SIO 224 Homework 8

1a) Non-dimensionalize the vector form of the Navier-Stokes equation which includes thermal buoyancy forces as the only body force. Assume the fluid is Boussinesq. Choose a characteristic pressure $PL^2/\rho\kappa^2$ to non-dimensionalize the pressure, and thermal diffusivity κ to non-dimensionalize both time and velocity. You may have to experiment a little to find an appropriate choice for scaling T. Your final momentum equation will look similar to the standard version except instead of the $\nabla^{*2}\mathbf{u}^*$ term being scaled by $1/Re$ it will instead be a dimensional group of ν/κ which is the Prandtl number. The buoyancy term will also be scaled by $Ra * Pr$.
1b) Divide your resultant momentum equation through by Pr, and assume that Pr is infinite. Write down this simplified version. Explain what this physically means in terms of a force balance.
1c) What does the Rayleigh number physically represent and what is a typical value for the Earth? Give you estimates of typical values for material properties of the Earth and other quantities, i.e. depth of the mantle, etc that are part of the Ra number.
1d) What values of Ra numbers correspond to transitions between different convective planforms (steady rolls, steady cellular, onset of plumes, time-dependent / chaotic)? Write them in terms of the critical Ra number (Ra/Ra_c).

2) You will calculate mantle flow based on tomography models of the Earth. You will need to install the virtual box software and then load a virtual machine into that. The virtual box software can be downloaded here (note that you need version 5.2.0 to work on macOS 10.13):
https: \\www.virtualbox.org \wiki \Download_Old_Builds_5_2
Use the command line from your terminal to obtain the virtual machine environment called 'seatree':
curl -C - -L -O http:\\www-udc.ig.utexas.edu \external \becker\ugesce\SEATREE64-2015.zip
you can follow the installation instructions from this website:
http:\\www-udc.ig.utexas.edu\external\becker\ftp\seatree_virtualbox_install.txt
and further information can be found here:
http:\\www-udc.ig.utexas.edu\external\becker\ugesce.html#download
Once you have the virtual machine running, find the application "hc" and run it. (Note that Netops has been warned that you might be visiting them for help!).

a) Run a reference model. There are several knobs to turn: 1) tomography model, 2) boundary condition, 3) scaling factor for Vs to density, 3) viscosity of the mantle. The last two of these can be depth dependent and you can modify the radial profiles using a GUI. The default values loaded in will be using the smean tomography model with free slip boundary conditions, constant density scaling factor of 0.2 and the viscosity profile of visc.sh08. When those values are set, click on 'Compute velocities'.

(i) Plot the model's predicted geoid and the observed geoid to view the geographical comparison.

(ii) What is the correlation (r) value to the observed geoid for degrees 1-20 and degrees 4-9 (it get's printed out on the bottom left and right of the predicted geoid plot)?

(iii) What are the values for the reference model and what do they mean in terms of mantle structure?

(iv) Plot the surface velocities (layer 38 - just to the bottom left of the plot should show z=0 meaning the surface) and give the rms value (just to the bottom right of the plot).

(v) Do the surface velocities look like the surface plate motions? Where do they match best and where do they match worst?

b) Choose 2 other tomography models and repeat the questions from Problem 2a(i-v). Think carefully about which models to choose. Briefly compare and contrast the results from those models with the reference model. Describe which model is best according to those criteria and try to give a reason why.

c) Vary the boundary conditions. (i) Using your preferred tomography model from Problem 2b, rerun the velocity calculation using the plate motions as the surface boundary condition. What is the new geoid correlation? What is the rms surface velocity (note this is for the actual Earth)? (ii) Using the free slip boundary condition, change the value of the constant density scaling factor from 0.2 to 0.15 and 0.25. What are the corresponding rms surface velocities for those density scalings? Describe why this is.

d) Edit the viscosity file using the GUI. (i) Add a low viscosity notch in the transition zone and run a model again with free-slip b.c. and scaling factor of 0.2. How big of a notch did you add (in terms of viscosity decrease and thickness)? Does this improve the correlation with the geoid? Does it improve the match with the rms surface velocity? (ii) Now move the viscosity notch to somewhere in the 100-300 km depth range. What is the effect of that now?