Math 128b – Spring 2014 – Homework set 2

Due Tuesday 2/4 in class before the lecture starts.

1. p.102, Excercise 5.

2. p.102, Excercise 9.

3. p.129, Excercise 9.

4. p.129, Excercise 12.

5. Apply your implementation of "naive" Gaussian elimination (from homework set 1) to $Ax = b$ with
$$A = \begin{pmatrix} 10^{-15} & 1 \\ 1 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$
What is the approximate solution? What is L and U? What is the condition number of A (use Matlab's cond(A,inf) to find it)? Apply Gaussian elimination with partial pivoting (in the form of Matlab's "\") to this system. Find $PA = LU$ (using Matlab's "lu" function). What is P, L and U?

6. Consider the following three operations: (i) swap one equation for another; (ii) add or subtract a multiple of one equation from another; (iii) multiply an equation by a nonzero constant. Prove that when each one is applied to a linear system of equations ($Ax = b$), then it leads to an equivalent linear system ($\tilde{A}x = \tilde{b}$), in the sense that the equivalent system has the same solution as the original one. Hint: What happens when you multiply $Ax = b$ from the left with a nonsingular matrix? Can you formulate the three operations in terms of matrix multiplications?

7. Let A be a real $n \times n$ matrix such that $A^T = A$. Show that if all n eigenvalues λ_j, $j = 1, \ldots, n$ are distinct ($\lambda_i \neq \lambda_j$ for $j \neq i$), then the eigenvectors are mutually orthogonal so that $v_i^T v_j = 0$ if $i \neq j$.