Math 128b – Spring 2014 – Homework set 9

Due Tuesday 4/8 in class before the lecture starts.

Problem 1

(a) Code our workhorse algorithm for computing eigenvalues and eigenvectors of a square matrix. First bring A into upper Hessenberg form (using Householder reflectors, you need to write your own code for this). Then apply shifted QR with inflation to find all eigenvalues of A (you can use Matlab's function "qr" for the required QR factorizations). Then apply one step of inverse power iteration to find the eigenvectors of A.

(b) Write code to compute the SVD of A. First compute eigenvalues and eigenvectors of

$$B = \begin{pmatrix} 0 & A^T \\ A & 0 \end{pmatrix},$$

using your code from (a). Then use the results (eigenvalues and eigenvectors of B) to construct the SVD of the matrix A.

Test your codes on the matrix

$$A = \begin{pmatrix} 7 & -33 & -15 \\ 2 & 26 & 7 \\ -4 & -50 & -13 \end{pmatrix},$$

and compare with Matlab's commands "eig(A)" and "svd(A)" give you. You should hand in your codes for the workhorse QR algorithm (including the step with putting A into upper Hessenberg form) and for your SVD, as well as the results of these codes when applied to A above.

Problem 2
Let A be an $n \times n$ symmetric matrix.

(a) Show that the eigenvalues of A are real.

(b) Show that the eigenvectors of A can be chosen real.

(c) Show that if all eigenvalues are distinct, then the eigenvectors of A are orthogonal (the assumption of distinct eigenvalues makes the proof easy, but the statement is true even if some eigenvalues are repeated).

(d) Show that A in upper Hessenberg form is tridiagonal.

(e) Express the SVD of A in terms of the eigenvalues and eigenvectors of A. In particular show that the singular values of A are the absolute values of the eigenvalues of A.

Problem 3
Let A be a real $m \times n$ matrix.

(a) Show that A^TA and AA^T have the same nonzero eigenvalues.

(b) Show that the nonzero singular values of A are the square roots of the eigenvalues of A^TA or AA^T.