Math 313-001
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Homework #1

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1-1

#28. $ax_1 + bx_2 = f$
$cx_1 + dx_2 = g$ $\rightarrow \begin{bmatrix} a & b & f \\ c & d & g \end{bmatrix}$

$\sim \begin{bmatrix} 1/a & b/a & f/a \\ c & d & g \end{bmatrix}$, multiply by $\frac{1}{a}$, since $a \neq 0$.

$\sim \begin{bmatrix} 1/a & b/a & f/a \\ 0 & d - \frac{cb}{a} & g - \frac{cf}{a} \end{bmatrix}$ Replace R2 by R2 + (-c) R1

In order for the system to be consistent for all g and f, $d - \frac{cb}{a} \neq 0$.
Thus, $d \neq \frac{cb}{a} \Rightarrow ad \neq bc$.

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#8. $\begin{bmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & -1 & 0 & -4 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & 1 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 4 \end{bmatrix}$

$\begin{cases} x_1 = -9 \\ x_2 = 4 \\ x_3 = r \quad (\text{free variable}) \end{cases}$

#18. $\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & -2 \\ 0 & h+15 & -7 \end{bmatrix}$

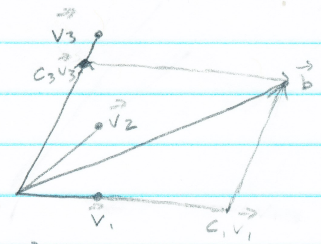
In order for the system to be consistent $h + 15 \neq 0$. So, $h \neq -15$.

1-3

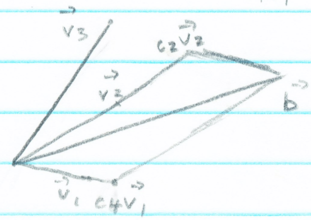
#13. $\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

This system is inconsistent, so b is <u>not</u> a linear combination of the columns of A.

#32



$c_1\vec{v}_1 + c_3\vec{v}_3 = \vec{b}$



$c_4\vec{v} + c_2\vec{v}_2 = \vec{b}$

Thus, the equation $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 = \vec{b}$ has at least two solutions.

(Note: Any two vectors that are not multiples of each other span the plane between them, in this case $\mathbb{R}^2$. Thus $\text{span}\{\vec{v}_1, \vec{v}_2\} = \mathbb{R}^2$ and $\text{span}\{\vec{v}_2, \vec{v}_3\} = \mathbb{R}^2$. And $\vec{b} \in \mathbb{R}^2$, so by the definition of span, $\vec{b}$ can be written as a linear combination of $\vec{v}_1$ and $\vec{v}_2$ and as a linear combination of $\vec{v}_1$ and $\vec{v}_3$. This is basically stating what we did graphically in words.)