Data assimilation and inverse problems – Homework Set 8

Particle filters

1. Consider a stochastic version of the Lorenz'63 model given by:

$$x_k = f(x_{k-1}) + \sqrt{\Delta t}\, v_k, \quad v_k \sim \mathcal{N}(0, I), \text{ iid},$$

   where

$$f(x_k) = x_k + \Delta t \begin{pmatrix} \sigma(x_k^2 - x_k^1) \\ x_k^1(\rho - x_k^3) - x_k^2 \\ x_k^1 x_k^2 - \beta x_k^3 \end{pmatrix}$$

   where $x_k = (x_k^1, x_k^2, x_k^3)^T$, $\sigma = 10$, $\beta = 8/3$, $\rho = 28$. Use a time step of $\Delta t = 0.01$.

   (a) You have observations of x^1 and x^3 every $\Delta T = 0.01$ time units. The observation noise is Gaussian with zero mean and covariance $R = I$. Implement a "standard" particle filter, an optimal particle filter, and an EnKF. How do these three methods compare?

   (b) You have observations of x^1 and x^3 every $\Delta T = 0.1$ time units. The observation noise is Gaussian with zero mean and covariance $R = I$. Implement a "standard" particle filter and an EnKF. How do these three methods compare?

2. Try a particle filter on the Lorenz'95 model (see HW 3). Does it "work"? How many particles do you need?