5. THE CRUSTAL MAGNETIC FIELD

Magnetic anomalies are the signals of the internal geomagnetic field left behind after the part generated by Earth's core has been removed from the observations. Most of the mantle and some of the lower portions of the crust are above the Curie point of magnetic minerals; thus the source of the magnetic anomalies must lie in the crust or uppermost mantle. We see this from the Lowes spectrum; if there were significant mantle sources, the spectrum would not die away exponentially, but more as a power law, just like the gravitational SH spectrum. Magnetic anomalies have of course played a very important role in earth sciences: the correlation of lineated magnetic anomalies roughly parallel to mid-oceanic ridge crests with the geomagnetic reversal time scale provided the final evidence for seafloor spreading.

Regional magnetic surveys, both on land and under the oceans used to be performed by taking spot readings of the magnetic field. Now, however, aircraft or shipboard surveys are commonly carried out with a towed proton precession magnetometer, which measures only the field intensity not its direction. In order to study magnetic anomalies a standard (usually IGRF) core field model, $\mathbf{B}_0$, is subtracted from the observations; this is often called the *regional magnetic field*. Suppose the sum of all contributions to the field is $\mathbf{B}$. The *total field anomaly* is the difference between the magnitudes of the observed and core magnetic fields:

$$\Delta T = |\mathbf{B}| - |\mathbf{B}_0|. \tag{150}$$

Now we will let $\Delta\mathbf{B}$ be the contribution to $\mathbf{B}$ of some anomalous magnetic source. Then

$$\mathbf{B} = \mathbf{B}_0 + \Delta\mathbf{B}. \tag{151}$$

We would like to know $\Delta\mathbf{B}$ to study the crustal source, so

$$\begin{aligned}\Delta T &= |\mathbf{B}_0 + \Delta\mathbf{B}| - |\mathbf{B}_0| \\ &= \sqrt{|\mathbf{B}_0|^2 + |\Delta\mathbf{B}|^2 + 2\Delta\mathbf{B}\cdot\mathbf{B}_0} - |\mathbf{B}_0|. \end{aligned} \tag{152}$$

Neglecting quantities $O|\Delta\mathbf{B}|^2/|\mathbf{B}_0|^2$ we find

$$\begin{aligned}\Delta T &= |\mathbf{B}_0|[1 + 2\Delta\mathbf{B}\cdot\mathbf{B}_0/|\mathbf{B}_0|^2]^{\frac{1}{2}} - |\mathbf{B}_0| \\ &= |\mathbf{B}_0|[1 + \Delta\mathbf{B}\cdot\mathbf{B}_0/|\mathbf{B}_0|^2] - |\mathbf{B}_0| \\ &= \Delta\mathbf{B}\cdot\mathbf{B}_0/|\mathbf{B}_0| = \Delta\mathbf{B}\cdot\hat{\mathbf{B}}_0. \end{aligned} \tag{153}$$

Thus $\Delta T \approx \Delta\mathbf{B}\cdot\hat{\mathbf{B}}_0$, that is, it is the component of the anomaly field in the direction of the regional field, provided the anomaly field is small in magnitude relative to the total field. The validity of this approximation depends on the size of $\Delta\mathbf{B}$ relative to $\mathbf{B}$. Typical crustal magnetic anomalies range in magnitude from a few nT to several thousand nT, but are usually less than 5,000 nT, so this provides an adequate representation for total field anomalies.

Is the total field harmonic? Approximately, to the extent that the regional field direction remains constant in the survey region. We can see this easily, writing $\mathbf{\Delta B} = -\nabla V$ with harmonic V,

$$\nabla^2 \Delta T = -\nabla^2(\nabla V \cdot \hat{\mathbf{B}}_0) = -\hat{\mathbf{B}}_0 \cdot \nabla\nabla^2 V = 0. \tag{154}$$

If $\mathbf{B}_0$ is not effectively constant, the algebra is messy and the subject not worth much effort.

As with satellite observations, the reason one prefers to measure the field magnitude $|\mathbf{B}|$ at sea is that there is no need to keep an accurately oriented platform, and total field data can be made to ± 1 nT in 60,000 nT with very robust instruments, and ± 0.1 nT with only a little more effort. As we asked in 3.4.1 we can inquire whether knowledge of ΔT is actually sufficient to describe the harmonic field of crustal sources. If $\mathbf{B}_0$ is effectively constant, and not horizontal, then knowledge of ΔT is fully equivalent to knowledge of $\mathbf{\Delta B}$ itself, since one can then construct $\mathbf{\Delta B}$ from ΔT. But as usual, things aren't quite so simple. One collects survey data on long lines in the oceans, and it really is not possible to know $\mathbf{B}$ everywhere on a surface. If instead of ΔT on a long line one actually has the vector data $\mathbf{\Delta B}$, then valuable information can be obtained about the accuracy of the measurements and other useful things, like across track lineation, by looking at the correlations between the components. See Parker and O'Brien, *J. Geophys. Res.* 102, pp 24815-24, 1997. There is a move towards measuring the vector field in some marine surveys, particularly on Japanese ships and on near-seafloor instruments.

5:1 Magnetic Permeability and Susceptibility

In chapter 2 we outlined the relationship between magnetic displacement **H** and magnetic induction **B**

$$\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M} \tag{155}$$

with **M** representing the magnetic polarization or magnetization of the material. In the region where we make measurements it is unnecessary to distinguish between **B** and **H** because there are no currents flowing and no magnetization. Inside the crust, however, **M** depends on the atomic and macroscopic properties of the material. Materials can acquire a component of magnetization in the presence of an external magnetic field (such as that generated in Earth's core). The so-called *induced magnetization* is often considered to be proportional in magnitude to and along the direction of the external field and

$$\mathbf{M} = \chi \mathbf{H} \tag{156}$$

and χ is called the *magnetic susceptibility*. Then we can write

$$\mathbf{B} = \mu_0(1 + \chi)\mathbf{H} = \mu \mathbf{H} \tag{157}$$

where μ is the *magnetic permeability*. In practice χ may be dependent on field intensity, negative or need to be represented by a tensor (magnetically anisotropic materials).

Although diamagnetism due to perturbations of electron orbits in an applied field and paramagnetism (perturbations of atomic magnetic moments) are important physical processes, these are insignificant contributors to the geomagnetic field.

The important contributions come from materials with atomic moments that interact strongly with each other as a result of quantum mechanical exchange interactions. These are called *ferrimagnetic* materials and they can carry either an *induced* or *remanent* magnetization. The total magnetization of a rock will result from the sum of these two contributions

$$\mathbf{M} = \mathbf{M}_i + \mathbf{M}_r = \chi\mathbf{H} + \mathbf{M}_r. \tag{158}$$

The stability and acquisition of magnetization depends on temperature: above the Curie temperature thermal perturbations destroy the spontaneous magnetization so that the only remaining magnetization is from diamagnetic or paramagnetic effects.

Magnetite (Fe_3O_4, Curie temperature 580°C) and its solid solutions with ulvospinel Fe_2TiO_4) are the most important magnetic minerals in crustal rocks, although hematite, pyrhotite also play a role in paleomagnetic studies.

5:2 Crustal Magnetic Models

The ultimate goal of a regional magnetic survey is to make some inferences about the spatial distribution and nature of the magnetic sources generating the anomalies and hence to draw conclusions about the geological processes active in the earth. The first step in understanding this problem is to calculate the fields from a model: that is for a given distribution of magnetization predict the expected observations; this is called solving the *forward problem*.

How can we do this? We start by considering the magnetic scalar potential Ψ at position $\mathbf{r}$ due to a point dipole $\mathbf{m}$ located at $\mathbf{s}$:

$$\Psi(\mathbf{r}) = \frac{\mu_0}{4\pi}\mathbf{m}\cdot\nabla_s\frac{1}{|\mathbf{r} - \mathbf{s}|}. \tag{159}$$

To find the potential for a distribution of magnetization, we consider it to be a sum of contributions from elemental dipoles (recall the magnetization vector $\mathbf{M}$ is just a density of dipole moment per unit volume):

$$\Psi(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_V \mathbf{M}(\mathbf{s})\cdot\nabla_\mathbf{s}\frac{1}{|\mathbf{r} - \mathbf{s}|}d^3\mathbf{s} \tag{160}$$

where V is the magnetized source region. Then

$$\mathbf{B} = -\nabla_\mathbf{r}\Psi(\mathbf{r}) = -\frac{\mu_0}{4\pi}\nabla_\mathbf{r}\int_V \mathbf{M}(\mathbf{s})\cdot\nabla_\mathbf{s}\frac{1}{|\mathbf{r} - \mathbf{s}|}d^3\mathbf{s}$$

$$= -\frac{\mu_0}{4\pi}\int_V \mathbf{M}(\mathbf{r})\cdot\nabla_r\nabla_s\frac{1}{|\mathbf{r} - \mathbf{s}|}\, d^3\mathbf{s}. \tag{161}$$

You will easily verify that

$$\nabla_r\nabla_s\frac{1}{|\mathbf{r} - \mathbf{s}|} = -\nabla_s\nabla_s\frac{1}{|\mathbf{r} - \mathbf{s}|}. \tag{162}$$

Now let's evaluate (162). By using the summation convention and keeping calm, it's quite easy. First some abbreviations:

let $\mathbf{R} = \mathbf{r} - \mathbf{s}$; $\partial_j = \partial/\partial s_j$ and also recall from Part I that $\partial_j s_k = \delta_{jk} = -\partial_j R_k$. Then

$$
\begin{aligned}
G_{ij} &= \partial_i \partial_j (R_k R_k)^{-\frac{1}{2}} = \partial_i [-\frac{1}{2}(R_k R_k)^{-3/2} \partial_j (R_k R_k)] \\
&= \partial_i [-\frac{1}{2}(R_k R_k)^{-3/2}(\partial_j R_k R_k + R_k \partial_j R_k)] \\
&= \partial_i [-\frac{1}{2}(R_k R_k)^{-3/2}(-\delta_{jk} R_k - R_k \delta_{jk})] = \partial_i [R_j (R_k R_k)^{-3/2}] \\
&= (\partial_i R_j)(R_k R_k)^{-3/2} \ + \ R_j \partial_i (R_k R_k)^{-3/2} \\
&= -\delta_{ij}(R_k R_k)^{-3/2} \ + \ R_j [-3/2 (R_k R_k)^{-5/2}](-2 R_k \delta_{ik}) \\
&= \frac{3 R_i R_j}{(R_k R_k)^{5/2}} \ - \ \frac{\delta_{ij}}{(R_k R_k)^{3/2}}.
\end{aligned} \tag{163}
$$

Substituting (163) into (161) and restoring vector notation gives us

$$
\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_V \left[\frac{3\mathbf{M}(\mathbf{s}) \cdot \mathbf{R}\mathbf{R}}{|\mathbf{R}|^5} \ - \ \frac{\mathbf{M}(\mathbf{s})}{|\mathbf{R}|^3} \right] \, d^3\mathbf{s} \tag{164}
$$

where you will recall $\mathbf{R} = \mathbf{r} - \mathbf{s}$. The expression in brackets is nothing more than the vector magnetic field at $\mathbf{r}$ from a point dipole, moment M at $\mathbf{s}$. Most geophysicists seem afraid of this formula and it doesn't appear very often in texts.

Finally if we have measured the total field anomaly and the regional core field has constant direction $\hat{\mathbf{B}}_0$, then

$$
\begin{aligned}
\Delta T &= \frac{\mu_0}{4\pi} \int_V \left[3\mathbf{M} \cdot \mathbf{R}\hat{\mathbf{B}}_0 \cdot \frac{\mathbf{R}}{|\mathbf{R}|^5} \ - \ \frac{\mathbf{M} \cdot \hat{\mathbf{B}}_0}{|\mathbf{R}|^3} \right] \, d^3\mathbf{s} \\
&= \frac{\mu_0}{4\pi} \int_V \mathbf{G}(\mathbf{r} - \mathbf{s}) \cdot \mathbf{M}(\mathbf{s}) \, d^3\mathbf{s}.
\end{aligned} \tag{165}
$$

Solving the forward problem thus comes down to evaluating the above integral for an appropriately shaped body. In practice this is usually done by making approximations to the shape of the body.

For compact regions such as seamounts (which are isolated submarine volcanos) a favorite plan is to divide the region V into a set of smaller elementary shapes, like cuboids, over which the integral can be done exactly, then sum. See Blakely's book for some references to this approach. Sometimes it is permissible to take $\mathbf{M}$ = constant in space. Then one can apply identities and get (165) into a surface integral with the divergence theorem. The surface can be conveniently approximated by a set of triangular faces, called a *tessellation*; see Parker, Shure, and Hildebrand, *Rev. of Geophys.* 25, pp 17-40, 1987.

In the early days of marine magnetic survey work it was discovered, as you will know, that in many places the magnetic anomaly pattern takes the form of a series of stripes, caused by reversals of the ancient geomagnetic field and linear emplacement at the ocean ridges. This geometry allows a simplification by assuming that the magnetization $\mathbf{M}$ is constant in one direction, usually identified with the y axis. Then (165) can be reduced to an integral of the x-z plane, and is much easier to do; the formula for a polygon shape is quite simple.

Often it can be assumed that the magnetic layer is very thin, so that one can ignore variations of $\mathbf{M}$ in the vertical z direction. You will appreciate that when the thin-layer assumption is made, (165) has the form of a convolution and then

Fourier methods can be invoked. This is very popular in marine magnetic work, both for the older single-profile data sets and the more modern surveys of an area. Again Blakely devotes a lot of space to this issue.

Exercise:

(a) As suggested, with **M** constant rewrite (165) in a form that you can apply the divergence theorem and reduce the integral to a surface form.

(b) Find out about Poisson's relationship. Explain what it is and when its use might be appropriate.

5:3 The Magnetic Annihilator and Runcorn's Theorem

At the risk of being repetitious, I want to restate that the object of the process of magnetic modeling is to learn about the state of the crust, first its magnetic state, and then, if we're lucky, other things too. In the last section we assumed that **M** was known and we calculated $\Delta\mathbf{B}$ from it, but in reality we don't know the magnetization and we do know the anomaly. We need to reverse the process, which is called solving the *inverse problem*. A major question in any inverse problem is that of uniqueness: does a (complete and exact) set of data determine the unknown magnetization, or is there ambiguity, even with ideal data? It turns out the inverse problem for magnetization is *ill-posed*, which means that there are infinitely many possible solutions to choose from, unless further restrictions or simplifying assumptions are brought in. You might perhaps have come to this conclusion from the equivalent source theorem for gravity, which applies equally well here: if $\mathbf{B} = -\nabla\Psi$ and all magnetic sources are confined to compact region V

$$\Psi(\mathbf{r}) = \frac{1}{4\pi}\int_{\partial V} d^2\mathbf{s}\;\left[\Psi\frac{\partial}{\partial n}\frac{1}{|\mathbf{r}-\mathbf{s}|} \;-\; \frac{1}{|\mathbf{r}-\mathbf{s}|}\frac{\partial\Psi}{\partial n}\right]. \tag{166}$$

This equation tell us that we can mimic the potential of a magnetized body by poles and dipoles on the surface, whatever the true interior distribution of **M** may be. The need for poles is a bit disturbing, however.

Here is another, perhaps more startling example. In (165) let the magnetization be $\mathbf{N} = \nabla q$ where $q(\mathbf{s})$ is *any* smooth function that vanishes on the boundary ∂V. It will be seen from (162) that $\mathbf{G} = \nabla p$ and $\nabla^2 p = 0$ provided $\mathbf{r}$ is outside V, which it always is. Then

$$\frac{4\pi}{\mu_0}\Delta T = \int_V \nabla p\cdot\nabla q\; d^3\mathbf{s} = \int_V [\nabla\cdot(q\nabla p) \;-\; q\nabla^2 p]\; d^3\mathbf{s}$$

$$= \int_{\partial V} q\nabla p\cdot\hat{\mathbf{s}}d^2\mathbf{s} = 0. \tag{167}$$

Thus the anomaly caused by this whole family of functions is zero. Hence if we have a magnetization **M** that matches observation, then so does $\mathbf{M}+\mathbf{N}$. The function **N** is an example of a magnetic *annihilator*, a function with no observable magnetic field, yet nonzero internal magnetization. Of course we have already encountered toroidal magnetic fields, whose currents in a sphere are also annihilator sources.

The existence of annihilators means that we cannot ever know what a magnetic source is based solely on its magnetic anomaly. The problem is ill-posed.

It was assumed in the marine world that if the magnetic layer was very thin, this problem would be eliminated. But Parker and Huestis (*J. Geophys. Res.,* 79, pp 1587-93, 1974) showed there was ambiguity even then, in the form of a single function which can be added in arbitrary amounts to any solution, without disturbing the agreement.

Finally, an important example of an annihilator was discovered by Keith Runcorn in studies of the moon's magnetic field (*Nature* 253, 1042, pp 701-3, 1975). Runcorn was convinced the moon had once had an internal dynamo which has ceased to operate because the lunar core became solid. Samples from the moon have proved to be magnetic, yet there appears to be almost no lunar magnetic field. Critics of Runcorn asked, If there is a general lunar magnetization from the ancient dynamo, why can't we see the field from these fossil sources? Here is Runcorn's surprising answer: If the magnetization of the moon is induced magnetization in a shell, no matter what form the internal dynamo field was like, the resulting magnetization would be an annihilator – no observable magnetic field even though the rocks could be strongly magnetic.

Here is a proof. Consider the induced lunar magnetization in the shell $c \le r \le b$: we suppose $\mathbf{M} = \kappa\nabla\Phi$ where $\kappa = -\chi/\mu_0$ assumed constant, and so $\nabla^2\Phi = 0$, since Φ is the scalar potential of the dynamo source in the moon's core. Now consider the scalar potential Ψ from the induced sources, outside the shell. Let $R = |\mathbf{r} - \mathbf{s}|$, then by (160):

$$\Psi(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_V \mathbf{M}\cdot\nabla\frac{1}{R}\,d^3\mathbf{s} = \frac{\mu_0}{4\pi}\int_V \kappa\nabla\Phi\cdot\nabla\frac{1}{R}\,d^3\mathbf{s}. \tag{169}$$

We can now apply our favorite vector identity and the divergence theorem:

$$\Psi = \frac{\mu_0\kappa}{4\pi}\int_V d^3\mathbf{s}\left[\nabla\cdot(\frac{1}{R}\nabla\Phi) \;-\; \frac{1}{R}\nabla^2\Phi\right]$$

$$= \frac{\mu_0\kappa}{4\pi}\left[\int_{S(b)} - \int_{S(c)}\right] d^2\mathbf{s}\,\frac{1}{R}\,\partial_r\Phi = \Psi_b \;-\; \Psi_c \tag{170}$$

where we have used the fact Φ is harmonic; notice in (170) that the normals on $S(c)$ and $S(b)$ point in opposite directions and so the two surface integrals must be subtracted. Let us look at the contribution Ψ_b from the outer surface $S(b)$; we introduce the SH expansion of $1/R$ from Part I (equation 138), and an expansion in c_l^m for the potential Φ:

$$\Psi_b = \frac{\mu_0\kappa}{4\pi}\int_{S(b)} d^2\mathbf{s}\left[\sum_{l,m}\frac{4\pi}{2l+1}\frac{b^l}{r^{l+1}}Y_l^m(\hat{\mathbf{r}})Y_l^m(\hat{\mathbf{s}})^*\right]\left[\sum_{l',m'} -\frac{l'+1}{b^{l'+2}}c_{l'}^{m'}Y_{l'}^{m'}(\hat{\mathbf{s}})\right]$$

$$= -\frac{\mu_0\kappa}{b^2}\sum_{l,m}\sum_{l',m'} c_{l'}^{m'}\frac{l'+1}{2l+1}\frac{1}{r^{l+1}}Y_l^m(\hat{\mathbf{r}})\int_{S(1)} Y_{l'}^{m'}(\hat{\mathbf{s}})Y_l^m(\hat{\mathbf{s}})^*\,b^2 d^2\hat{\mathbf{s}}$$

$$= -\mu_0\kappa\sum_{l,m}\frac{l+1}{2l+1}\frac{c_l^m}{r^{l+1}}Y_l^m(\hat{\mathbf{r}}). \tag{171}$$

The remarkable thing about (171) is that it is independent of the radius of the surface b; so the same answer will be obtained for Ψ_c, the integral over $S(c)$. But $\Psi = \Psi_b - \Psi_c$, and so $\Psi(\mathbf{r})$ is identically zero for all $\mathbf{r}$.

This argument is applicable to the Earth to some degree. The induced crustal magnetization from a uniform medium in a shell will produce no observable anomaly. You will easily see this generalizes to any series of shells, so χ can vary with depth, and the annihilator property persists. Thus the crustal fields we see are due to lateral variability, changes in layer thickness, and other departures from uniformity. Of course the Earth's crust is so heterogeneous we are not surprised to see very large anomalies almost everywhere. Why would one expect the upper regions of the moon to be less heterogeneous than the Earth's crust?

5:4 Results – Magnetic Anomalies Everywhere

The major triumph for magnetic anomalies has been the discovery of the Vine-Mathews magnetic stripes, the discovery of sea-floor spreading, and the mapping of the ages of the ocean basins. Reversal of direction results in a very strong, short wavelength contrast in magnetization which gives rise to intense magnetic anomalies (200-2,000 nT) in the scale range 10-100 km at the sea surface. Traditionally these anomalies have been modeled by thin layers (500 - 1,000 m thick) with blocks of magnetization where the direction is constant, except for changes in sign, and the intensity is constant too. This kind of model never fits the observations exactly, but does a reasonably good job if geological ages are needed. If one wishes to get more detail, and allow more flexibility in the models the annihilator remains a problem. The next step in sophistication is to allow the models to vary in intensity with x, and to make the layer follow the topographic variations. The magnetic annihilator then has a large scale component and another that varies like the bathymetry. Adding or subtracting this function can make the magnetization change sign and appear more-or-less correlated with topographic relief. When one is seeking geomagnetic intensity histories, the ambiguity is troublesome.

One of the happy exercises carried out on profile data is to apply spatial linear filters to change the apparent dip of the magnetization vector. It can be shown that there is a filter which after application to an anomaly results in the profile that would have been observed at the north pole, with vertical magnetization. This activity is called *reduction to the pole* (see Blakely for more). The idea here is that magnetizations measured or acquired in different latitudes yield anomalies that differ widely in appearance even if the underlying block pattern is similar; reduction to the pole makes profiles much easier to compare, and also offers some information about the latitude of formation of that piece of crust.

Another (but less successful) application of geomagnetism to marine anomalies is the analysis of the anomalies from seamounts. The major piece of information required here is the average direction of magnetization, which gives the usual paleomagnetic clues about where the volcano was formed. To overcome a serious ambiguity problem the first studies simply assumed the magnetization vector within the seamount was quite constant, both in direction and in intensity. Now with only three parameters to fit nonuniqueness disappears. Of course the constant-$\mathbf{M}$ models don't fit the observations very well at all, but marine geologists have generally ignored this difficulty. I have invented various methodologies for improving the fits and estimating the error in the resulting direction. One approach that worked reasonably well recognized the observation from drilling the oceanic crust that direction of $\mathbf{M}$ is much more nearly constant than $|\mathbf{M}|$, which varies by more than one order of magnitude in a single body. So we ask, What uni-directional magnetizations fit the given anomaly pattern, if any? It turns out that in some cases, quite a close clustering of directions will fit the anomaly, thus providing a direction and uncertainty. For details see Parker, R. L., A theory of ideal bodies for seamount magnetism, *J. Geophys.*

Res. B10, pp 16101-12, 1991.

Magnetic anomalies observed near the surface over land are generally smaller in amplitude than the marine ones. This is assumed to be because continental rocks are on the whole comprised of much less magnetic types: granitic, metamorphic and sedimentary rocks are orders of magnitude less magnetizable, and have no thermoremanent component. But aeromagnetic surveys are relatively cheap and are routinely used to locate and delineate ore bodies. At the longest wavelength ($>$ 1,000 km) however, as mapped by satellite, the continents have larger magnetic anomalies than the oceans. It is assumed that this is because on the longest scales induced magnetization is at work, and the extra thickness of the continental crust (or greater depth to Curie isotherm) gives the continents an advantage.