

```

A=reshape([1,0,0,-1,1,-1,0,0,1,0,0,0,0,1,-1,0,0,1,0,0,0,0,1,-1,0,0,1,0,0,0,0,1],4,8)'
b=[1.631,1.204,2.947,3.186,1.735,-2.778,-1.449,1.321]';

[u,s,v]=svd(A);

st=s';
for i=1:4
st(i,i)=1/st(i,i);
end;

sol2=v*st*u'*(b);

% look at errors:

[A*sol2-b]

std(A*sol2-b)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% NOW another approach

P=A*((A'*A)\A')
IP=eye(8)-P;

% You prove  $I - P = I - U(:,1..m)*U'(:,1..m)$ 

% Notice, idempotent:

IP^2-IP

% Now onward

[v,e]=eig(IP);

diag(e)

% notice 4 nonzero eigenvalues

% inf norm solution: pin  $e_{\{1,2,4,6\}}$  at:

ev=[-0.00475,.00475,0,.00475,0,.00475,0,0]'

rhs=-IP*(b+ev);

% remaining free variables - use LS (not required, could choose other)
% for a change, use the qr decomp (return to later, see matlab qr).

IPs=IP(:,[3,5,7,8]);

    r=qr(IPs);
    xs = r\r\'(IPs'*rhs)
    res = rhs - IPs*xs;
    iter = r\r\'(IPs'*res);
    xs = xs + iter

% Takes many iterations, converges to:

    0.00000308764268
    0.00325308764268
   -0.00349691235732
   -0.00024691235732

% vs. IPs\rhs in matlab, which gives

    0.00025000000000

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```
0.0035000000000000
-0.0032500000000000
0
```

```
% sum of squares of first: 2.287195049146928e-05
% of second: 2.287500000000000e-05
```

```
ev(3)= 0.00000308764268;
ev(5)= 0.00325308764268;
ev(7)= -0.00349691235732;
ev(8)= -0.00024691235732;
```

```
% check projection now:
```

```
IP*(b+ev)
```

```
% Yes -- b+errors now has no projection in null space of A
```

```
[u,s,v]=svd(A);
```

```
sol=v*st*u'*(b+ev);
```

```
[A*sol,b+ev]
```

```
% compare errors of the two data fits:
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```
[ev,A*sol2-b]
```

```
% compare misfit in L2 basis:
```

```
std(ev)
std(A*sol2-b)
```