Math 128b – Spring 2014 – Homework set 4

Due Tuesday 2/18 in class before the lecture starts.

1. Consider the linear $n \times n$ system $Ax = b$ with $n = 100,000$,

$$A = \begin{pmatrix} 3 & -1 & & & \\ -1 & 3 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 3 & -1 \\ & & & -1 & 3 \end{pmatrix},$$
$$b = (2, 1, 1, \cdots, 1, 1, 2)^T.$$

   (a) Use $x_0 = (0, 0, \ldots, 0)^T$ as your initial guess and solve the above system using the Jacobi method. Stop the iteration when the ∞-norm of the residual is less than 10^{-3}.

   (b) Use $x_0 = (0, 0, \ldots, 0)^T$ as your initial guess and solve the above system using the Gauss-Seidel method. Stop the iteration when the ∞-norm of the residual is less than 10^{-3}.

   (c) Use $x_0 = (0, 0, \ldots, 0)^T$ as your initial guess and solve the above system using the conjugate gradient method. Stop the iteration when the ∞-norm of the residual is less than 10^{-3}.

   You should use Matlab's sparse matrix package to do these calculations. You should hand in: your code for Jacobi and Gauss-Seidel methods as well as one figure that shows the ∞-norm of the residual as a function of the iteration number for all three methods.

2. p. 116, Exercises 2.5: 5