THE LIFE TECTONIC
Rheology of the Mantle and Plates (part 1):
Deformation mechanisms and flow rules of mantle minerals
Topics covered in this class

• Rheology of Earth (viscous limit)
• Fluid Dynamics for geological phenomena
• Composition of the Earth
• Thermodynamics and high-pressure mineral physics
• Seismological structure of the mantle
• Geochemical structure of the mantle
• Dynamic processes of the Earth (plumes, slabs, thermochemical piles)
• Heat and mass transport in the deep Earth (convection, thermal history)
• Energetics of the core (magnetic field generation)
Where to turn to for more help..

- *Mantle Convection in Earth + Planets*  
  Schubert, Turcotte, and Olson (2001)

- *Numerical Geodynamic Modelling*  
  Gerya (2009)


- *Treatise on Geophysics V. 7, Ch. 2*

- Papers by those lucky people in the Rheology fan club (partial list only):
  - numerical modelers: Podladchikov, Solomatov, Burov, Gerya, Bercovici, Tackley, Yuen
  - experimentalists: Karato, Kolhstedt, Hirth, Jackson
What is rheology?

- Rheology is the physical property that characterizes deformation behavior of a material (solid, fluid, etc).

- Rheology of Earth materials includes elasticity, viscosity, plasticity, etc.

- For the deep Earth: mantle is fluid on geological timescales so we focus on its viscosity.

- For tectonic plates: still viscous on geological timescales, but the effective viscosity is a subject of debate.

\[
\sigma = \frac{E\varepsilon}{\text{solid mechanics}}
\]
\[
\sigma = 2\eta\dot{\varepsilon} \quad \text{fluid mechanics}
\]
What is viscosity?

• constitutive relation between stress and strain-rate (deformation rate)

• in the continuum description, it is the analog of the elastic moduli which relate stress and strain

• measure of a fluid’s ability to flow

• diffusivity of momentum
Definition of creep

- movement of crystal defects
- point defects - extra atoms or vacancies
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  - two types of dislocations: “edge” and “screw”

  - each described by parallel or normal Burgers vector \((b^*)\)
Definition of creep

- movement of crystal defects
  - point defects - extra atoms or vacancies
  - line defects - dislocations which represent a rearrangement of atomic bonds
    - two types of dislocations: “edge” and “screw”
    - each described by parallel or normal Burgers vector (b*)
  - creep will occur through whichever mechanism requires least amount of energy
**Diffusion creep**

point defects move by *diffusion*

- through the crystal matrix (Nabarro-Herring)
Diffusion creep

point defects move by *diffusion*

- through the crystal matrix (Nabarro-Herring)
- along the grain boundaries (Coble)
Diffusion creep
Dislocation creep

line defects move by *dislocation*

- two types of dislocations: “edge” and “screw”
- any line defect can be represented by linear combination of the two (simply add Burgers vectors)
- line dislocations have two types of motion: glide and climb
- independent of grain-size (point of difference with diffusion creep)
Glide process of dislocation creep

- glide motion stays within glide plane
Climb process of dislocation creep

- climb dislocation occurs outside the glide surface
Deformation maps

- for any given stress + temperature, one mechanism will be weaker (and preferred) over all others

\[ \dot{\varepsilon} = \dot{\varepsilon}_{\text{disl}} + \dot{\varepsilon}_{\text{diff}} \]

- map assumes constant grain size

![Deformation mechanism map for olivine, with grain size 1 mm, showing iso-strain rate contours (after Frost and Ashby, 1982).](image)
Creep mechanisms in the mantle

Billen, Annual Rev. Geophys., 2008
Flow rule

- relates the deformation (strain rate) to the applied deviatoric stress through a viscosity

- deformations add in series (viscosities add in parallel)

\[ \sigma = \eta \dot{\varepsilon} \]
\[ \dot{\varepsilon} = \frac{1}{\eta} \sigma \]
\[ \dot{\varepsilon}_{tot} = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \ldots \]
\[ \frac{1}{\eta_{eff}} = \frac{1}{\eta_1} + \frac{1}{\eta_2} + \ldots \]
Flow rule

• for isotropic fluids, we can describe the flow rules with an effective strain rate, an effective stress, and an effective viscosity

\[ \sigma_{II} = \sqrt{\sigma : \sigma} \]
\[ \dot{\varepsilon}_{II} = \sqrt{\dot{\varepsilon} : \dot{\varepsilon}} \]
\[ \sigma_{II} = \eta_{eff} \dot{\varepsilon}_{II} \]

• 2nd invariants of deviatoric stress and strain rates are scalars

• in practice, we know the strain-rates from the velocity field rather than stress, so viscosity is normally rewritten in terms of strain-rate

NOTE: contractions of these tensors usually have a 1/2 term times the sum of the squares of the components (when assuming co-axial compression / pure shear)
Rheology: land of jargon and confusion

- **Newtonian** strictly means **linear** viscosity, but is commonly used to refer to **diffusion** creep with stress exponent \( n=1 \)

- **Non-Newtonian** usually refers to **non-linear** rheology resulting from **dislocation** creep that is **stress dependent** through **power law** on stress with an exponent \( n=3.5, \) or historically \( n=3 \)

- Example: diffusion creep is thermally activated and pressure dependent, and thus has exponential sensitivity to \( T,P \) but is sometimes referred to as “Newtonian” even though it is a non-linear \( f(T,P) \)

Tackley, 2000
Comparison of deformation processes

- any non-linear rheology can be represented by an effective linear rheology
- the effective viscosity is simply the slope of the line drawn from the origin to the stress-strainrate curve
- example of effective viscosity for several different rheologies that may be important for the mantle + plates
- think a little more about these curves stress vs. effort, efficiency vs. effort, and total productivity vs. effort

Tackley, 2000
Arrhenius dependence

- one can use thermodynamics to describe the sensitivity of diffusion when it is thermally activated

\[ D = D_0 \exp \left[ -\frac{(E_A + P V_A)}{RT} \right] \]

- the diffusion of vacancies, etc has an exponential dependence on T,P

- this can also be understood in terms of the homologous temperature

- the deformation resulting from the diffusion creep is the strain rate and is inversely proportional to viscosity

\[ \dot{\varepsilon} = \frac{1}{\eta} \sigma \]

- the Arrhenius term describes the exponential behavior of viscosity

\[ \eta(T, P) = \eta_0 \exp \left[ \frac{(E_A + P V_A)}{RT} \right] \]

- also valid for dislocation creep
General flow rule for mantle material

\[ \dot{\varepsilon} = A \left( \frac{b^*}{d} \right)^m \left( \frac{\sigma}{\mu} \right)^n \exp \left[ -\frac{(E_A + PV_A)}{RT} \right] \]
General flow rule for mantle material

\[ \dot{\varepsilon} = A \left( \frac{b^*}{d} \right)^m \left( \frac{\sigma}{\mu} \right)^n \exp \left[ -\frac{(E_A + P V_A)}{R T} \right] \]

\[ \dot{\varepsilon} = A \left( \frac{b^*}{d} \right)^m \left( \frac{\sigma}{\mu} \right) \left( \frac{\sigma}{\mu} \right)^{n-1} \exp \left[ -\frac{(E_A + P V_A)}{R T} \right] \]
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\]

\[
\frac{1}{\eta} = \frac{A}{\mu} \left( \frac{b^*}{d} \right)^m \left( \frac{\sigma}{\mu} \right)^{n-1} \exp \left[ -\frac{(E_A + PV_A)}{RT} \right]
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\[ \dot{\varepsilon}_{\text{tot}} = \dot{\varepsilon}_{\text{diff}} + \dot{\varepsilon}_{\text{disl}} \]

\[ \frac{1}{\eta_{\text{eff}}} = \frac{1}{\eta_{\text{diff}}} + \frac{1}{\eta_{\text{disl}}} \]

• note: diffusion and dislocation creep have different activation enthalpies
What is “n” for dislocation creep?

\[ \dot{\varepsilon}_{tot} = \dot{\varepsilon}_{diff} + \dot{\varepsilon}_{disl} \]

- note: different y-intercept corresponds to different activation enthalpies which is due to the influence of water

_Hirth and Kohlstedt, 2003_
Extrapolating to mantle conditions

\[ V^* = 11 \times 10^{16} \text{ m}^3/\text{mol} \]

\[ V^* = V^*(T,P) \]

\[ T_p = 1350 \degree C \]

\[ C_{OH} = 1000 \text{ H/10}^6 \text{Si} \]

\[ \sigma = 0.3 \text{ MPa} \]

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Hirth and Kohlstedt, 2003
Just add water...

- contour lines are strain rates in (1/s) - dislocation is horizontal because it is independent on grain size but diffusion creep shows grain size dependence

- much lower stresses required to achieve same strain rate when water is present (note different y-axis) which translates into lower activation enthalpy
Just add water...

- contour lines are strain rates in (1/s) - dislocation is horizontal because it is independent on grain size but diffusion creep shows grain size dependence

- much lower stresses required to achieve same strain rate when water is present (note different y-axis)

- translates into lower activation energy and volume ($E^*$ and $V^*$)

\[
\dot{\varepsilon}(C_{OH}) = A \left( \frac{b^*}{d} \right)^m \left( \frac{\sigma}{\mu} \right)^n \exp \left[ -\frac{(E_A^* + P V_A^*)}{RT} \right]
\]
Grain size sensitive creep

- GSS creep relies on combination of mechanisms: grain boundary sliding (GBS) and basal slip along the easiest slip system (easy)

\[ \dot{\varepsilon} = \dot{\varepsilon}_{\text{disl}} + \dot{\varepsilon}_{\text{diff}} + \frac{1}{\dot{\varepsilon}_{\text{gbs}}} + \frac{1}{\dot{\varepsilon}_{\text{easy}}} \]

- GSS operates at small grain size only relevant to diffusion creep

- accommodates large amounts of strain without deforming crystals (referred to as superplasticity)

- GSS important process in lower mantle with small grain size (Karato et al., Science, 1995) through extrapolation to high P, low strainrate

Drury, 2005
Recrystallization

- dynamic process changing the distribution of grain sizes
- grain size reduction (large grains fracture) and growth (small grains fuse together)
- microstructure important feedback with dislocation creep
- time-dependent process

linear (n=1)  
without recrystallization

power law (n=3.5)  
with recrystallization

Jessell et al, EPSL, 2005
Grain growth in the lower mantle

- recrystallization for material going down through 660 phase change
- diffusion creep likely dominant mechanism for lower mantle but if the grain size is sufficiently small, then superplasticity could be important mechanism
Harper-Dorn creep

- low-temperature plasticity OR Peierls stress mechanism*

\[ \dot{\varepsilon} = A \left( \frac{b^*}{d} \right)^m \left( \frac{\sigma}{\mu} \right)^n \exp \left[ -\frac{(E_A + P V_A)}{RT} \left( 1 - \frac{\sigma}{\sigma_p} \right)^q \right] \]

*read about Peierls other contributions to science here: https://en.wikipedia.org/wiki/Rudolf_Peierls

- applicable to the interior of a subducted slab (below: 4 cm/yr and 10 cm/yr)

Karato et al, PEPI, 2001
Generalized flow rule for a slab

\[ \dot{\varepsilon}_{tot} = \dot{\varepsilon}_{diff} + \dot{\varepsilon}_{disl} + \dot{\varepsilon}_{H-D} \]

\[ \frac{1}{\eta_{eff}} = \frac{1}{\eta_{diff}} + \frac{1}{\eta_{disl}} + \frac{1}{\eta_{H-D}} \]

- diffusion, dislocation, Harper-Dorn creep mechanisms work independently, so in general, only a single power-law \((m,n,q)\) is in effect at any given time

- usual values for exponents \((m,n,q)\) are \((2.5,1,0) + (0,3.5,0) + (0,2,2)\)

- each mechanism has a different value for \(A; E\) and \(V\) depend on water content

\[ \dot{\varepsilon} = A \left( \frac{b^*}{d} \right)^m \left( \frac{\sigma}{\mu} \right)^n \exp \left[ -\frac{(E_A + P V_A)}{RT} \left( 1 - \frac{\sigma}{\sigma_p} \right)^q \right] \]
Viscoplasticity

- most materials have finite strength described by limiting stress and materials cannot support stresses in excess of their yield stress

- upon reaching their yield stress they deform through plastic flow (a solid beam starts to act like toothpaste)
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- actually, care must be taken to guarantee one is actually on the yield surface and remains on it (i.e. use harmonic avg at your own risk)

\[
\eta = \begin{cases} 
\frac{\sigma_{II}}{\dot{\varepsilon}_{II}} & (\sigma < \sigma_{yield}) \\
\eta_{eff} = \frac{\sigma_{yield}}{\dot{\varepsilon}_{II}} & (\sigma \geq \sigma_{yield})
\end{cases}
\]
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• visco-elasto-plastic behavior can be written as generalized flow rule with an associated flow for each of the viscous, elastic, and plastic parts
Strength envelope for an oceanic plate

- maximum stress that is supported is equivalent to a yield stress
- three layer lithosphere: brittle crust, strong core, ductile underside
- based on extrapolating the deformation behavior of crystals (microscale) to that of a rock scale (macroscale)
- role of large scale faults and tectonic fabric (mesoscale)
- likely mechanism in strong core is Peierls creep (low strainrate, high stress)

Kohlstedt et al, JGR, 1995

SIO 224: Internal Constitution of the Earth, Rheology of the Mantle and Plates
Anisotropy

- dislocation creep ($n=3.5$) and dynamic recrystallization during deformation generate crystal alignments and lattice preferred orientation (LPO) of crystals.

- significant seismic anisotropy observed in the upper mantle -> implies dislocation creep is the dominant mechanism.

- lack of anisotropy seen in lower mantle -> diffusion creep is dominant.

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