Thermal history and the evolution of the Earth's core

1. Introduction

Thermal history calculations concerning the evolution of the core have been done for many years. Those calculations done before 2000 could typically accomodate an ancient inner core, have a whole outer core that is vigorously convecting and so is adiabatic and homogenous, and deliver a reasonable amount of heat to the mantle. As we demonstrate below, the growing inner core is an important energy source for powering the dynamo, and, if it is a young feature of the Earth, driving the dynamo by just cooling the Earth becomes problematic.

Two things happened to change this comfortable state of affairs. First, estimates of the melting temperature of iron at the ICB have gone up. As we discussed earlier in class, that melting temperature is now experimentally determined to be 6230 ± 500K and is in good agreement with ab-inito calculations. This is about 1000K larger than previous estimates. This impacts not only the temperature at the CMB but also the temperature gradient there if convection occurs all the way through the outer core because the adiabatic temperature gradient is proportional to temperature:

$$\frac{dT}{dr}_{ad} = -\frac{\alpha g T}{C_p} = -\frac{g T \gamma}{\phi}$$

This means that the heat flow into the mantle goes up. Revised thermal history calculations find that the rate of secular cooling has to be much larger which leads to an age for the inner core of less than 1By and a substantial amount of heat going into the mantle.

The second thing that changed was that ab-initio calculations and laboratory measurements indicated a much higher value for the thermal conductivity of liquid iron and iron alloys. Actually, the electrical conducivity (or its reciprocal, resistivity) is easier to compute, and assuming electron effects dominate, the two conductivities are related by the Widemann-Franz law:

$$k = \frac{LT}{\rho} = LT\sigma$$

where ρ is the resistivity, k is the thermal conductivity, σ is the electrical conductivity and L is the Lorentz number ($L = 2.44 \times 10^{-8}$ W Ω/K^2). At high values of resistivity, an effect called "saturation" occurs. This happens in all transition metals as the mean free path between electron scattering events becomes comparable to the inter-atomic spacing. The paper by Gomi et al (2013) discusses this. and they give

$$\frac{1}{\rho_{tot}} = \frac{1}{\rho_{ideal}} + \frac{1}{\rho_{sat}}$$

where ρ_{ideal} is the resistivity of the alloy in the absence of saturation effects. As ρ_{ideal} gets very large, this gives $\rho \rightarrow \rho_{sat}$. Multiplying the above equation by LT gives

$$k_{tot} = k_{ideal} + k_{sat}$$

where k_{sat} is typically twice k_{ideal}. Using these equations allows them to model some shock wave data on iron-silicon alloys and the saturation terms are key to fitting the data (red curve in figure on the next page).

Davies et al (2015) give an overview of the effect of high conductivities on the thermal evolution of the core. They consider some Fe-O-Si alloys for which there are extensive ab-initio calculations. Such calculations suggest that the behavior of S in such alloys is very similar to Si. Furthermore, both S and Si are relatively easy to incoporate into the solid inner core but oxygen is almost completely excluded. It is the exclusion of oxygen that gives rise to the density jump at the ICB. This density jump is key to some

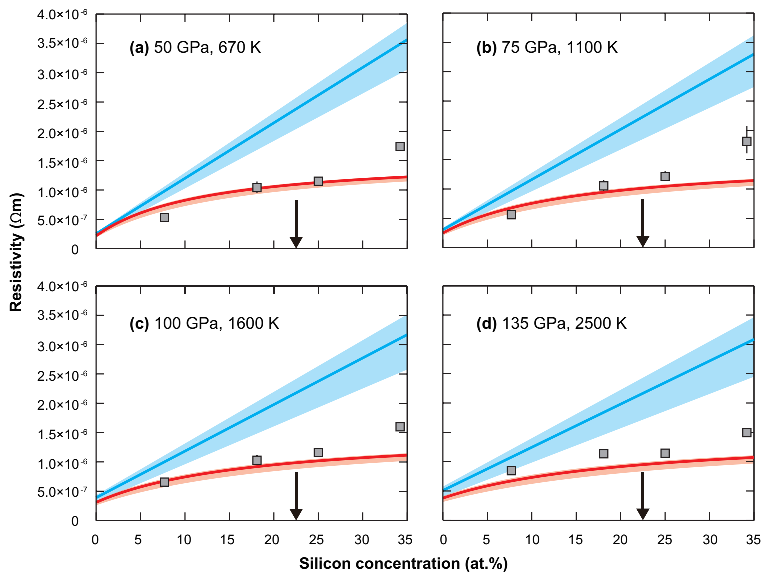

Fig. 5. Shock-wave data for resistivity of iron-silicon alloy (square symbol, Matassov, 1977) compared to our estimates with (red, Eq. (12)) and without (blue, Eq. (10)) the saturation effect at (a) 50 GPa and 670 K, (b) 75 GPa and 1100 K, (c) 100 GPa and 1600 K, and (d) 135 GPa and 2500 K. Note that the saturation model better agrees with the previous measurements, particularly for Si concentrations that are relevant for the outer core (χ_{Si} = 22.5 shown by arrows). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

aspects of the evolution models and we discussed this early on in the course. Davies et al consider three separate values, 0.6 g/cc (the value in PREM), 0.8 g/cc (the value found by Masters and Gubbins), and a fairly extreme value of 1.0 g/cc. These three density jumps lead to three separate models of core alloy which are adjusted (along with the temperature) to fit the properties of the outer core. The resulting temperatures and thermal conductivities are shown below.

The thermal conductivities are a factor of two to three higher than previous values and have a huge impact on thermal history calculations. Note that all recent studies find a thermal conductvitiy of 80 to 110 W/m/K at the CMB and 140-160 W/m/K at the ICB. Previous values ranged from 28 to 46 W/m/K.

Paleomagnetic observations show that the Earth's magnetic field has persisted for at least 3.5 Byr with little apparent change in its strength. One of the main goals of a thermal history calculation is to figure out how this is achieved. To evaluate the power requirements of a dynamo, we need to know how much heat is generated through Ohmic dissipation. This is the dominant dissipative mechanism in the core (except for heat conduction). Unlike the mantle, viscous dissipation is thought to be small since the viscosity is so small.

Note that estimating the amount of Ohmic dissipation going on in the core is hard because most of it is going on at short wavelengths which are sufficiently attenuated at the Earth's surface that they are below the signal from the crustal magnetic field. Furthermore, the core could hold a large toroidal field which is invisible to us but would contribute to the Ohmic dissipation. In what follows we use numerical dynamo models to make a conservative estimate of the dissipation.

The amount of power needed to power a dynamo can provide useful constraints on the thermal history of the Earth – and particularly on the history of inner core growth. To do this problem, we need to consider the equations which control the global balance of energy *and* entropy. The reason for this is that dissipation does not enter into the global energy balance and only occurs in the entropy balance. We initially consider the case where there are no compositional effects and all terms are purely thermal.

Here, we average over a time scale that is long compared with the dynamo process but short compared with the time scale of the evolution of core processes. We assume tht convection mixes the core to a basic state of hydrostatic equilibrium, adiabaticity, and compositional homogeneity. Radial variations in the thermodynamic properties far exceed lateral variatioms due to the convective process, so we ignore the departure from the radial variations of the thermodynamic parameters.

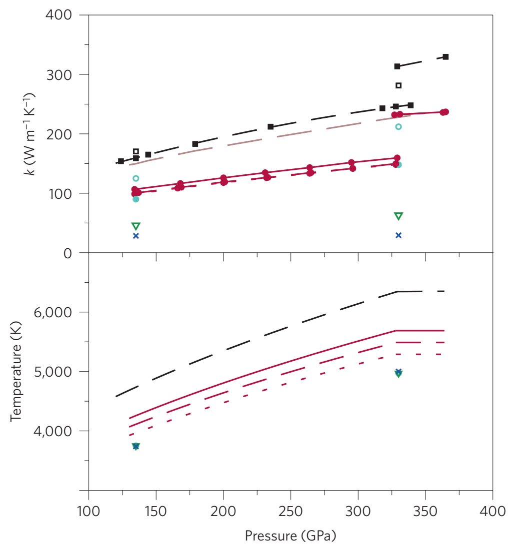

Figure 1 | Comparison of thermal conductivity estimates (top) and adiabatic temperature profiles (bottom) from different studies. The core chemistry models in Table 1 are shown in black (100%Fe; ref. 24) and red (82%Fe–8%O–10%Si, solid line[25]; 79%Fe–13%O–8%Si, long-dashed line[25]; 81%Fe–17%O–2%Si, short-dashed line[80]). Data from two other recent studies are shown for pure Fe (open black squares[26], brown dashed line[23] using the volume–temperature data of Pozzo *et al.*[24]), a mixture of 76.8%Fe–23.2%O (open aqua circles[26]) and a mixture of 77.5%Fe–22.5%Si (filled aqua circles[26]). Two older estimates of k are shown by the open green triangles[29] and blue crosses[30]. Inner core values were obtained from calculations on solid mixtures[27].

2. Global energy balance – no compositional effects

The global energy balance is given by

$$Q = \int_S \mathbf{q} \cdot dS = -\int_S k\nabla T \cdot dS = \int \rho h \, dV - \int \rho \frac{De}{Dt} \, dV - \int \rho \mathbf{v} \cdot \nabla\psi \, dV - \int_S p\mathbf{v}.dS$$

T is temperature, k is thermal conductivity, h is the local heat generation by radioactive sources, ρ is the density, $\mathbf{v}$ is the local velocity, p is pressure and e is internal energy. In the absence of chemical effects, the gravitational term comes only from compression and slow contraction of the core so $-\rho\nabla\psi \simeq \nabla p$. Also

$$de = Tds + \frac{p}{\rho^2} d\rho$$

$$\int \rho \frac{De}{Dt} \, dV = \int \rho T \frac{Ds}{Dt} \, dV + \int \frac{p}{\rho} \frac{D\rho}{Dt} \, dV = \int \rho C_p \frac{DT}{Dt} \, dV - \int \alpha T \frac{Dp}{Dt} \, dV + \int \frac{p}{\rho} \frac{D\rho}{Dt} \, dV$$

$$\int \rho \frac{De}{Dt} \, dV = \int \rho C_p \frac{DT}{Dt} \, dV - \int \alpha T \frac{Dp}{Dt} \, dV - \int p \nabla \cdot \mathbf{v} \, dV$$

Note that, in the absence of compositional effects, the pressure terms cancel:

$$\int p\nabla \cdot \mathbf{v} \, dV - \int \rho \mathbf{v} \cdot \nabla\psi \, dV - \int_S p\mathbf{v}.dS \simeq \int \left(p\nabla \cdot \mathbf{v} + \mathbf{v} \cdot \nabla p - \nabla \cdot (p\mathbf{v}) \right) \, dV = 0$$

So

$$Q = \int \rho h \, dV - \int \rho C_p \frac{DT}{Dt} \, dV + \int \alpha T \frac{Dp}{Dt} \, dV = Q_R + Q_S + Q_P$$

The inclusion of the latent heat of solidification of inner core material can be done by considering the latent heat as an anomaly in C_p at the ICB. Thus, when evaluating terms Q_S becomes $Q_S + Q_L$. and

$$Q = Q_R + Q_S + Q_L + Q_P$$

The form of Q_L is discussed below and it is found that Q_P is small.

3. Global entropy balance – no compositional effects

The global entropy balance is given by

$$\int \rho \frac{Ds}{Dt}\, dV = -\int \frac{\nabla \cdot \mathbf{q}}{T}\, dV + \int \frac{\rho h}{T}\, dV + \int \frac{\phi}{T}\, dV$$

where ϕ is the combined Ohmic and viscous heating (though Ohmic heating dominates). Now

$$\frac{\nabla \cdot \mathbf{q}}{T} = \mathbf{q} \cdot \frac{\nabla T}{T^2} + \nabla \cdot \frac{\mathbf{q}}{T} = -k \left(\frac{\nabla T}{T}\right)^2 + \nabla \cdot \frac{\mathbf{q}}{T}$$

so

$$\int \frac{\rho C_p}{T} \frac{DT}{Dt}\, dV - \int \frac{\alpha T}{T} \frac{Dp}{Dt}\, dV = \int \frac{\rho h}{T}\, dV + \int \frac{\phi}{T}\, dV - \frac{Q}{T_s} + \int k \left(\frac{\nabla T}{T}\right)^2\, dV$$

where T_s is the temperature at the surface of the core. Substituting the above equation for Q and rearranging gives:

$$\int \frac{\phi}{T}\, dV + \int k \left(\frac{\nabla T}{T}\right)^2\, dV = \int \left(\rho h - \rho C_p \frac{DT}{Dt} + \alpha T \frac{Dp}{Dt}\right)\left(\frac{1}{T_s} - \frac{1}{T}\right)\, dV$$

$$E_\phi + E_k = E_R + E_S + E_L + E_P$$

Note that all the terms on the right hand side are multiplied by an "efficiency factor" which can be quite small. The pressure term E_P is also small.

4. Evaluating the integrals

The temperature gradient is assumed adiabatic:

$$T(r) = T_s \exp\left(\int_r^c \frac{g\gamma}{\phi}\, dr\right)$$

Note that the change in exponent over time as the core cools will be small so

$$\frac{1}{T}\frac{DT}{Dt} \simeq \frac{1}{T_s}\frac{dT_s}{dt} \simeq \frac{1}{T_i}\frac{dT_i}{dt}$$

and can be taken out of the integrals. As an example:

$$Q_S = -\int \rho C_p \frac{DT}{Dt}\, dV = -C_p \frac{1}{T_s}\frac{dT_s}{dt} I_s \quad \text{where} \quad I_s = \int \rho T\, dV$$

The latent heat term needs a little care as the rate at which the inner core grows is dependent on the difference between the adiabatic and melting temperature gradients

$$Q_L = 4\pi r_i^2 L \rho(r_i) \frac{dr_i}{dt} \quad \text{where} \quad \frac{dr_i}{dt} = \frac{1}{(dT_m/dp - dT/dp)} \frac{T_i}{\rho g} \frac{1}{T_s} \frac{dT_s}{dt}$$

This can be written as

$$4\pi r_i^2 L\rho(r_i) C_r \frac{dT_s}{dt}$$

where

$$C_r = \frac{1}{(dT_m/dp - dT/dp)} \frac{T_i}{\rho g} \frac{1}{T_s}$$

Note that this allows the energy and entropy equations to be written in the simple form:

$$Q_S = \left(\bar{Q}_s + \bar{Q}_L\right) \frac{dT_s}{dt} + Q_R$$

where $Q_L = \bar{Q}_L dT_s/dt$ etc. Similarly,

$$E_\phi + E_k = \left(\bar{E_S} + \bar{E_L}\right) \frac{dT_s}{dt} + E_R$$

where the barred quantities can be computed from the basic core model. If we choose all the quantities in the basic core model, and the surface heat flow and the radioactive heating (possibly zero) we can solve for the cooling rate and then integrate these equations back in time. We can monitor if E_ϕ stays viable.

5. Dissipation and diffusion

E_k can be estimated using the adiabatic temperature gradient and is about $1\ \times 10^9$ W/K. Note that this assumes that convection occurs all the way up to the surface of the core which may not be true. It is possible that the core is sub-adiabatic near the surface but this requires a discussion of compositional effects.
The dissipation term is almost all ohmic heating (viscous dissipation is thought to be small in the core.) So

$$E_\phi = \int \frac{\mathbf{j}\cdot\mathbf{j}}{\sigma T}\, dV$$

This is difficult to estimate since the toroidal field in the core is invisible and short wavelength fields are attenuated at the surface to be well below the signal from crustal sources. Using model dynamos gives something like $4\ \times 10^8$ W/K – similar to the diffusion term. This is what is used in the Davies calculation but is conservative. It is very difficult to drive a dynamo over the age of the Earth with thermal effects alone.

6. Convective efficiency – a simple example

It is possible to do a convective efficiency calculation in simple situations (though we are more interested in model results). Let $\phi = \mathbf{j}\cdot\mathbf{j}/\sigma$ so

$$\Phi = \int \phi\, dV \quad \text{and} \quad E_\phi = \int \frac{\phi}{T}\, dV$$

Consider the case when we have only radioactive heating and no cooling then

$$Q = Q_R \quad \text{and} \quad E_\phi + E_k = E_R = \int \rho h \left(\frac{1}{T_s} - \frac{1}{T}\right) dV$$

Now E_k is positive so

$$\frac{\Phi}{T_{max}} \le E_\phi \le E_R \le Q\left(\frac{1}{T_s} - \frac{1}{T_{max}}\right)$$

Therefore we can define an "efficiency"

$$\eta = \frac{\Phi}{Q} \leq \frac{T_{max}}{T_s} - 1$$

In principle, this could be greater than one but in practice the maximum temperature will be found at the ICB and the maximum efficiency will be about 25%. It turns out that all the terms considered above have the same low efficiency factors. An exception is the gravitational energy release due to slow separation of light elements into the outer core as the inner core grows. To investigate this, we need to add chemical effects.

7. Adding chemical effects

Consider the core as a two component system – this is simple to extrapolate to a multi-component system as long as the individual systems don't interact. c is the mass fraction of solute. Expressions for the internal energy and entropy change are:

$$de = Tds + \frac{p}{\rho^2}d\rho + \mu dc$$

where s is entropy, and μ is the chemical potential. The continuity equation for c is given by

$$\rho\frac{Dc}{Dt} = -\nabla \cdot \mathbf{i}$$

where $\mathbf{i}$ is the mass flux transport by diffusion. The entropy is a function of p, T, and c so

$$s = s(p, T, c)$$

$$\frac{Ds}{Dt} = \left(\frac{\partial s}{\partial p}\right)_{T,c}\frac{Dp}{Dt} + \left(\frac{\partial s}{\partial T}\right)_{p,c}\frac{DT}{Dt} + \left(\frac{\partial s}{\partial c}\right)_{p,T}\frac{Dc}{Dt}$$

$$\frac{Ds}{Dt} = -\frac{\alpha}{\rho}\frac{Dp}{Dt} + \frac{C_p}{T}\frac{DT}{Dt} + \left(\frac{\partial s}{\partial c}\right)_{p,T}\frac{Dc}{Dt}$$

8. Gravitational term due to growth of inner core

We now get gravitational contributions as the light element is rejected from the inner core over time. Consider the gravitational term

$$\nabla \cdot (\rho\psi\mathbf{v}) = \mathbf{v} \cdot \nabla(\rho\psi) + \rho\psi\nabla \cdot \mathbf{v} = \rho\mathbf{v} \cdot \nabla\psi - \psi\frac{\partial\rho}{\partial t}$$

so

$$Q_G = -\int_\infty \rho\mathbf{v}.\nabla\psi\, dV = -\int_\infty \nabla \cdot (\rho\psi\mathbf{v})\, dV - \int_\infty \psi\frac{\partial\rho}{\partial t}\, dV$$

$$Q_G = -\int \psi\frac{\partial\rho}{\partial t}\, dV = \int \rho\psi\alpha_c\frac{Dc}{Dt}\, dV \quad \text{where} \quad \alpha_c = \frac{-1}{\rho}\left(\frac{\partial\rho}{\partial c}\right)_{P,T}$$

Now the global energy equation becomes

$$Q = Q_R + Q_S + Q_L + Q_P + Q_G + Q_H$$

where Q_H is related to the heat of reaction (but is small). Note that we can write Dc/Dt in the outer core terms of the cooling rate

$$\frac{Dc}{Dt} = \frac{4\pi r_i^2 \rho(i)}{M_{oc}} C_r \Delta c \frac{dT_s}{dt}$$

where Δc is the compositional jump between the inner and outer core (mostly due to oxygen).

There is a new sink of entropy due to diffusion of material which can be shown to have the form

$$E_\alpha = \int \frac{\mathbf{i} \cdot \mathbf{i}}{\alpha_D T} \, dV$$

where α_D is a diffusion coefficient. This term can include some barodiffusion effects (diffusion of light elements down the pressure gradient) but these don't seem to be a significant contribution to the entropy balance. The global entropy balance becomes

$$E_\phi + E_k + E_\alpha = E_R + E_S + E_L + E_P + E_H + \frac{Q_G}{T_s}$$

Note that the last term has no "efficiency factor" and so is more important in the entropy balance. With all these approximations we can write the global energy and entropy equations as

$$Q_S = \left(\bar{Q}_s + \bar{Q}_L + \bar{Q}_G\right) \frac{dT_s}{dt} + Q_R$$

and

$$E_\phi + E_k + E_\alpha = \left(\bar{E_S} + \bar{E_L} + \bar{E_G}\right) \frac{dT_s}{dt} + E_R$$

9. Results

The top figure on the next page shows the results of calculations which assume that E_ϕ is zero before the inner core grows then fixes the CMB heat flow after that. This ensures that the whole outer core convects in accord with the assumptions made. The calculations are mainly sensitive to the density jump at the ICB and the thermal conductivity. The upper curves show results for a low thermal conductivity and show that a dynamo is possible with low values of Q_{CMB} (5 to 7 TW) but the higher conductivities require values of 9 to 13 TW. The highest value corresponds to the lowest value of density jump since more cooling is required in this case

The next figure shows inner core age versus CMB temperature 3.5 Byr ago. The low thermal conductivity calculations have low heat flow into the mantle and relatively low CMB temperatures implying that little mantle melting was going on in the distant past. The high thermal conductivity solutions have very high ancient temperatures, very young inner cores, and substantial ancient heat flows into the mantle. This implies substantial partial melting of the mantle in the past. The highest temperatures are associated with the lowest density jumps which require the greatest cooling. Note also that adding radioactivity has little effect on the answer.

Note that if the heat flow at the surface is less than the heat flow conducted down the adiabat, a stable subadiabatic layer will be formed. Calculations which include this effect give a layer perhaps 100km thick and a slightly reduced heat flow of 13 TW. If such a layer existed, downwelling at the top of the core would not occur. Unfortunately, studies of core flow using the secular variation of the magnetic field are unable to resolve if downwelling is occurring or not.

10. Other possible sources of energy/entropy

The solutions discussed above are sufficiently constrained and extreme that alternative scenarios have been suggested. One way is to posit that magnesium entered the core, either as oxide or silicate, in the early history of the Earth. At ambient conditions, magnesium and metallic iron are immiscible but equilibration at high temperatures (above 3000K) in the aftermath of giant impacts could allow one to two weight percent to

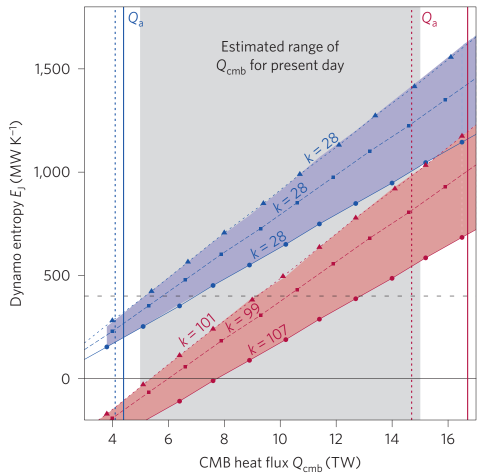

Figure 2 | Present-day core energy budget. Models with high values of the thermal conductivity k use the red profiles in Fig. 1 that have been calculated for $\Delta\rho = 0.6$ (solid lines), 0.8 (long-dashed lines) and 1.0 g cm^{-3} (short-dashed lines); models in blue use $k = 28$ W m^{-1} K^{-1} for each $\Delta\rho$. Other parameters are given in Table 1. Vertical lines indicate ranges for the heat Q_a lost down the core adiabat (colour and linetype again denote k and $\Delta\rho$ respectively). The horizontal black dashed line indicates a plausible estimate for E_J (ref. 65). Dynamo action requires $E_J > 0$. The grey shaded region indicates present-day estimates of CMB heat flow[62,63]. For $Q_{cmb} < Q_a$ any convection in the uppermost core is driven compositionally against thermal stratification.

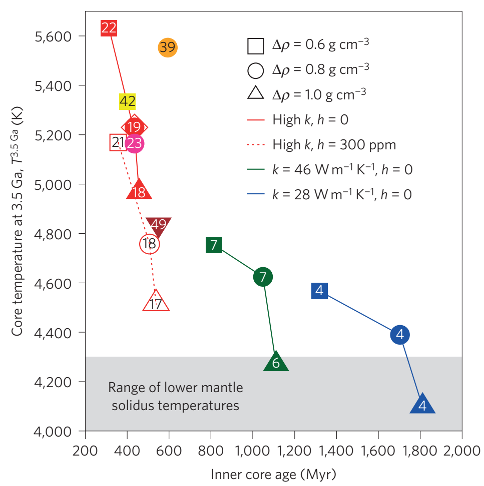

Figure 3 | Core thermal evolution. Numbers inside each symbol give CMB heat flow (in TW) at 3.5 Ga. High k models use the red profiles in Fig. 1 that have been calculated for each $\Delta\rho$; models in blue and green use the same k for each $\Delta\rho$. Models joined by lines use $E_J = 0$ before inner core formation, after which Q_{cmb} is set constant to ensure the outer core remains just superadiabatic. Results from other recent studies are shown in yellow[68], pink[63], orange[70] and maroon[69]. The inverted triangle denotes that $\Delta\rho$ did not enter into this formulation. Open diamond denotes the reference case in Fig. 4.

enter the core.m This will be followed precipitation caused by rapidly decreasing Mg solubility as the core cools. The density difference between the precipitate and the core is about an order of magnitude greater that the density jump at the ICB and it is interesting to note that precipitating a 10km thick layer at the CMB is energetically equivalent to crystallizing the whole inner core.. A core evolution model by O'Rourke and

Stevenson is shown in the final figure and the differences between the dashed line and the solid lines show the dramatic effect of precipitation. It should be noted that this model is controversial with experimental results on Mg partitioning that suggest that the needed strong temperature dependence is not observed.

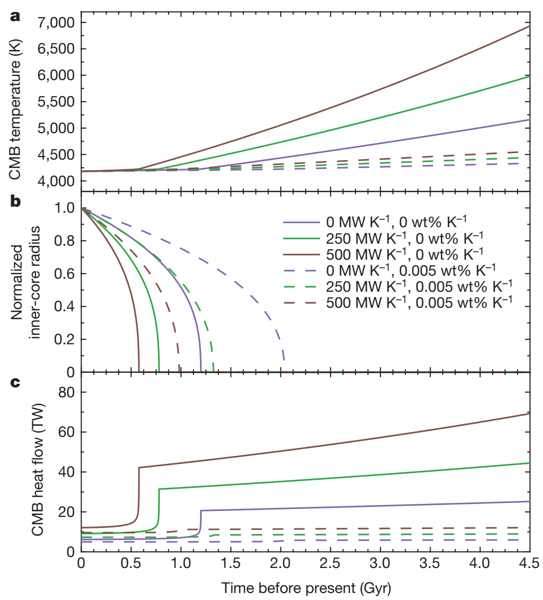

Figure 3 | Thermochemical evolution of the core for various rates of precipitation and entropy production associated with ohmic dissipation. Assuming that the core always produces a constant amount of entropy required to sustain the dynamo, we calculate the implied CMB temperature (**a**), inner-core radius relative to the present (**b**), and CMB heat flow (**c**). Gyr, billion years.