SIO 224 Homework 6

1) Summarize the geochemical evidence for a region enriched in radioactive elements which is not sampled by the global ridge system. Where do you think such a region is likely to be? Explain your answer

2) There are essentially two major classes of mechanisms of mantle material deformation. What are they and what kinds of flow laws do they lead to? What are the critical factors that control the effective viscosity in both cases? Which class is likely to dominate in regions of high stress (or high deformation rate) and which class is likely to dominate in a low stress region.

3) Calculate mantle vicosity as a function of depth assuming that

$$\eta \propto \exp\left(\frac{E_a + pV_a}{RT}\right)$$

Use $E_a = 523kJ/mol$ and $V_a = 1.34 \times 10^{-5} m^3/mol$. Normalize the viscosity to the value 10^{21} Pa s at a depth of 150 km. Assume a single rheological law applies over the entire mantle and that all rheological parameters and the stress associated with flow are constant with depth. Are your values of viscosity consistent with the conclusion from post glacial rebound studies that viscosity does not change substantially across the mantle? If not, which of your assumptions would you change to get a viscosity that is more nearly constant with depth?

Hint: Use the approximation that $g(r)$ is roughly constant in mantle so that $dg/dr \simeq 0$. This allows you to determine $\rho(r)$ so you can compute the hydrostatic pressure as a function of depth and the adiabatic temperature can be computed as a function of density assuming a contant Gruneisen ratio ($\simeq 1.5$). You will have to choose a foot temperature for the mantle adiabat which should give a reasonable temperature at the pressure of the 660km discontinuty.

4a) Non-dimensionalize the Navier-Stokes equation which includes Coriolis forces as the only body force.

$$\rho \frac{D\mathbf{v}}{Dt} + 2\rho\mathbf{\Omega} \times \mathbf{v} == -\nabla p + \eta \nabla^2 \mathbf{v}$$

Choose a characteristic pressure $\rho\Omega LV$ to non-dimensionalize the pressure. Your final equation will look similar to the non-rotational reference frame version except another dimensional group appears in front of the Coriolis force term (and a similar term in front of the pressure term). The dimensional group is $1/Ro$ and it describes the importance of rotational effects relative to inertial effects (Ro is the Rossby number). Write down the non-dimensionalized equation and the dimensional group you derived for the Ro.

4b) Redo the non-dimensionalization again but this time choose a different characteristic lengthscale for the gradient operator, i.e. $\nabla^* = H\nabla$. This time, normalize the equation by the dimensional group next to the Coriolis term first and rewrite the Navier-Stokes equation. At this intermediate step, you should recognize the dimensional group that scales the inertial term. However, there is another new dimensional group in front of the viscous term. This is the Ekman number, Ek, which can be written more compactly using the kinematic viscosity, $\nu = \eta/\rho$. The Ekman number describes the importance of viscous forces relative to Coriolis forces. Write down the non-dimensionalized equation and the dimensional group you derived for the Ek.

4c) As a final step, normalize the entire equation from part (b) by the Rossby number. The point of doing this is that it should recover the scaling for how important the viscous forces are relative to the inertial. All the terms in the resulting equation should look nearly identical to the equation you derived in part (a), with the exception of the viscous term which now has a scaling of Ek/Ro. Starting from Ek/Ro, algebraically manipulate that expression into one for Re (e.g. Re times some other factor) and this will give the relationship between Ek, Ro, and Re.