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## Research Interests: Inverse theory, geomagnetism, spectral analysis, electromagnetic sounding

Bob Parker has worked in a number of his research areas during the 2007 academic year, including the inverse theory of geomagnetic sounding with his student Ashley Medin and Steven Constable, work partially described in the 2006 Annual Report. We completed the study and obtained for the first time firm bounds derived from geomagnetic time series on the average conductivities in three regions: the upper mantle, the transition zone, and the top of the lower mantle. Our results (published in *PEPI*) preclude the presence of significant amounts of water in the transition zone conjectured in the geodynamics literature. Work on the two-dimensional extension of the inversion technology continues.

Another broad topic occupying Parker this year with several specific applications is estimation of power spectra from time series, or spatial profiles. Modern methods of spectral estimation are based on the *multitaper method* introduced in 1982 by David Thomson, now at Queens University. The basic idea is that many independent estimates are computed from the record by applying a set of orthogonal weight functions (called tapers), finding the periodogram, and performing a weighted average. In the now classic approach devised by Thomson the tapers are computed to minimize broad-band bias, which is the tendency for power from strong peaks to spread into neighboring frequency intervals of lower power; this undesirable tendency is called "spectral leakage." To achieve maximum suppression of random variability, the multitaper must average the true spectrum over a designated frequency interval, introducing a second form of bias which flattens out the strong peaks. Parker worked with student German Prieto, and colleagues David Thomson and Frank Vernon to study this issue. We have found that by estimating the covariance matrix of the multitaper spectra it is possible to calculate two derivatives of the spectrum with respect to frequency, and in this way estimate the true curvature of the spectrum which is normally lost during the standard averaging procedure. Thus the peaks can be recovered, and the true spectrum more faithfully rendered. This work is described in a paper submitted to the Geophysical Journal International.

The multitaper method is a form of nonparametric estimation, intended for situations in which no simple equation s known to describe the shape of the power spectrum. Sometimes, however, a simple mathematical expression works very well. Such a situation arises in a study underway by Professor David Sandwell and his student Joseph Becker who are making maps of seafloor roughness down to the one-kilometer scale based on ship tracks. In some areas where the coverage is very dense one discovers that the power spectrum of long straight profiles of bathymetry is quite accurately described by the equation

$$P(k_x) = \frac{b}{(1 + k_x^2/k_0^2)^{\mu - \frac{1}{2}}}$$

where the three constants b,  $k_0$  and  $\mu$  depend on the region; here  $k_x$  is the along-track wavenumber, or reciprocal wavelength. Mixing of the ocean waters by tidal motions is strongly influenced by the bottom roughness, and a global assessment of this effect is currently impossible because of sparse coverage of the ocean bathymetry. Sandwell and Becker aim to fill this gap, but to do so they must rely in many places on a few scattered ship tracks on which to base an estimate of  $P(k_x)$ ; see Figure 1a. We need not only values for the three unknowns in the equation, but some estimate of the accompanying uncertainties in the estimates, and for these purposes the nonparametric approach is not optimal. Parker has shown





*Figure 1*: (a) Ship tracks available for analysis in a 175-km square in the Pacific; (b) The unsmoothed power spectrum and best-fitting parametric model spectrum; (c) Blue: histogram of residuals, yellow histogram after stretching.

that to obtain reliable values with uncertainties it is better not to calculate a smooth spectrum as a starting point. Instead, we form estimates at many frequencies with large individual variances as in Fig 1b, and let the functional form of P to do the smoothing. The parameters in the equation are found by a nonlinear least-squares fit, but the distribution function of the residuals is far from Gaussian, causing bias. To remedy that defect, the spectrum (and the model) are subjected to cube-root stretching before fitting: this mapping transforms the usual chisquared error distribution into an excellent approximation of the normal distribution; see Figure 1c. Other problems we have dealt with include combine short-wavelength estimates from many short pieces of track, and calculating the distortions caused by track curvature. With these various ideas we are able to obtain reliable seafloor spectral parameters in almost every one degree square of the oceans.

## **Relevant Publications**

Medin, A., Parker, R., and Constable, C., Making sound inferences from geomagnetic sounding, *Phys. Earth & Planet. Int.*, v 160, 51-9, doi:10.1016/j.pepi.2006.09.001, 2007.

Prieto, G. A., Parker, R. L., Thomson, D. J., and Vernon, F. L., The Quadratic multitaper spectrum, *Geophys. J. Internat.*, submitted Nov 2006.