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Research Interests: Inverse theory, geomagnetism, spectral analysis, electromagnetic induction.

While his student Ashley Medin continued to work on the more difficult, general inverse problem of electromagnetic induction in two dimensions, Bob Parker devoted some effort to studying the very simplest 1- and 2-D systems in order to calibrate and validate her general numerical computer codes. In magnetotelluric (MT) surveys the complex function of frequency, the admittance $c = E_y \,/\, \mathrm{i}\omega B_x$ is measured at the Earth's surface when a horizontal time-periodic magnetic field induces currents in the ground. In the simplest case electrical conductivity in the Earth is vertically stratified, that is $\sigma = \sigma(z)$, and is terminated with a perfect conductor at the known depth h. The first question we ask is, What are the allowed values of c for such a structure, if the only other thing we know about $\sigma(z)$ is that it is positive? The surprisingly simple answer is that c must always lie within a semicircular region in the complex plane, with diameter h on the real axis and the permissible zone below it, called the *green zone*. As ω increases from zero, the admittance of any bounded one-dimensional conductivity structure traces a smooth curve inside the green zone, starting at $c = h$, and ending at $c = 0$; see Figure 1.

Next consider the following minimalist inverse problem: we measure c at exactly one frequency; what can we say with certainty about $\sigma(z)$? A common way of selecting from among the infinite number of possible conductivities is to pick the one with the smallest L_2 norm, σ with the smallest value of

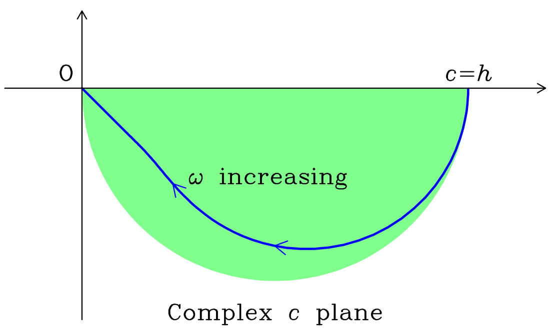
$$\| \sigma \|_2 = \left(\int_0^h \sigma(z)^2 \, dz \right)^{1/2} . \tag{1}$$

This strategy, *regularization*, is often adopted to stabilize numerically delicate inverse problems with many data, but the most elementary problem appears to have escaped previous attention. By the classical variational method, we find the Euler-Lagrange differential equation for the complex electric field:

$$E'' = \begin{cases} \mathrm{i}\omega\mu_0 |\lambda| \, \mathrm{Re}(E^2 \mathrm{e}^{\mathrm{i}\phi}) \, E, & \mathrm{Re}(\lambda E^2) \ge 0 \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

where λ is a complex parameter and $\phi = \arg \lambda$. The norm minimizing conductivity is given by $\sigma = \mathrm{Re}(\lambda E^2)$ when positive, and zero otherwise. From these equations we can discover the solution to the 1-D inverse problem with the smallest L_2 norm that matches a single admittance value. The results are summarized in Figure 2 which

Figure 1: The green zone and the locus of admittances for a uniform layer.



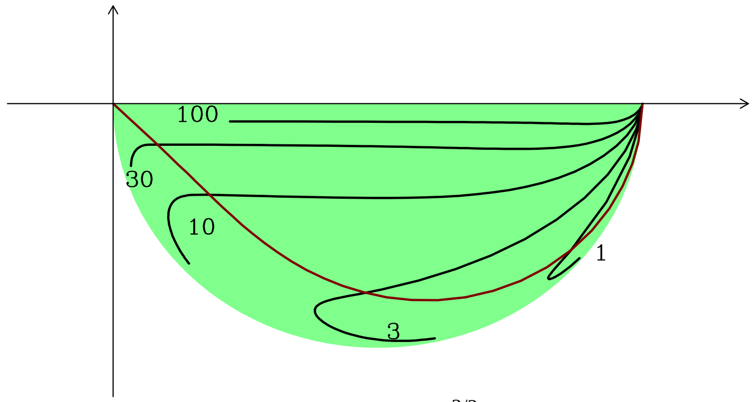


Figure 2: Contours of the smallest $\|\sigma\|_2 \omega \mu_0 h^{3/2}$ in the complex c plane. Below the red line $\sigma > 0$ throughout; above it there is an insulating layer at the top.

shows the value of the smallest possible norm at each allowed admittance. These solutions have been used as checks on the regularized 2-D numerical codes.

For quality control of MT measurements, it has been suggested that at each observation site it should be possible to construct a 1-D conductivity structure compatible with the admittances measured at that site: points lying far from the 1-D response are treated with caution and may be rejected. This is known to be only approximately correct, but the discrepancies between the best-fitting 1-D profile and a given set of data belonging to a 2-D structure have always been found to be tiny. Parker has discovered a class of 2-D systems in which the deviation can be extremely large: a thin conducting sheet with horizontally varying conductance $\tau(x)$ (vertically integrated conductivity) above an insulating layer, of thickness h lying over a perfect conductor. The system is excited by a horizontal magnetic field in the y direction in *transverse electric* (TE) mode induction. In the example illustrated below

$$\tau(x) = \tau_0(1 + a\, e^{-|x|/b}) \tag{3}$$

with $a = 1$ and $b = ½h$; the admittances are measured at $x = 0$. If the measured admittances are to be compatible with a 1-D profile, the trajectory of $c(\omega)$ should remain inside the green zone as we discussed earlier; see Figure 1. However, for this 2-D conducitivity model, the admittance lies outside the zone for every frequency except zero and infinity. Different choices of a and b yield even larger violations, but it is not known at this time if they can be made arbitrarily large. In any case, the idea that all 2-D structures have approximate 1-D responses at a single station must be reconsidered. A paper on this work is in preparation.

Figure 3: Admittances at $x = 0$ from a thin sheet conductor with conductance given by (3) and $a = 1$, $b = ½h$. The dimensionless frequency $\Omega = \omega\mu_0 h \tau_0$.