SIO 227B – Practical 2

Write a subroutine to propagate the $\mathbf{y}$ vector for toroidal modes from the bottom of a homogeneous spherical shell to the top of a homogeneous shell. In other words, compute $\mathbf{P}(r, r_0)$ where

$$\mathbf{y}(r) = \mathbf{P}(r, r_0)\mathbf{y}(r_0)$$

Check your answer by modeling the mantle as a single spherical shell and computing toroidal mode frequencies (*i.e.*, those frequencies for which $y_2(a) = 0$) for some low values of l. If you choose the density and rigidity appropriately, your answers should be similar to the more realistic calculations given in the paper by Masters and Widmer which contains a table of mode frequencies computed for a realistic Earth model.

Now write a subroutine to compute the $\mathbf{y}$ vector at the top of a homogeneous sphere (this is simpler than the shell because the solution must be regular at the origin). This subroutine allows you to start toroidal mode calculations at the center of the Earth. Compute the toroidal mode frequencies for a sphere with the average properties of the inner core.

Now construct a three-layer model of the Earth consisting of your average mantle and average inner core and an outer core with a small but finite rigidity (set the shear velocity to a few hundred meters per second). Compute the modes of this model in the same part of ω/l space as you did the above calculations. Explain what you see, particularly with regard to the frequency spacings of the modes at fixed l. Which of these modes do you think would be observable at the surface?

(Subroutine sphb2.f is available in mode.dir for computing spherical Bessel functions and might be useful. Here is a description of the call:

```
        subroutine sphb2(j,y,jp,yp,x,l)
c Calculate spherical Bessel functions of the 1st and 2nd kind with real
c arguments. Does only one l
c*** all variables are double precision except for l which is integer
c on input
c x = arg of Bessel function (not zero)
c l = angular order
c on output.
c j contains the spherical Bessel function of order l
c y contains the spherical Bessel function of order l
c jp=dj/dx
c yp=dy/dx
c Continued fraction technique for computing spherical Bessel functions in
c Mie scattering, see : W. Lentz ,Applied Optics, vol.15, #3, March, 1976
```

If you don't like Fortran programming, you can probably do this in Matlab – but don't ask Guy for help!)

In the class directory (subdirectory mode.dir) you can run a program called "minos" (documentation on how to run it is in minos.doc). The program computes the eigenfrequencies and eigenvectors of all kinds of modes. You can view the eigenvectors and/or energy densities using the program "eigpix" which can also be run from this directory. Use the program to compute an example of each of the following kinds of modes and plot where they occur on the ω/l plot:

1) A Stoneley mode on the inner core boundary
2) A Stoneley mode on the mantle core boundary
3) A mantle shear mode (makes up surface waves)
4) A PKP equivalent mode
5) A $PKIKP$ equivalent mode
6) A J equivalent mode (otherwise called a "core mode").
7) A PcP or ScS equivalent mode

Note that you can also use the Q and group velocity of a mode to help you identify its character. In particular, modes which sample into the core tend to be high Q and J modes will have the same Q as the model value for Q_μ in the inner core. The group velocity of waves trapped on the mantle-core boundary is about 8.4 km/s as can be determined from the slope of the ω/l curve. PcP and ScS equivalent modes have very low group velocities (why?).