
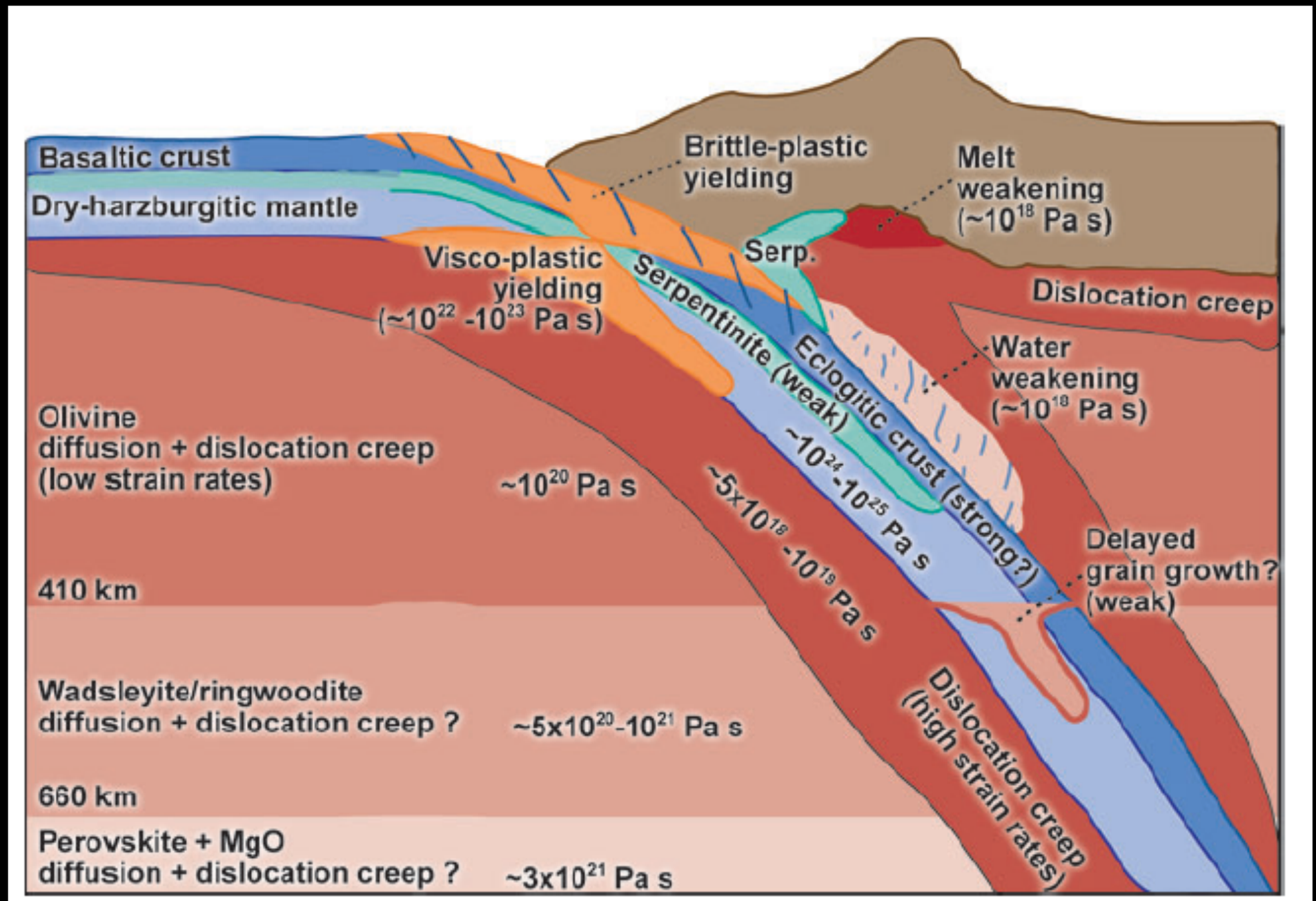


Fig. 3 Deformation mechanism map for olivine, with grain size 1 mm, showing iso-strain rate contours (after Frost and Ashby, 1982).

Creep mechanisms in the mantle



Billen, Annual Rev. Geophys., 2008

Flow rule

- relates the deformation (strain rate) to the applied deviatoric stress through a viscosity
- deformations add in series (viscosities add in parallel)

$$\sigma = \eta \dot{\varepsilon}$$

$$\dot{\varepsilon} = \frac{1}{\eta} \sigma$$

$$\dot{\varepsilon}_{tot} = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \dots$$

$$\frac{1}{\eta_{eff}} = \frac{1}{\eta_1} + \frac{1}{\eta_2} + \dots$$

Flow rule

- for isotropic fluids, we can describe the flow rules with an effective strain rate, an effective stress, and an effective viscosity

- 2nd invariants of deviatoric stress and strain rates are scalars

- in practice, we know the strain-rates from the velocity field rather than stress, so viscosity is normally rewritten in terms of strain-rate

$$\sigma_{II} = \sqrt{\sigma : \sigma}$$

$$\dot{\varepsilon}_{II} = \sqrt{\dot{\varepsilon} : \dot{\varepsilon}}$$

$$\sigma_{II} = \eta_{eff} \dot{\varepsilon}_{II}$$

NOTE: contractions of these tensors usually have a 1/2 term times the sum of the squares of the components (when assuming co-axial compression / pure shear)

Arrhenius dependence

- one can use thermodynamics to describe the sensitivity of diffusion when it is thermally activated

$$D = D_0 exp\left[-\frac{(E_A + P\,V_A)}{RT}\right]$$

- the diffusion of vacancies, etc has an exponential dependence on T,P

- this can also be understood in terms of the homologous temperature

$$D = D_0 exp\left[-g\frac{T_m\,(P)}{T}\right]$$

- the deformation resulting from the diffusion creep is the strain rate and is inversely proportional to viscosity

$$\dot{\varepsilon} = \frac{1}{\eta}\sigma$$

- the Arrhenius term describes the exponential behavior of viscosity

- also valid for dislocation creep

$$\eta(T,P) = \eta_0 exp\left[\frac{(E_A + P\,V_A)}{RT}\right]$$

General flow rule for mantle material

$$\dot{\varepsilon} = A\left(\frac{b^*}{d}\right)^m \left(\frac{\sigma}{\mu}\right)^n exp\left[-\frac{(E_A + P\,V_A)}{R\,T}\right]$$

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$$\dot{\varepsilon} = A\left(\frac{b^*}{d}\right)^m \left(\frac{\sigma}{\mu}\right)\left(\frac{\sigma}{\mu}\right)^{n-1} exp\left[-\frac{(E_A + P\,V_A)}{R\,T}\right]$$

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$$\frac{1}{\eta} = \frac{A}{\mu}\left(\frac{b^*}{d}\right)^m \left(\frac{\sigma}{\mu}\right)^{n-1} exp\left[-\frac{(E_A + P\,V_A)}{R\,T}\right]$$

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$$\eta = \frac{\mu}{A}\left(\frac{d}{b^*}\right)^m \left(\frac{\mu}{\sigma}\right)^{n-1} exp\left[\frac{(E_A + P\,V_A)}{R\,T}\right]$$

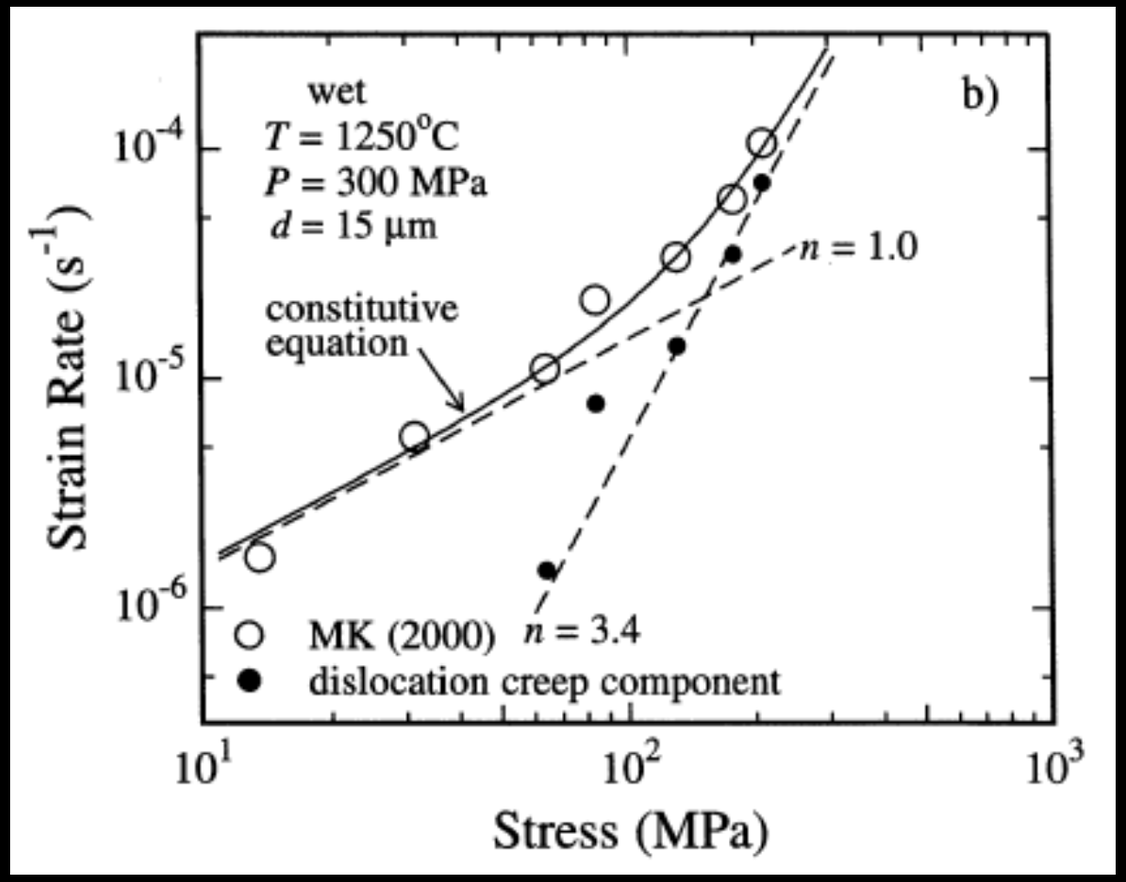
$$\dot{\varepsilon}_{tot} = \dot{\varepsilon}_{diff} + \dot{\varepsilon}_{disl}$$

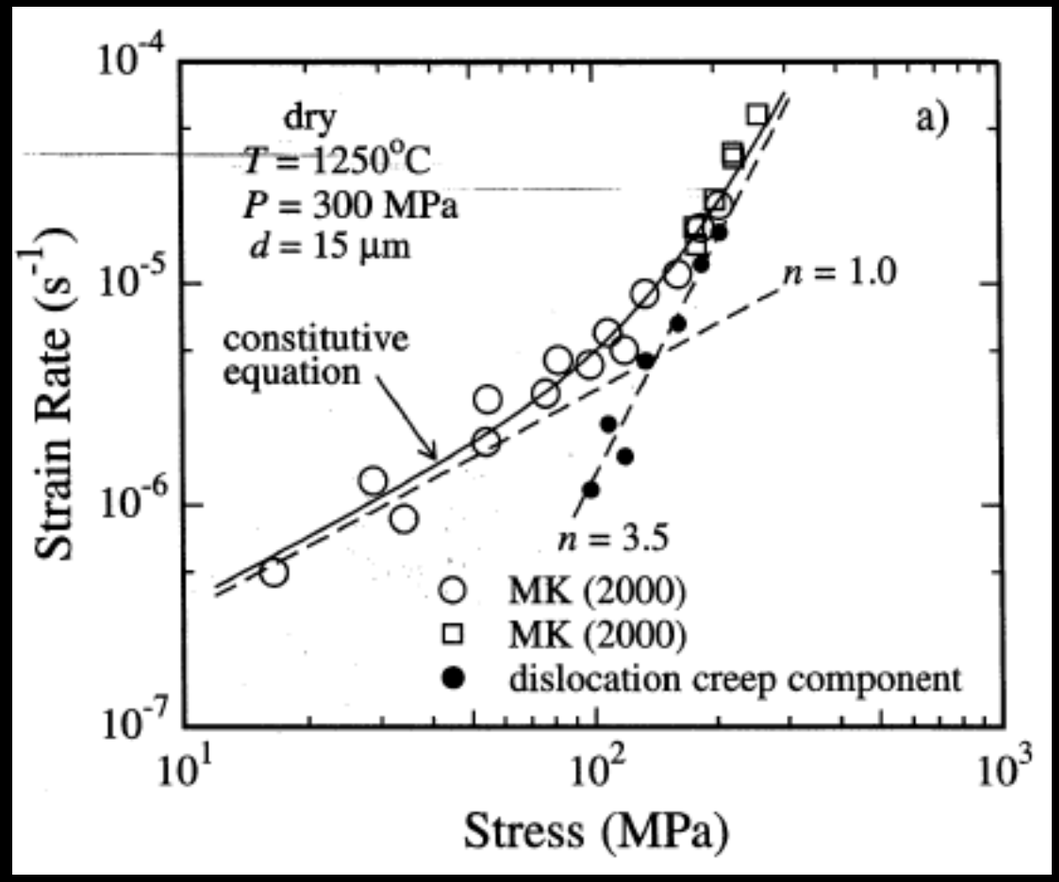
$$\frac{1}{\eta_{eff}} = \frac{1}{\eta_{diff}} + \frac{1}{\eta_{disl}}$$

- note: diffusion and dislocation creep have different activation enthalpies
- usual values for (m,n) are (2.5,1) for diffusion, (0,3.5) for dislocation

What is “n” for dislocation creep?

$$\dot{\varepsilon}_{tot} = \dot{\varepsilon}_{diff} + \dot{\varepsilon}_{disl}$$

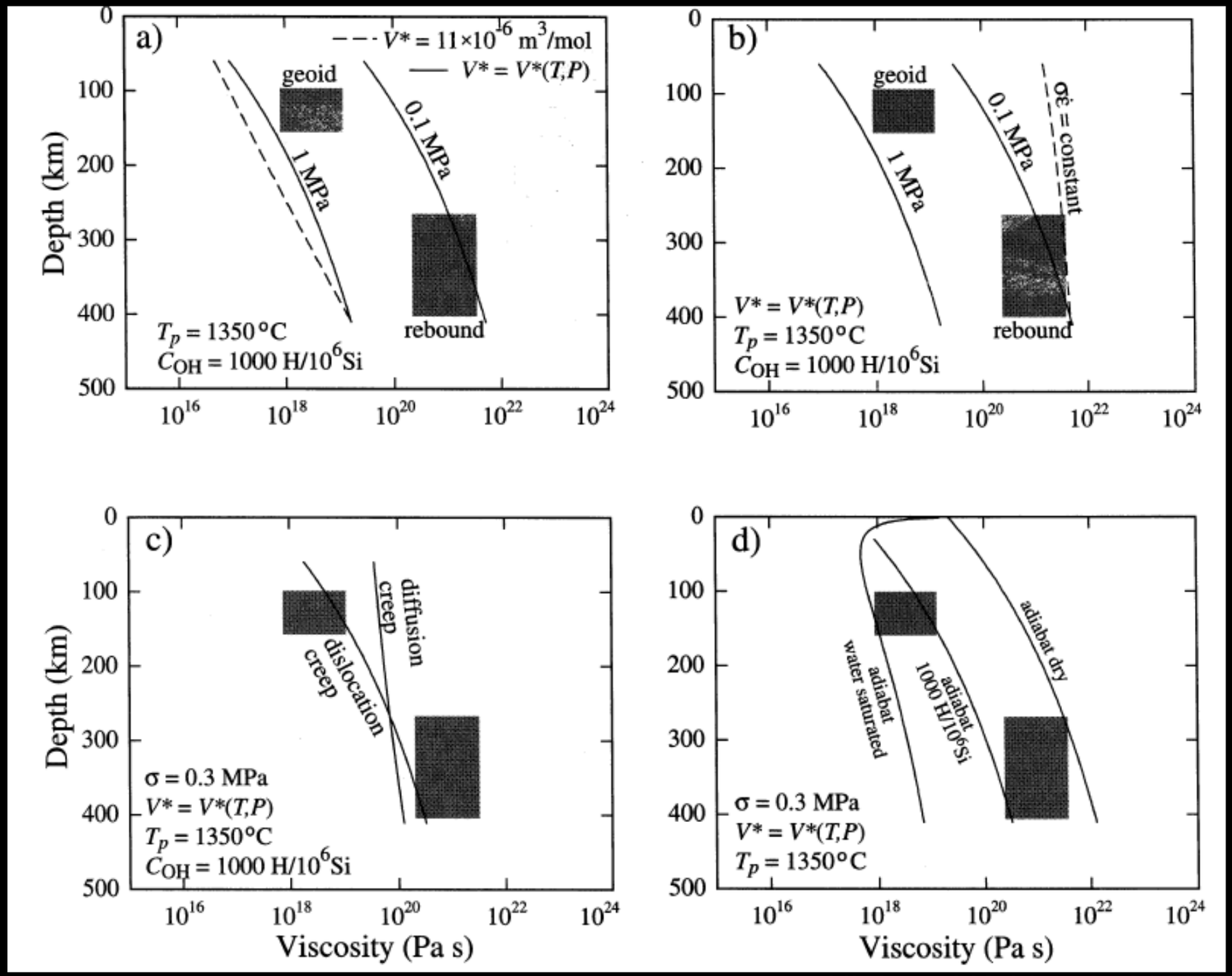




Hirth and Kohlstedt, 2003

- **note: different y-intercept corresponds to different activation enthalpies which is due to the influence of water**

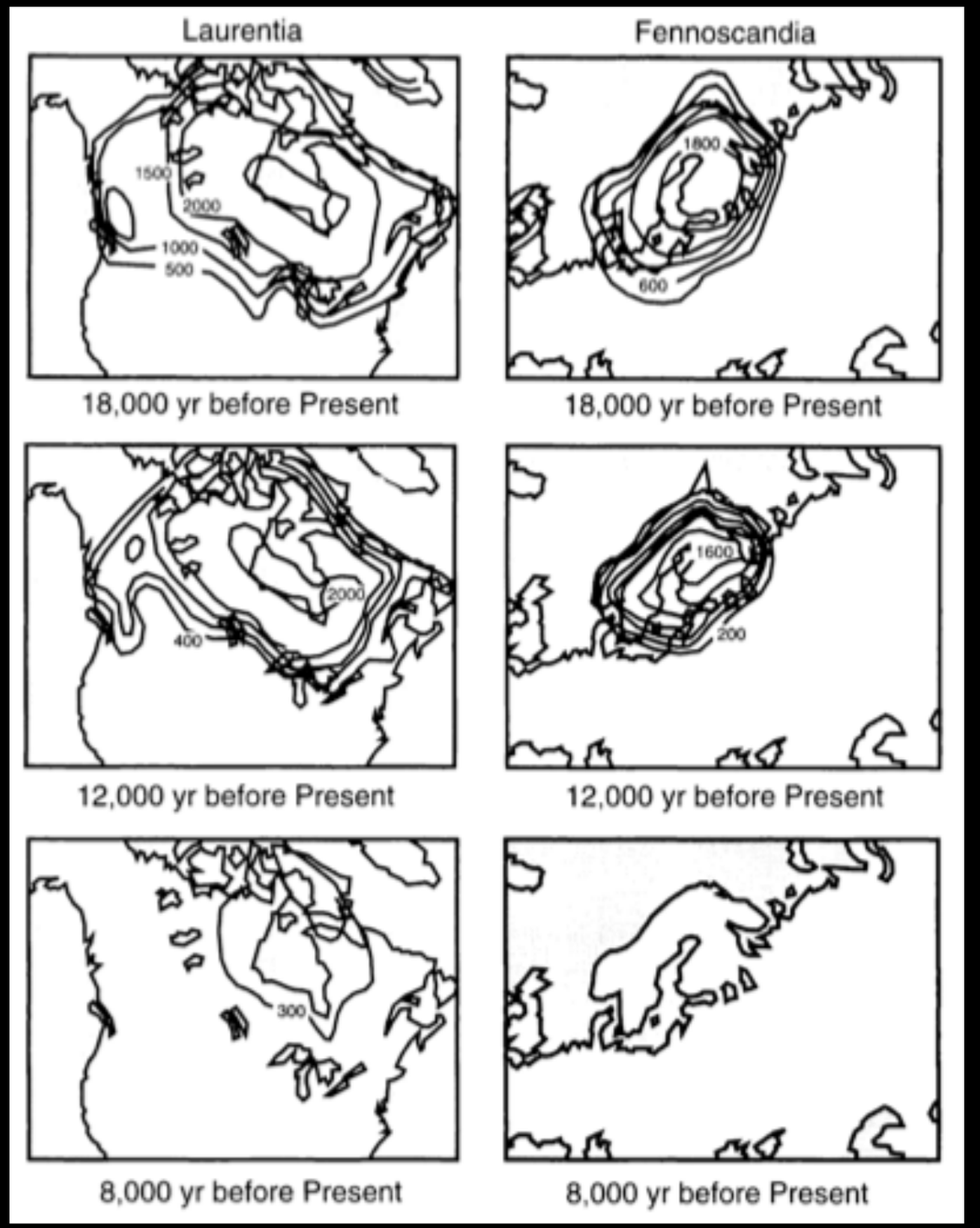
Extrapolating to mantle conditions



Hirth and Kohlstedt, 2003

What is the viscosity of the mantle?

- **early work by Haskell (1935) used post-glacial rebound of Fennoscandian uplift**



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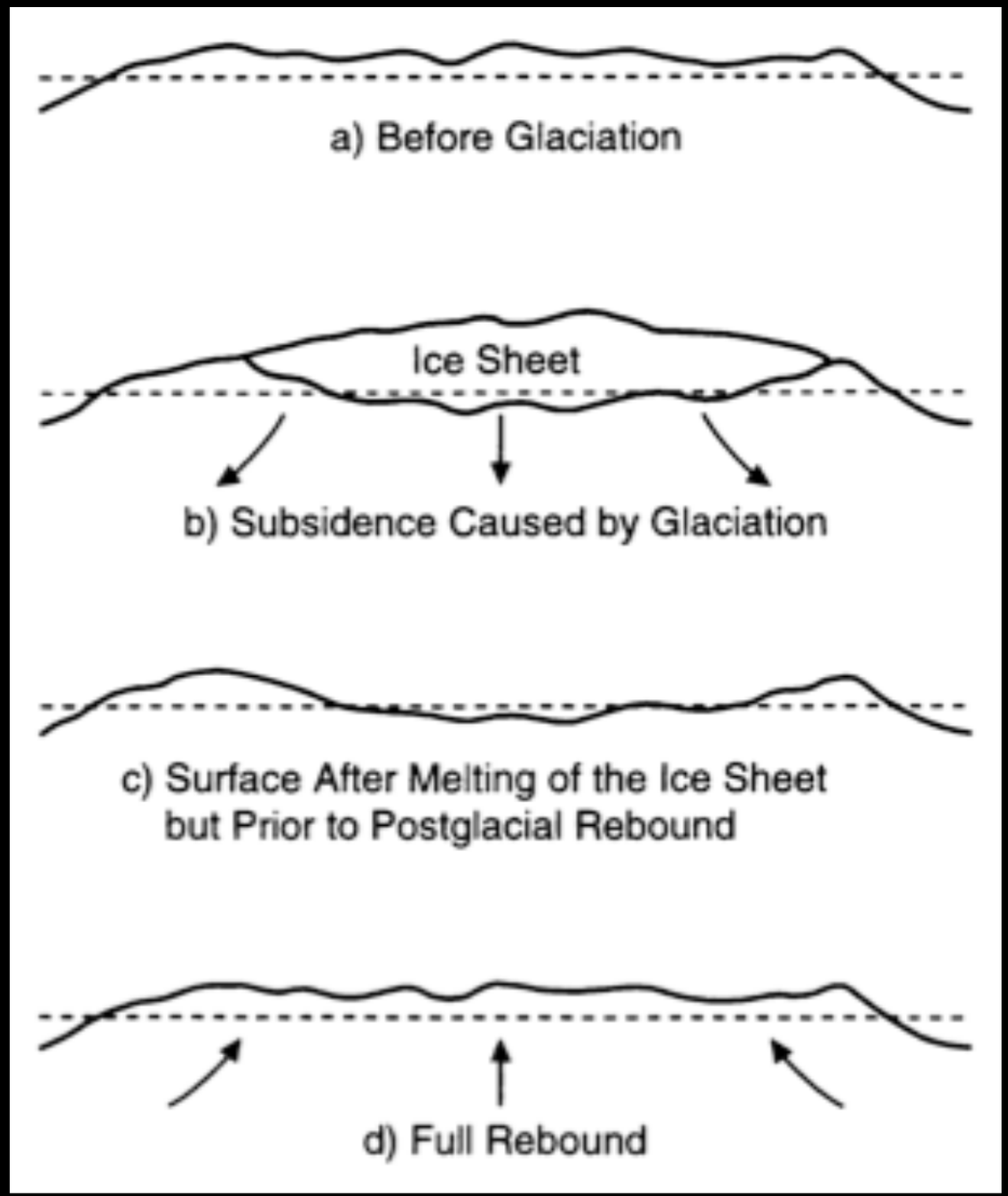

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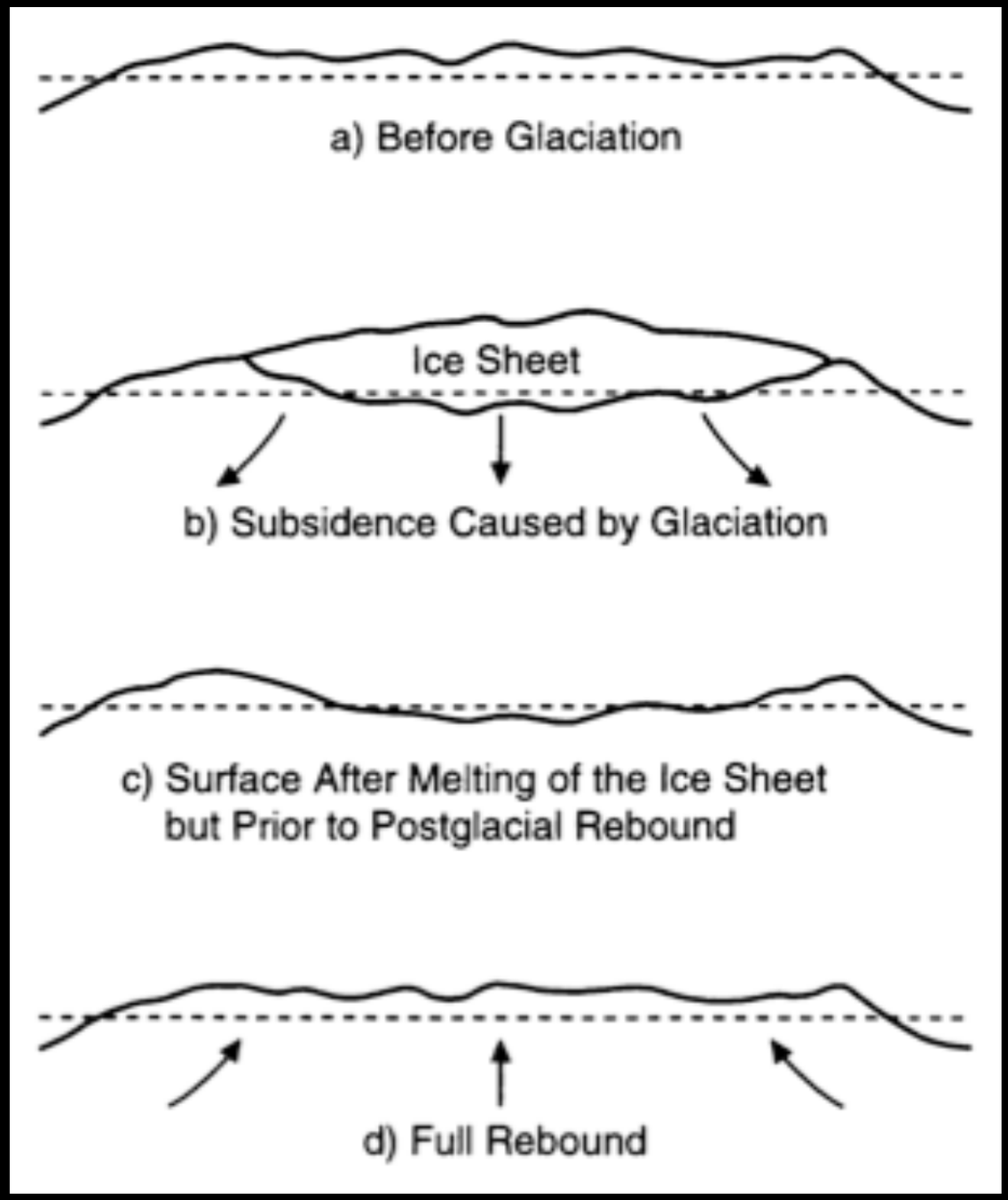

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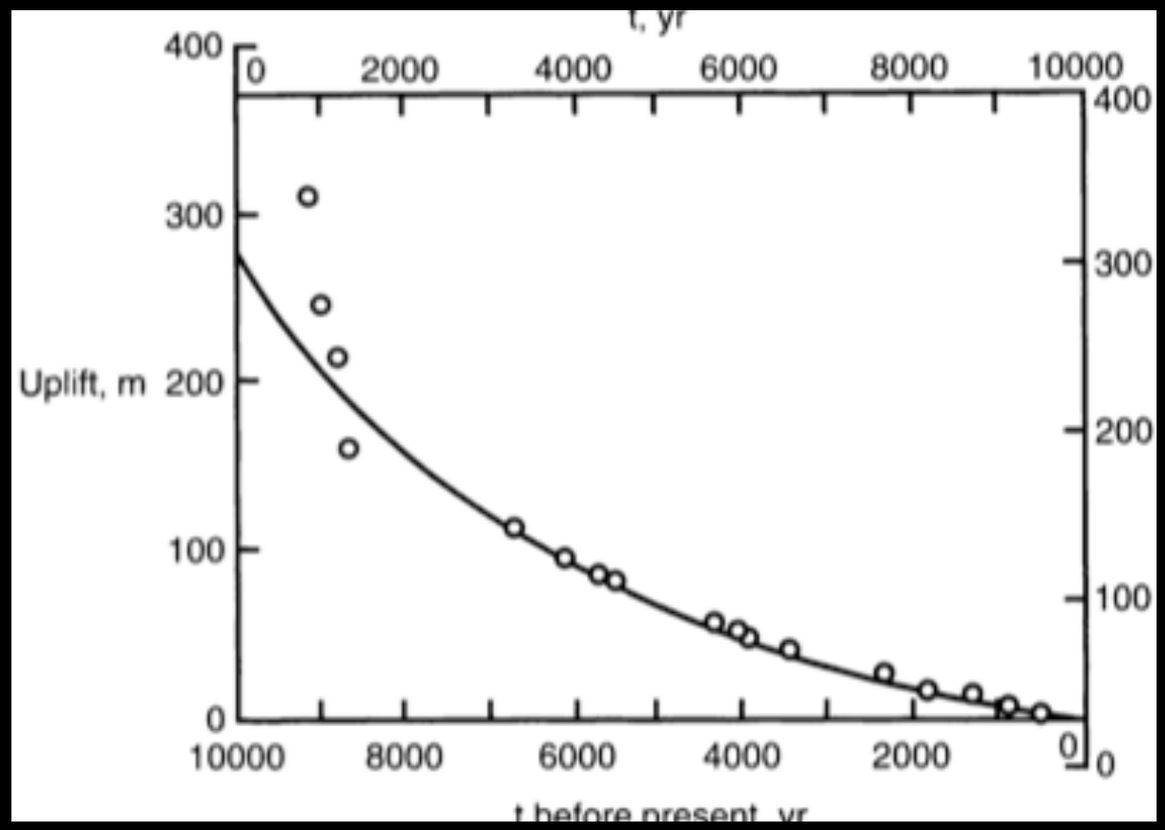
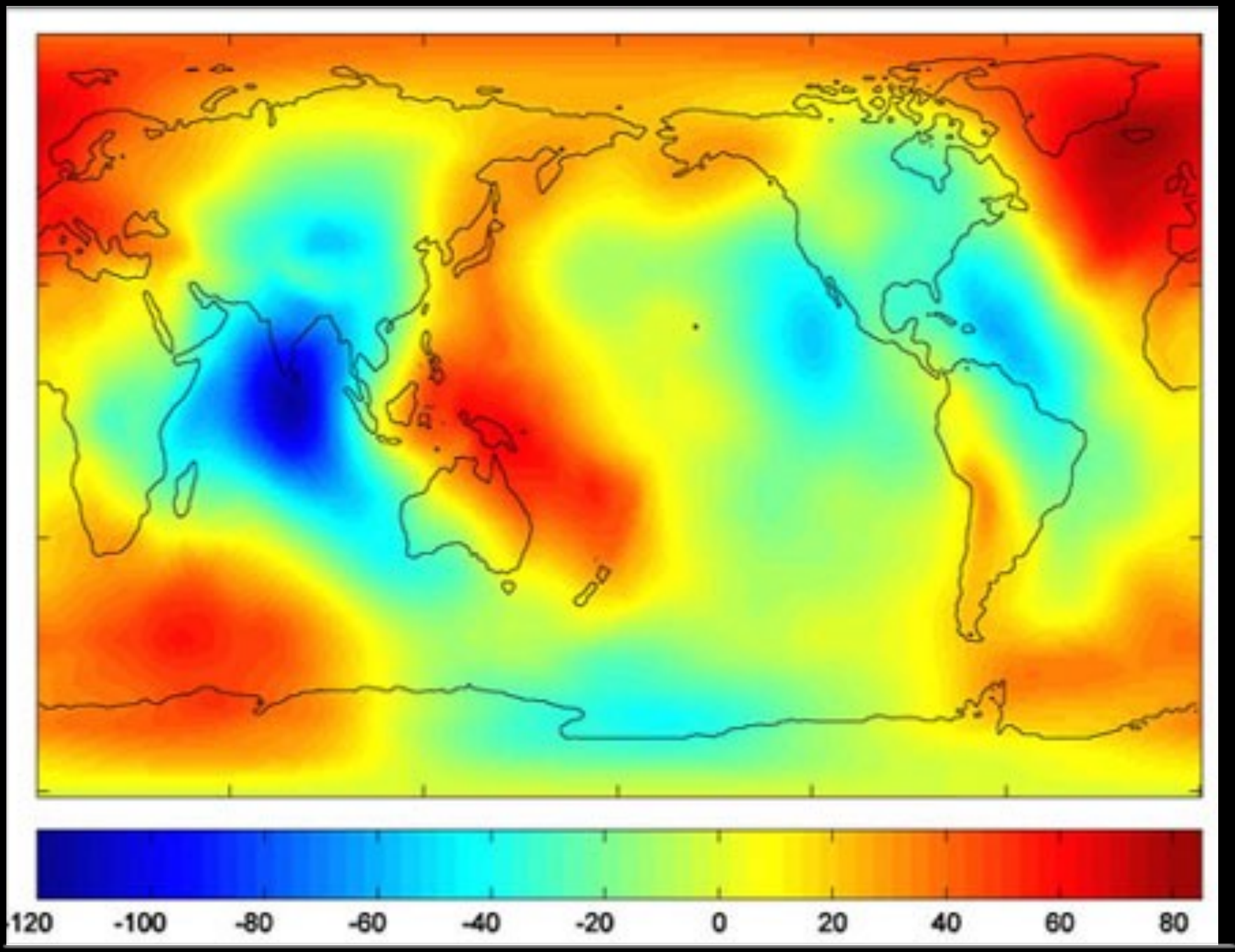
- response of a viscous half-space to removal of a surface load
- Haskell constraint gives an absolute value but is depth averaged over range of sensitivity
- Haskell viscosity = 1e21 Pa-s
- later studies by Peltier, Ranalli, Lambeck

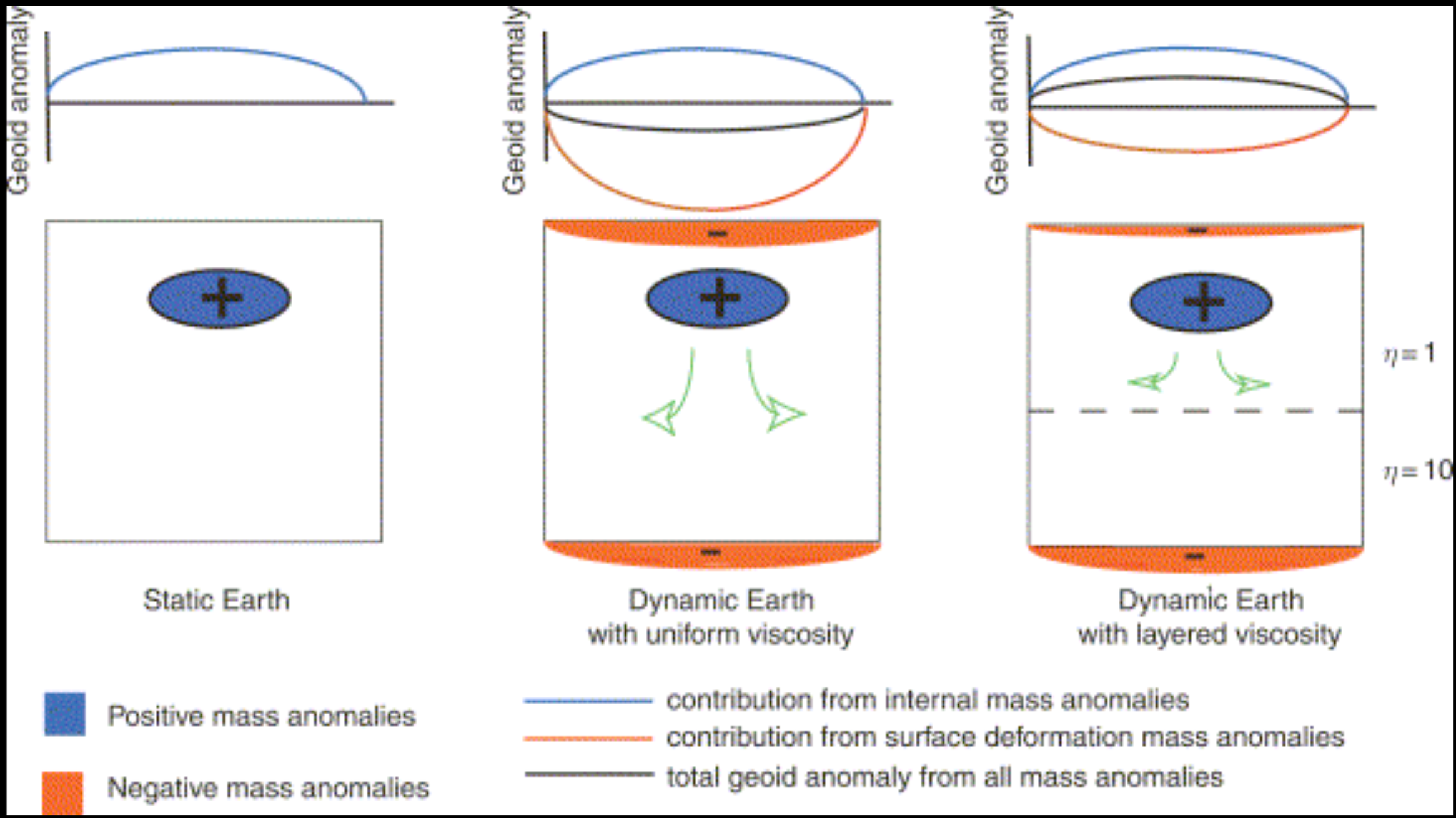


$$\tau_r = \frac{4\pi\eta}{\rho_m \lambda}$$

The geoid



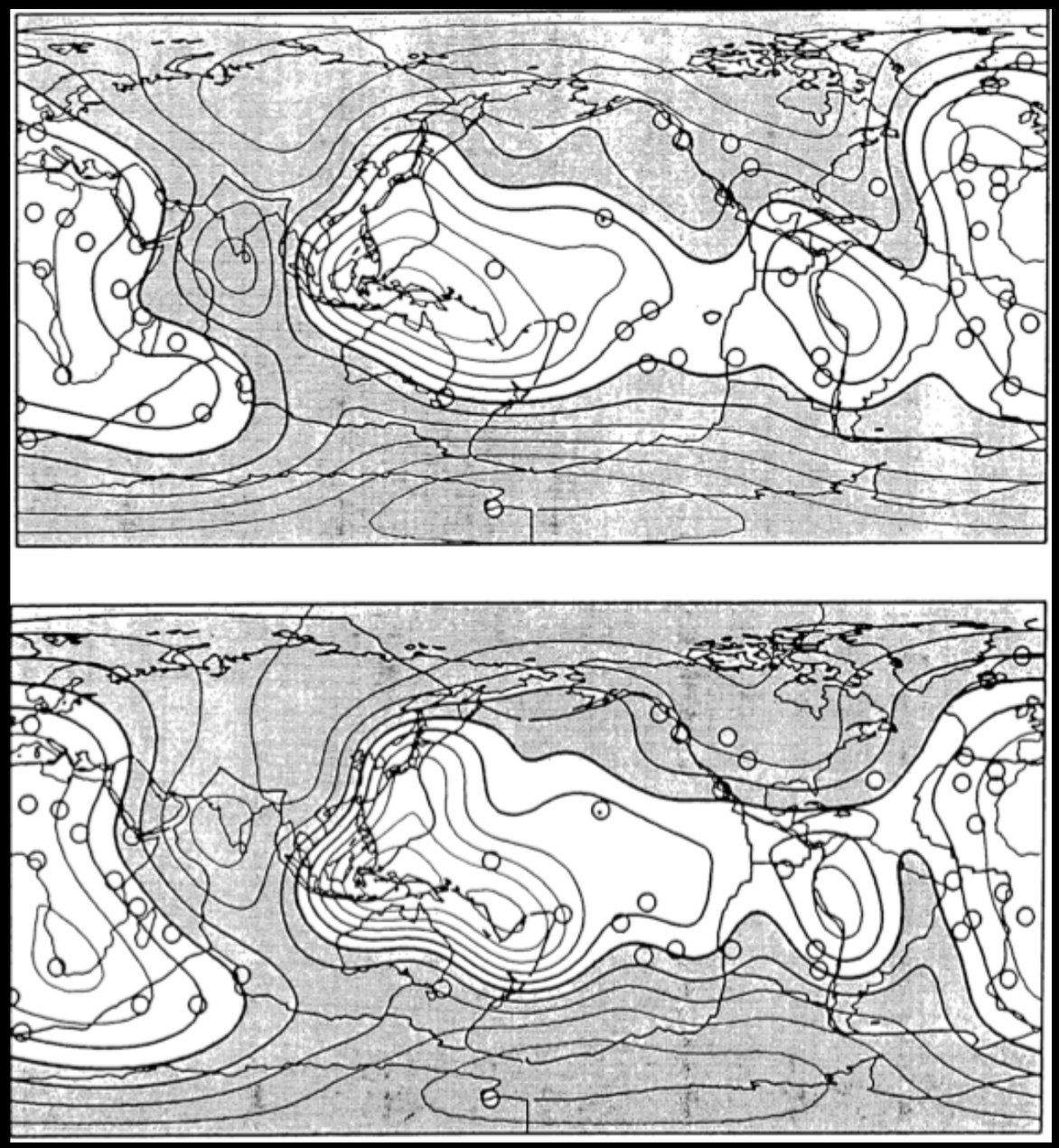
Earth's dynamic geoid



King, Treatise on Geophys, V.7 Ch 8, 2007

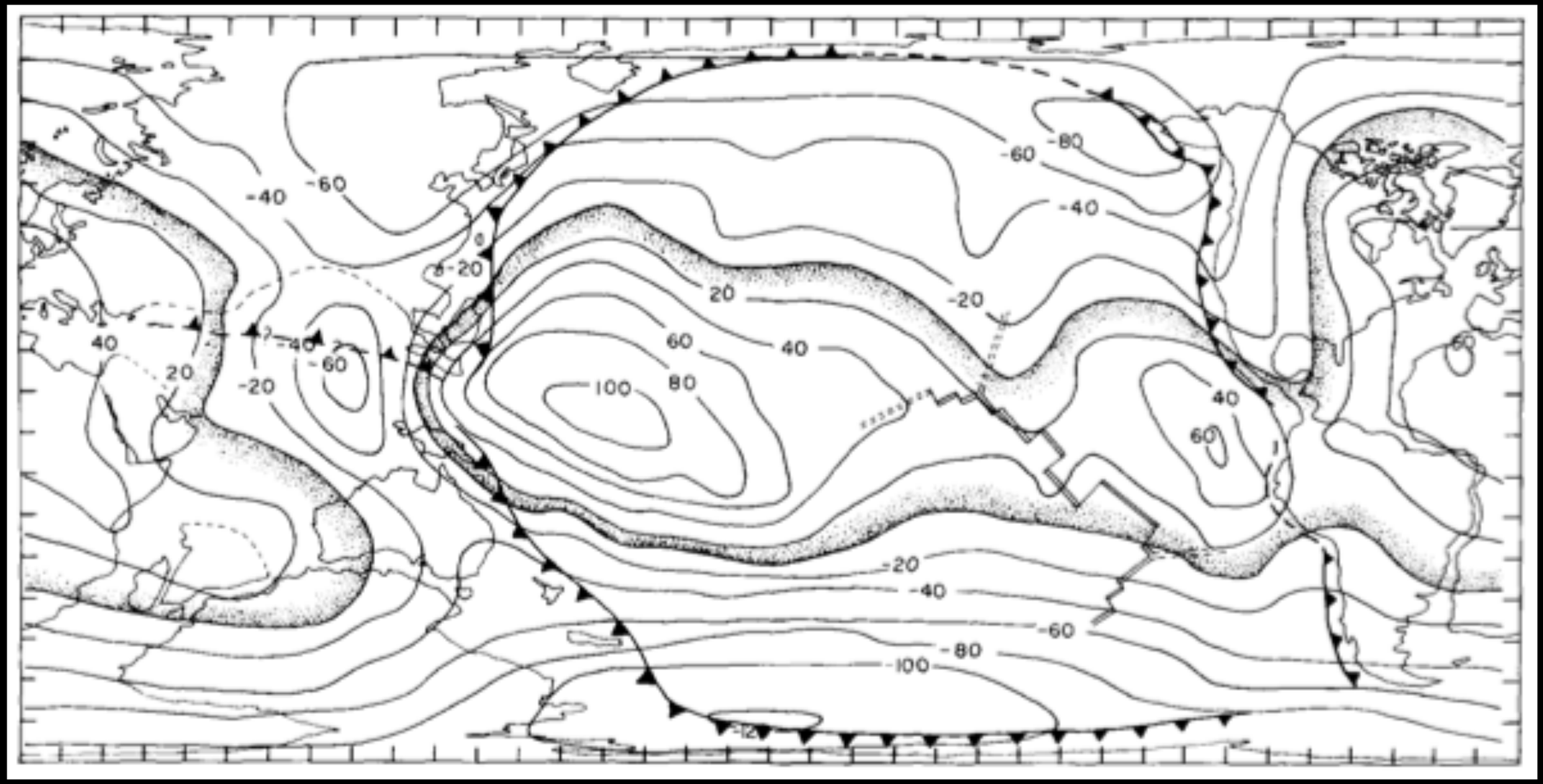
Earth's dynamic geoid: large scale

- flow driven by seismic heterogeneity
- 2 layer model; no absolute scale
- contributions of dynamic topography very important
- lower mantle 30x more viscous
- agrees well with history of subduction and location of past supercontinent
- dominated by long wavelengths of geoid (l=2-3)



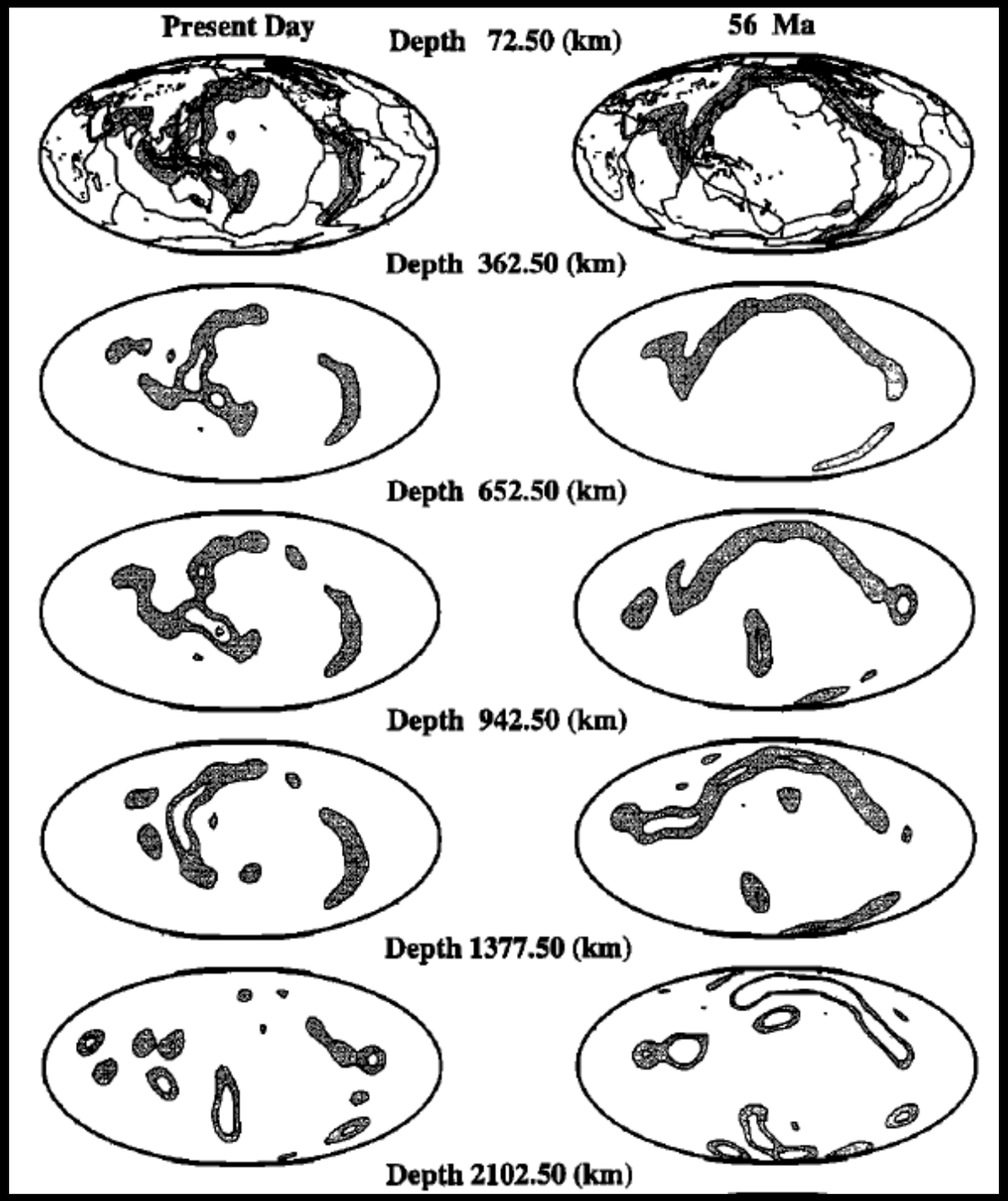
Hager and Richards, 1989

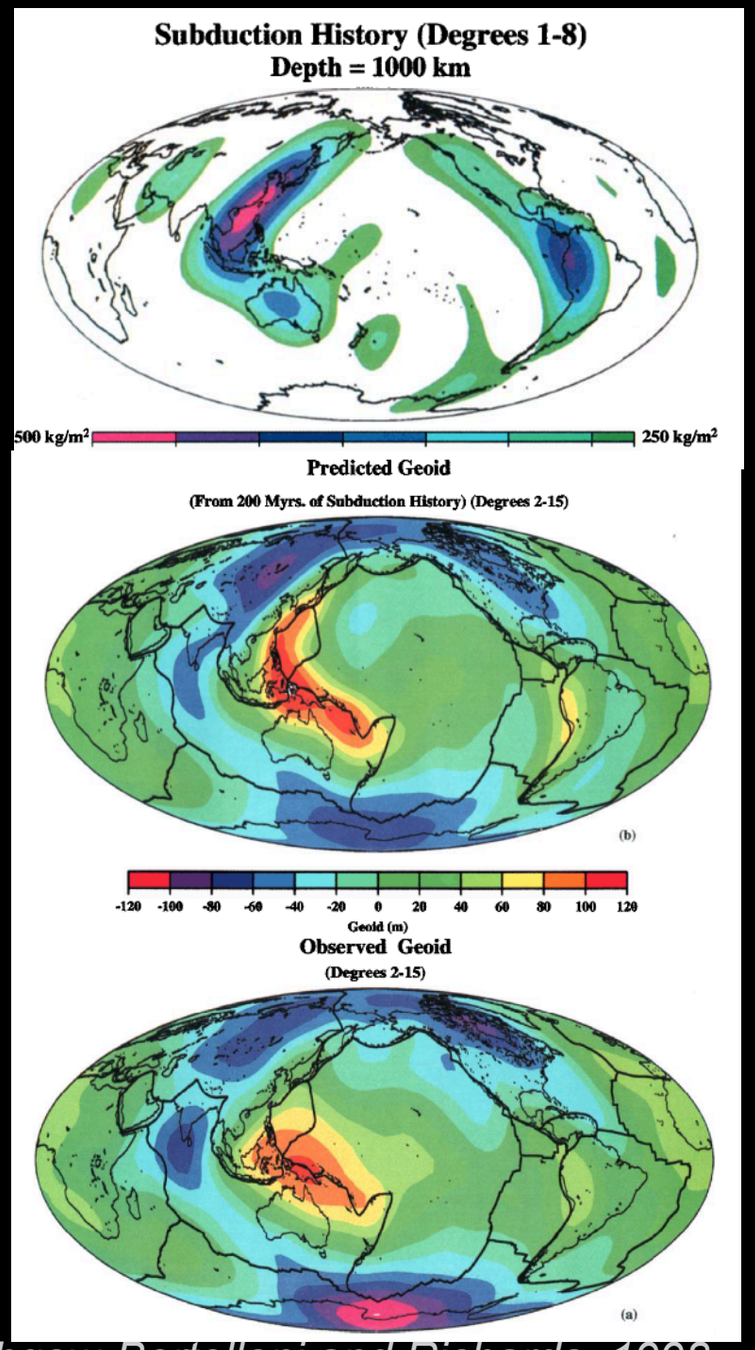
Large scale geoid and Pangaea



Chase and Sprowl, 1982

Dynamic geoid: history of subduction





Lithgow-Bertelloni and Richards, 1998

The geoid: over the Atlantic and Pacific

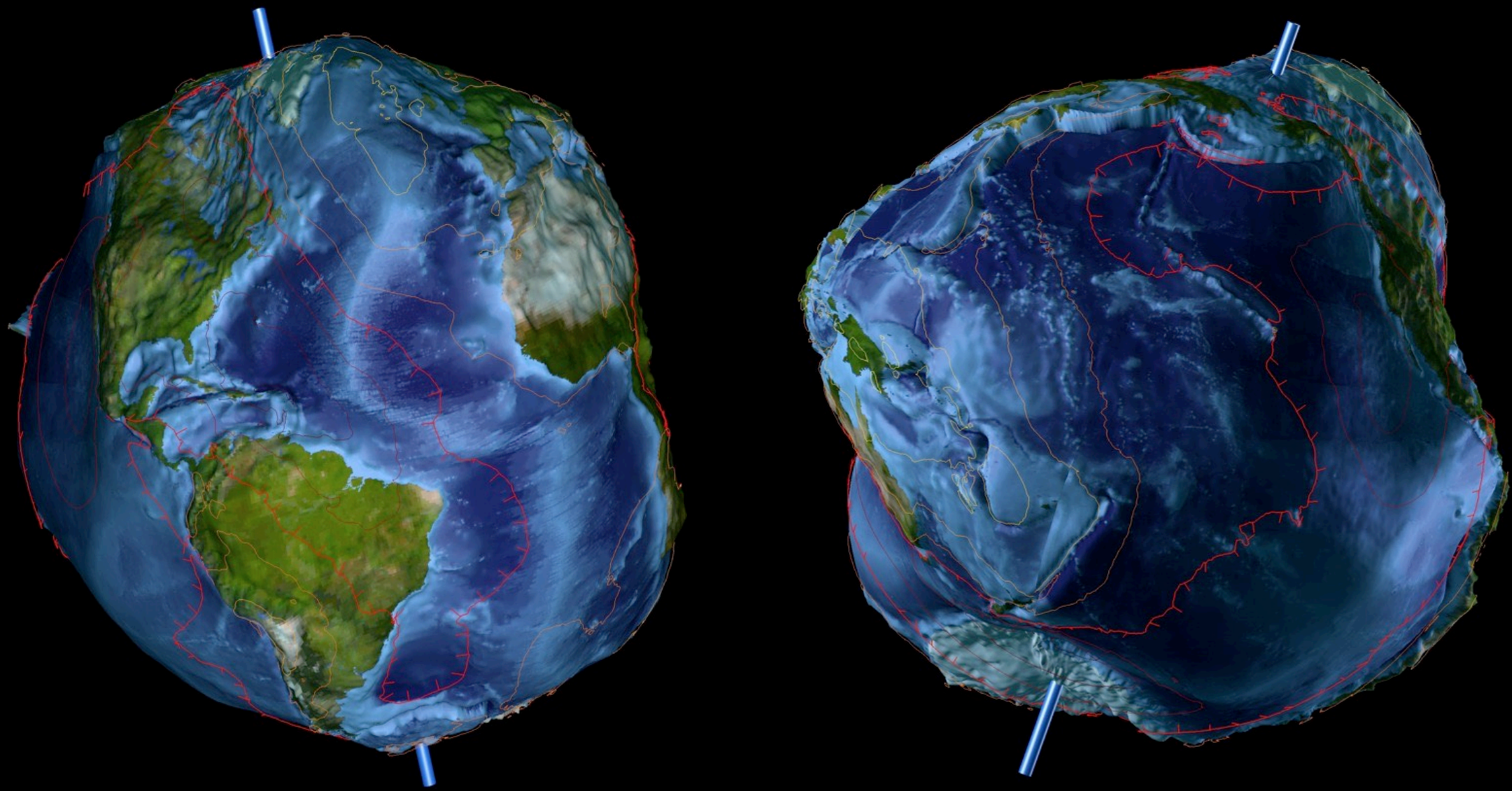
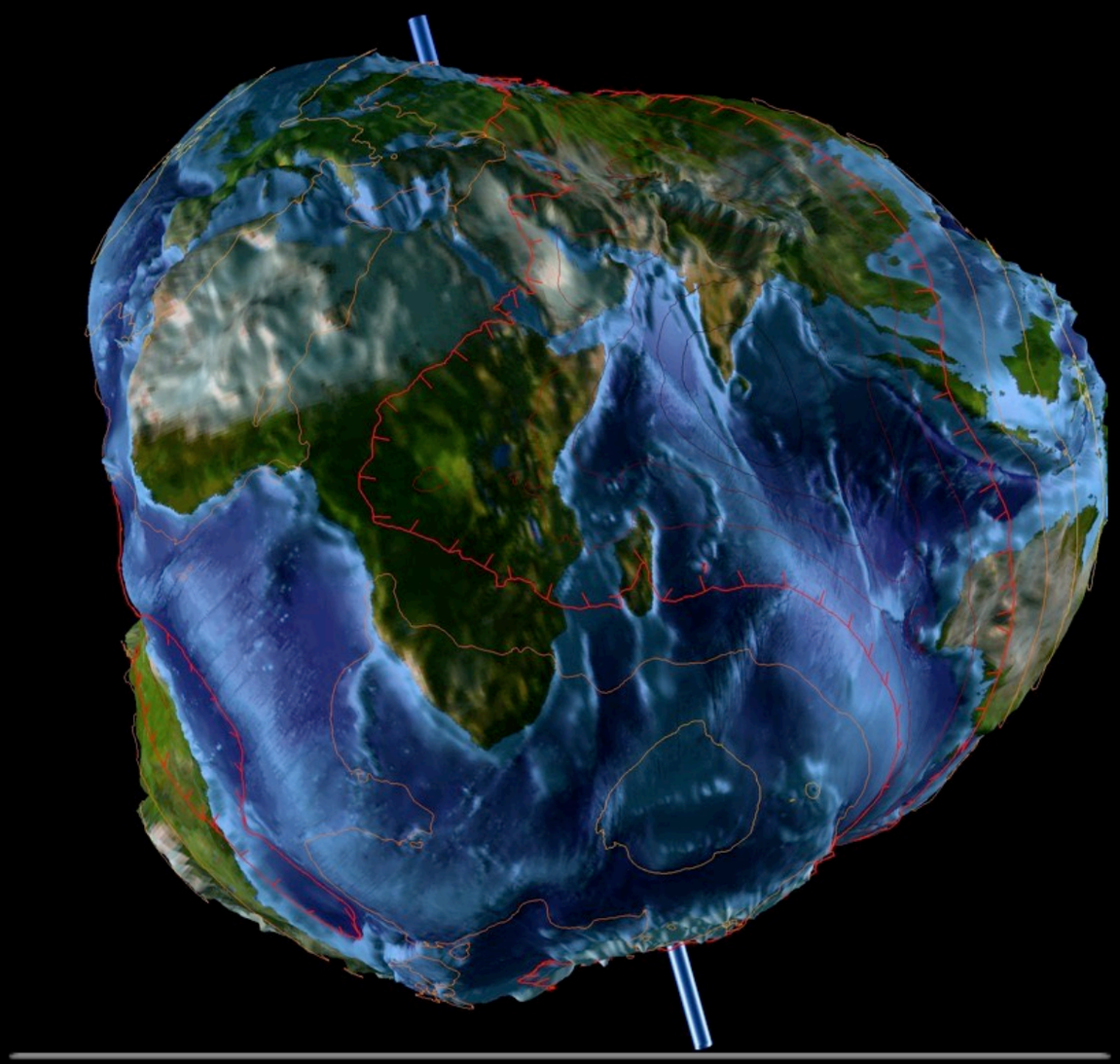
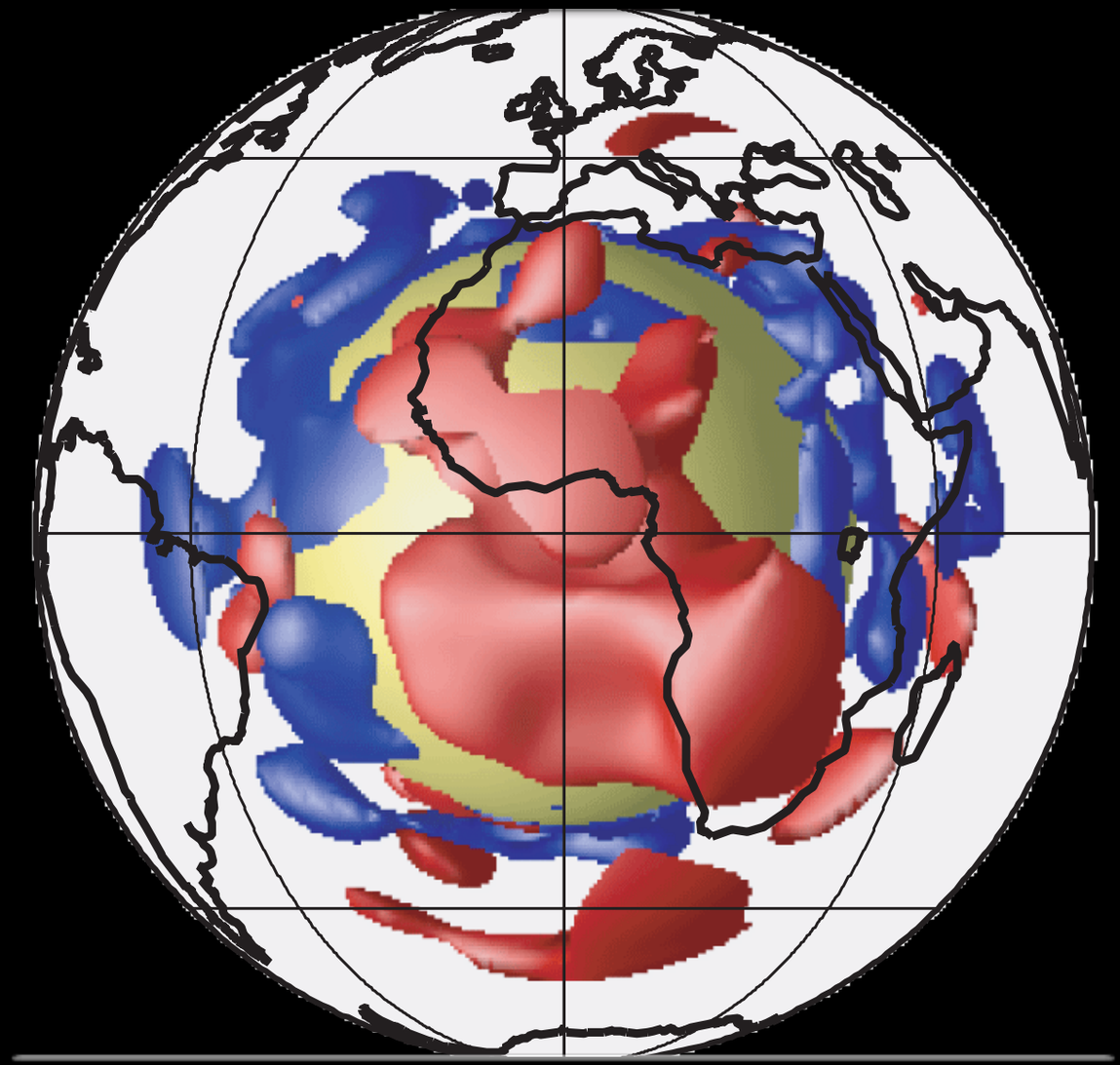


image by M. Sandiford

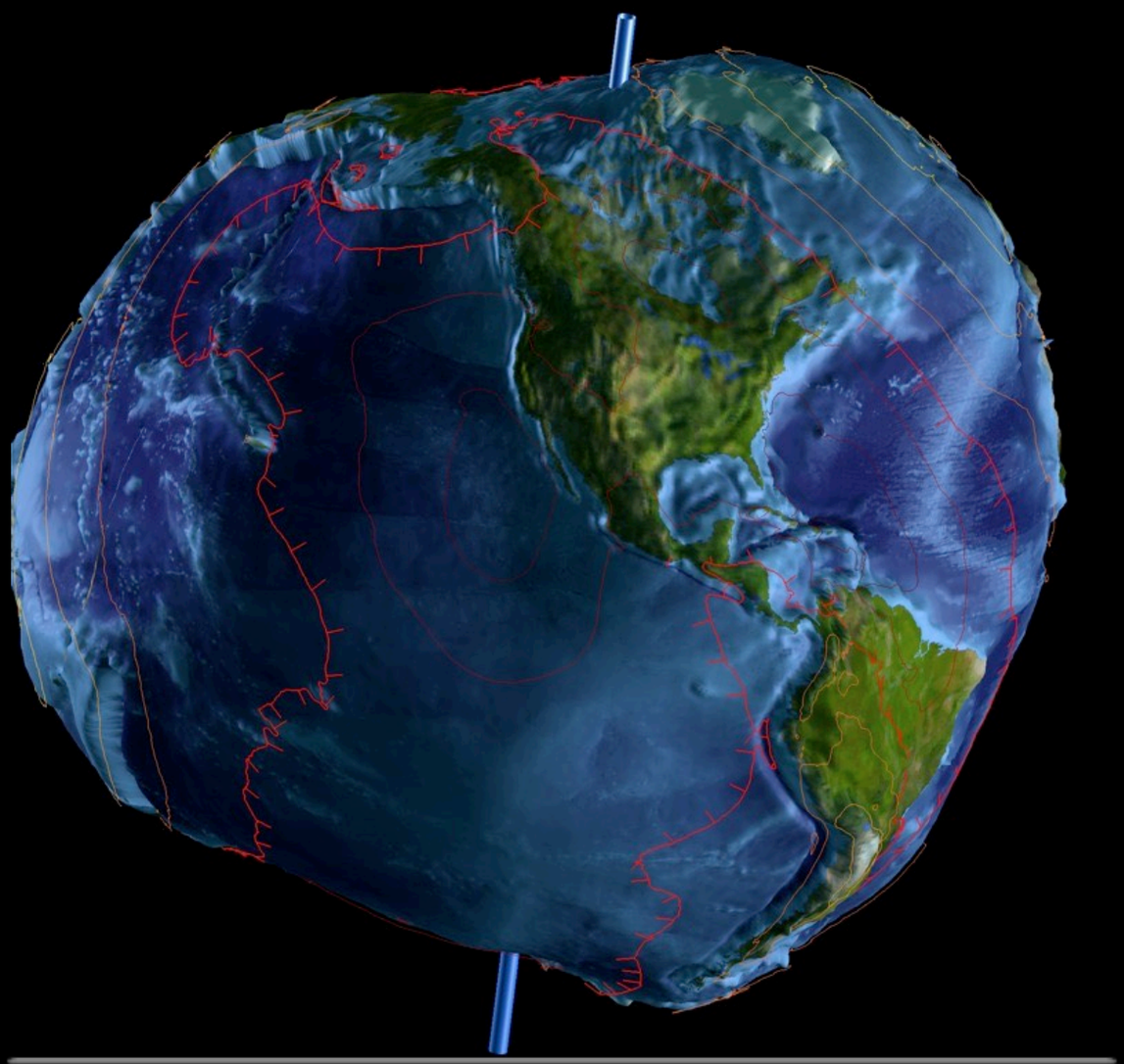
The geoid: over Africa

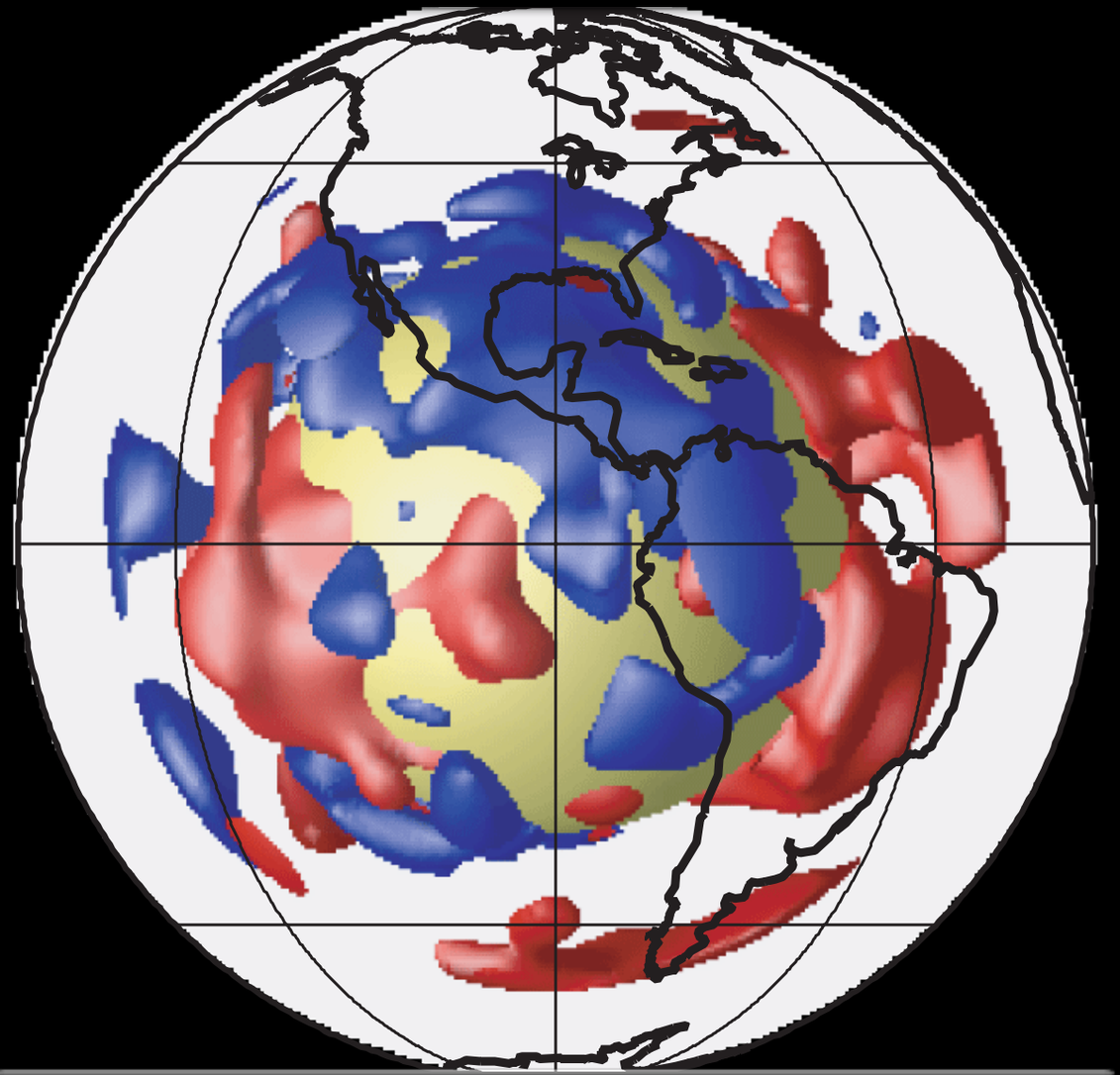




geoid by M. Sandiford, tomography by Garnero

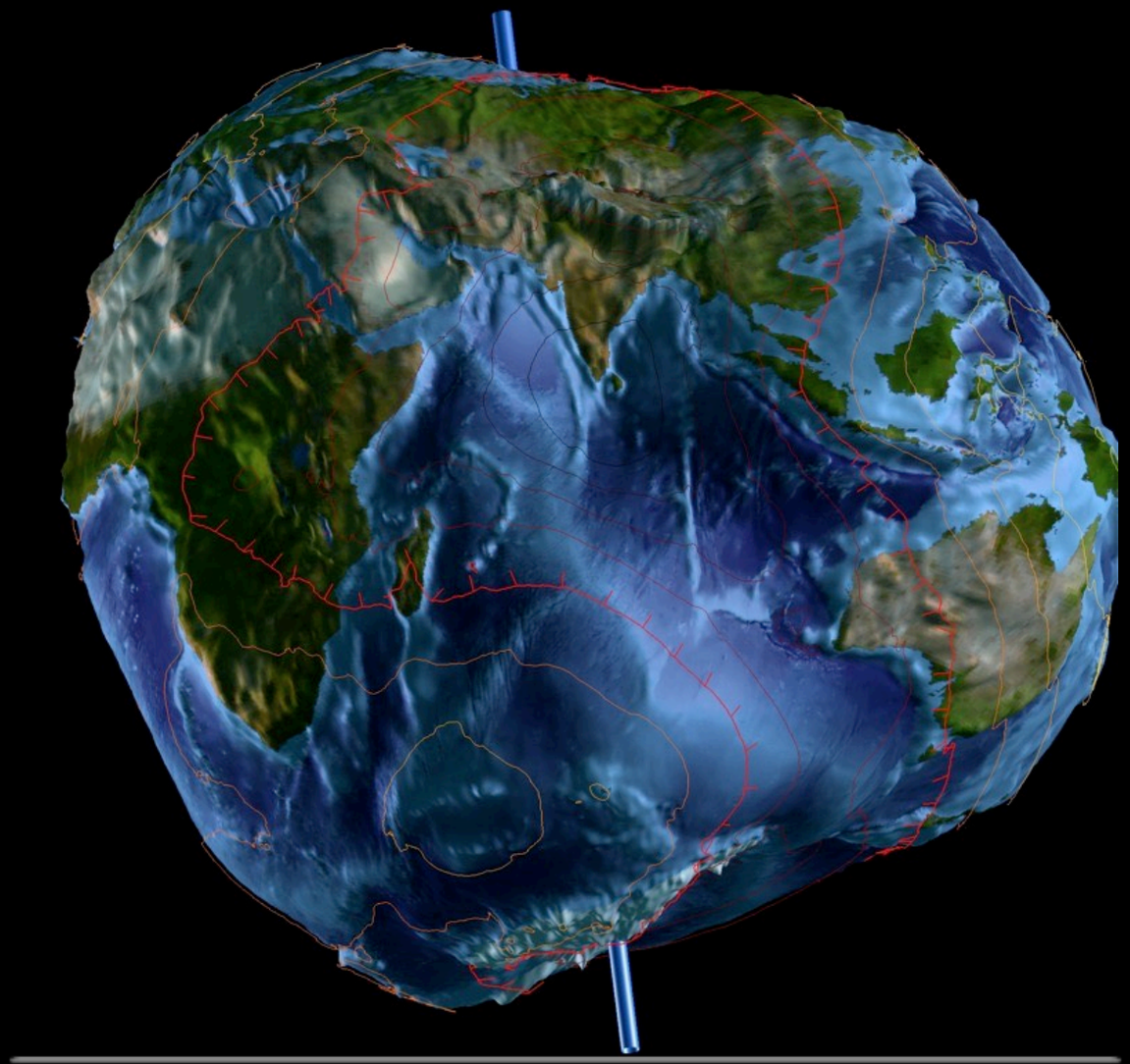
The geoid: over N. America

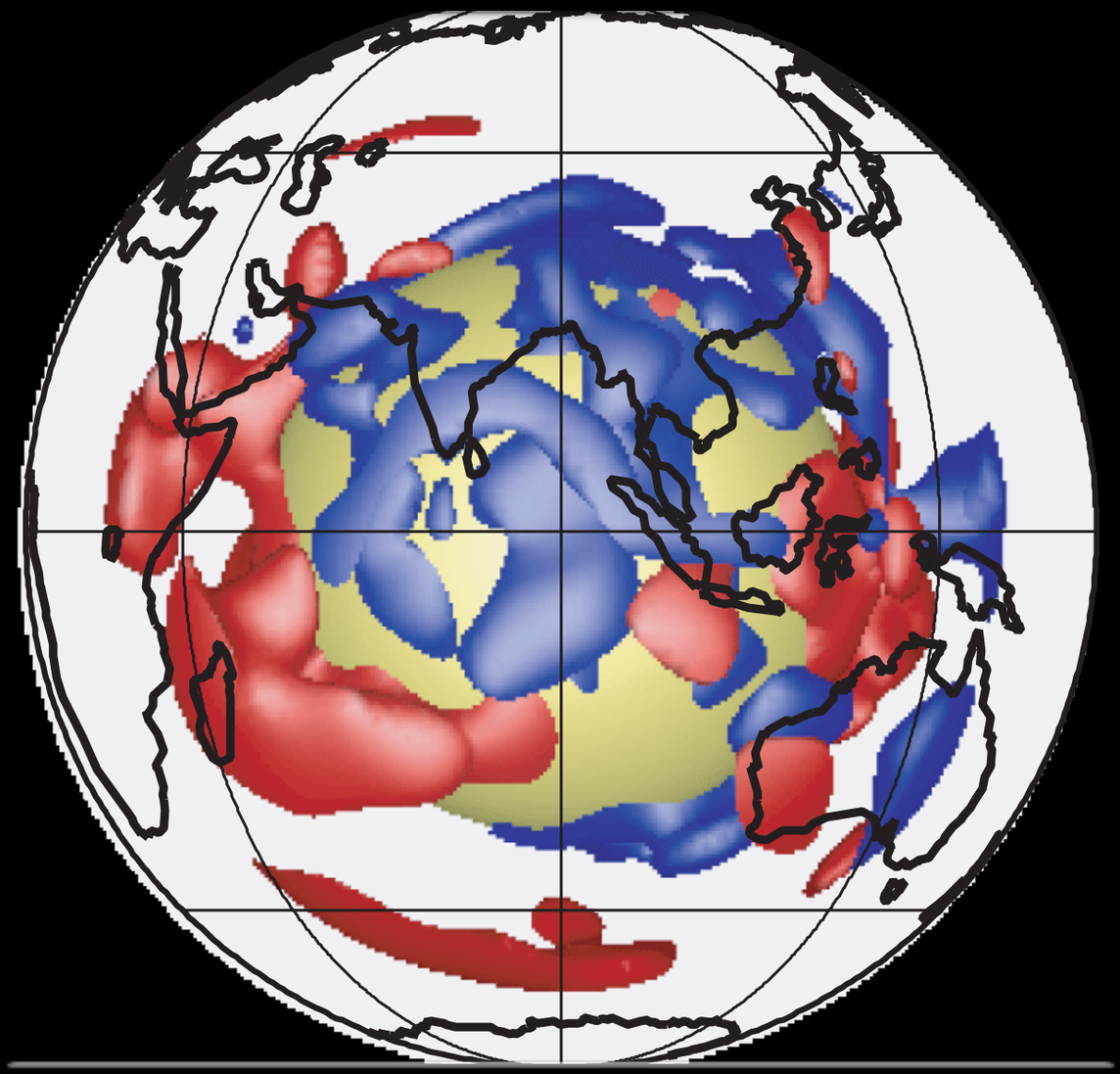




geoid by M. Sandiford, tomography by Garnero

The geoid: over India

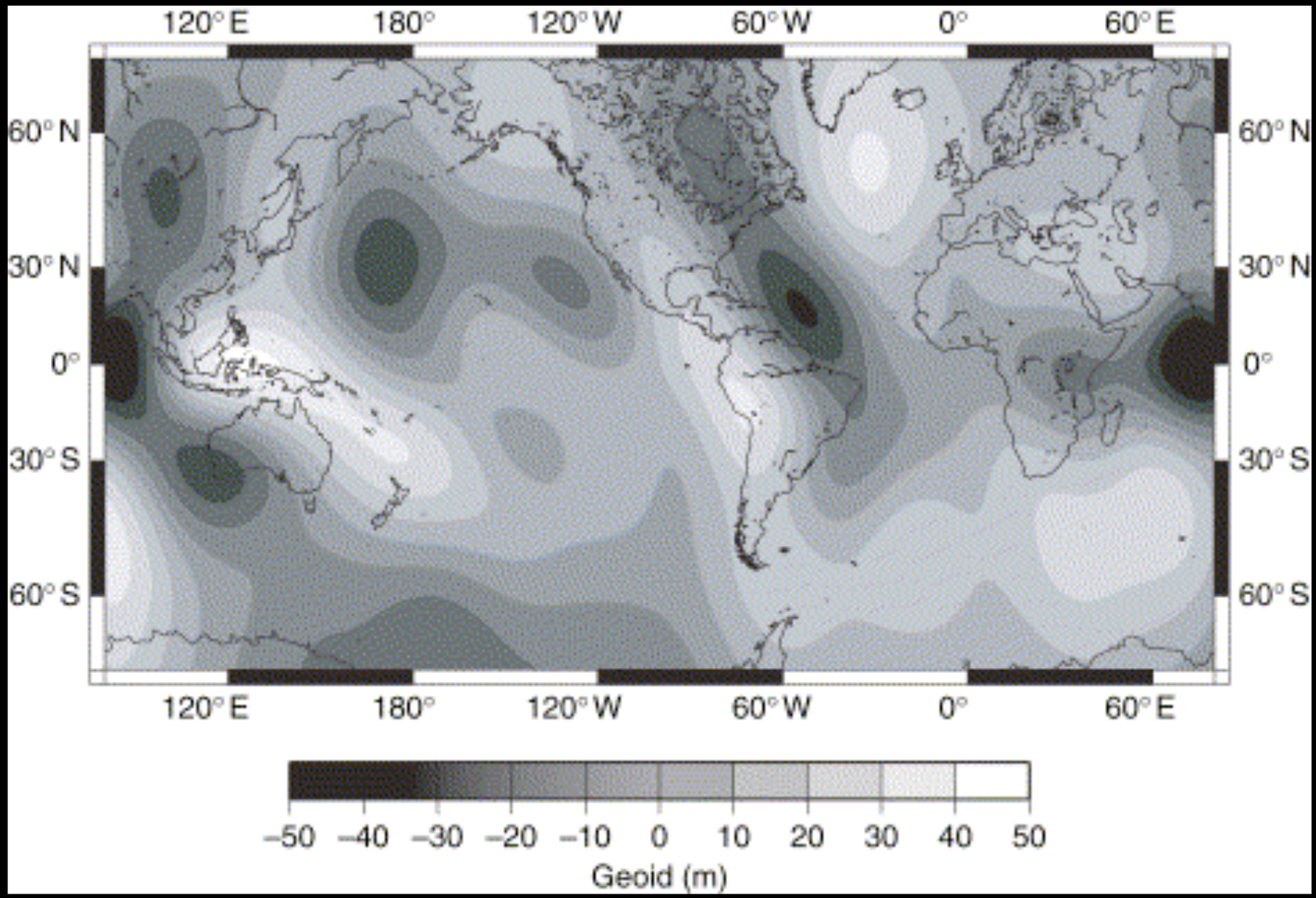




geoid by M. Sandiford, tomography by Garnero

Earth's dynamic geoid: medium-scale

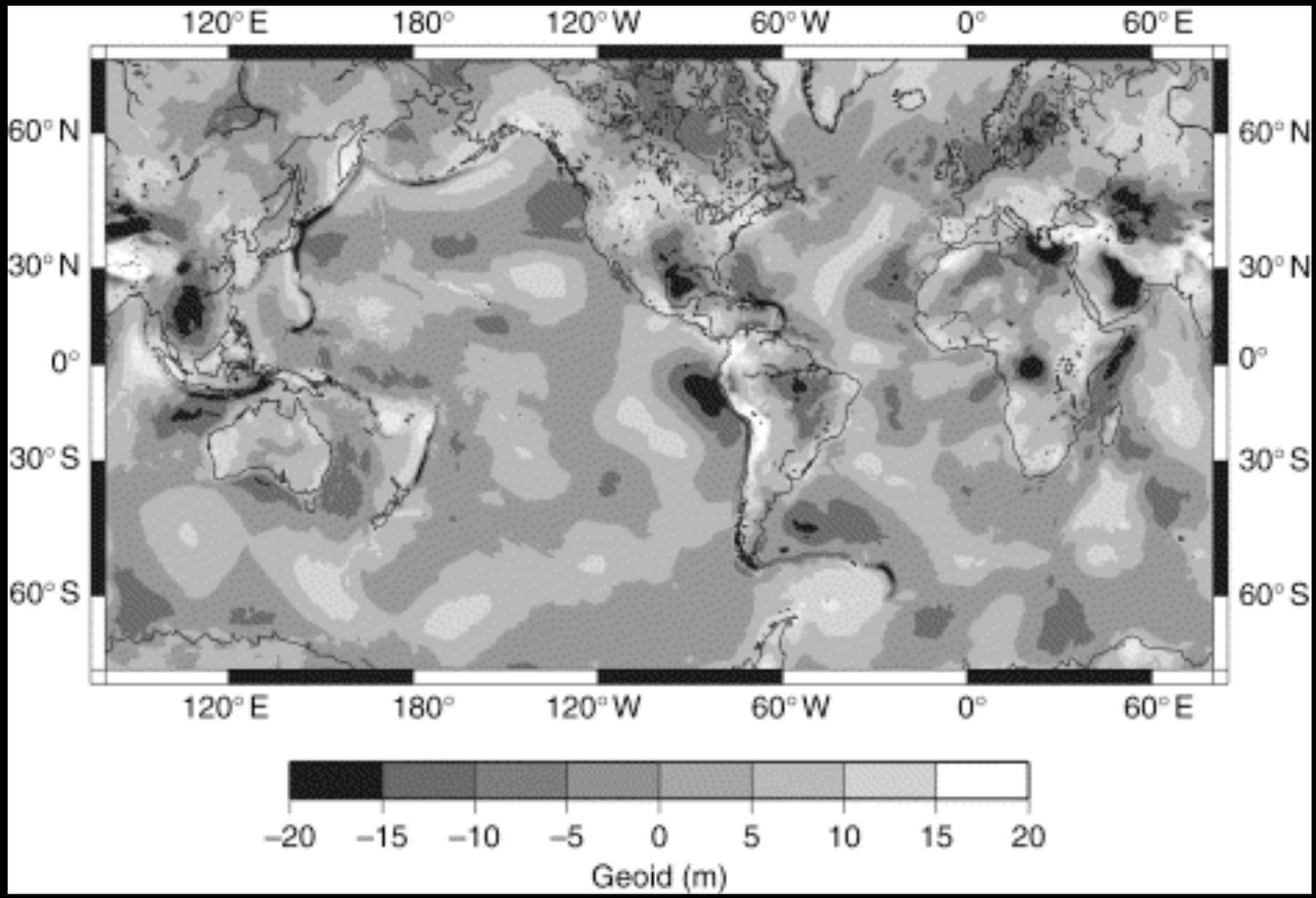
- **band-pass filtered (degrees 4-9) agrees well with upper mantle slabs**



after Lemoine et al., 1997

Earth's dynamic geoid: small-scale

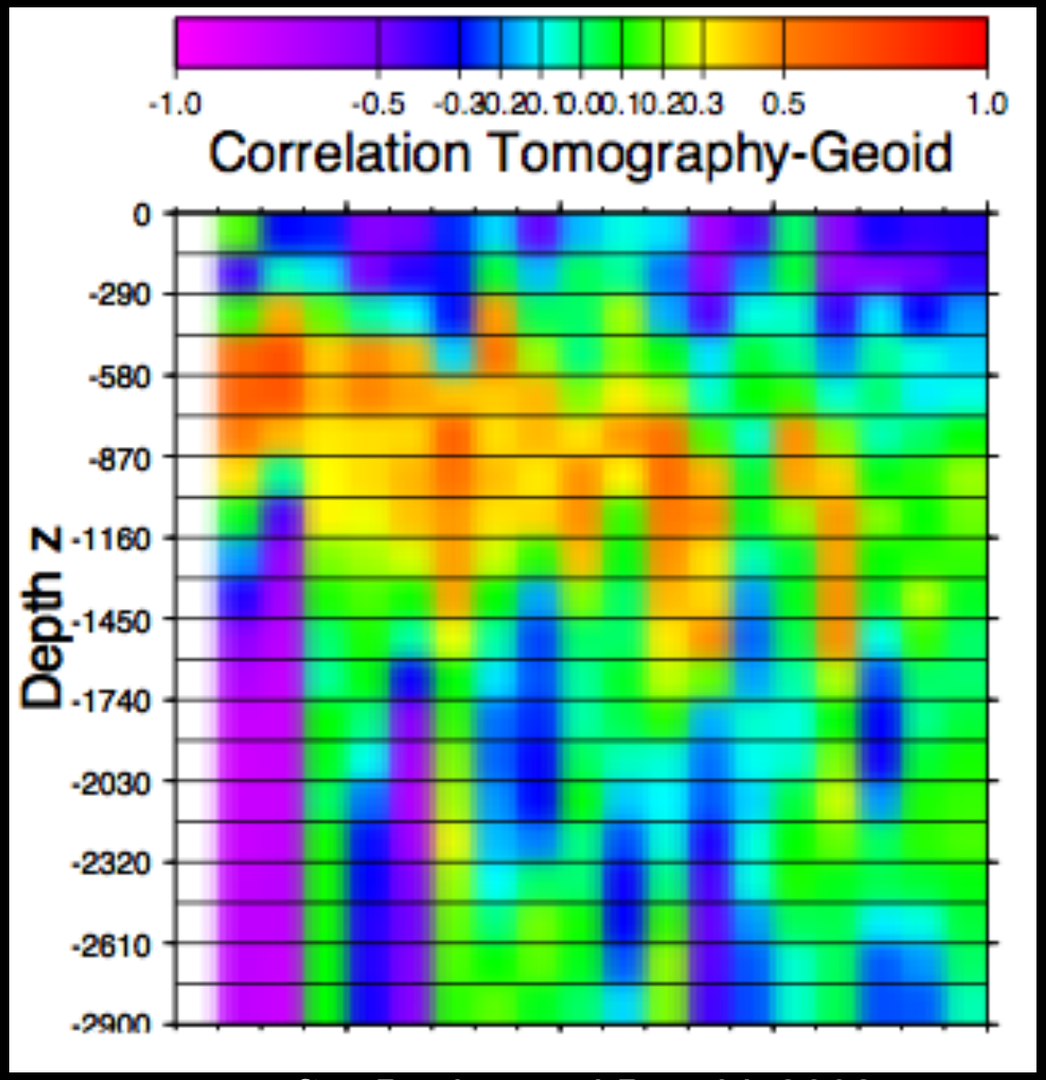
- **only small degrees (< 5000 km) more agreement with present day topography**



after Lemoine et al., 1997

Geoid kernels for 2 layer mantle

- using s-mean model (avg of all s-wave tomography models)
- robust feature of constraining geoid with 2 layers is 100x factor is too large



after Becker and Boschi, 2002

Geoid kernels for 2 layer mantle

- using s-mean model (avg of all s-save tomography models)
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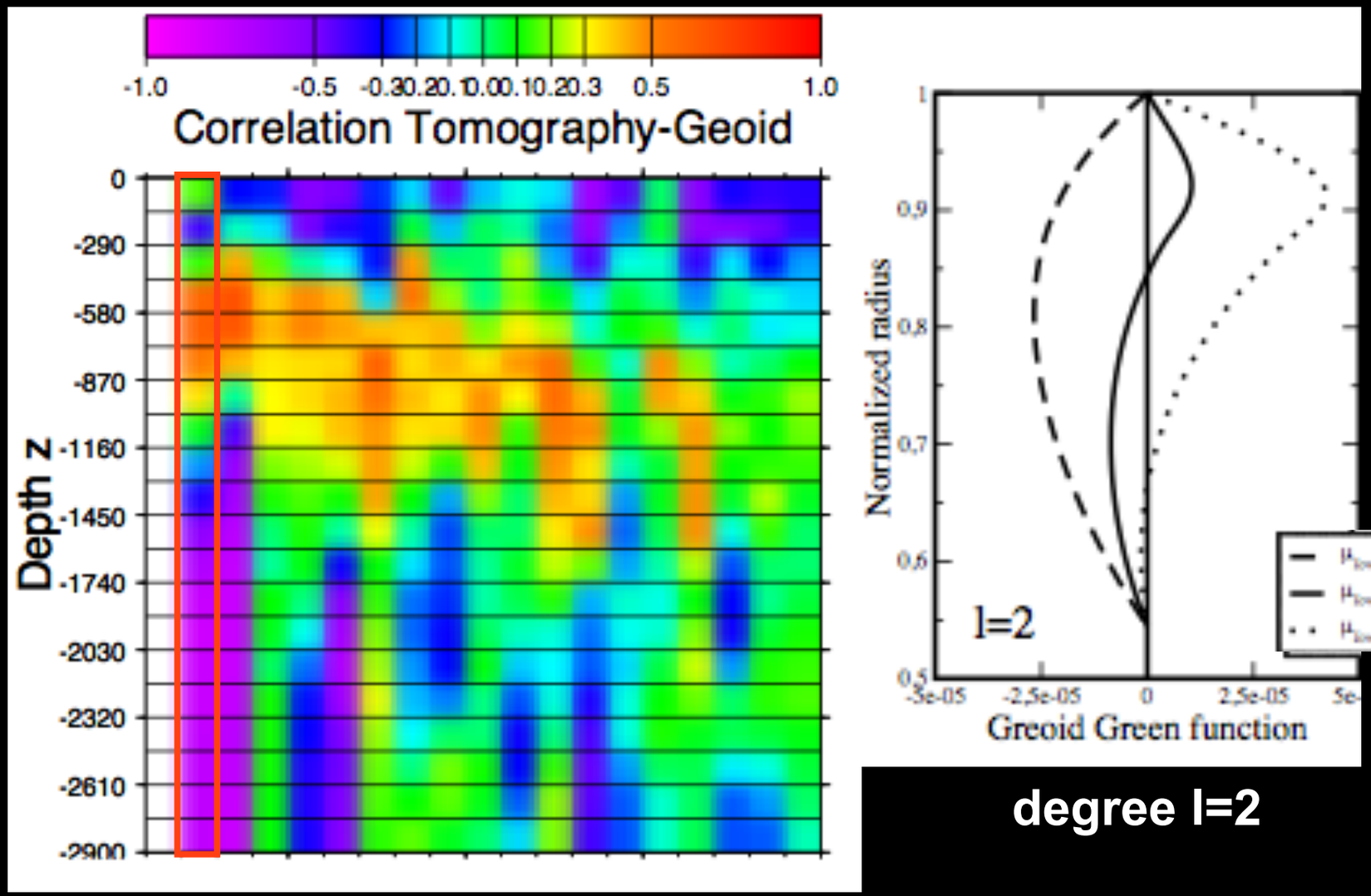

degree l=2

after Becker and Boschi, 2002

Geoid kernels for 2 layer mantle

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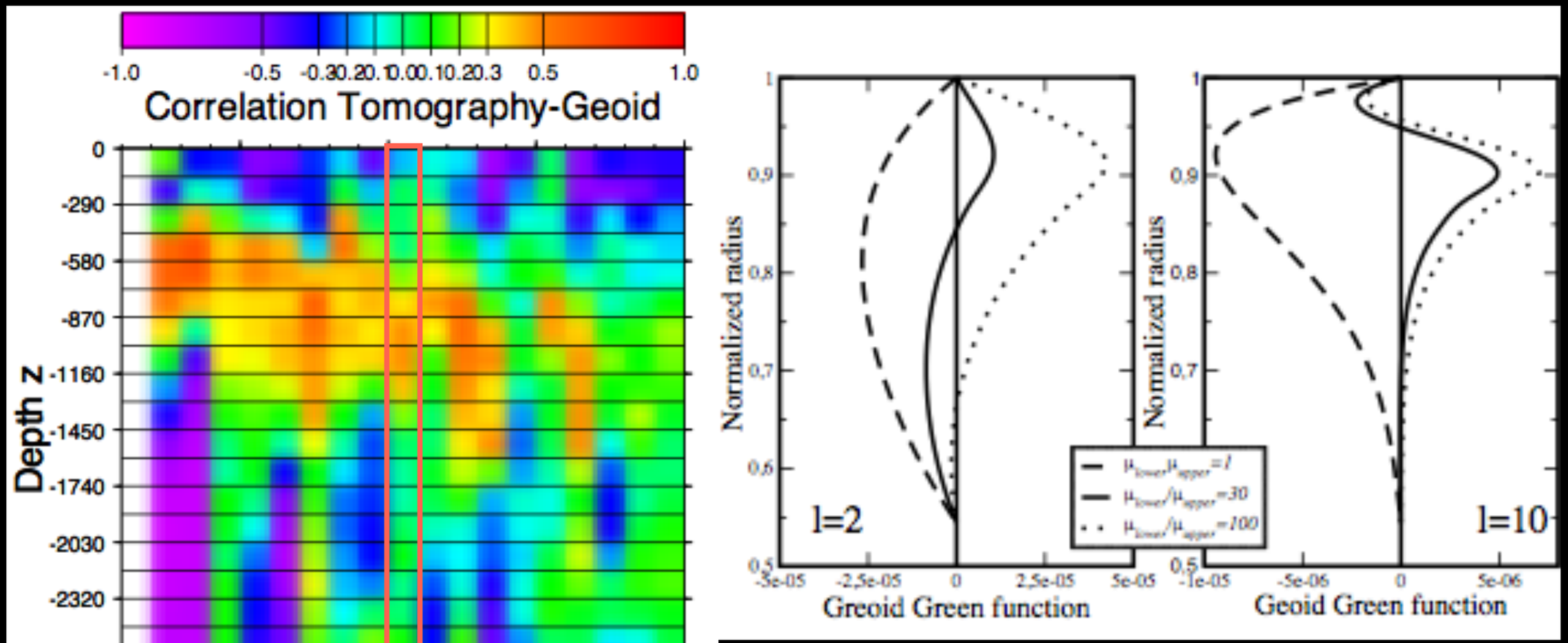
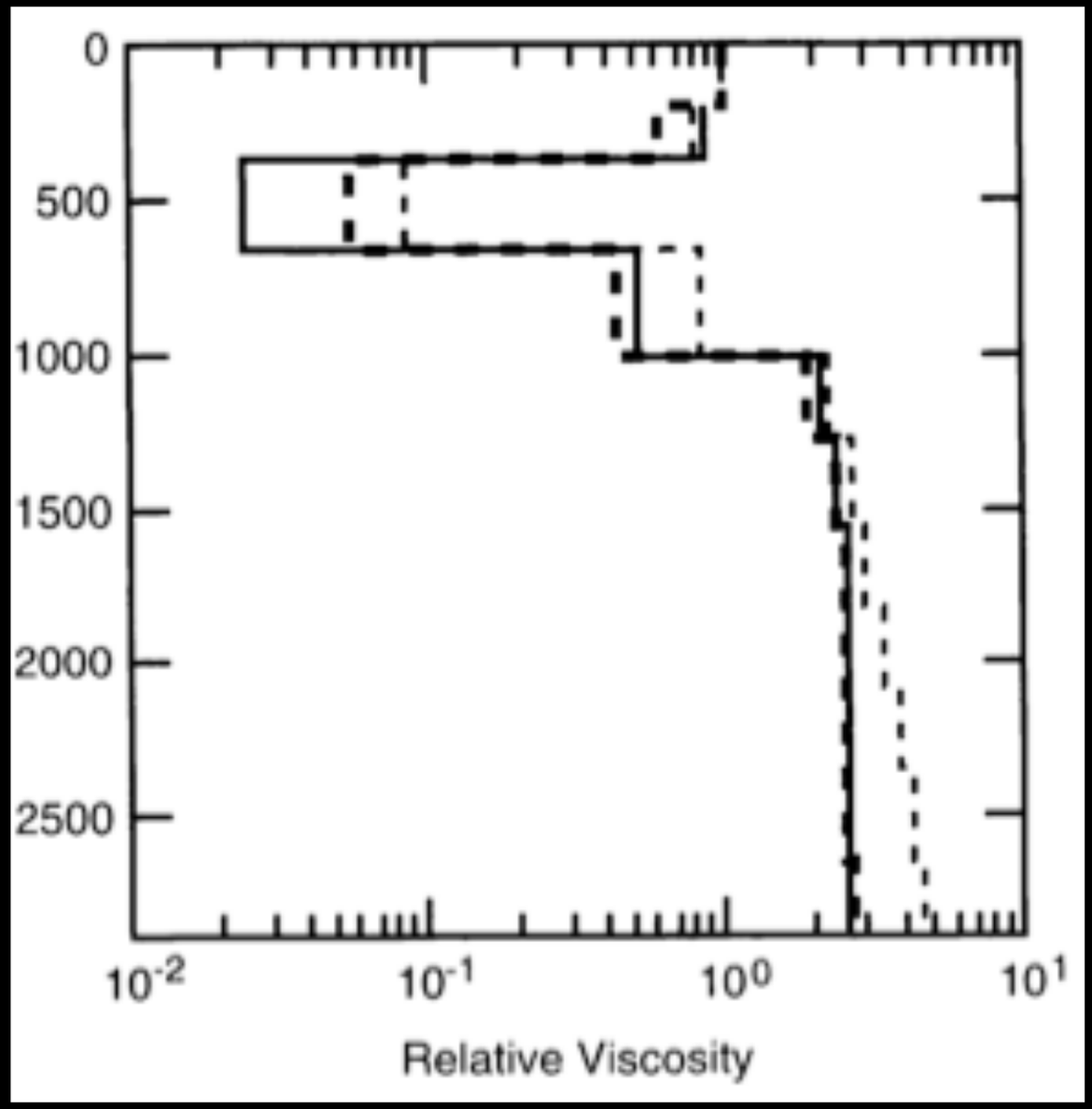


degree l=2 **degree l=10**

after Becker and Boschi, 2002

Survey of η(r)

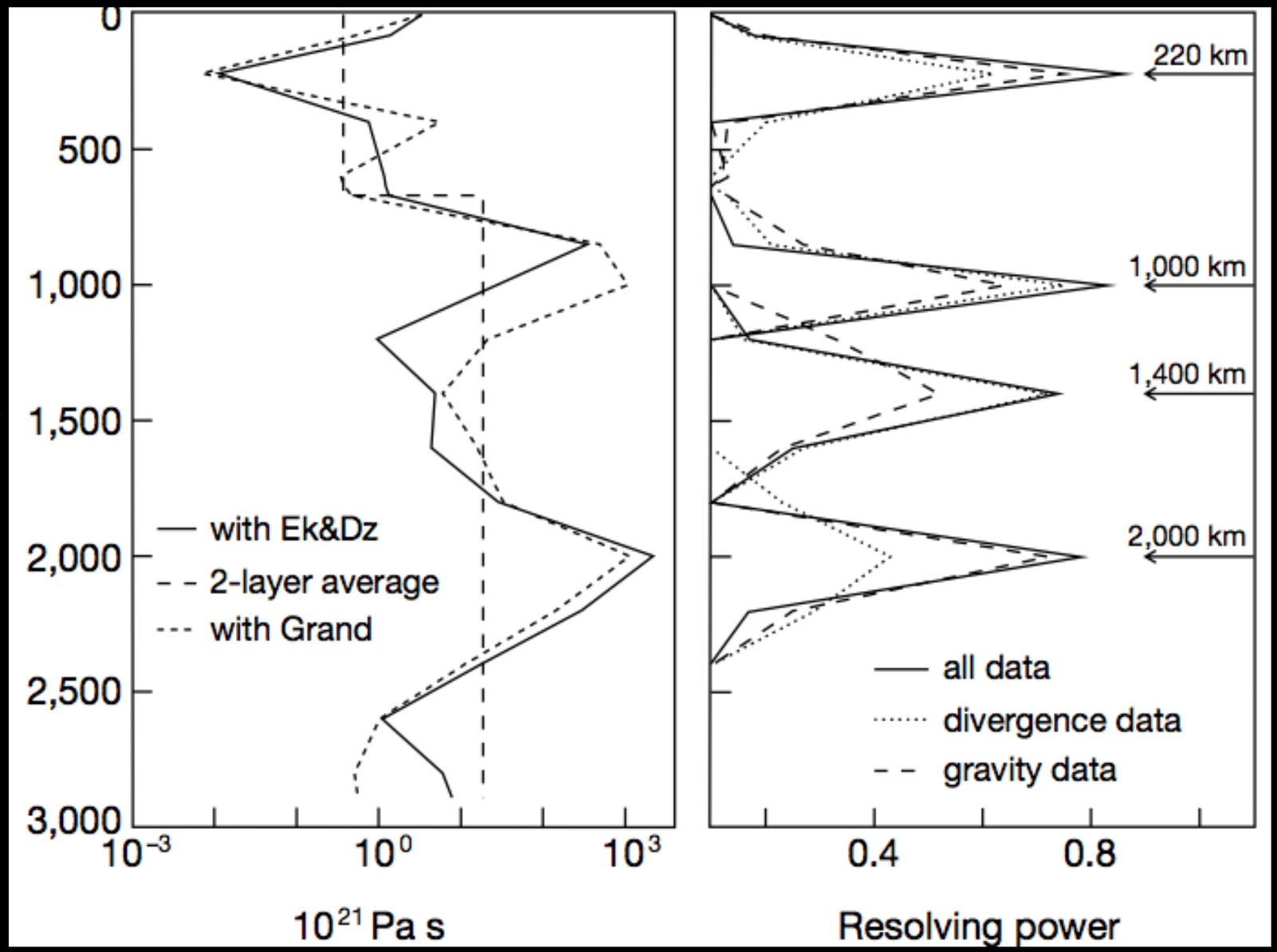
- using an S-wave tomography model
  - convert $^{\Delta}v_s$ to $^{\Delta}\rho$ to drive flow
  - 11 layer model; scale factor 1e21
- also see low viscosity transition zone
- preferred model (heavy dashed line)
- about a factor 10x increase at 660 km
- another factor 4x increase at 1000 km
- weakness is in arbitrary way of scaling velocities to densities and uncertainties in tomography model



King and Masters, 1992

Survey of η(r)

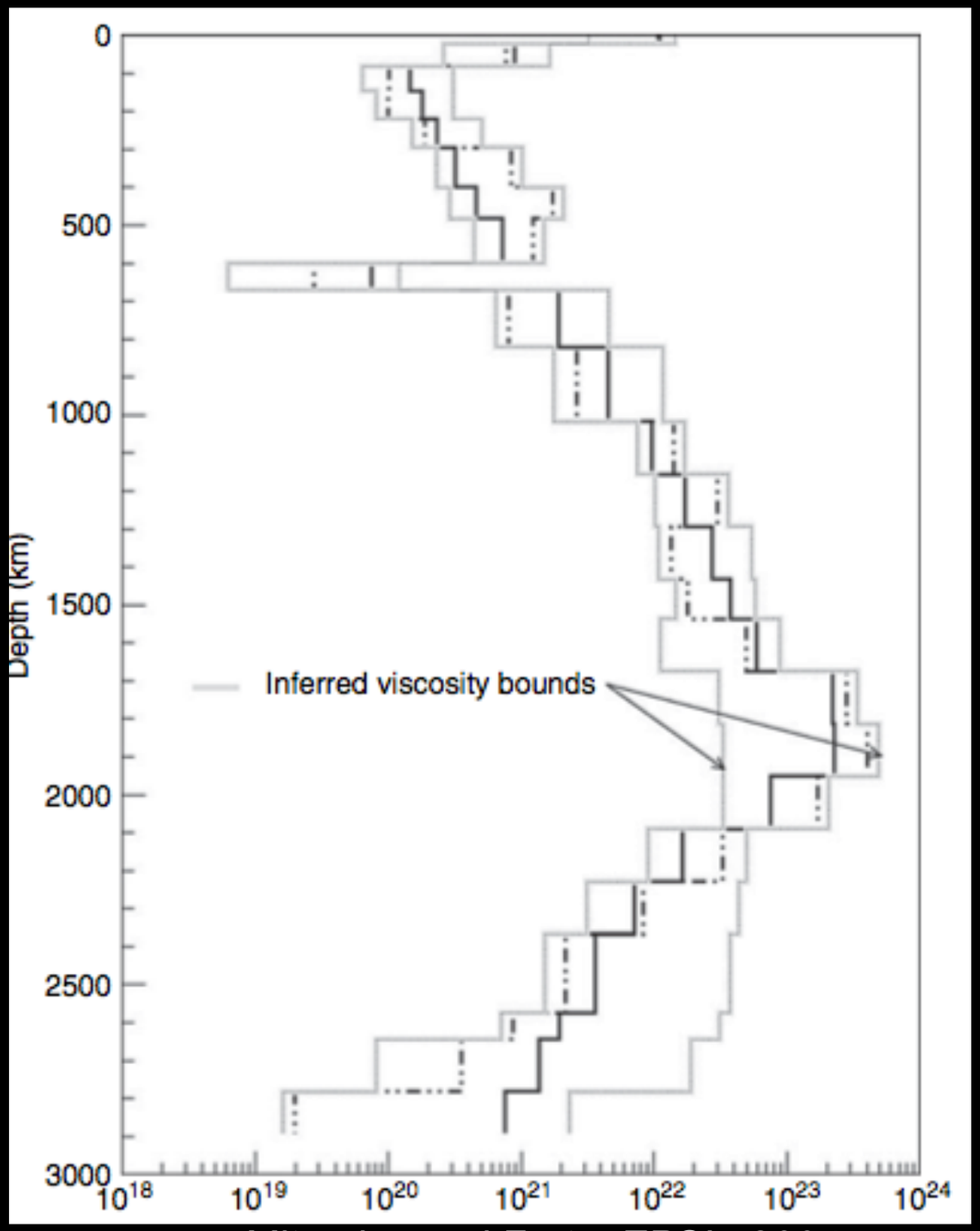
- joint inversion of tomography and geodynamic constraints (plate motions + CMB ellipticity + geoid
- low-viscosity channel
- two distinct viscosity hills observed - jumps at 660 and 2000 km
- nature of 660 viscosity jump unconstrained (step function or more gradual ramp are both allowed)
- attempt to find optimal scaling parameters for converting $^{\Delta}v_s$ to $^{\Delta}\rho$



Forte and Mitrovica, Nature, 2001

Survey of η(r)

- incorporates new constraints from global isostatic adjustment data as well as plate motions + geoid + excess CMB ellipticity
- preferred model solid line, 25 layers
- robust constraint on upper mantle viscosity 5.e20 Pa-s averaged from (100-550 km depth)
- best fit includes low viscosity notch <u>above</u> 660 km - added *a priori*
- claim all previously published viscosity profiles are wrong due to not including GIA - need joint inversion



Mitrovica and Forte, EPSL, 2004