Data assimilation and inverse problems – Homework Set 7

Importance sampling

1. Let x be the parameter you want to estimate. Assume a Gaussian prior, $p(x) = \mathcal{N}(1,1)$. You have an observation given by

$$y = x^3 + \eta \quad \eta \sim \mathcal{N}(0, 0.1^2).$$

The posterior distribution you want to sample is given by

$$p(x|y) = \exp(-F(x)), \quad F(x) = \frac{1}{2}(x-1)^2 + \frac{1}{2}\left(\frac{y - x^3}{0.1}\right)^2.$$

An experimentalist collected for you the measurement $y = 2.5$.

Draw samples from the posterior distribution of this data assimilation problem using

(a) a Gaussian proposal;

(b) a multivariate-t proposal distribution;

(c) a proposal generated by random maps.

Comment on and compare the effective sample size $N_{\text{eff}} = N_e/\rho,\ \rho = E[w^2]/E[w]^2$ for each of the three methods.

2. Use importance sampling to draw samples from the distribution

$$p(\theta) \propto \exp\left(-F(\theta)\right), \quad \text{where } F(\theta) = 10^{-2}\,||\theta - [5;5]||^4 + 0.2\,\sin(5||\theta||).$$

Note that θ is a vector with two components. Draw a triangle plot of your samples. Compare with a plot of p (evaluate p over a grid for θ).