On the anelastic contribution to the temperature dependence of lower mantle seismic velocities

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Abstract

There is little agreement regarding the importance of anelastic contributions to the temperature dependence of seismic wave velocities. Some studies consider the anelastic contribution to be negligible, whereas others argue that it may be as important as the purely elastic contribution. In order to evaluate these claims, we review physically plausible models for mantle relaxation processes and construct a model of activation enthalpy based on solid state physics as well as on recent experimental results. We then deduce self-consistent 1-D $Q$ profiles and evaluate the anelastic contribution to the temperature dependence of $V_s$ and $V_p$. Physically plausible values of activation enthalpies yield anelastic contributions that increase with depth and account for at least 20% of the velocity temperature derivatives at 2600 km. We show that anelasticity may significantly moderate temperature variations in hot (slow velocity) zones. This suggests that inferred lateral changes in temperature, as well as arguments regarding the presence of chemical heterogeneity, and/or temperature induced phase transformations, are likely to be significantly affected by anelastic effects in the mantle. We also compute values of the dimensionless parameters $R_\varphi$ and $R_\rho$, and $R_{\varphi/s}$. We find that anelasticity can either increase or decrease $R_{\varphi/p}$, but the magnitude of the correction, of the order of 15%, is smaller than the uncertainty in the elastic value. Anelasticity cannot change the sign of $R_{\varphi/s}$ and $R_{\rho/s}$, but it can decrease their values by as much as 50%. However, given current uncertainties in seismic and mineral data, the effects of anelasticity on the dimensionless parameters do not bring any quantitative constraints on lateral variations of temperature, chemical composition and/or mineral phase.

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1. Introduction

Tomographic 3-D images of seismic velocities in the lower mantle are now available on both global and regional scales. Comparisons of P- and S-wave lateral variations, such as given by $R_{s/p} = \delta \ln V_s / \delta \ln V_p$, may provide new constraints on lower mantle structure and evolution. The first studies of this kind showed that the value of $R_{s/p}$ in the lower mantle is positive and larger than 2.0 (Jordan and Lynn, 1974; Souriau and Woodhouse, 1987). Using more accurate seismic data, Robertson and Woodhouse (1995, 1996) and Su and Dziewonski (1997) concluded that $R_{s/p}$ is likely to increase with depth, from a value close to 2.0 at 800 km to values of 2.6–2.8 in the lowermost
mantle. Kennett et al. (2001) deduced a constant value of approximately 2, while Masters et al. (2000) obtained results similar to Robertson and Woodhouse (1996). Regional studies by Saltzer et al. (2001) and Tkalić and Romanowicz (2002) showed that $R_{s/p}$ values may differ in regions with slabs from those without slabs. Su and Dziewonski (1997) and Masters et al. (2000) also revealed a possible anti-correlation between variations of bulk sound and shear velocities in the lowermost mantle, i.e. that $R_{p/s} = \delta \ln \rho / \delta \ln V_s$ is likely to be negative. No significant correlation between bulk sound and shear velocity variations was found by Saltzer et al. (2001). Kennett and Gorbatov (2004) performed a joint inversion of P and S travel times and found the bulk sound and shear velocities to be anti-correlated in the lowermost mantle, in agreement with Resovsky and Trampert (2003).

Masters et al. (2000) argued that lateral seismic variations associated with both high values of $R_{s/p}$ and negative values of $R_{p/s}$, have a combined thermal and chemical origin. This hypothesis highlights the fact that the seismic anomalies are intimately related to the style and mixing efficiency of mantle convection. Thus, precise mapping of density anomalies associated with mantle dynamics, may provide additional constraints on the origin of lateral heterogeneities in the mantle. However, lateral density heterogeneities are not well constrained and little consensus exists regarding their amplitudes (Ishii and Tromp, 2001; Romanowicz, 2001). Resovsky and Trampert Resovsky and Trampert (2003) obtained negative values of $R_{p/s} = \delta \ln \rho / \delta \ln V_s$ at depths below 2000 km, raising the possibility that density variations are anti-correlated with shear velocity variations. If confirmed, this observation would add evidence for lateral chemical heterogeneity in the deep mantle since negative values of $R_{p/s}$ cannot be thermally generated in an otherwise homogeneous medium (e.g. Karato and Karki, 2001).

These findings motivated several efforts to perform 3-D interpretations of observed seismic anomalies in terms of temperature and chemical composition (Forte and Mitrovica, 2001; Trampert et al., 2001; Deschamps and Trampert, 2003; Saltzer et al., 2004; Trampert et al., 2004). With the exception of Forte and Mitrovica (2001), these authors assumed that the lower mantle behaves as a purely elastic medium. However, based on extrapolations of low pressure mineralogical properties, Karato (1993) argued that attenuation of seismic waves may significantly affect their temperature sensitivities in the lower mantle. He predicted that the anelastic contribution exceeds the purely elastic derivatives of shear velocities at depths below 2000 km, and hence that anelasticity might significantly moderate the estimated magnitude of lateral chemical and thermal heterogeneities in the deep mantle. However, a decade later, two studies published by Trampert et al. (2001) and Karato and Karki (2001) arrived to very different conclusions regarding the importance of anelasticity. The former claimed that the effects of attenuation on temperature derivatives in the lower mantle are negligible since they are of the order of the uncertainties in the elastic temperature derivatives and, consequently, that the observed $R_{s/p}$ ratio is not significantly affected by attenuation. In contrast, Karato and Karki (2001) claimed that anelastic effects are non-negligible and large enough to account for the observed values of $R_{s/p}$ without invoking lateral chemical heterogeneities. Hence considerable uncertainty exists concerning the magnitude of chemical heterogeneity in the lower mantle, as well as the extent to which anelasticity should be taken into account in the determination of this magnitude.

In this report, we describe our attempt to better constrain the impact of anelasticity on the temperature sensitivity of seismic velocities. We focus on the temperature derivatives rather than the various dimensionless parameters: As we reiterate in our concluding remarks, anelasticity does not significantly affect the ability of these parameters to resolve the presence of lateral chemical heterogeneity in the lower mantle. We first review physical models for the relaxation processes in the lower mantle, and use recent experimental data in order to self-consistently relate these models to 1-D seismic profiles of the shear quality factor $Q_s$. We then compute the resulting anelastic corrections to the temperature derivatives of seismic wave velocities in the lower mantle. We find that these corrections increase with depth and are likely a significant fraction of the total temperature derivative.

### 2. Lateral variations of seismic velocities and corrections for anelasticity

At a given depth $z$, assuming that no phase transformations occur, the observed lateral variations of seismic velocities, $\Delta \ln V(\omega, z)$, can be expressed in terms of lateral variations of temperature $dT$ and composition $d\chi$ as follows:

$$d\ln V(\omega, z) = \left[ \frac{\partial \ln V(\omega, z)}{\partial T} \right]_{\chi, P} dT + \left[ \frac{\partial \ln V(\omega, z)}{\partial \chi} \right]_{T, P} d\chi.$$  \hspace{1cm} (1)

Given a specific model of mantle bulk composition, and relevant mineral-physics data, one can evaluate the
two partial derivatives and express seismic observations in terms of lateral variations of temperature and composition (see e.g. Stacey, 1998; Trampert et al., 2001; Deschamps and Trampert, 2003; Trampert et al., 2004).

2.1. Attenuation in the lower mantle

Since attenuation of elastic waves, and the concomitant dispersion processes, are present in the mantle, the two partial derivatives in Eq. (1) are in general functions of frequency. Physical models of relaxation mechanisms (such as diffusion and dislocation creep) that may be responsible for shear wave attenuation and velocity dispersion, can be found in various textbooks (e.g. Shewmon, 1963; Nowick and Berry, 1972). The relevance and applications of these models to the observed seismic attenuation have been developed and widely discussed by several authors over the last three decades (see e.g. Jackson and Anderson, 1970; Liu et al., 1976; Kanamori and Anderson, 1977; Minster and Anderson, 1981; Karato and Spetzler, 1990). Dominant mantle relaxation processes are believed to be related to delayed atomic motions in response to applied stresses or strains (e.g. relaxation by atomic diffusion or by motions of structural defects). In such cases, the relaxation time is inversely proportional to the mobility of atoms or defects and is therefore strongly temperature dependent (e.g. Jackson and Anderson, 1970). The characteristic time of the relaxation \( \tau \) can be written as

\[
\tau = \tau_0 \exp \left[ \frac{G^*(P, T)}{RT} \right],
\]

where \( G^* \) is the activation Gibbs free energy (in general a function of pressure \( P \) and temperature \( T \)) and \( R \) is the universal gas constant.

In the Earth’s mantle, a spectrum of relaxation processes is almost certainly present. Liu et al. (1976) and Kanamori and Anderson (1977) have developed an absorption band model based on the standard linear solid. Within the absorption band, the attenuation (usually quantified by the quality factor \( Q \) defined as a loss of energy per cycle: \( 1/Q = \Delta E/2\pi E \)) was assumed to be independent of frequency. The constant \( Q \) model was revisited by Minster and Anderson (1981) who assumed that the relaxation processes are related to dislocation creep. Considering a distribution of anelastic relaxation times in the form \( D(\tau) \sim \tau^{-1} \), where \( \alpha \) is a phenomenological constant with material-dependent values, they concluded that the quality factor \( Q \) may be frequency dependent within the absorption band. When the attenuation is small, the asymptotic behavior of \( Q \) can be described by a power-law in the form \( Q \propto (\omega \tau)^{\alpha} \).

Values of \( \alpha \) relevant for mantle materials have not yet been accurately determined. By comparing attenuations of the Chandler wobble, solid Earth tides, and the gravest normal mode (\( \rho S_2 \)), Anderson and Minster (1979) determined an \( \alpha \) of 0.2–0.3, whereas Smith and Dahlen (1981) deduced a value between 0.1 and 0.2. Using seismic and normal mode observations Anderson and Given (1982) obtained a value close to 0.15 but noted that this parameter was poorly constrained. Several investigations, relying on measurements of body waves in the frequency range 0.01–8 Hz, found values of \( \alpha \) varying from 0.1 to 0.6 Sipkin and Jordan, 1979; Clements, 1982; Ulug and Berckhemer, 1984; Flanagan and Wiens, 1998; Cheng and Kennett, 2002; Shito et al., 2004) However, most of these studies targeted the upper mantle region. They also included high-frequency data beyond the high-frequency corner of the absorption band (i.e. above \( \sim 1 \) Hz, e.g. Sipkin and Jordan, 1979) where \( \alpha \) is supposed to be close to 1. Therefore, the obtained \( \alpha \) may be biased towards higher values than those expected within the absorption band.

Parameter \( \alpha \) for mantle minerals has also been studied experimentally. Karato and Spetzler (1990) argued that its value lies between 0.2 and 0.4. Recently, Jackson et al. (2005) obtained an \( \alpha \) of 0.28±0.01 for a fine-grained olivine sample at a pressure of 300 MPa and temperature of 1200 °C. Since this study has been performed on an upper mantle mineral close to its melting temperature, the results may not be directly applicable to the lower mantle.

The activation Gibbs free energy \( G^* \) can be written in terms of the activation energy, \( E^* \), activation volume, \( V^* \), and activation entropy, \( S^* \), as

\[
G^* = H^* - TS^* = E^* + PV^* - TS^*.
\]

Using the power-law assumption and Eq. (2), the quality factor \( Q \) can then be written as follows

\[
Q = Q_0 \omega^2 \exp \left( \frac{\alpha H^*}{RT} \right) = Q_0 \omega^2 \exp \left( \frac{\alpha E^* + \alpha PV^*}{RT} \right),
\]

where \( Q_0 \) is a pre-exponential term that includes the dependence on activation entropy \( S^* \). Since it is reasonable to assume that \( S^* \) is temperature independent (e.g. Shewmon, 1963), \( Q_0 \) is a constant. Its value depends on material properties, such as grain size, and its determination remains an important experimental challenge (e.g. Jackson, 2000). Note that when \( \alpha = 0 \),
Eq. (4) reduces to the frequency independent model introduced by Kanamori and Anderson (1977). In this case, inferred lateral variations of $Q$ (Romanowicz, 1994, 1998) cannot be explained with lateral temperature variations.

2.2. Effects of attenuation

The effect of attenuation on the temperature derivative of seismic velocities was first studied by Karato (1993). Assuming a frequency dependent model of $Q$, the temperature derivative can be expressed as:

$$\frac{\partial \ln V(\omega, z)}{\partial T} \bigg|_{z=0} = \frac{1}{\pi} \frac{F(z)}{Q(\omega, z, T) RT^2}$$

(5)

where $V^\circ(z)$ is the unrelaxed (high-frequency limit) wave velocity and

$$F(z) = \frac{z \pi}{2} \cot \frac{z \pi}{2}.$$

The same analytical form is used for P and S waves by substituting $V_p$ and $Q_p$, or $V_s$ and $Q_s$, respectively. For a given frequency, the amplitudes of the anelastic corrections to the elastic temperature derivatives of seismic velocities are affected by a strongly non-linear interplay between the activation enthalpy $H^*$, the temperature $T$, and material-dependent parameters $\alpha$ and $Q_0$.

Attenuation can also affect the dependence of seismic velocities on chemical composition (the second term in Eq. (1)). There are, however, no data that can usefully constrain a physical model. Therefore, in what follows we focus on estimates of the magnitude of temperature effects, as given by Eq. (5).

2.3. Models of $H^*$

Little is known about the values and depth dependence of activation energies in the lower mantle. Due to technical difficulties, experimental studies have so far been limited to simple materials and pressures not exceeding those in the transition zone. The results of these studies have been reviewed by Béjina et al. (2003).

Most of the few studies that address the lower mantle are limited to magnesiowüstite. Getting et al. (1997) obtained an activation energy $E^*$ of 230 kJ/mol at ambient pressure. Yamazaki and Irfüne (2003) studied Fe–Mg interdiffusion in magnesiowüstite up to 35 GPa. They obtained activation energies $E^*_{\text{Mg}}$ and $E^*_{\text{Fe}}$ of 226±32 kJ/mol and 113±74 kJ/mol, respectively.

They also obtained an activation volume $V^*_{\text{Mg}}$ of 1.8±1.2 cm$^3$/mol. Holzapfel et al. (2003) obtained an activation energy of 253±50 kJ/mol and an activation volume of 3.3±0.1 cm$^3$/mol. The diffusion of oxygen in MgO has been studied by van Orman et al. (2003) who obtained a value of 3.3±2.4 cm$^3$/mol for $V^*_{\text{O}}$. Ito and Cohen (1997) used molecular dynamics simulations to estimate values the zero pressure activation energy and volume of 140 kJ/mol and 2.5 cm$^3$/mol, respectively. A high value of $E^*_{\text{O}}=400$ kJ/mol was measured by Hensler and Cullen (1968) for steady-state dislocation creep. We note however that our understanding of the relaxation mechanisms does not allow us to simply associate low values of $E^*$ with diffusion and high values with dislocation movements. For example, Karato (1998) argued that motion of pre-existing geometrical kinks may require low values of $E^*$, similar to those measured by (Getting et al., 1997). On the other hand, when relaxation is mediated by nucleation of dislocation kinks, the activation energies are higher.

Energetics of relaxation processes in silicate perovskites are not accurately constrained. Webb et al. (1999) measured viscoelastic characteristics of CaTiO$_3$ and SrTiO$_3$ and obtained activation energies $E^*$ between 500 and 700 kJ/mol. Yamazaki et al. (2000) studied silicon self-diffusion in MgSiO$_3$ at pressures of 25 GPa and deduced values of $E^*$ of 336 kJ/mol and 311 kJ/mol for lattice and grain-boundary diffusion processes, respectively. Under similar conditions, Holzapfel et al. (2005) obtained a value of 414±62 kJ/mol for interdiffusion of Fe–Mg in silicate perovskite.

Molecular dynamics simulation (Wright and Price, 1993) gave activation energies significantly higher: $E^*>900$ kJ/mol. However, as discussed by (Béjina et al., 2003), numerical simulations yield significantly different results in cases with and without oxygen vacancies. A direct comparison of these results with experiments performed on natural samples is thus difficult. Finally, very little is known about values of activation volumes in perovskites.

Sammis et al. (1981) used elastic models of defect structures, developed by Zener (1942) and Keyes (1963), to derive upper and lower estimates of the activation volume for creep mechanisms. The two limiting values are given by $V^*_{\text{min}}=V^*_{\text{K}}$, $V^*_{\text{max}}=V^*_{\text{K}}$, where $V^*_{\text{K}}$ can be computed from the following expressions:

$$V^*_{\text{K}} = (E^* + PV^*_{\mu}) \left( \frac{\partial \ln \mu}{\partial P} \right)_T \left[ 1 - \frac{1}{K_T} \left( \frac{\partial \ln \mu}{\partial \ln T} \right)_P - \beta T \right]^{-1}$$

(7)
and

\[ V'_{K} = (E' + PV'_K) \left[ \left( \frac{\partial \ln K_T}{\partial P} \right)_T - \frac{1}{K_T} \right] \left[ \frac{1}{\left( \frac{\partial \ln T}{\partial P} \right)_T} - \beta T \right]^{-1} \]  

(8)

where \( K_T \) and \( \mu \) are the bulk and shear moduli, and \( \beta \) is the coefficient of thermal expansion. This model was revisited by Poirier and Liebermann (1984) to study the depth dependence of the activation volume in the lower mantle. Similarly to (Sammis and Smith, 1981) they obtained a decrease of about 50% in \( V' \) between the top and bottom of the lower mantle. Fig. 1a shows values of lower and upper limits of the activation volume as functions of depth, estimated by using the geotherm proposed by Brown and Shankland (1981), and thermoelastic properties reported by (Mattern et al., 2005; Matas et al., in press). Since the absolute values of activation volumes are scaled by the adopted values of \( E' \) (see Eqs. (7) and (8)), we only show results for \( E' = 200 \) kJ/mol. With the same value of \( E' \), activation volumes predicted for MgSiO₃ perovskite (solid lines in Fig. 1a) are lower by about 30% than those obtained for MgO, mostly due to the differences in elastic properties.

Fig. 1b depicts resulting activation enthalpies \( H' \) for magnesiowüstite and silicate perovskite as a function of depth. As discussed above, values of \( E' \) are not accurately determined. We therefore use values of 200 and 500 kJ/mol to bracket the range of experimental observations. Two limiting cases can thus be defined: first, a model with \( E' = 200 \) kJ/mol and the lower bound \( V' \) obtained for MgSiO₃ (hereafter denoted as a model “MinEn”). A second model, which we call “MaxEn”, has \( E' = 500 \) kJ/mol and the upper bound \( V' \) associated to MgO. Note that a depth-independent \( H' \) can be reasonably obtained only for a zero activation volume.

The activation enthalpy can also be estimated with the empirical relation proposed by Weertman (1970), who based it on observed correlations between the activation enthalpy and the melting temperatures of various metals. The so-called homologous temperature model relates the activation enthalpy \( H' \) to the melting temperature \( T_m \) through a simple linear law:

\[ H' = g R T_m \]  

(9)

where \( g \) is a proportionality constant characteristic of the material. This model has been applied in earlier studies (Karato, 1993; Karato and Karki, 2001; Forte and Mitrovica, 2001) to estimate the role of attenuation in the lower mantle. Yamazaki and Karato (2001) used the melting temperatures obtained by Zerr and Boehler (1994) for MgO and Wang (1999) for MgSiO₃, along with the expression \( dH' / dP = g R (dT_m / dP) \), obtained...
from Eq. (9), to obtain activation volumes of 2.2 and 3.0 cm³/mol, respectively. Although these values are generally consistent with the experimental observations on MgO discussed above, the homologous temperature model should be used with caution. The melting temperature of the mantle and its dependence on depth are poorly known. The uncertainty in \( T_m \) of MgO at the base of the mantle is at least of the order of 2000 K: experimental studies indicate a melting temperature of about 5200 K (e.g. Zerr and Boehler, 1994) whereas molecular dynamics modeling gives values exceeding 7500 K (Cohen and Gong, 1994; Cohen and Weitz, 1998). Consequently, activation enthalpies predicted with Eq. (9) vary widely. Choosing \( g = 10 \) (e.g. Yamazaki and Karato, 2001), activation enthalpies \( H^* \) (800 km) and \( H^* \) (2600 km) obtained with the experimental melting curve (Zerr and Boehler, 1994), are 315 and 420 kJ/mol, respectively, and they reach 500±50 and 670±50 kJ/mol, respectively, using the computational results from (Cohen and Gong, 1994; Cohen and Weitz, 1998).

We note that estimates of \( H^* \) based on Zerr and Boehler’s measurements (1994) are consistent with those obtained from defect-diffusion model with \( E^* = 230 \) kJ/mol discussed above. On the other hand, the estimates based on molecular dynamics are broadly similar to those corresponding to relaxation processes with \( E^* = 400 \) kJ/mol. One should keep in mind, however, that there is no evidence that the correlation between activation enthalpy and the melting temperature observed for metals, also exists for mantle minerals (oxides, silicates). For example, measurements for binary oxides show no such correlation (Freer, 1980; Poirier, 2000), and thus seriously undermine the physical significance of the proportionality factor \( g \) in Eq. (9). The rough agreement discussed above is therefore due to our arbitrary choice of \( g = 10 \) and is not based on a physically justified model. We therefore do not use this relation in the present study.

2.4. Seismic 1-D radial profiles of \( Q \)

Although the seismic attenuation in the Earth’s mantle has been studied over the last 40 yr, its accurate determination remains one of the important challenges in the Earth sciences (Romanowicz and Durek, 2000). Most seismological studies have concentrated on shear attenuation, \( Q_s = Q_\mu \). There is no evidence for significant compressional attenuation in the mantle and very high values (if not infinite) of \( Q_\kappa \) are usually assumed. In this case, \( Q_s \) can be simply related to \( Q_p \) using the Poisson solid model: \( Q_p = 3/4(V_p/V_s)^2 Q_s \) (Anderson and Hart, 1978). Several 1-D \( Q_s \) profiles have been proposed over the last three decades: PREM (Dziewonski and Anderson, 1981); QL6 (Durek and Ekstrom, 1996); SL8 (Anderson and Hart, 1978); QLM9 (Lawrence and Wysession, 2006); QM1 (Widmer et al., 1991); AK135-f (Montagner and Kennett, 1996). As

![Graph showing radial profiles of quality factor](image-url)
can be seen in Fig. 2, tight constraints on lower mantle attenuation have not emerged. One can however argue that $Q_s$ is bracketed between 300 and 500 at the top of the lower mantle and does not change by more than a factor of two between the depth of 660 km and the top of the $D''$ layer. This conclusion is reinforced by a recent study by Resovsky et al. (2005).

Using Eq. (4), the quality factor $Q_s$ in the lower mantle can be modeled in a manner that is thermodynamically consistent with a particular model of activation volume and a given thermal structure. The frequency dependence of $Q_s$ can be approximated by adopting a non-zero value for $\alpha$. Although its value has not yet been accurately determined (see the discussion above), a value of 0.3 is consistent with experimental as well as seismic estimates. We show below that varying $\alpha$ between 0 and 0.5 does not affect the essence of our results. The pre-exponential factor $Q_0$ is chosen so that the resulting $Q_s$ is bracketed between 300 and 500 at 700 km, as suggested by seismic models (see Fig. 2). The value of $Q_0$ is recomputed for every model of activation enthalpy and thermal structure.

Seismic values of $Q_s$ cannot be, however, directly compared with model values: Lateral temperature variations in the lower mantle are likely to yield large lateral variations of $Q_s$ due its exponential dependence on temperature. As pointed out by Karato and Karki (2001), it is not appropriate to use constant seismic $Q_s$ values in Eq. (5) since the seismic values are averages over the lateral temperature variations. They argue that if the amplitude of the temperature variations is $\delta T$, then the correct estimates of $(\partial \ln V/\partial T)_\delta$ can be obtained from Eq. (5) by integrating the second term on the right-hand side from $T-\delta T$ to $T+\delta T$, rather than simply multiplying by $2\delta T$. Doing so is equivalent to using the following expression for $1/Q$ in Eq. (5)

$$
\frac{1}{Q} = \left(\frac{1}{Q}\right)_{\text{seismic}} \frac{1}{2x\xi} \left[ \exp \left( \frac{x}{1+x\xi} \right) - \exp \left( -\frac{x}{1-x\xi} \right) \right]
$$

(10)

where $x = \delta T/T$ and $\xi = \alpha H^*/RT$. This expression differs from that of Karato and Karki (2001) (see their Eq. (27)), which is valid only for infinitesimal values of $\delta T/T$. In Fig. 3, we compare values given by Eq. (10) with those obtained by Karato and Karki (2001) for various values of $x$ and $\xi$. Although we confirm that the seismically obtained values of $Q_s$ may be overestimated, our correction factor is significantly lower than that predicted by Karato and Karki (2001). For instance, when $\xi=5$ (corresponding for example to combination of the following values: $H^*=263$ kJ/mol, $\alpha=0.3$, $T=1900$ K or $H^*=346$ kJ/mol, $\alpha=0.3$, $T=2500$ K) and $\delta T/T=0.5$, Karato and Karki (2001) obtained a correction factor of 2.40 whereas we only predict an enhancement by a factor of 1.06.

To summarize, to obtain a model distribution of $Q_s$, we first calibrate the pre-exponential factor $Q_0$ by using Eq. (10) and a seismically observed value of $Q_s$ at the depth of 700 km ($=Q_{\text{ref}}$). Then we use Eq. (4) to determine the appropriate depth dependence of $Q_s$ for each model of activation enthalpy and thermal structure.

![Fig. 3. Ratio of exact (this study, Eq. (10)) and approximate (Eq. (27) in Karato and Karki, 2001) correction factor used to compare values of $Q_s$ computed from physical models with those obtained from seismology.](image-url)
Finally, we use Eq. (10) again to compare the computed $Q_s$ with available seismic observations. Fig. 4 depicts predicted radial profiles of $Q_s$ given by models MinEn and MaxEn (solid lines), as they would be “seen” seismically. Values of $\alpha$ and $\delta T/T$ are set to 0.3 and 0.4, respectively. The $Q_s$ profiles obtained with model MinEn decrease slowly with depth and are similar to seismic models QM1 and AK135-f. Model MaxEn with $Q_{ref}=300$, yields a $Q_s$ profile that increases to a maximum in the deep mantle in a manner similar to seismic model SL8. These comparisons show that a rather simple physical model is able to yield good qualitative and even semi-quantitative agreement with seismic observations. However, when $Q_{ref}=500$, values of $Q_s$ in the deep mantle exceed 1000. Since such high values are not seismically observed, a combination of high $E^*$ together with a high $Q_{ref}$ would appear to be incompatible with lower mantle rheology. Similarly inconsistent with seismic observations are models based on a constant activation enthalpy (i.e. $V^{*}=0$ cm$^3$/mol), shown by the dashed lines.

2.5. Anelastic contribution to temperature derivatives

Having constructed a physical model of $Q_s$ that is generally consistent with the (admittedly broad) range of seismically observed values, we now use Eq. (5) to estimate the anelastic contribution to the temperature derivatives of seismic velocities. Fig. 5 shows the elastic temperature derivatives (first term in Eq. (5)) obtained for a pyrolitic composition with methods described in Matas et al. (in press). These derivatives are very similar to those computed by Trampert et al. (2001). We also show the uncertainties in the computed derivatives, as derived by Trampert et al. (2001) from experimental uncertainties in the pertinent elasticity data.

Fig. 6 shows the ratio of the anelastic contribution to the total temperature derivatives for shear velocities, expressed as percentages

\[ X_s = 100 \frac{A_s}{V_s T + A_s} \tag{11} \]

where

\[ A_s = \frac{1}{\eta} \frac{F(\alpha) H^*}{Q_p RT^2} \left( \frac{\partial \ln V_p^u}{\partial T} \right)_{\xi, \lambda} \tag{12} \]

The results for compressional velocities, $X_p$, obtained by replacing $s$ with $p$, are shown by dashed lines in Fig. 6. The values of $\alpha$ and $\delta T/T$ are set to 0.3 and 0.4, respectively. Clearly anelasticity enhances the temperature dependence of both shear and compressional velocities. If seismic attenuation is caused by relaxation processes controlled by low values of $H^*$ (such as predicted by the model MinEn), the enhancement for shear velocities is about 10% at 700 km and reaches 20% at the base of the mantle. Varying $Q_{ref}$ between 300 and 500 changes these results by less than 10%. The effects appear to be significantly larger, exceeding 50% throughout the mantle if relaxation processes are characterized by high activation enthalpies (such as those in model MaxEn). Though the anelastic contributions to compressional velocities are slightly smaller, the...
effects are significant and essentially mirror those on the shear velocities.

Fig. 6 also demonstrates that the main source of uncertainty in the significance of the anelastic effects is due to insufficient constraints on the physics of absorption, as represented by the activation enthalpy, rather than the uncertainties in the elastic derivatives, which contribute errors of the order of a few %.

Uncertainty in the lower mantle geotherm (see e.g. Jeanloz and Morris, 1986; Matas et al., in press) is another source of errors. Therefore we also computed activation volumes and enthalpies for a geotherm with larger temperature gradients than those of the Brown and Shankland geotherm. For example, taking the temperature at 2600 km to be 3300 K instead of 2400 K, the anelastic contribution based on model...
MinEn increases by 8%. A similar result would be obtained using a constant $H^*$ model and Brown and Shankland’s geotherm, thus indicating a significant tradeoff between the activation volume and the temperature profile.

The above computations were performed with specific values of the parameter $\alpha$ (0.3) and the amplitude of lateral temperature variations $\delta T/T$ (0.4). These values are also subject to uncertainties. In Fig. 7, we show the effect of varying $\alpha$ between 0 and 0.5, i.e. in a range that spans physically plausible values of $\alpha$ within the absorption band. The anelastic effects are not very sensitive to uncertainties in $\alpha$ and $\delta T/T$ when the activation energies are low (model MinEn). For larger enthalpies (model MaxEn), the effects are quite sensitive to $\alpha$, and also to $\delta T/T$ when $\alpha$ exceeds 0.2. However, the amplitude of anelastic contributions increase significantly for $\alpha$ above 0.3. For $\alpha$ below

Fig. 7. Effect of $\alpha$ on the relative magnitude of anelastic effects. ($Q_{ref}$ is set to 400). The estimated anelastic contribution is sensitive to $\alpha$ and $\delta T/T$ only when absorption processes are controlled by large enthalpies.

Fig. 8. Magnitude of lateral temperature anomalies obtained from seismic anomalies $\delta \ln V_s$ of $\pm 2\%$ as a function of activation enthalpy at the depth of 2600 km. Anelasticity may significantly decrease the amplitude of hot anomalies (solid lines) whereas the cold anomalies (dashed lines) are affected only slightly. Dot-dashed lines show the temperature variations obtained from a purely elastic model. $\alpha=0.3$; $H^*$ and $Q_s$ are recomputed self-consistently with $\delta T/T$. The shaded areas depict uncertainties resulting from variations in the seismic $Q_s$ between 300 and 500 at this depth.
this value, it decreases by only a small amount. It thus appears that the anelastic contributions to temperature derivatives are significant irrespective of the specific values of $\alpha$ and $\delta T/T$.

In order to illustrate the potential significance of the anelastic contributions to tomographic interpretations, we compute lateral temperature variations (positive and negative) inferred from ±2% seismic velocity anomalies assuming a range of activation enthalpies. Fig. 8 shows that anelasticity may significantly decrease the amplitude of hot temperature anomalies (solid lines) compared to those obtained from a purely elastic interpretation (dot-dashed lines). For example, assuming a reference seismic quality factor $Q_{ref}$ of 300 at the depth of 2600 km, positive temperature anomalies are decreased by 25% when $H^*(2600 \text{ km}) = 300 \text{ kJ/mol}$, i.e. the inferred $\Delta T$ is 550 K instead of the 730 K obtained in the elastic limit. In hot (slow velocity) zones, the quality factor $Q_s$ is relatively low even for high values of $H^*$ and, thus, the effect of anelasticity remains strong. The effect is less important in cold (fast velocity) zones, mainly because of a strongly non-linear interplay between the activation enthalpy $H^*$, temperature $T$, and the resulting quality factor $Q_s$. Thus, negative temperature anomalies are decreased by only 12% when $H^* = 300 \text{ kJ/mol}$. At low temperatures, high values of $H^*$ yield high $Q_s$, and, thus, the anelastic corrections become small and vanishes for $H^*$ above 600 K/mol. Note that since the elastic temperature derivatives are functions of temperature, the hot and cold elastic limits differ by about 100 K. The uncertainties due to variations of the reference seismic $Q_s$ between 300 and 500 (shaded areas in Fig. 8) are smaller than the difference between the two elastic limits. The fact that anelasticity causes velocity changes due to high temperature anomalies (in upwelling mantle areas) to be larger than those due to cold temperature anomalies (in down-going slabs) was noted earlier in a study of upper mantle $Q$ tomography Romanowicz, 1994. The net effects illustrated by Fig. 8 strongly suggest that inferred lateral changes in temperature, as well as arguments regarding the presence of chemical heterogeneity, and/or temperature induced phase transformations, are likely to be significantly affected by anelastic effects in the mantle.

2.6. Anelastic contribution to $R_{s/p}$, $R_{\phi/s}$, and $R_{\rho/s}$

Although we wish to focus on the temperature derivatives, we also estimate the consequences of anelasticity for the dimensionless ratios $R_{s/p} = \delta \ln V_s/\delta \ln V_p$, $R_{\phi/s} = \delta \ln V_{\phi}/\delta \ln V_s$, and $R_{\rho/s} = \delta \ln \rho/\delta \ln V_s$. As we discuss in the concluding remarks, while anelasticity can significantly change the values of these ratios, little is gained in the ability of these parameters to resolve the presence or absence of lateral chemical heterogeneity in the lower mantle.

Using Eq. (5) for $V_s$ and for $V_p$, and assuming that bulk attenuation is negligible, one can write the
relationship between elastic and anelastic values of $R_{s/p}$ as follows:

$$R_{s/p}^{anel} = R_{s/p}^{el} + \frac{A_p}{V_{p,T} + A_p} (L - R_{s/p}^{el})$$

(13)

where

$$L = \frac{3}{4} \left( \frac{V_p}{V_s} \right)^2 .$$

(14)

It can be seen from these relations that anelasticity may either increase or decrease $R_{s/p}$. If $R_{s/p}^{anel} < L$ then anelasticity increases $R_{s/p}$, while versa. The elastic limit of $R_{s/p}$ can be written as (see e.g. Karato and Karki, 2001):

$$R_{s/p}^{el} = \frac{L}{(L - 1) \frac{\delta_s}{1 - \delta_s} + 1}$$

(15)

where $\delta_s = (\frac{\partial \ln K_s}{\partial \ln \rho})_p$ and $\Gamma = (\frac{\partial \ln \mu}{\partial \ln \rho})_s$. Since $L > 1$ and $\Gamma > 1$, $R_{s/p}^{el} < L$ only if $\delta_s > 1$. In other words, as $\delta_s$ approaches 1, $R_{s/p}^{el}$ increases and the anelastic correction to $R_{s/p}$ decreases. When $\delta_s = 1$ the anelastic correction vanishes, independently of the activation enthalpy model used, and $R_{s/p}$ reaches its maximum elastic value.

As pointed out by Trampert et al. (2001), current uncertainties in the purely elastic temperature derivatives (see Fig. 5) result in rather large uncertainties in the elastic value of $R_{s/p}^{el}$. For instance, an uncertainty of $0.4 \cdot 10^{-5}$ km s$^{-1}$ K$^{-1}$ in the temperature derivatives of seismic velocities yields an uncertainty of $\pm 0.3$ in $R_{s/p}^{el}$ at 800 km, and $\pm 0.7$, at 2600 km. Since the development of a robust model for $R_{s/p}^{el}$ is beyond the scope of this study, we estimate the amplitudes of anelastic corrections using seismically plausible values of $R_{s/p}^{el}$ in the lower mantle. In Fig. 9, we show the contribution to $R_{s/p}$ due to anelasticity for two values of $R_{s/p}^{el}$ (2.0 and 2.5), taken as constant throughout the lower mantle. For $R_{s/p}^{el} = 2.0$, the anelastic contribution is rather small: it varies between 0.05 and 0.3 assuming various models of activation enthalpies discussed above and values of $\alpha$ between 0.2 and 0.4. The anelastic correction never exceeds 15% of the elastic value. When $R_{s/p}^{el} = 2.5$, the correction is negligible because $L$ is also close to 2.5. We note that the correction is negative at depths shallower than 1500 km, where $L < 2.5$.

A comparable analysis of the relationships between the elastic and anelastic values of $R_{p/s}$ and $R_{\rho/s}$ shows that both are affected in an identical manner

$$R_{\phi,p/s}^{anel} = R_{\phi,p/s}^{el} \left( 1 - \frac{A_s}{V_{s,T} + A_s} \right) = R_{\phi,p/s}^{el} \left( 1 - \frac{X_s}{100} \right) .$$

(16)

Since $X_s$ is positive, anelasticity decreases the magnitude of both dimensionless parameters. The decrease ranges from 20% for model MinEn, to 50% for model MaxEn.

3. Concluding remarks

We have attempted to shed some light on the significance of anelastic contributions to the lateral temperature derivatives of seismic velocities, and some dimensionless parameters that may play a significant role in establishing the presence or absence of chemical heterogeneities in the lower mantle. Previous studies have found these contributions to be either negligible (Trampert et al., 2001), or as important as the purely elastic derivatives (Karato and Karki, 2001). These divergent views are made possible by the large uncertainties associated with calculated elastic temperature derivatives, as well as poorly constrained estimates of anelastic parameters. However, if mantle materials have a distribution of relaxation times that yield a seismic $Q_s$ as given by Eq. (4), then the anelastic contribution in Eq. (5) no longer depends on $Q_s$ and $H^*$. Independently, as effectively assumed by Trampert et al. (2001). A constant $H^*$ requires a zero activation volume, which appears highly unlikely, and a constant $Q_s$ profile is incompatible with a constant $H^*$ model for reasonable temperature profiles. If we accept Eqs. (4) and (5) as valid representations of the physics of mantle attenuation, then our estimates indicate that anelastic contributions to the temperature derivatives are almost certainly significant, especially in the deepest lower mantle (see Fig. 6). In this sense, we agree with Karato and Karki (2001), except for the magnitude of the effect, which they most likely overestimated. Our calculated anelastic corrections approach purely elastic values in magnitude only if relaxation is dominated by mechanisms with a high value of $E^*$ in the deepest mantle (Fig. 6). Relaxation effects might be small enough to be negligible only in the shallowest lower mantle, and then only for small activation energies. It thus appears that it is imprudent to ignore anelasticity in the process of interpreting mantle tomography in terms of 3-D thermal and compositional models.

The role played by anelasticity in the dimensionless parameters $R_{s/p}$, $R_{\phi,s}$, and $R_{\rho/s}$ may not be as significant as one might think. Large values of $R_{s/p}$ and negative values of $R_{\phi,s}$ and $R_{\rho/s}$ have been cited as possible evidence for lateral chemical heterogeneity and/or temperature induced phase transformations (e.g. Su and Dziewonski, 1997; Masters et al., 2000; Saltzer et al., 2001; Resovsky and Trampert, 2003). Keeping in
mind the experimental and theoretical uncertainties associated with the estimated anelastic effects, as well as the seismic uncertainties on $R_{e/p}$, it is difficult to reject the possibility that even the purely elastic model, without lateral chemical heterogeneity, might account for the large seismic $R_{e/p}$ values. Addition of the anelastic corrections may raise values of $R_{e/p}$ in the deepest mantle and, consequently, increase the possibility of explaining seismic observation by thermal variations alone. However, as pointed out by Trampert and van der Hilst (2005), even if the observed $R_{e/p}$ is consistent with thermal variations alone, it does not necessarily mean that chemical variations are not present since it is possible for chemical heterogeneity to lower $R_{e/p}$.

In contrast to the currently poor resolving ability of $R_{e/p}$, negative values of $R_{\phi/s}$ or $R_{p/s}$ would decisively indicate the presence of lateral chemical heterogeneity or temperature-induced phase transformations. Anelasticity can change the magnitude of these parameters, but not their sign. The elastic limit of $R_{\phi/s}$ and $R_{p/s}$ can be expressed in terms of $\delta_S$ and $\Gamma$ as

$$R_{\phi/s} = \frac{\delta_S - 1}{\Gamma - 1} \quad \text{and} \quad R_{p/s} = \frac{2}{\Gamma - 1}.$$  \hspace{0.5em} (17)

Since $\Gamma$ is always significantly larger than one (e.g. Karato and Karki, 2001), negative values of $R_{\phi/s}$ or $R_{p/s}$ as observed by Resovsky and Trampert (2003), could be used as evidence for the presence of chemical variations. Negative $R_{\phi/s}$ values lead to similar conclusions, but the situation is more complex because elastic values of $\delta_S$ are not well constrained in the lowermost mantle. However, neither experimental nor theoretical studies suggest that $\delta_S$ is likely to be lower than 1 (Agnon and Bukowinski, 1990; Isaak et al., 1992; Karki et al., 2001). Anelasticity could affect $\delta_S$ only if compressional attenuation is not negligible but, since $Q_s$ appears to be very high in the lower mantle, the effect, if any, would be very small.

To summarize, we find that anelasticity most likely makes an important contribution to lateral variations in seismic velocities, and should therefore be included in attempts to interpret tomographic models in terms of thermal and mineral variations, and in comparisons of tomography with geodynamic models. However, in the absence of robust experimental and theoretical constraints on the nature of relaxation processes in the lower mantle, the magnitude of the anelastic effects can only be obtained by comparing seismically observed values of $Q_s$ with those predicted from self-consistent models based on solid state physics. For instance, estimates of activation volumes in the lower mantle could be obtained if both the depth dependence and frequency dependence of seismically derived models of $Q_s$ were better constrained. In principle, $R_{e/p}$, $R_{\phi/s}$, and $R_{p/s}$ could be used with other seismic data to resolve the magnitude of lateral variations in temperature, composition and mineral phase. However, given the present uncertainties in seismic and mineral data, the role of these parameters is much more limited. Anelasticity does not change the fact that the implications of large values of $R_{e/p}$ remain ambiguous. Similarly, presently only the signs of $R_{\phi/s}$ and $R_{p/s}$ constitute robust indicators of the presence or absence of mineral heterogeneities, regardless of the magnitude of anelastic effects.

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