

Rheology of the Mantle and Plates (part 1):

# Deformation mechanisms and flow rules of mantle minerals

# Topics covered in this class

- **Rheology of Earth (viscous limit)**
- **Fluid Dynamics for geological phenomena**
- **Composition of the Earth**
- **Thermodynamics and high-pressure mineral physics**
- **Seismological structure of the mantle**
- **Geochemical structure of the mantle**
- **Dynamic processes of the Earth (plumes, slabs, thermochemical piles)**
- **Heat and mass transport in the deep Earth (convection, thermal history)**
- **Energetics of the core (magnetic field generation)**

# Where to turn to for more help..

- ***Mantle Convection in Earth + Planets***  
Schubert, Turcotte, and Olson (2001)
- ***Numerical Geodynamic Modelling***  
Gerya (2009)
- **Hirth and Kohlstedt (2003)**
- **Regenaur-Lieb and Yuen (2003)**
- ***Treatise on Geophysics V. 7, Ch. 2***
- **Papers by those lucky people in the Rheology fan club (partial list only):**
  - **numerical modelers:**  
Podladchikov, Solomatov, Burov, Gerya, Bercovici, Tackley, Yuen
  - **experimentalists:**  
Karato, Kohlstedt, Hirth, Jackson

# What is rheology?

- Rheology is the physical property that characterizes deformation behavior of a material (solid, fluid, etc)
- Rheology of Earth materials includes elasticity, viscosity, plasticity, etc
- For the deep Earth: mantle is fluid on geological timescales so we focus on its viscosity
- For tectonic plates: still viscous on geological timescales, but the effective viscosity is a subject of debate

$$\sigma = E\varepsilon \quad \text{solid mechanics}$$

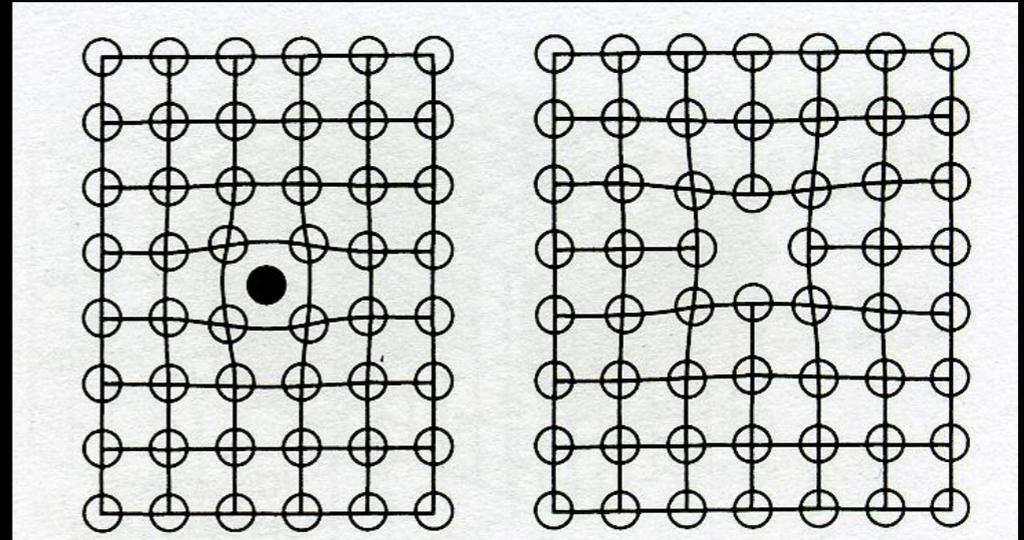
$$\sigma = 2\eta\dot{\varepsilon} \quad \text{fluid mechanics}$$

# What is viscosity?

- **constitutive relation between stress and strain-rate (deformation rate)**
- **in the continuum description, it is the analog of the elastic moduli which relate stress and strain**
- **measure of a fluid's ability to flow**
- **diffusivity of momentum**

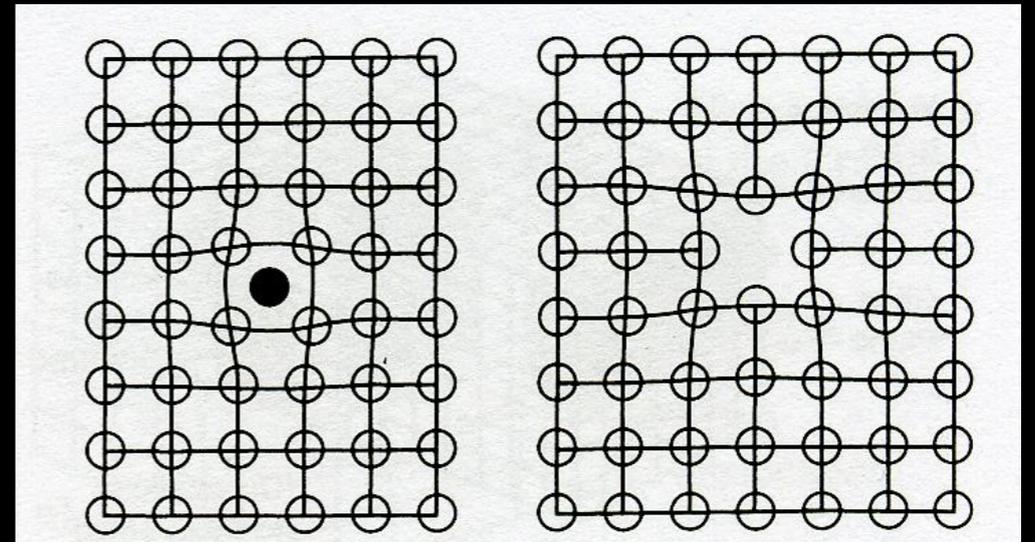
# Definition of creep

- movement of crystal defects
  - point defects - extra atoms or vacancies



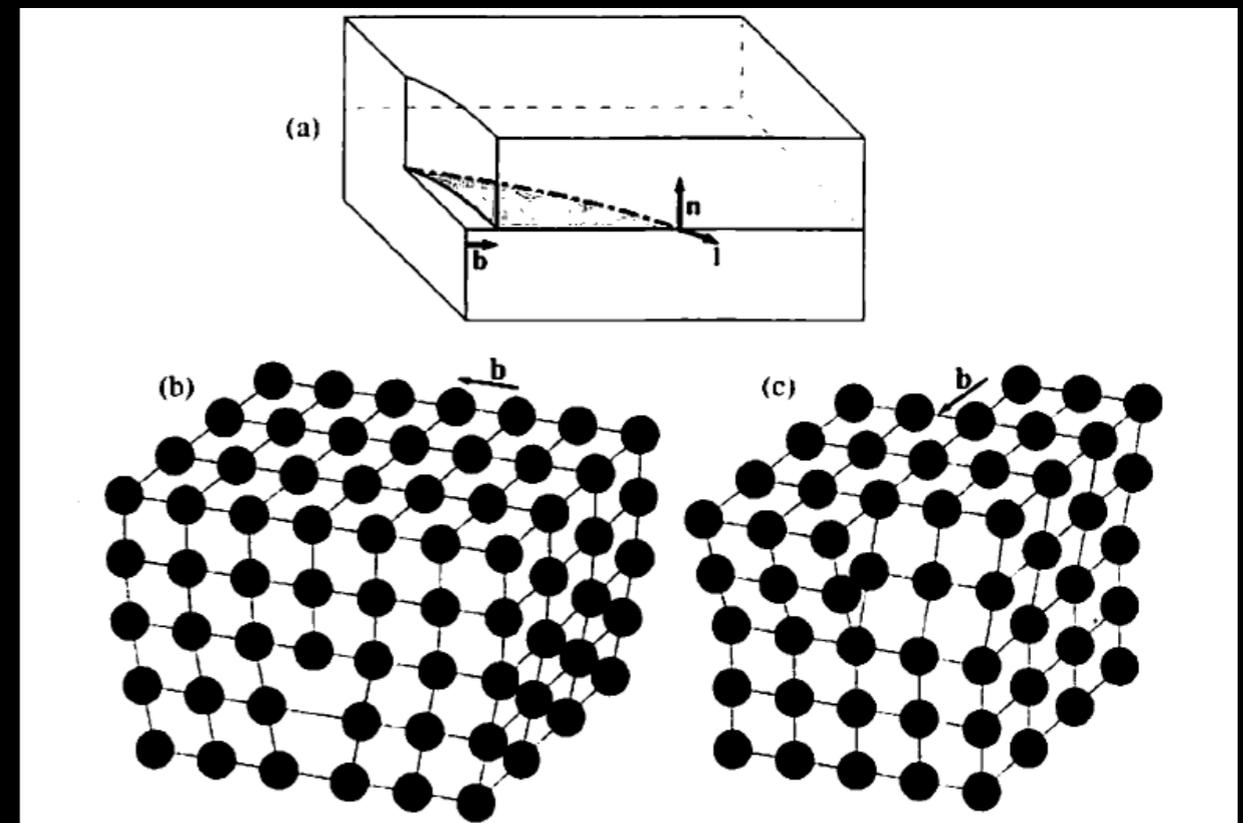
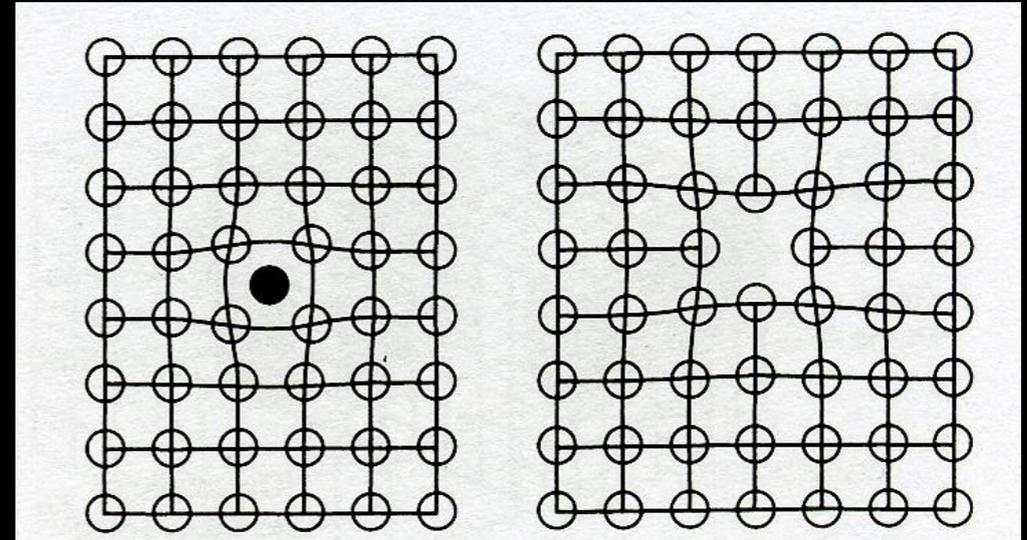
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  - **line defects - dislocations which represent a rearrangement of atomic bonds**



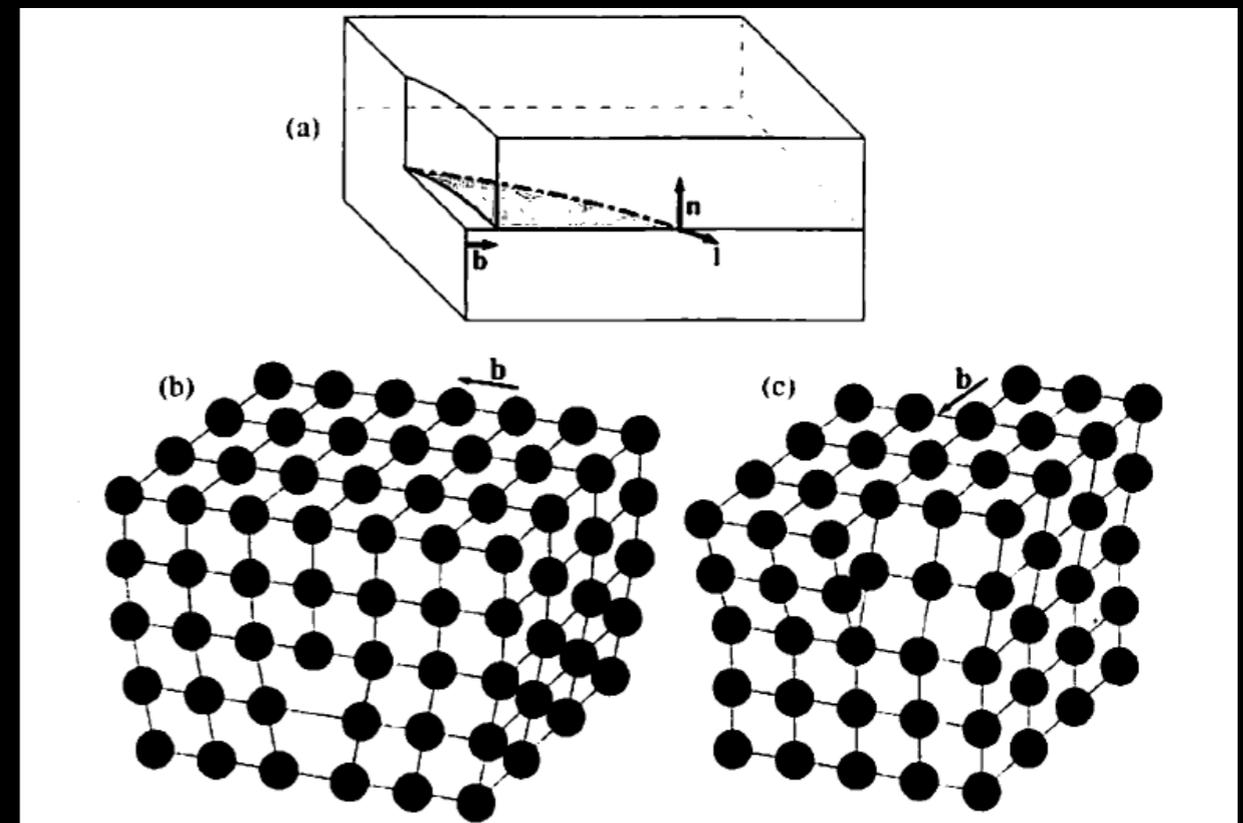
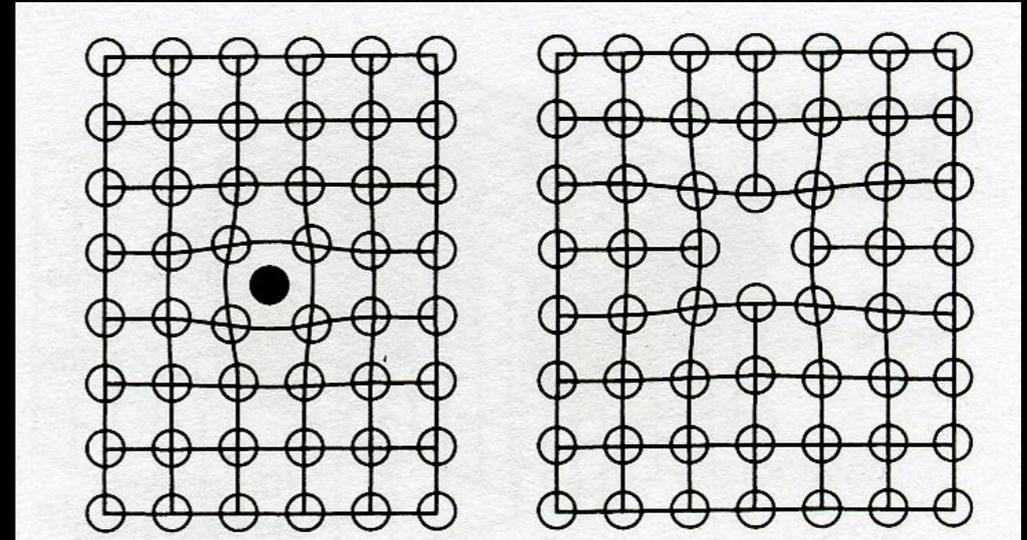
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    - two types of dislocations: “edge” and “screw”
    - each described by parallel or normal Burgers vector ( $b^*$ )



# Definition of creep

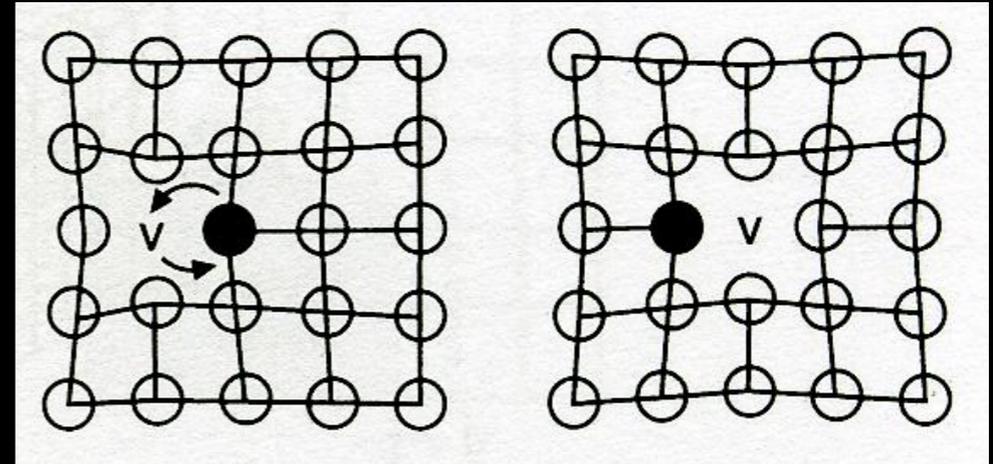
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  - line defects - dislocations which represent a rearrangement of atomic bonds
    - two types of dislocations: “edge” and “screw”
    - each described by parallel or normal Burgers vector ( $b^*$ )
- creep will occur through whichever mechanism requires least amount of energy



# Diffusion creep

point defects move by *diffusion*

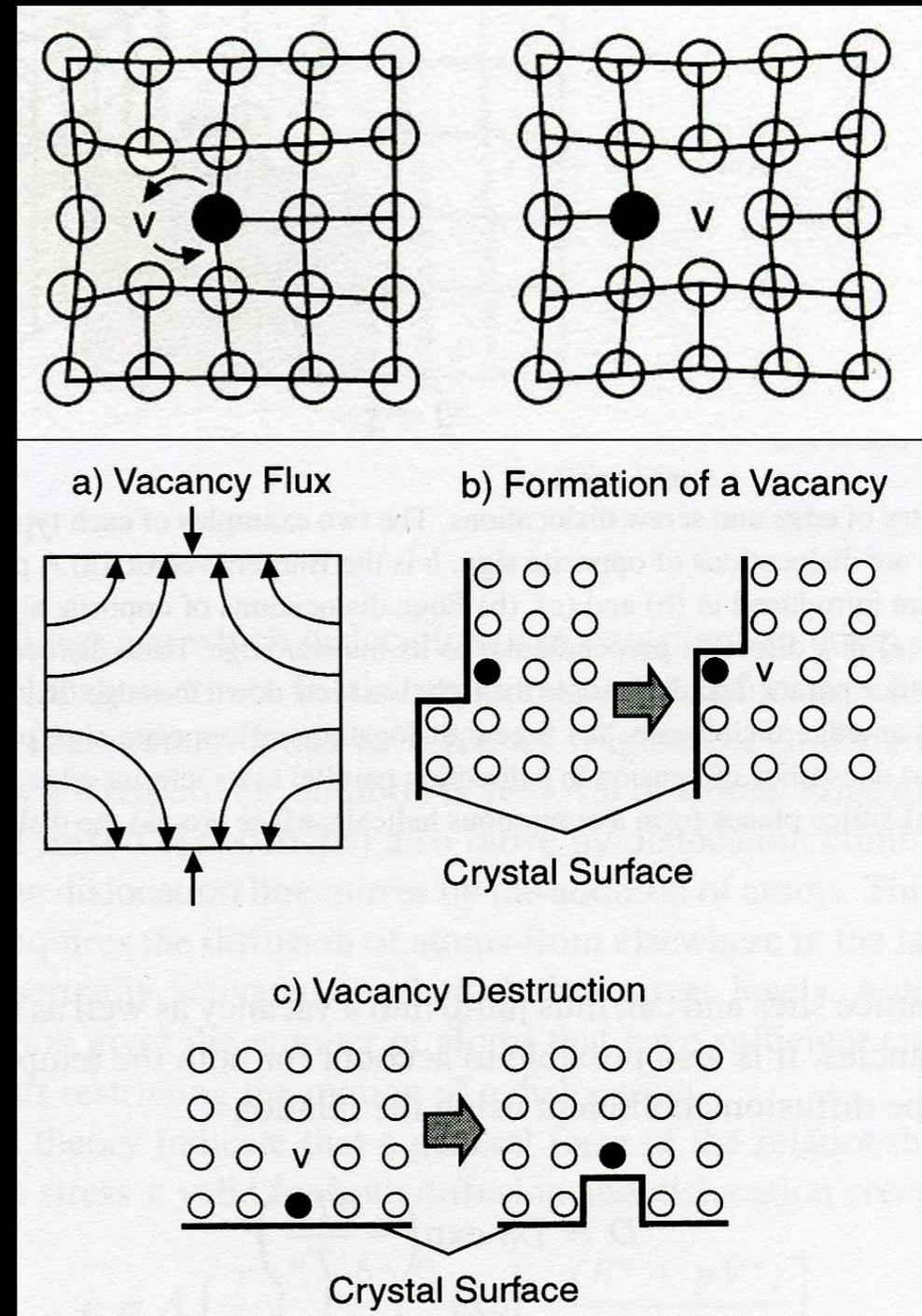
- through the crystal matrix (Nabarro-Herring)



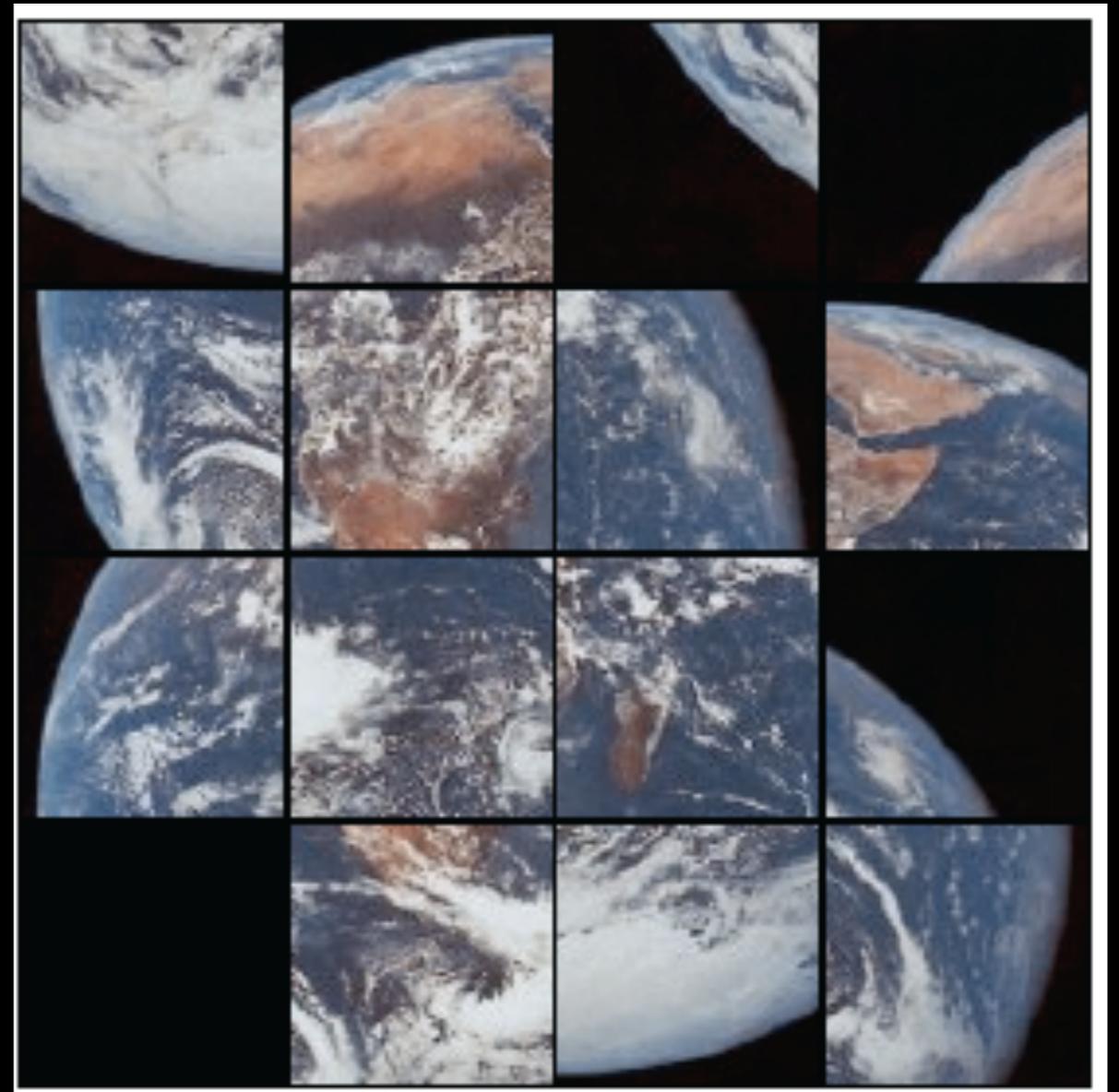
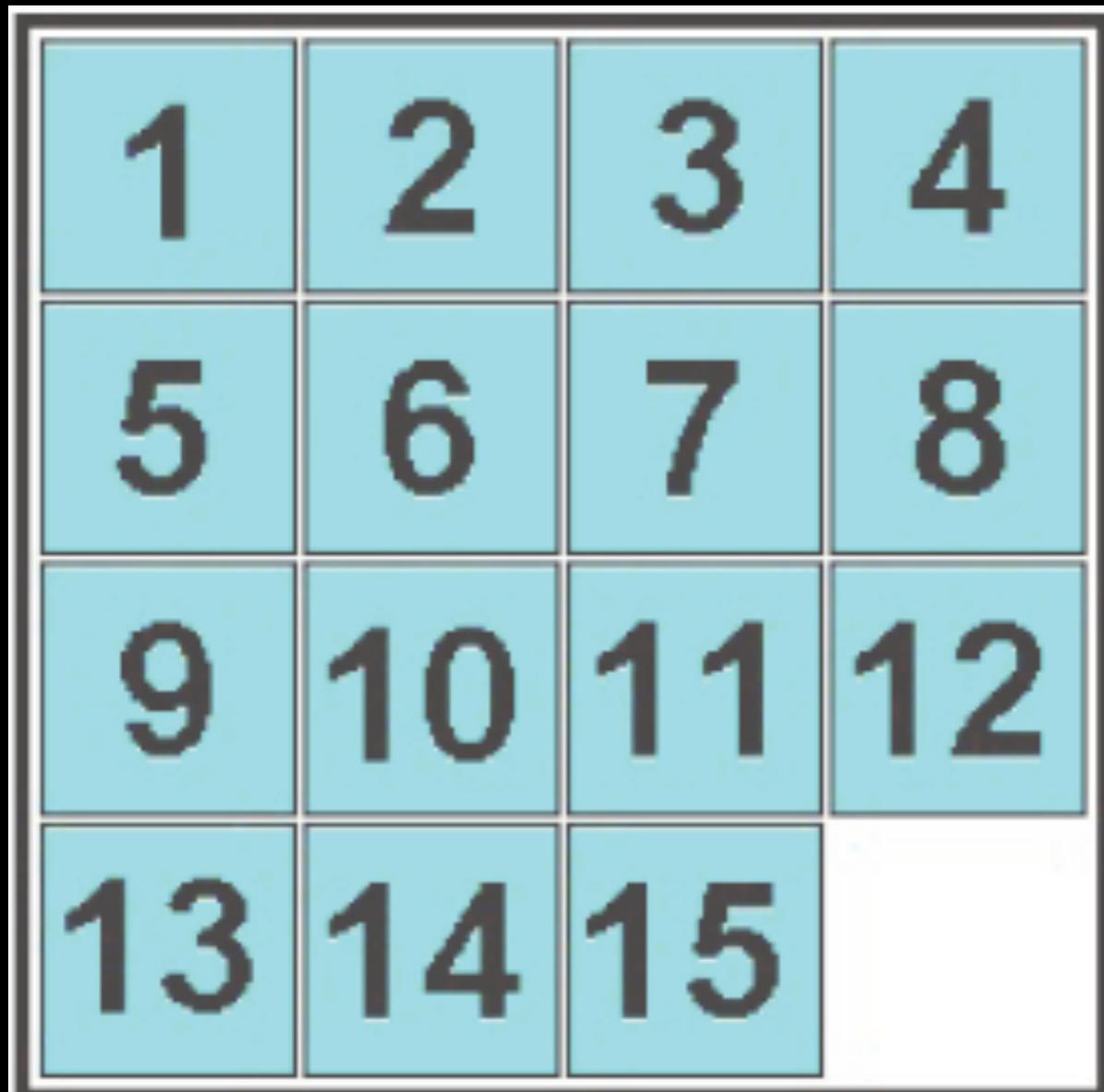
# Diffusion creep

point defects move by *diffusion*

- through the crystal matrix (Nabarro-Herring)
- along the grain boundaries (Coble)



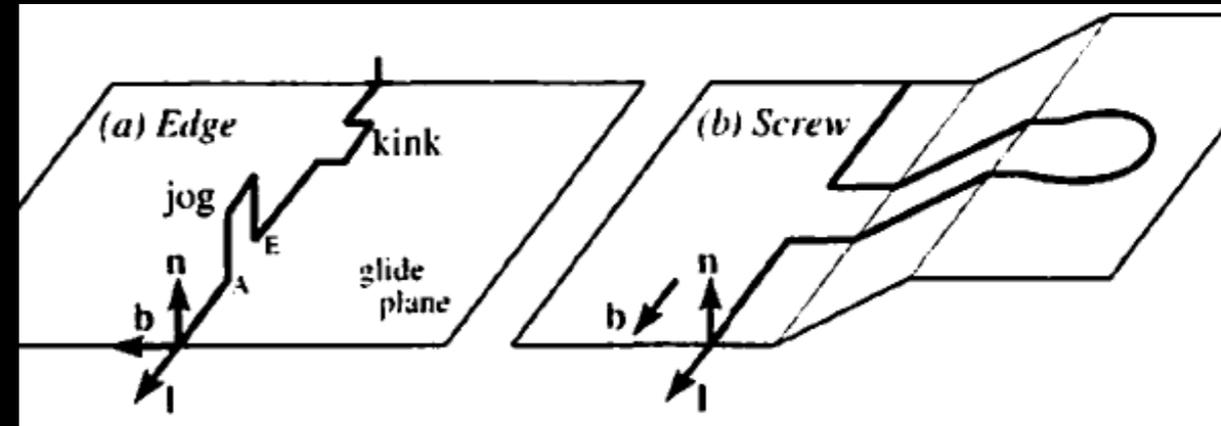
# Diffusion creep



# Dislocation creep

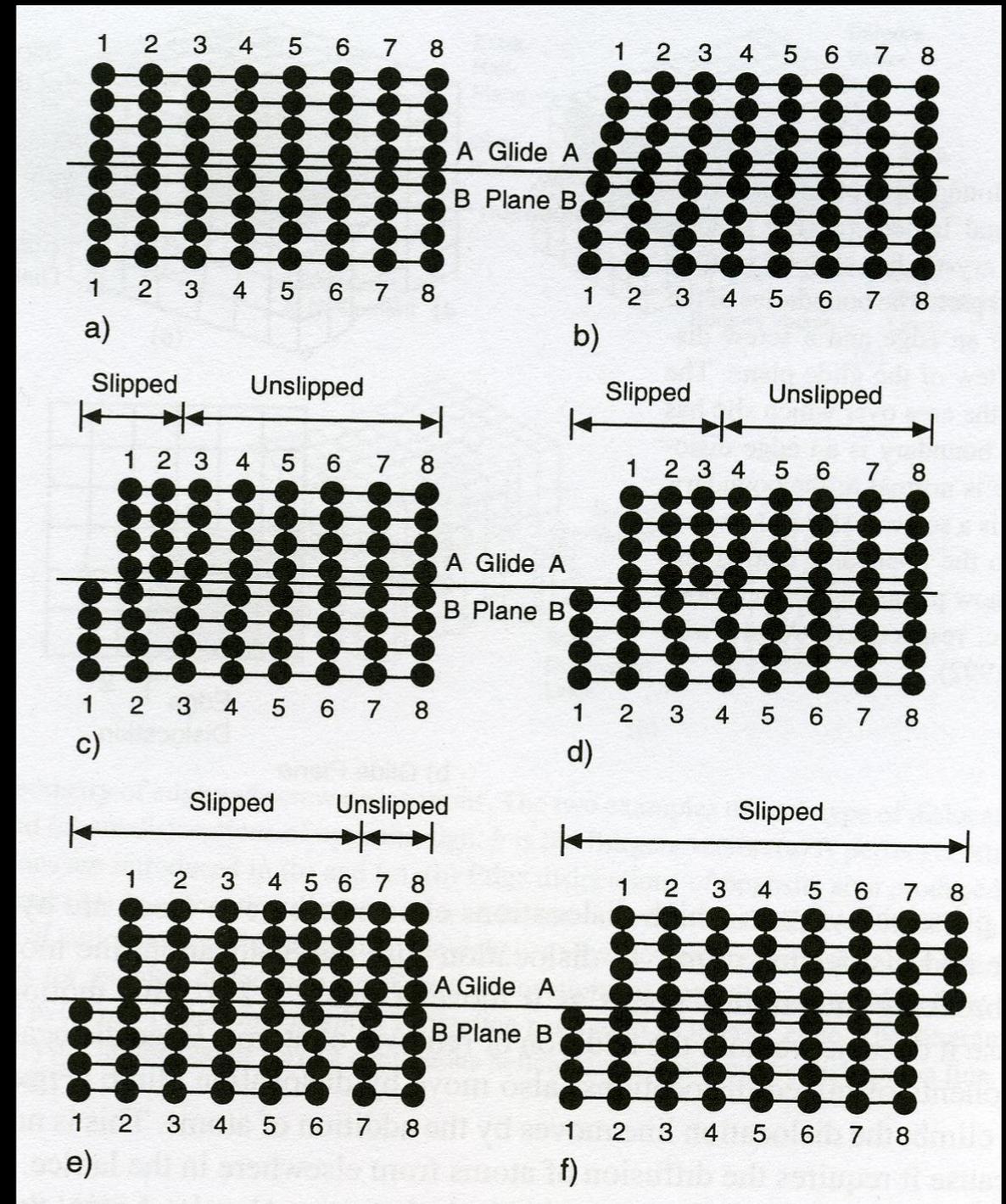
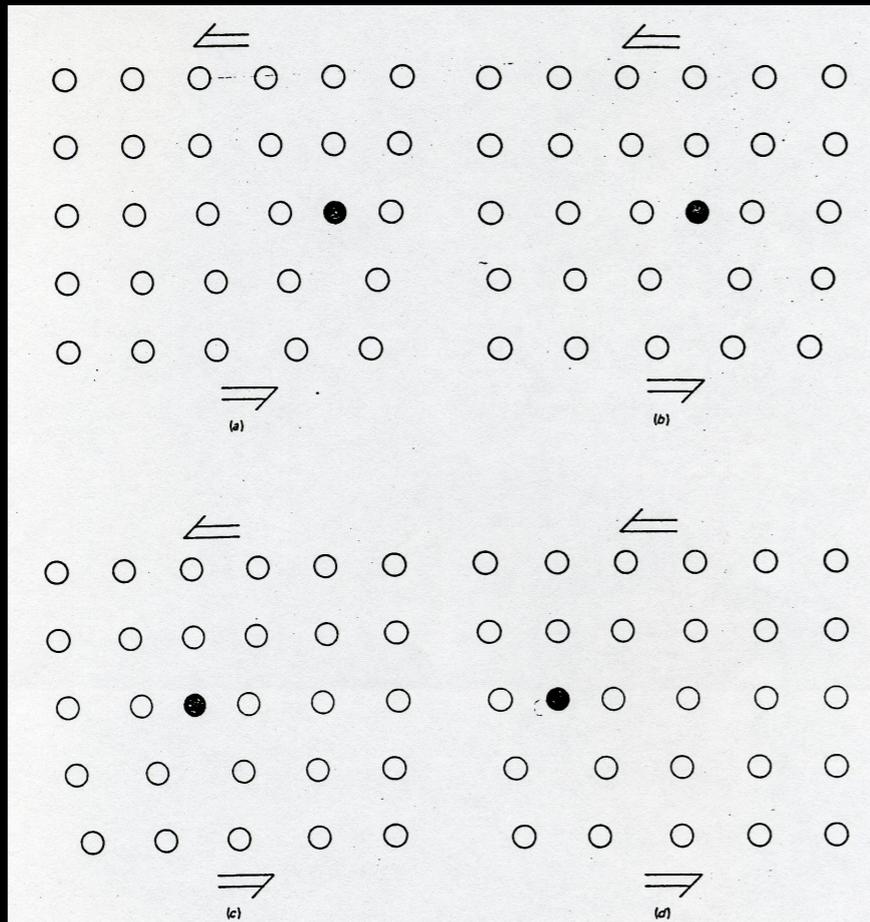
line defects move by *dislocation*

- two types of dislocations: “edge” and “screw”
- any line defect can be represented by linear combination of the two (simply add Burgers vectors)
- line dislocations have two types of motion: glide and climb
- independent of grain-size (point of difference with diffusion creep)



# Glide process of dislocation creep

- glide motion stays within glide plane





# Deformation maps

- for any given stress + temperature, one mechanism will be weaker (and preferred) over all others

$$\dot{\epsilon} = \dot{\epsilon}_{\text{disl}} + \dot{\epsilon}_{\text{diff}}$$

- map assumes constant grain size

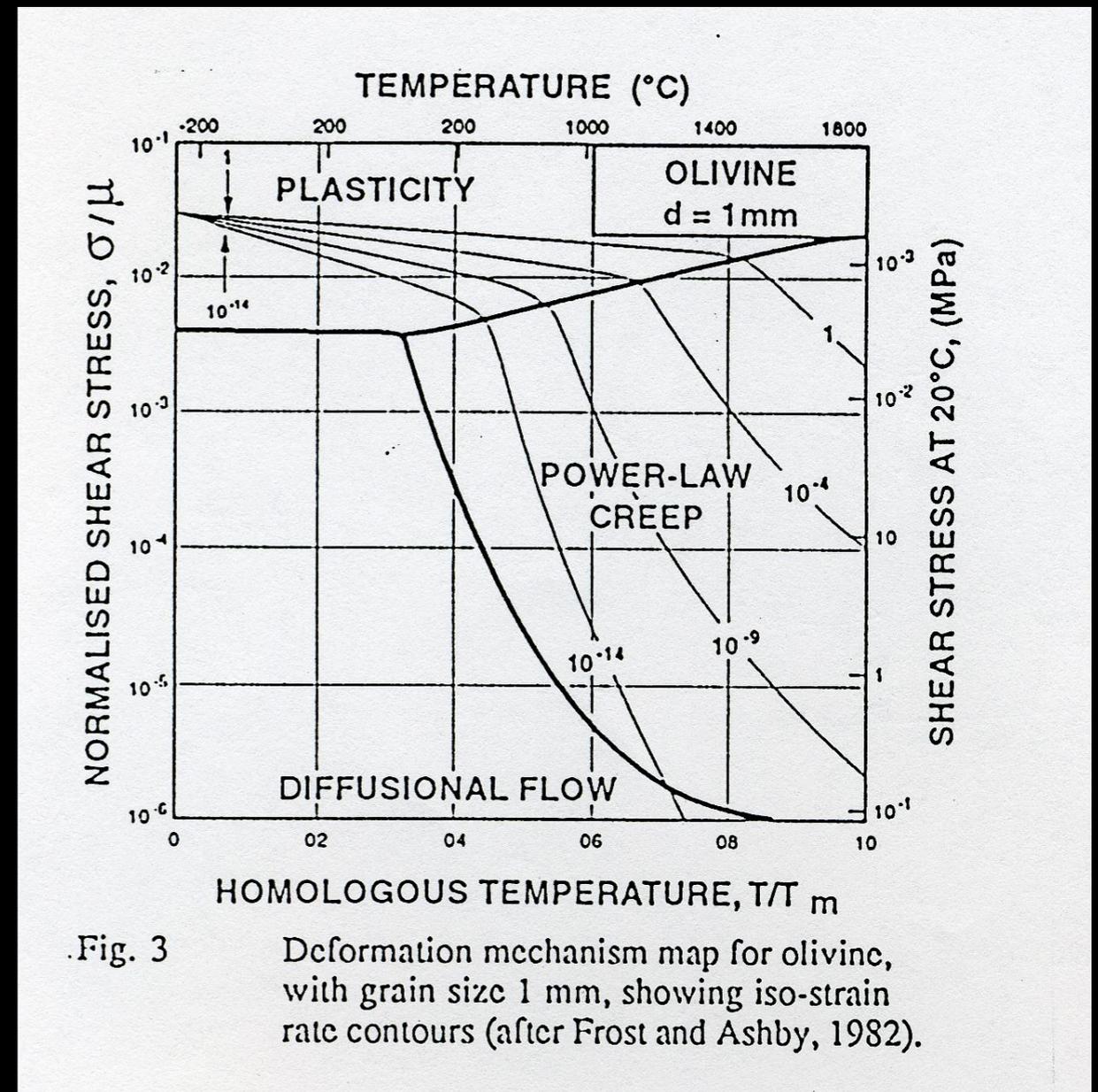
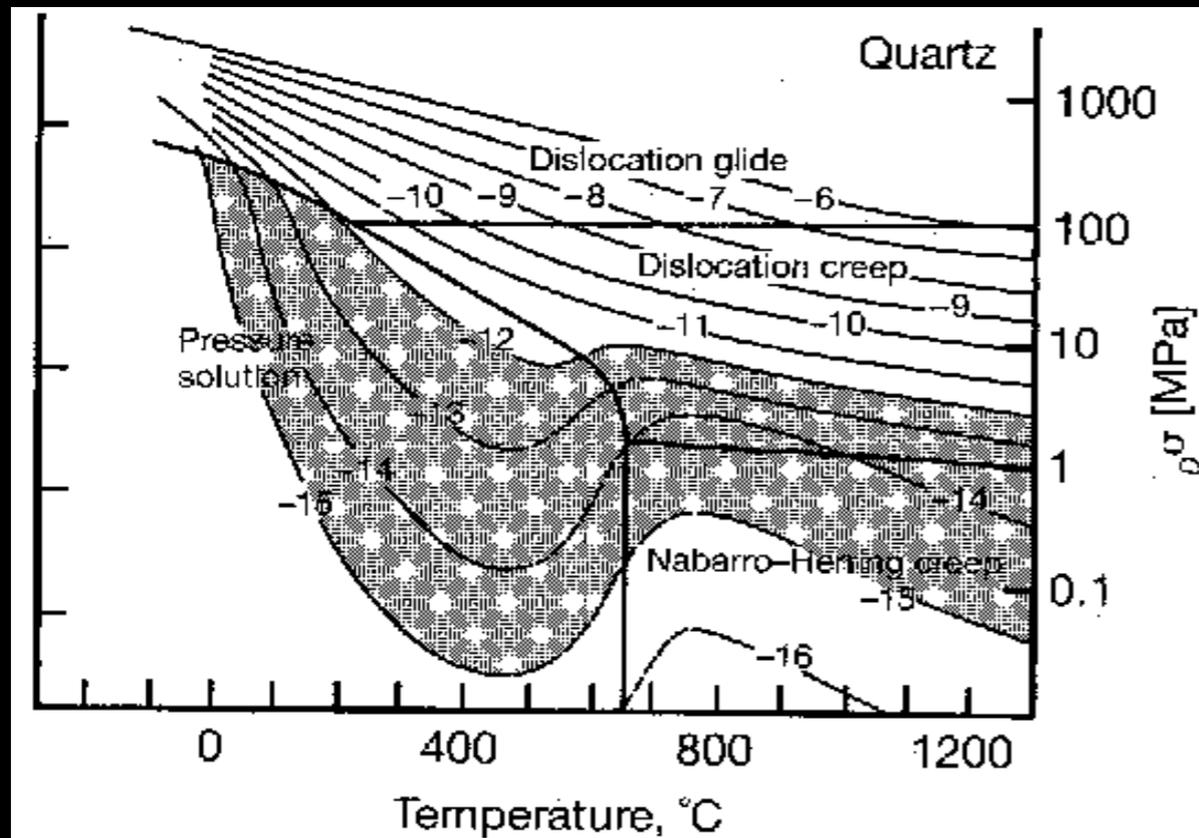
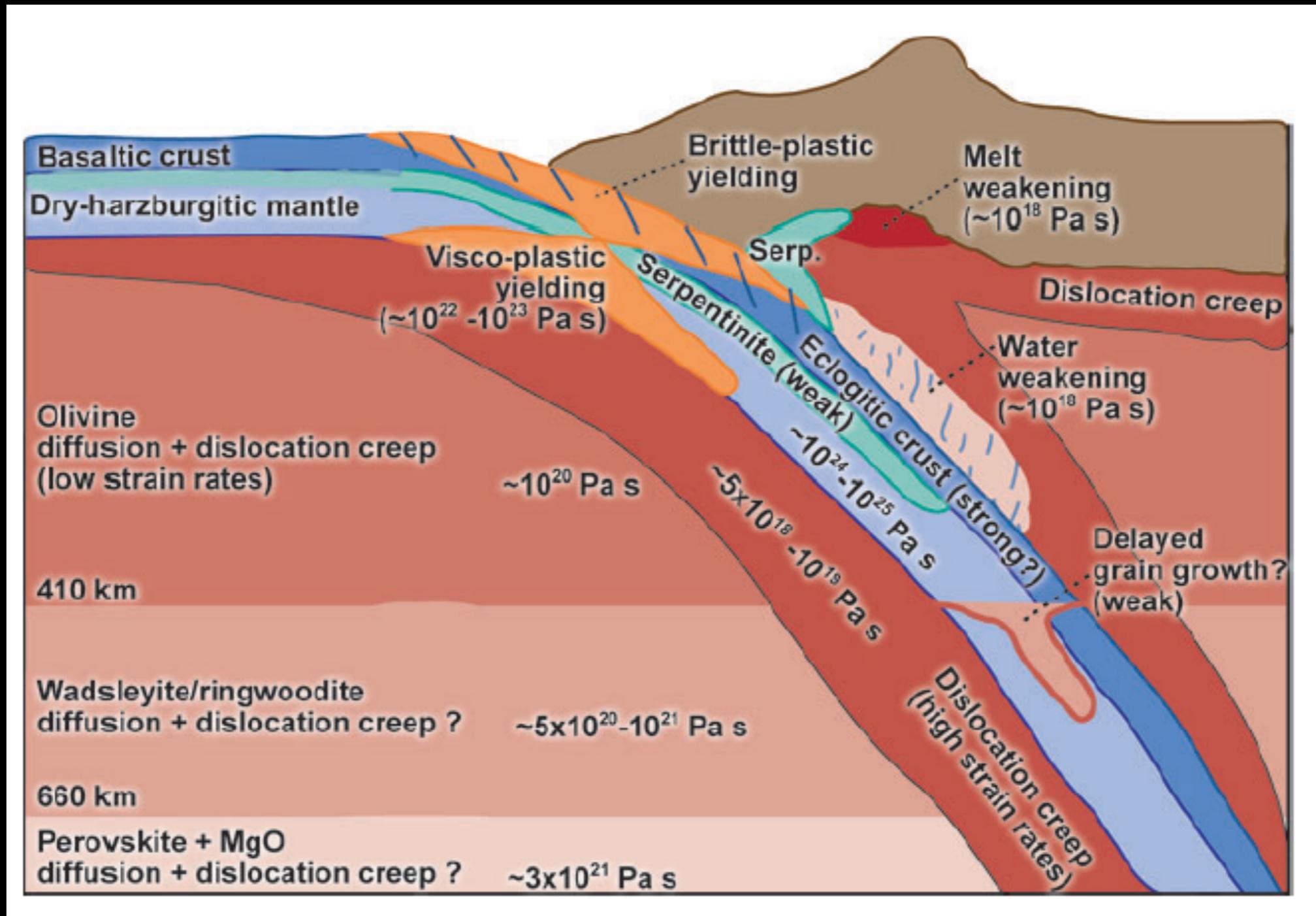


Fig. 3 Deformation mechanism map for olivine, with grain size 1 mm, showing iso-strain rate contours (after Frost and Ashby, 1982).

# Creep mechanisms in the mantle



Billen, Annual Rev. Geophys., 2008

# Flow rule

- relates the deformation (strain rate) to the applied deviatoric stress through a viscosity
- deformations add in series (viscosities add in parallel)

$$\sigma = \eta \dot{\epsilon}$$

$$\dot{\epsilon} = \frac{1}{\eta} \sigma$$

$$\dot{\epsilon}_{tot} = \dot{\epsilon}_1 + \dot{\epsilon}_2 + \dots$$

$$\frac{1}{\eta_{eff}} = \frac{1}{\eta_1} + \frac{1}{\eta_2} + \dots$$

# Flow rule

- for isotropic fluids, we can describe the flow rules with an effective strain rate, an effective stress, and an effective viscosity
- 2nd invariants of deviatoric stress and strain rates are scalars
- in practice, we know the strain-rates from the velocity field rather than stress, so viscosity is normally rewritten in terms of strain-rate

$$\sigma_{II} = \sqrt{\sigma : \sigma}$$

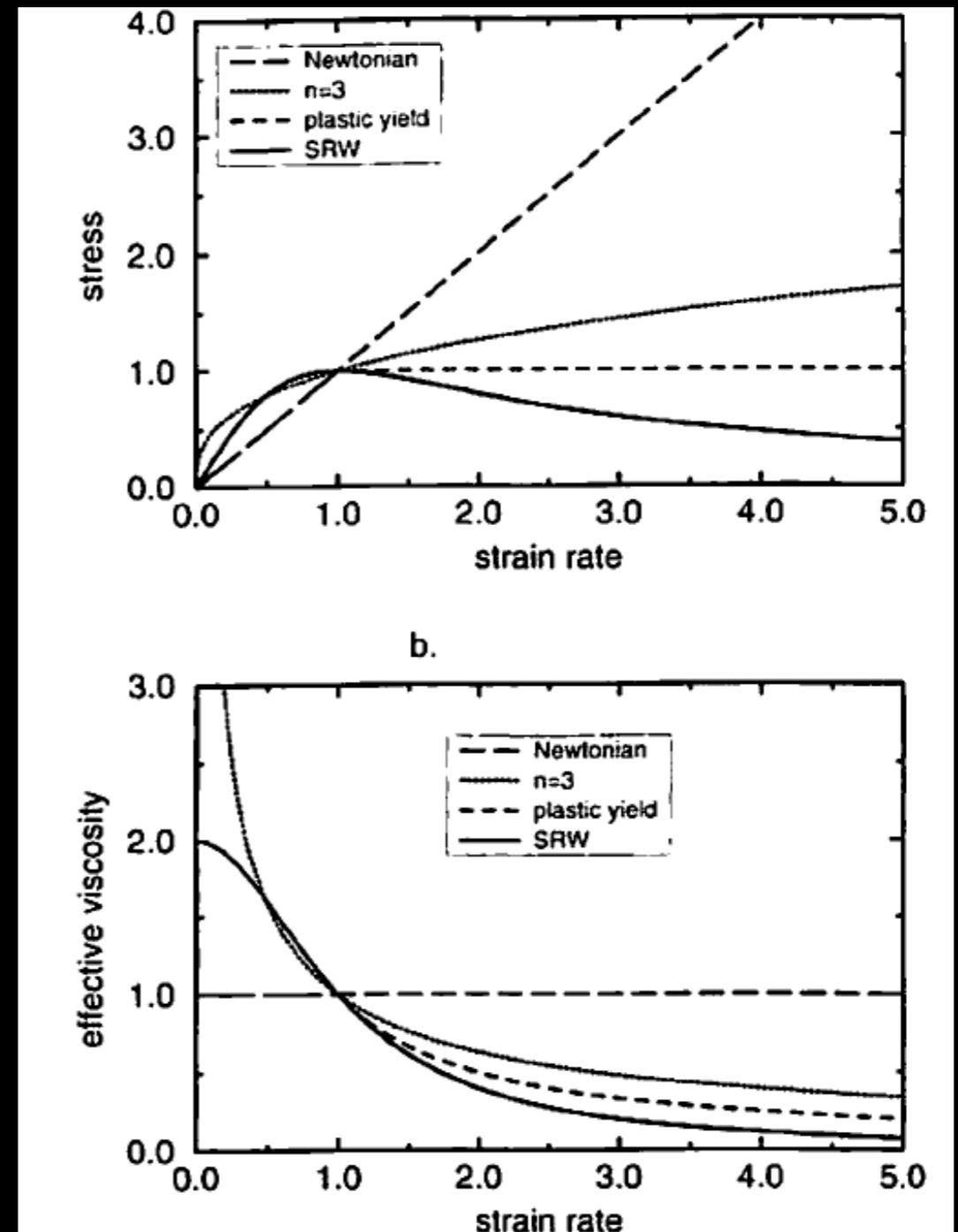
$$\dot{\epsilon}_{II} = \sqrt{\dot{\epsilon} : \dot{\epsilon}}$$

$$\sigma_{II} = \eta_{eff} \dot{\epsilon}_{II}$$

**NOTE:** contractions of these tensors usually have a 1/2 term times the sum of the squares of the components (when assuming co-axial compression / pure shear)

# Rheology: land of jargon and confusion

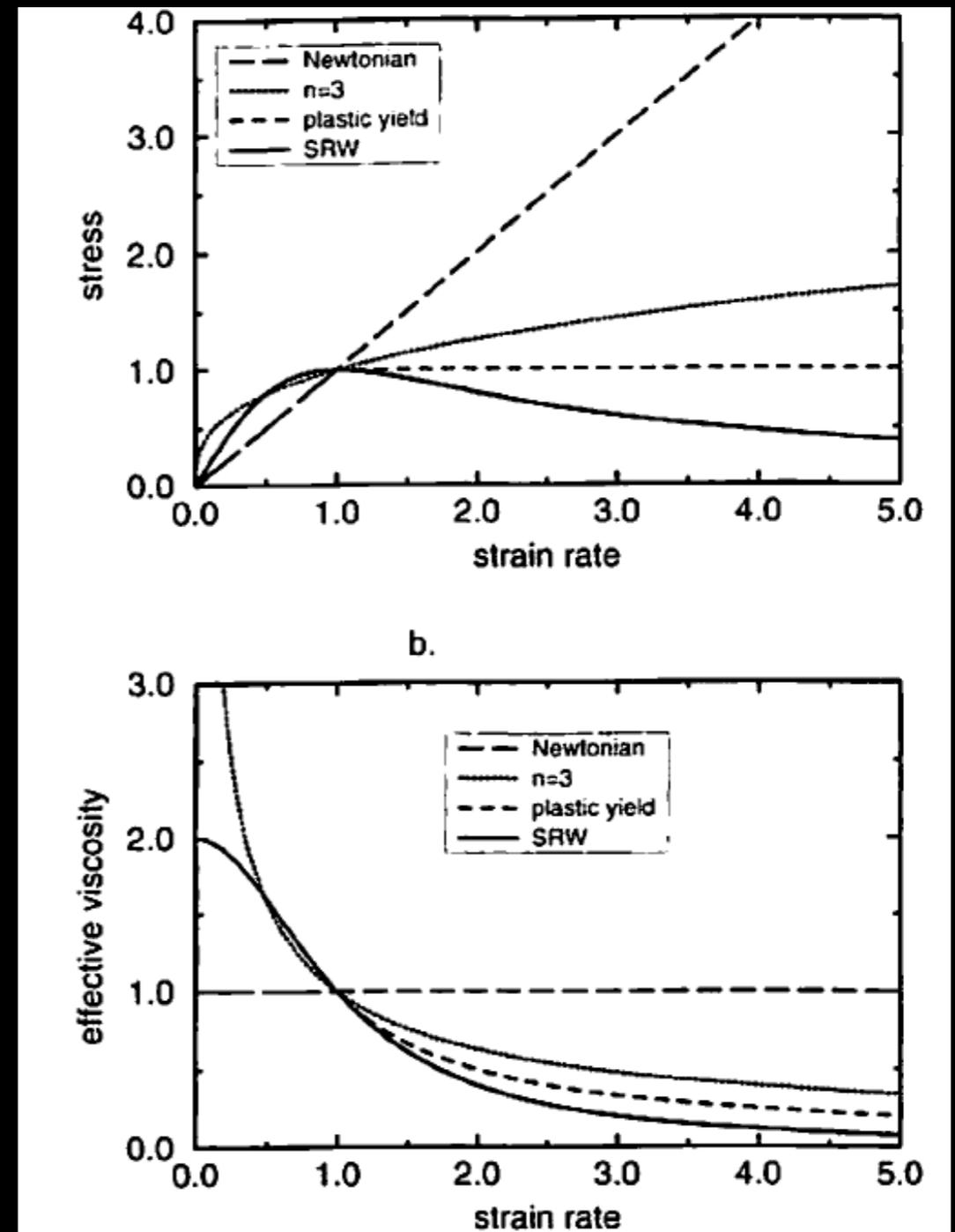
- **Newtonian** strictly means *linear* viscosity, but is commonly used to refer to *diffusion* creep with stress exponent ( $n=1$ )
- **Non-Newtonian** usually refers to *non-linear* rheology resulting from *dislocation* creep that is *stress dependent* through *power law* on stress with an exponent ( $n=3.5$ , or historically  $n=3$ )
- **Example:** diffusion creep is thermally activated and pressure dependent, and thus has exponential sensitivity to  $T,P$  but is sometimes referred to as “Newtonian” even though it is a non-linear  $f(T,P)$



Tackley, 2000

# Comparison of deformation processes

- any non-linear rheology can be represented by an effective linear rheology
- the effective viscosity is simply the slope of the line drawn from the origin to the stress-strainrate curve
- example of effective viscosity for several different rheologies that may be important for the mantle + plates
- think a little more about these curves stress vs. effort, efficiency vs. effort, and total productivity vs. effort



Tackley, 2000

# Arrhenius dependence

- one can use thermodynamics to describe the sensitivity of diffusion when it is thermally activated
- the diffusion of vacancies, etc has an exponential dependence on T,P
- this can also be understood in terms of the homologous temperature
- the deformation resulting from the diffusion creep is the strain rate and is inversely proportional to viscosity
- the Arrhenius term describes the exponential behavior of viscosity
- also valid for dislocation creep

$$D = D_0 \exp \left[ -\frac{(E_A + P V_A)}{RT} \right]$$

$$D = D_0 \exp \left[ -g \frac{T_m(P)}{T} \right]$$

$$\dot{\epsilon} = \frac{1}{\eta} \sigma$$

$$\eta(T, P) = \eta_0 \exp \left[ \frac{(E_A + P V_A)}{RT} \right]$$

# General flow rule for mantle material

$$\dot{\varepsilon} = A \left( \frac{b^*}{d} \right)^m \left( \frac{\sigma}{\mu} \right)^n \exp \left[ -\frac{(E_A + P V_A)}{RT} \right]$$

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$$\dot{\varepsilon} = \frac{A}{\mu} \left( \frac{b^*}{d} \right)^m \sigma \left( \frac{\sigma}{\mu} \right)^{n-1} \exp \left[ -\frac{(E_A + P V_A)}{RT} \right]$$

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$$\frac{1}{\eta} = \frac{A}{\mu} \left( \frac{b^*}{d} \right)^m \left( \frac{\sigma}{\mu} \right)^{n-1} \exp \left[ -\frac{(E_A + P V_A)}{RT} \right]$$

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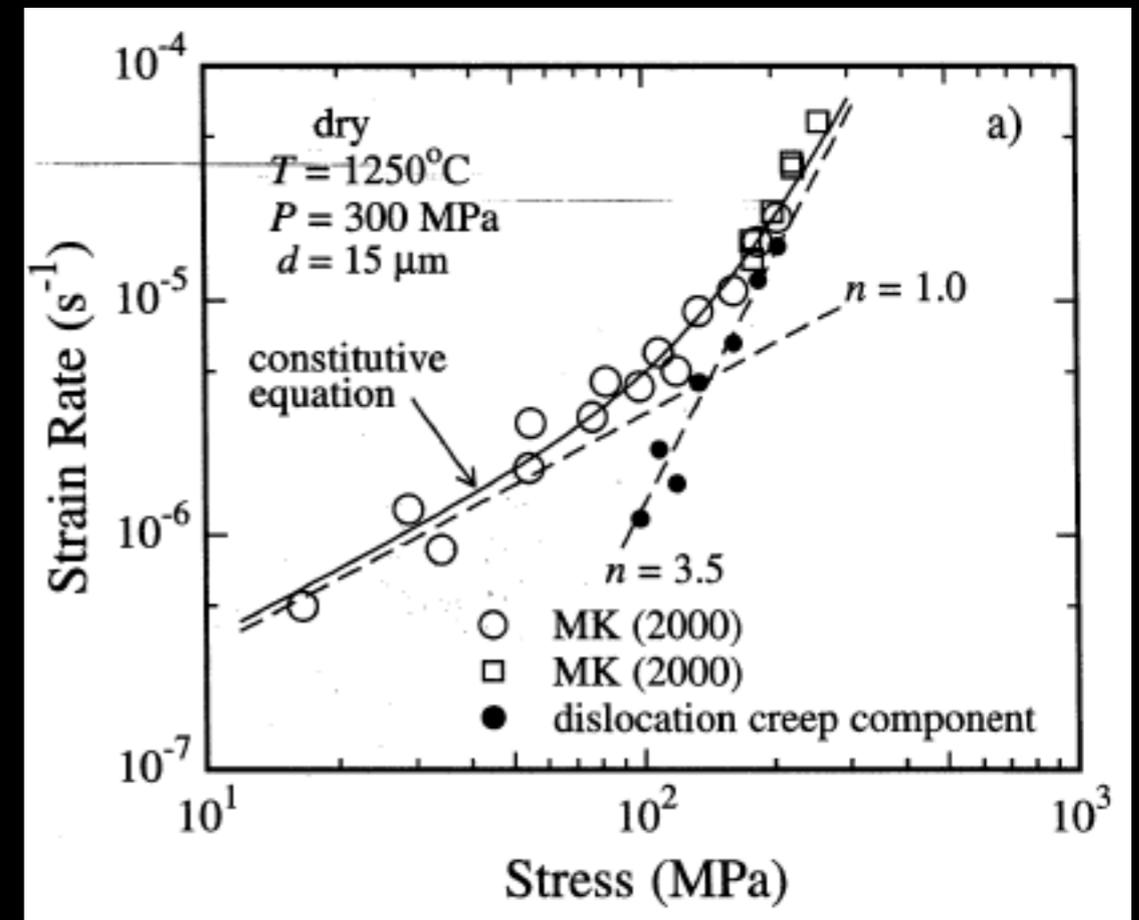
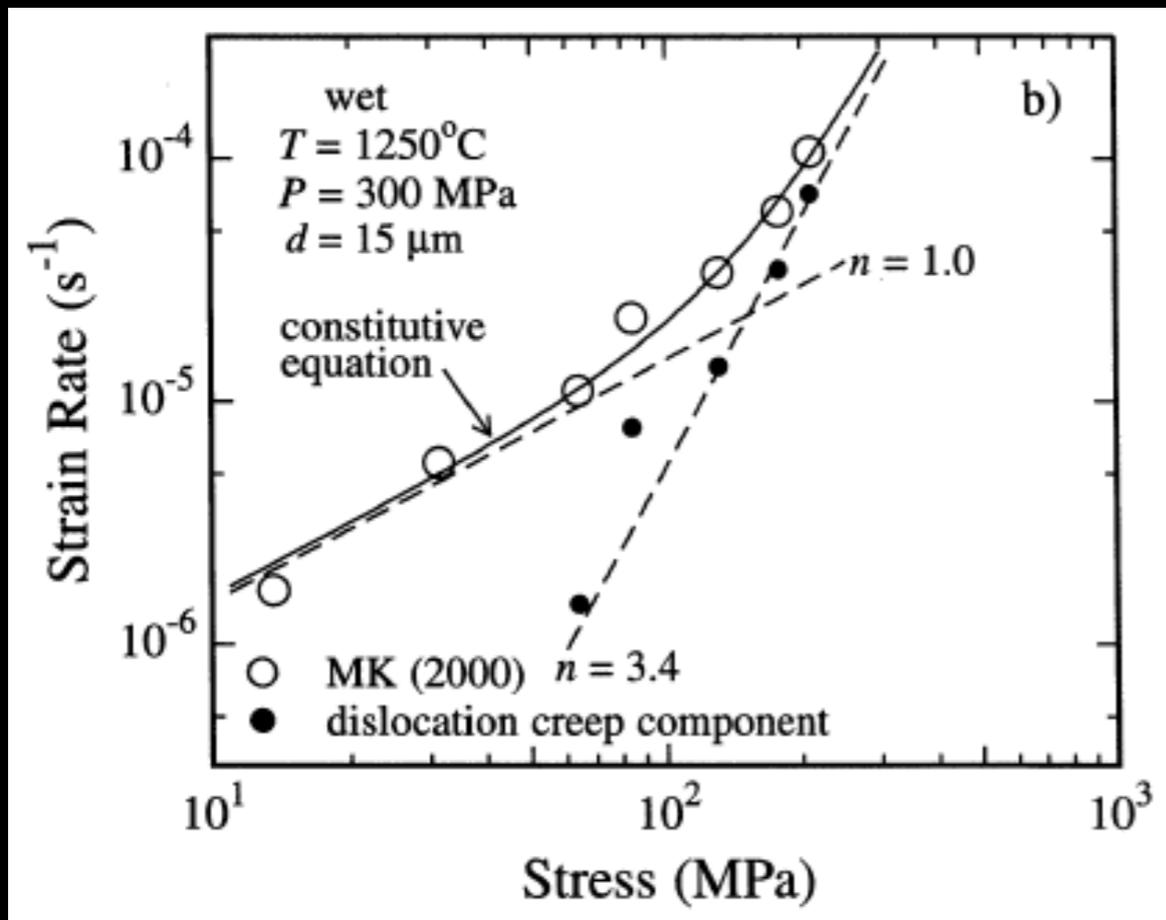
$$\dot{\epsilon}_{tot} = \dot{\epsilon}_{diff} + \dot{\epsilon}_{disl}$$

$$\frac{1}{\eta_{eff}} = \frac{1}{\eta_{diff}} + \frac{1}{\eta_{disl}}$$

- **note: diffusion and dislocation creep have different activation enthalpies**

# What is “n” for dislocation creep?

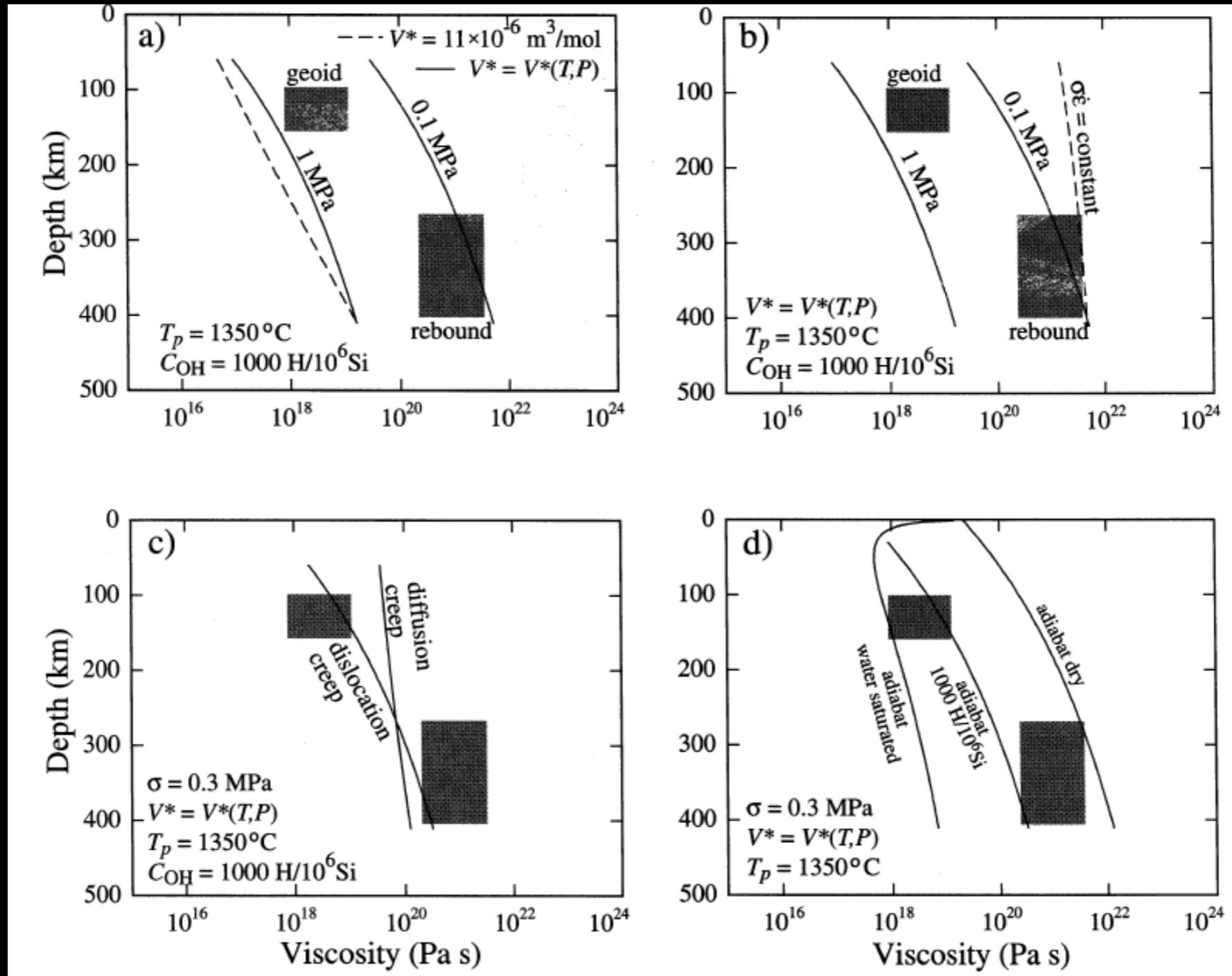
$$\dot{\epsilon}_{tot} = \dot{\epsilon}_{diff} + \dot{\epsilon}_{disl}$$



*Hirth and Kohlstedt, 2003*

- note: different y-intercept corresponds to different activation enthalpies which is due to the influence of water

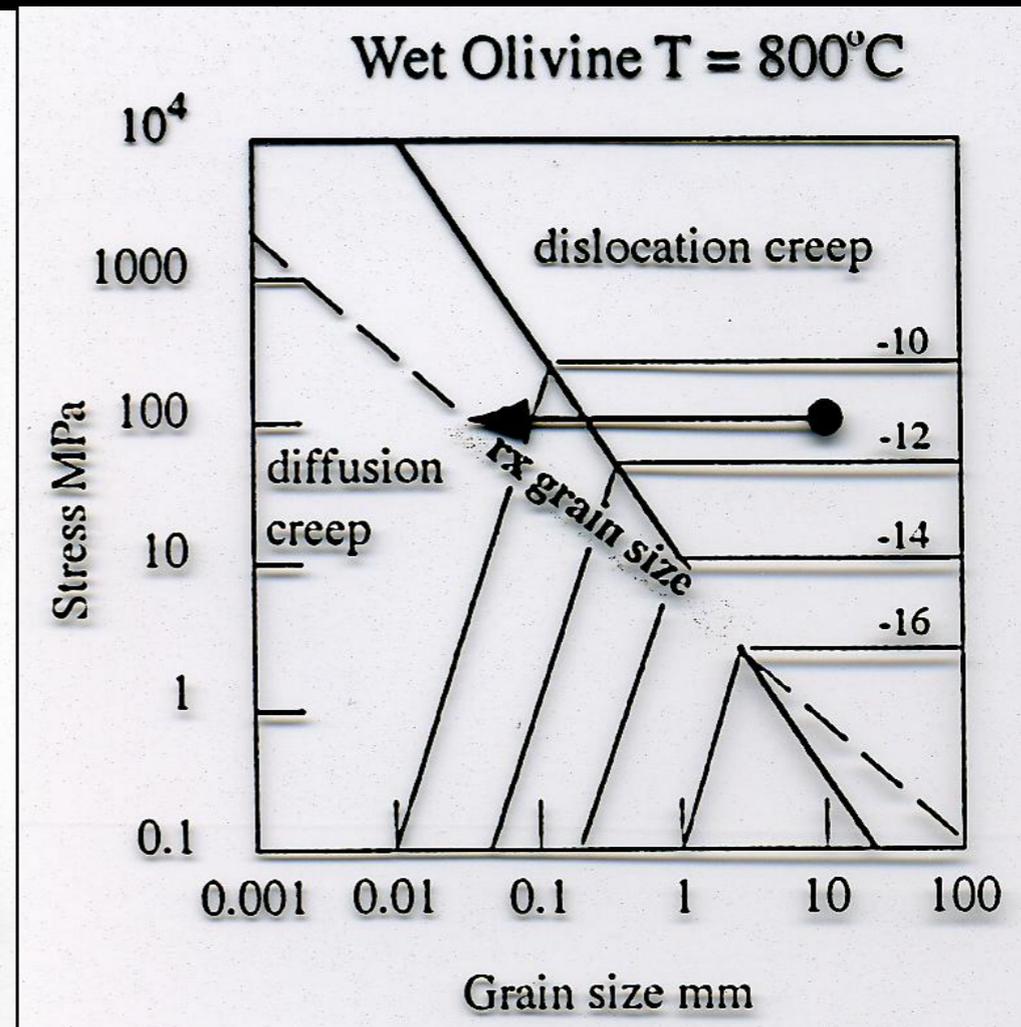
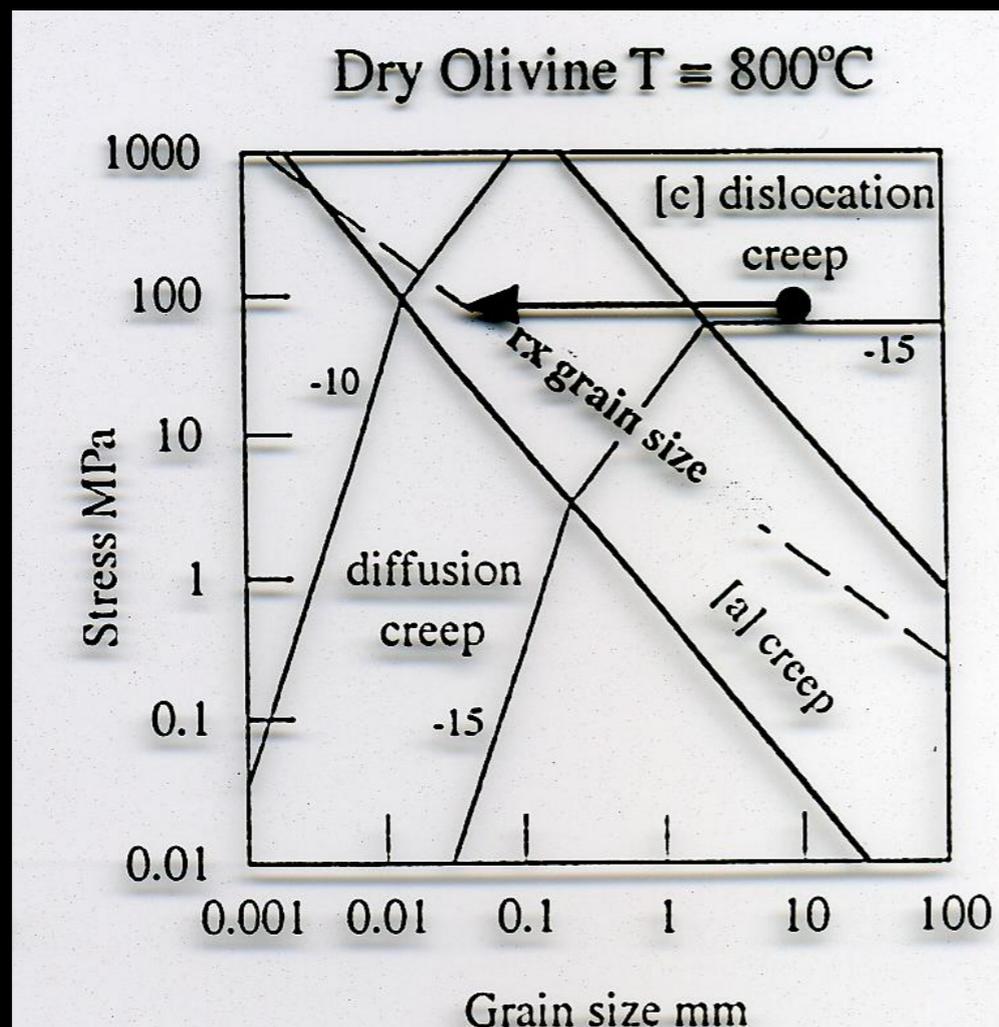
# Extrapolating to mantle conditions



Hirth and Kohlstedt, 2003

# Just add water...

- contour lines are strain rates in (1/s) - dislocation is horizontal because it is independent on grain size but diffusion creep shows grain size dependence
- much lower stresses required to achieve same strain rate when water is present (note different y-axis) which translates into lower activation enthalpy



# Just add water...

- contour lines are strain rates in (1/s) - dislocation is horizontal because it is independent on grain size but diffusion creep shows grain size dependence
- much lower stresses required to achieve same strain rate when water is present (note different y-axis)
- translates into lower activation energy and volume ( $E^*$  and  $V^*$ )

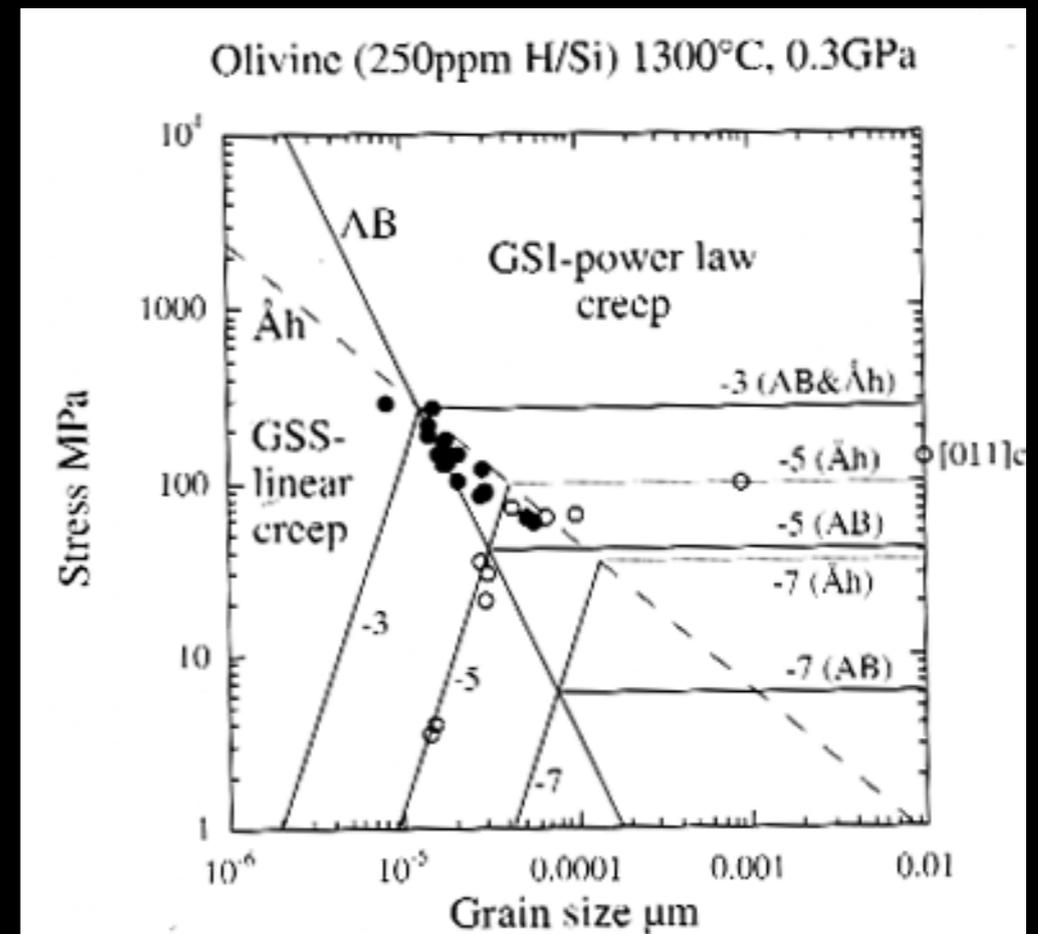
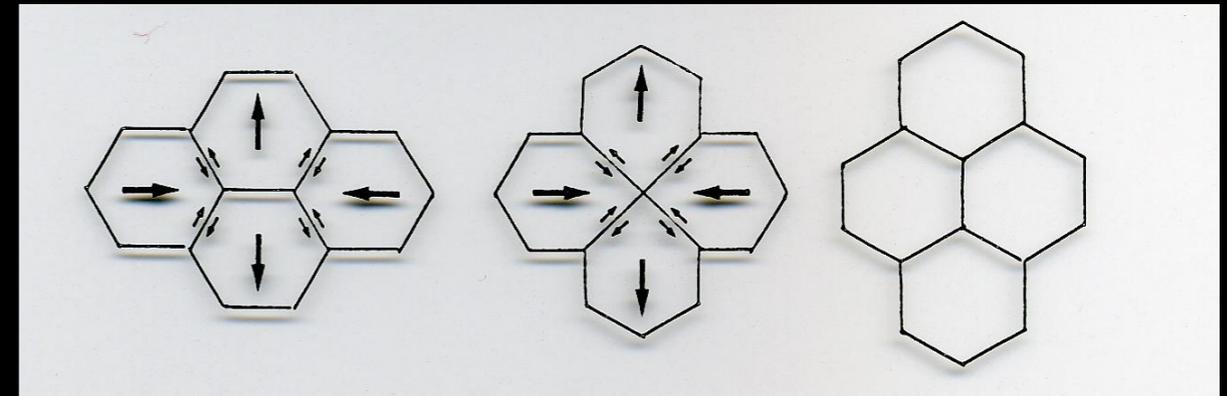
$$\dot{\epsilon}(C_{OH}) = A \left( \frac{b^*}{d} \right)^m \left( \frac{\sigma}{\mu} \right)^n \exp \left[ -\frac{(E_A^* + P V_A^*)}{RT} \right]$$

# Grain size sensitive creep

- GSS creep relies on combination of mechanisms: grain boundary sliding (GBS) and basal slip along the easiest slip system (easy)

$$\dot{\epsilon} = \dot{\epsilon}_{\text{disl}} + \dot{\epsilon}_{\text{diff}} + [1/\dot{\epsilon}_{\text{gbs}} + 1/\dot{\epsilon}_{\text{easy}}]^{-1}$$

- GSS operates at small grain size only relevant to diffusion creep
- accommodates large amounts of strain without deforming crystals (referred to as superplasticity)
- GSS important process in lower mantle with small grain size (Karato et al., Science, 1995) through extrapolation to high P, low strainrate



Drury, 2005

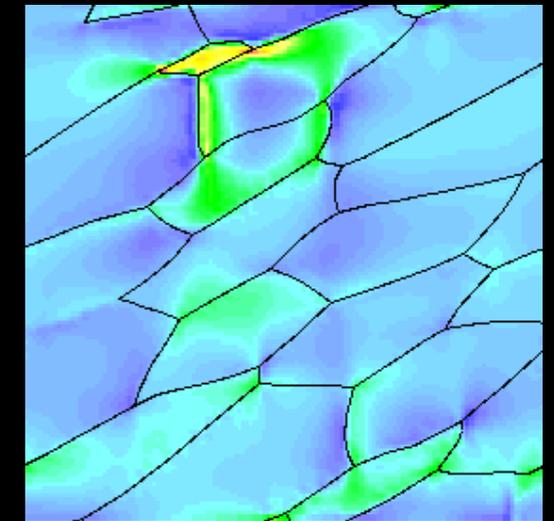
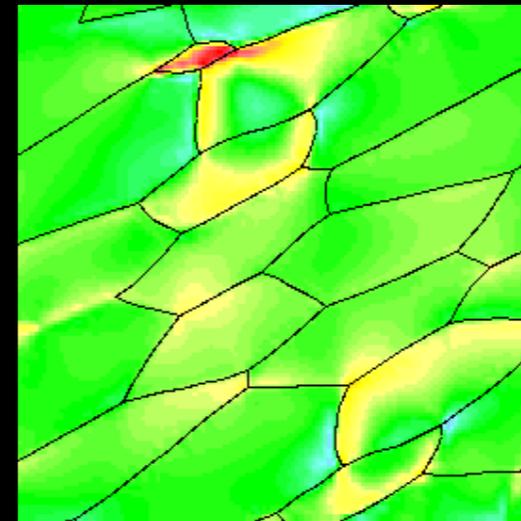
# Recrystallization

- dynamic process changing the distribution of grain sizes
- grain size reduction (large grains fracture) and growth (small grains fuse together)
- microstructure important feedback with dislocation creep
- time-dependent process

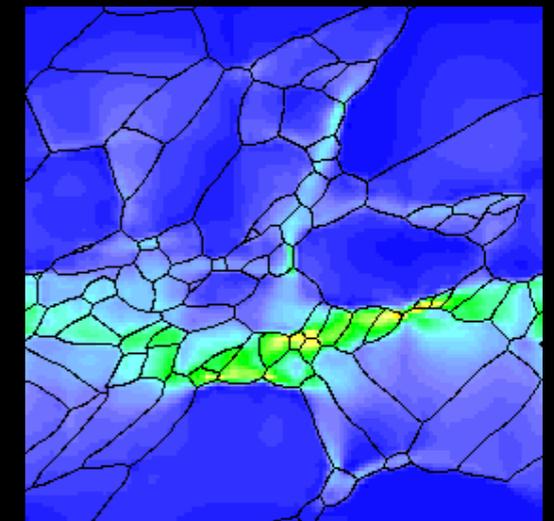
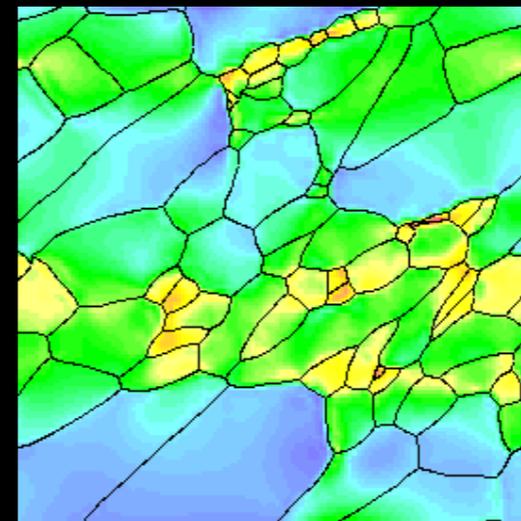
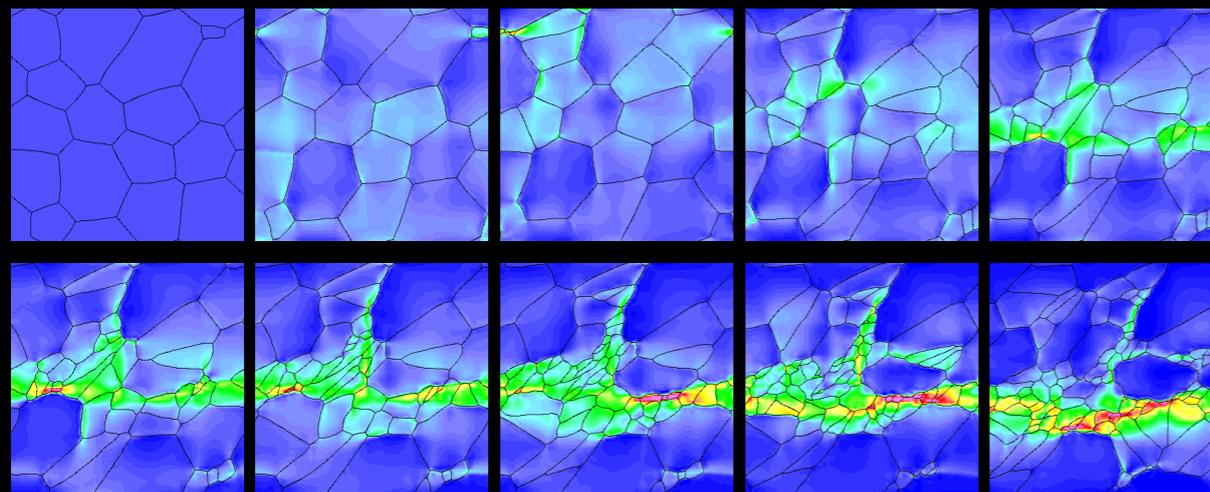
linear ( $n=1$ )

power law ( $n=3.5$ )

without recrystallization



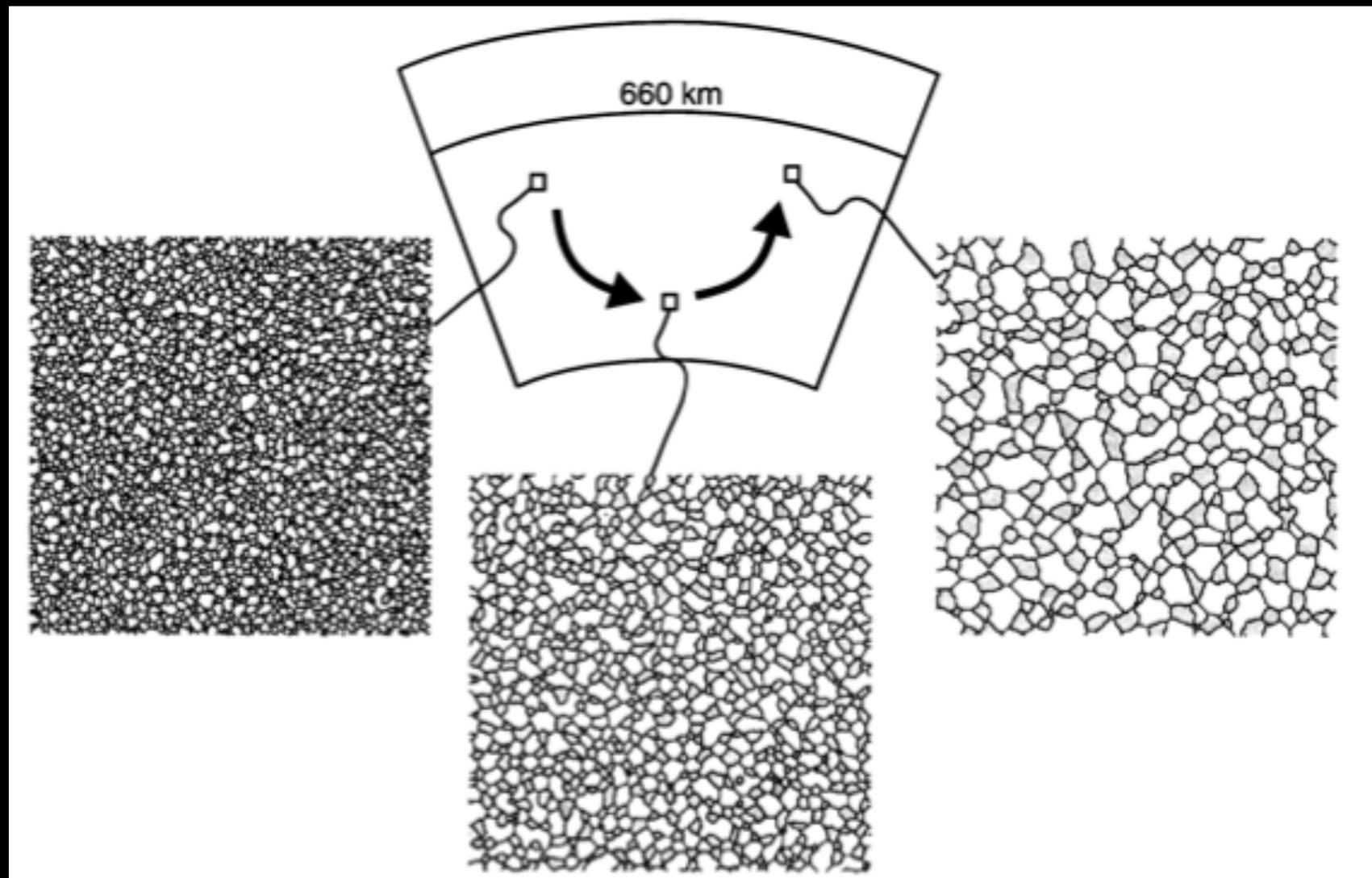
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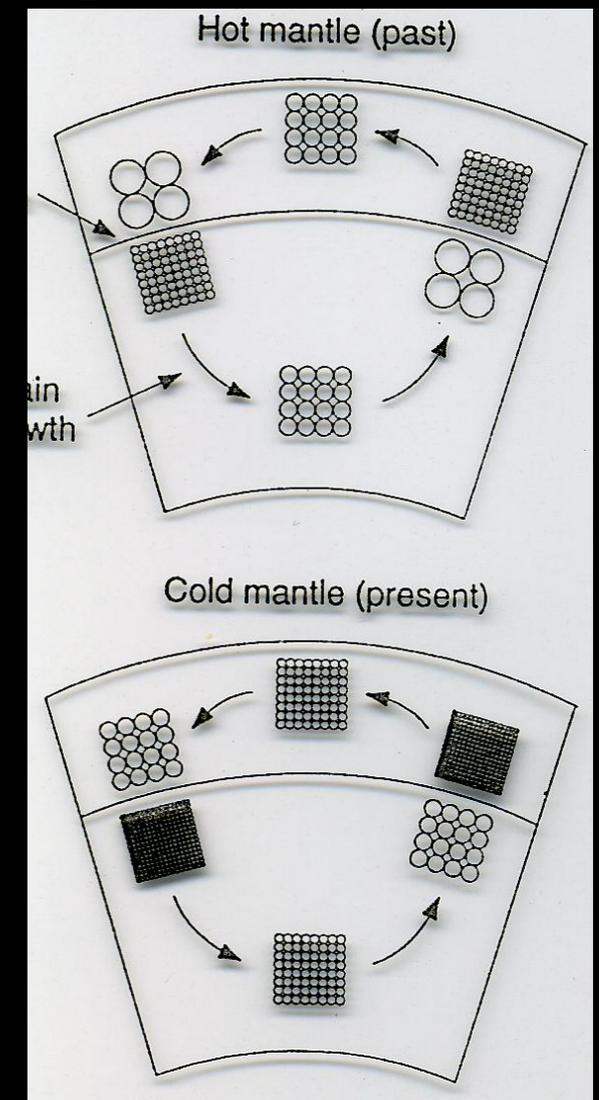
*Jessell et al, EPSL, 2005*

# Grain growth in the lower mantle

- recrystallization for material going down through 660 km phase change
- diffusion creep likely dominant mechanism for lower mantle but if the grain size is sufficiently small, then superplasticity could be important mechanism



Solomatov, *EPSL*, 2001



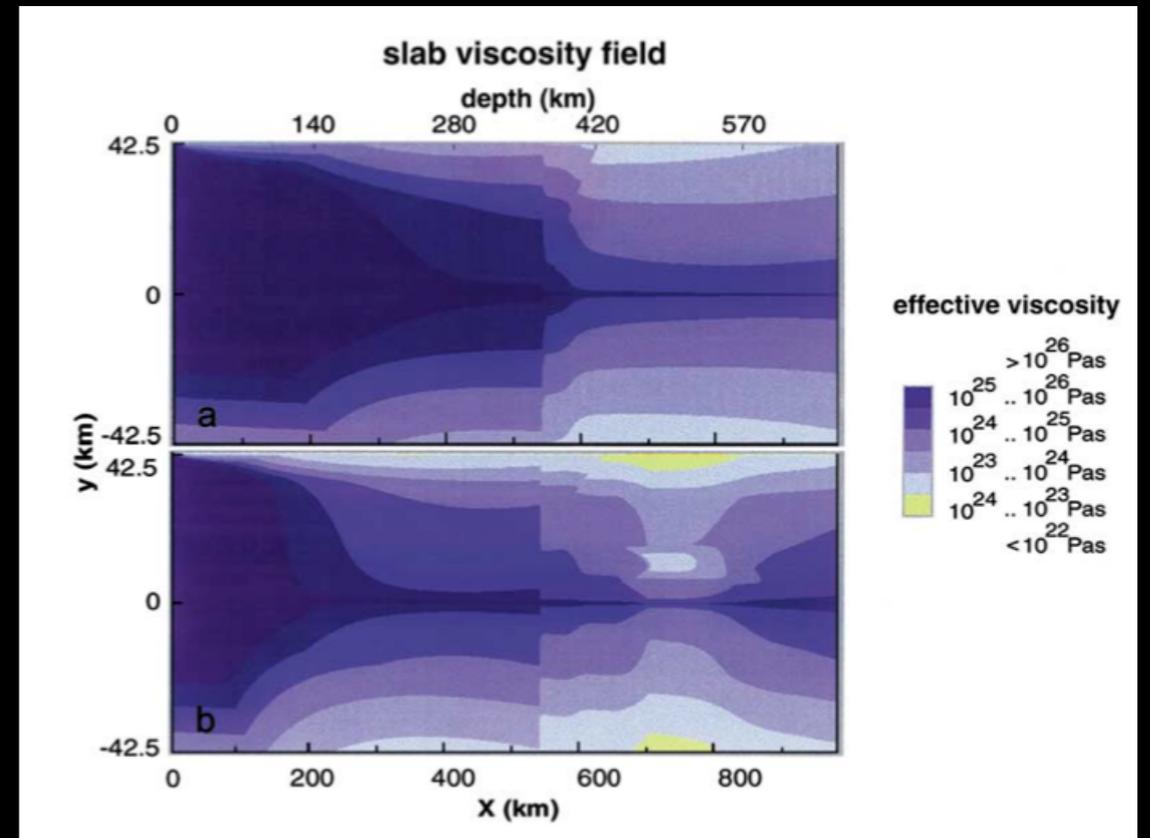
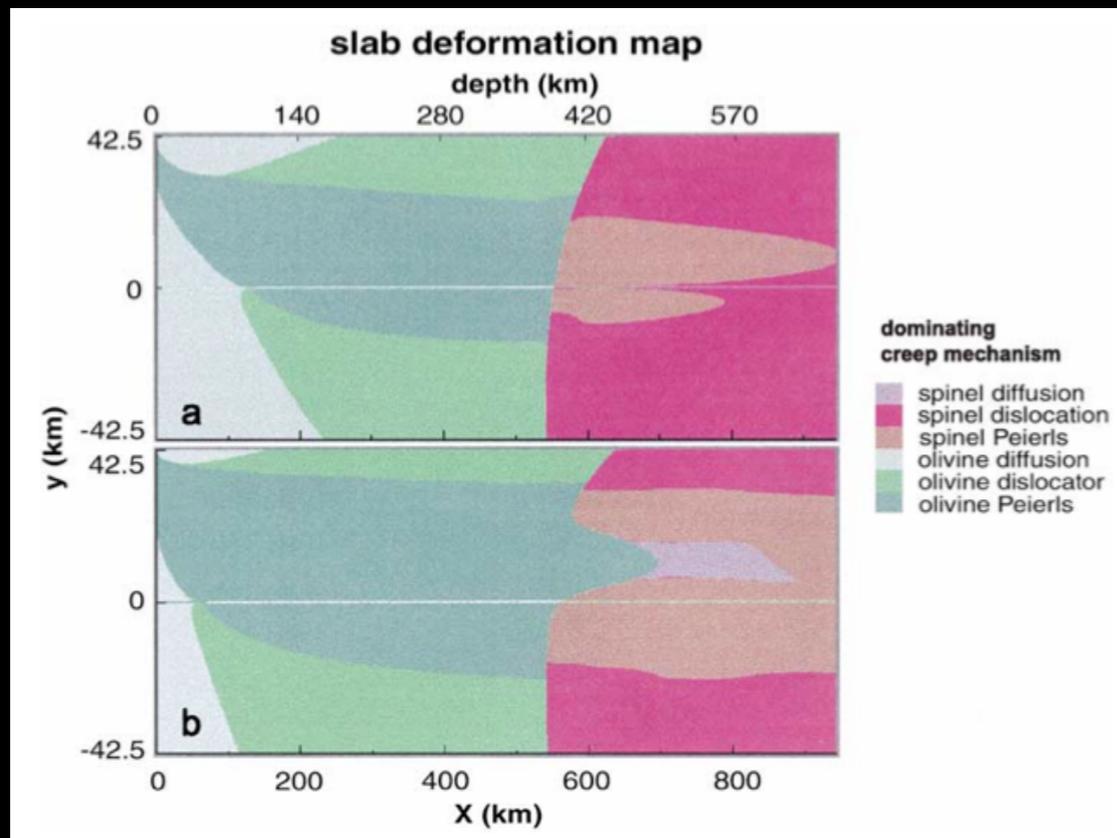
# Harper-Dorn creep

- low-temperature plasticity OR Peierls stress mechanism\*

$$\dot{\epsilon} = A \left( \frac{b^*}{d} \right)^m \left( \frac{\sigma}{\mu} \right)^n \exp \left[ -\frac{(E_A + P V_A)}{RT} \left( 1 - \frac{\sigma}{\sigma_p} \right)^q \right]$$

\*read about Peierls other contributions to science here: [https://en.wikipedia.org/wiki/Rudolf\\_Peierls](https://en.wikipedia.org/wiki/Rudolf_Peierls)

- applicable to the interior of a subducted slab (below: 4 cm/yr and 10 cm/yr)



Karato et al, PEPI, 2001

# Generalized flow rule for a slab

$$\dot{\epsilon}_{tot} = \dot{\epsilon}_{diff} + \dot{\epsilon}_{disl} + \dot{\epsilon}_{H-D}$$

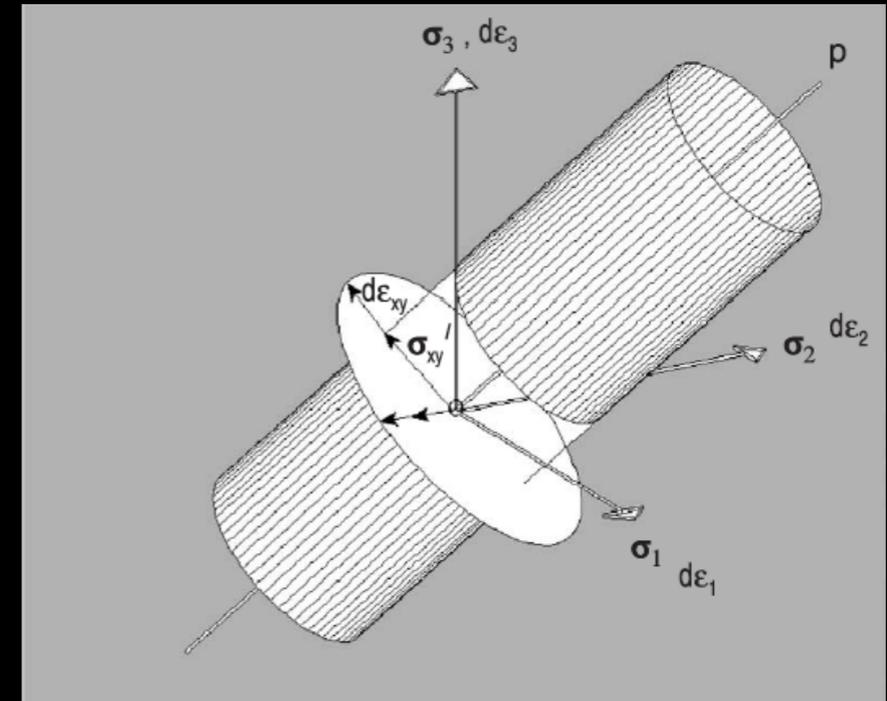
$$\frac{1}{\eta_{eff}} = \frac{1}{\eta_{diff}} + \frac{1}{\eta_{disl}} + \frac{1}{\eta_{H-D}}$$

- diffusion, dislocation, Harper-Dorn creep mechanisms work independently, so in general, only a single power-law (m,n,q) is in effect at any given time
- usual values for exponents (m,n,q) are (2.5,1,0) + (0,3.5,0) + (0,2,2)
- each mechanism has a different value for A; E and V depend on water content

$$\dot{\epsilon} = A \left( \frac{b^*}{d} \right)^m \left( \frac{\sigma}{\mu} \right)^n \exp \left[ -\frac{(E_A + P V_A)}{RT} \left( 1 - \frac{\sigma}{\sigma_p} \right)^q \right]$$

# Viscoplasticity

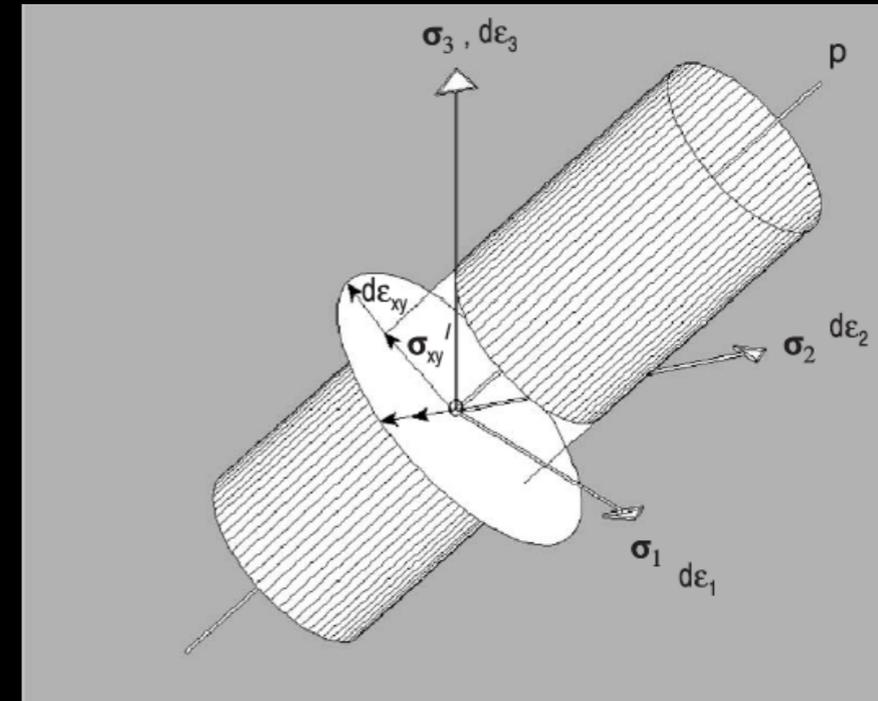
- most materials have finite strength described by limiting stress and materials cannot support stresses in excess of their yield stress
- upon reaching their yield stress they deform through plastic flow (a solid beam starts to act like toothpaste)



*von Mises yield envelope*

# Viscoplasticity

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- actually, care must be taken to guarantee one is actually on the yield surface and remains on it (i.e. use harmonic avg at your own risk)

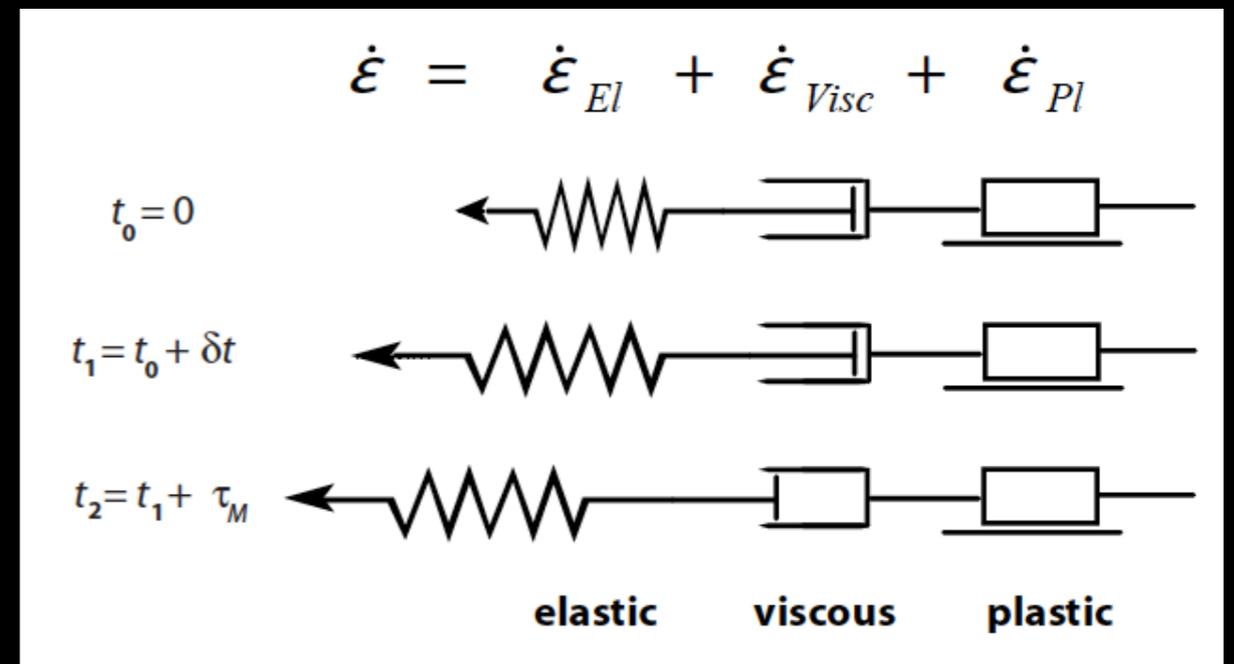


*von Mises yield envelope*

$$\eta = \begin{cases} \eta = \frac{\sigma_{II}}{\dot{\epsilon}_{II}} & (\sigma < \sigma_{yield}) \\ \eta_{eff} = \frac{\sigma_{yield}}{\dot{\epsilon}_{II}} & (\sigma \geq \sigma_{yield}) \end{cases}$$

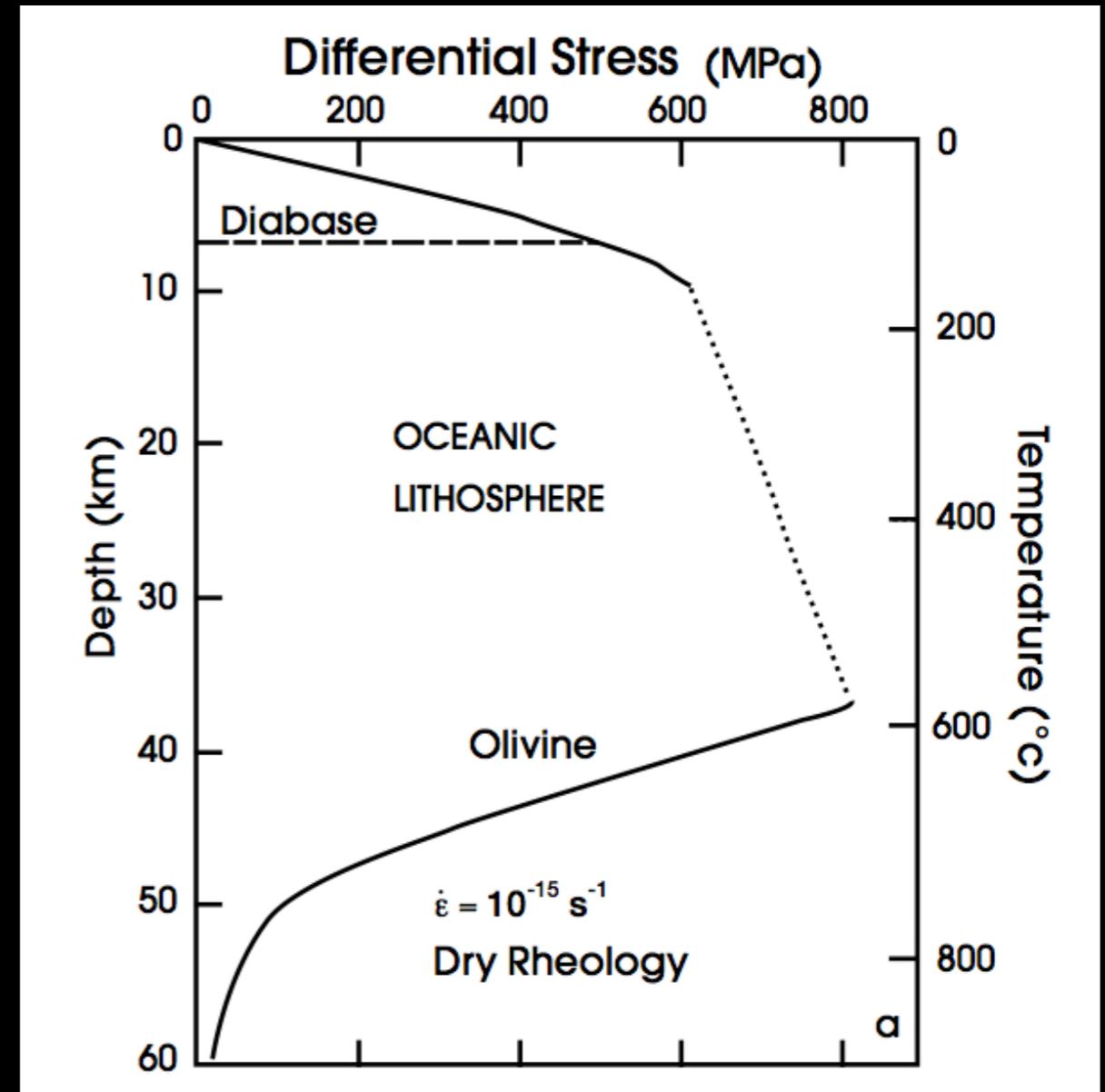
# Viscoplasticity

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- visco-elasto-plastic behavior can be written as generalized flow rule with an associated flow for each of the viscous, elastic, and plastic parts



# Strength envelope for an oceanic plate

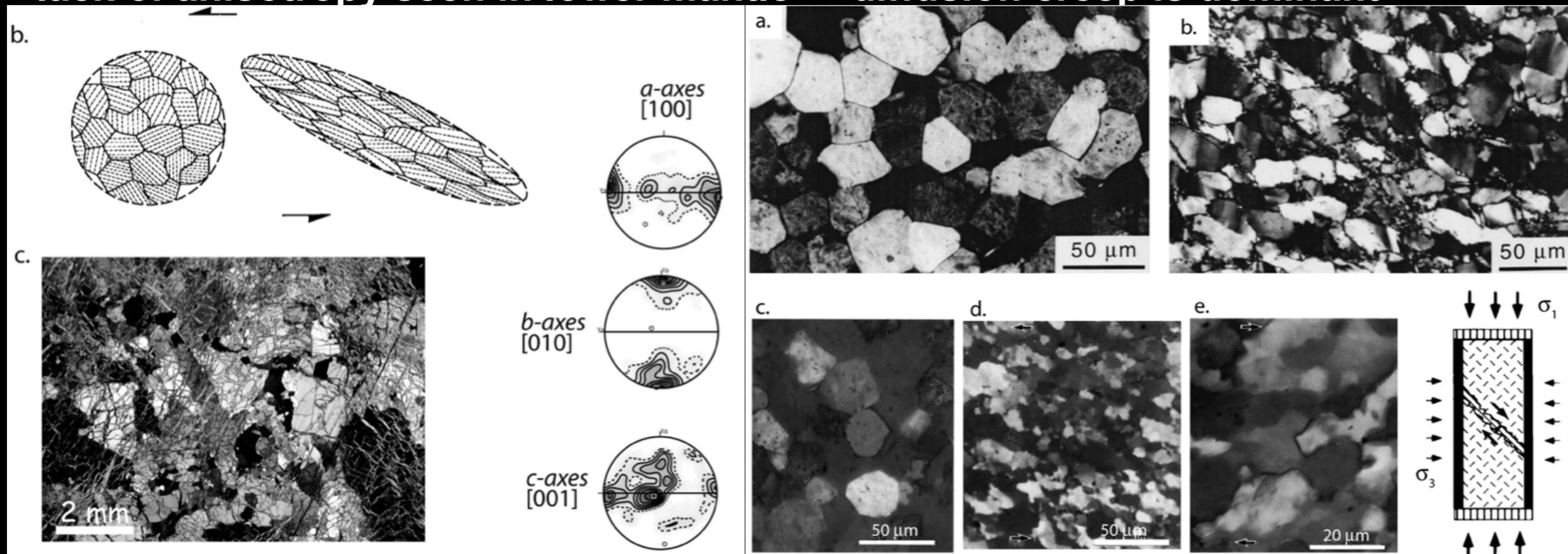
- maximum stress that is supported is equivalent to a yield stress
- three layer lithosphere: brittle crust, strong core, ductile underside
- based on extrapolating the deformation behavior of crystals (microscale) to that of a rock scale (macroscale)
- role of large scale faults and tectonic fabric (mesoscale)
- likely mechanism in strong core is Peierls creep (low strainrate, high stress)



*Kohlstedt et al, JGR, 1995*

# Anisotropy

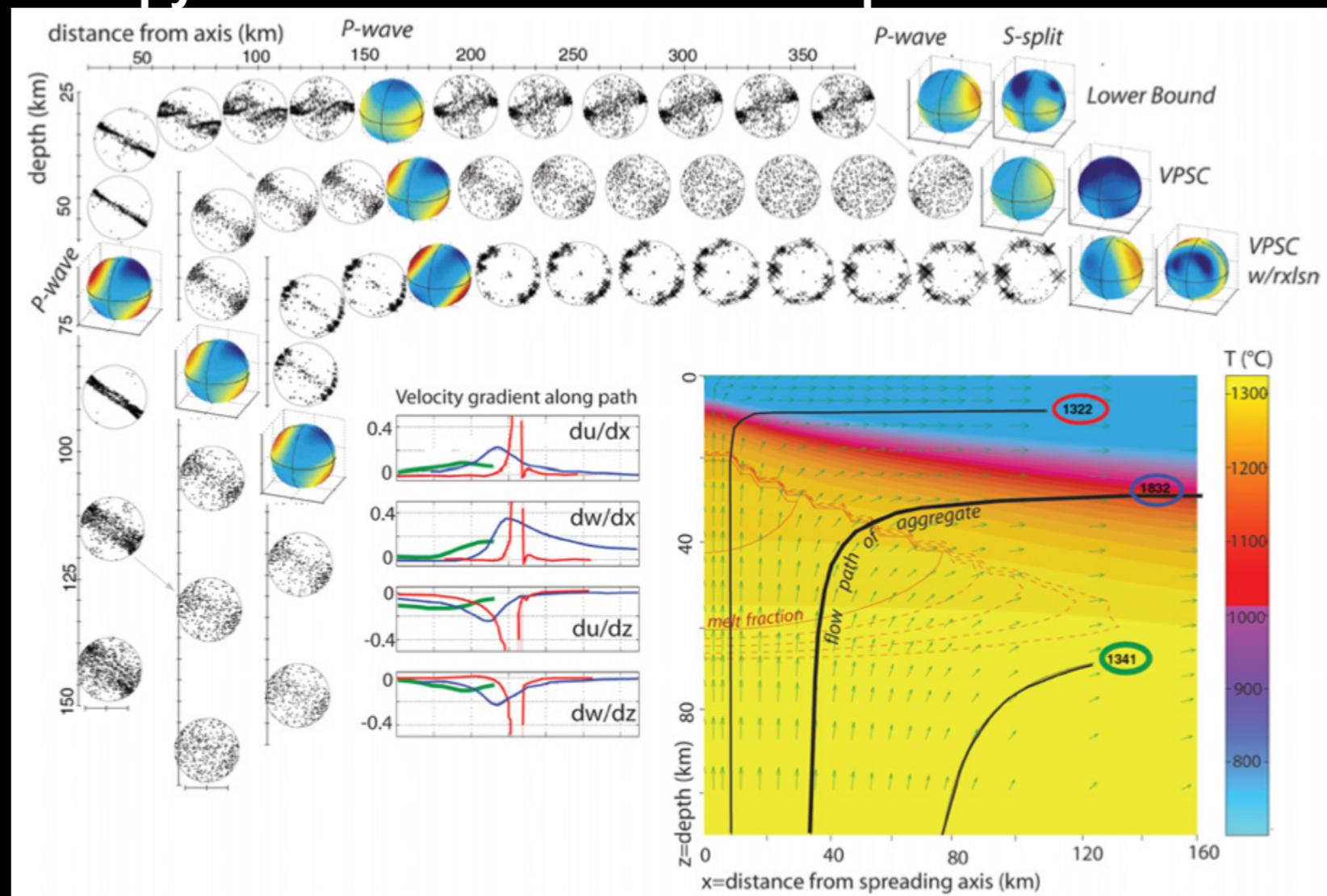
- dislocation creep ( $n=3.5$ ) and dynamic recrystallization during deformation generate crystal alignments and lattice preferred orientation (LPO) of crystals
- significant seismic anisotropy observed in the upper mantle -> implies dislocation creep is the dominant mechanism
- lack of anisotropy seen in lower mantle -> diffusion creep is dominant



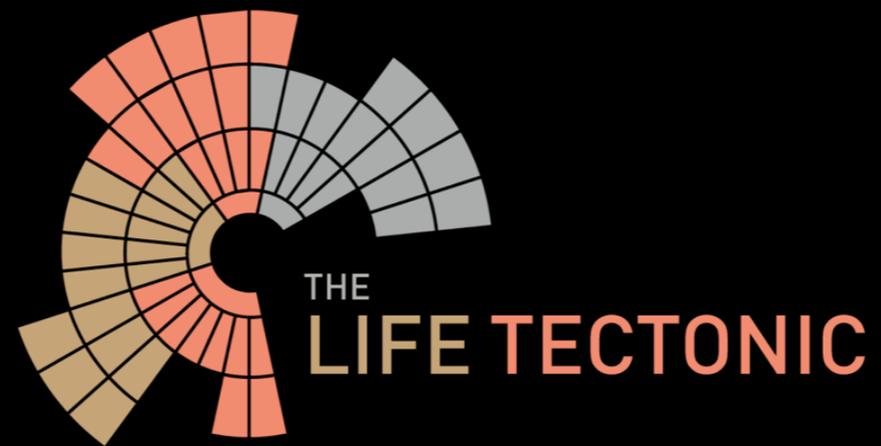
Blackman, Rep. Prog. Phys, 2007

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Blackman, Rep. Prog. Phys, 2007



[www.thelifetectonic.com](http://www.thelifetectonic.com)



*Thank you! Questions??*