Robert L. Parker
Professor of Geophysics, Emeritus
Email: rlparker@ucsd.edu
Phone: 42475

Research Interests: Inverse theory, geomagnetism, spectral analysis, electromagnetic induction.

This year Bob Parker has continued looking into problems associated with electromagnetic induction. A seemingly simple technical question has required some surprisingly heavy machinery to solve. In calculating the electromagnetic fields due to a controlled source in a realistic model it is useful to find the response from a layered medium, which provides a starting point for more detailed, finite-element calculations; in particular, the singular behavior near the transmitter is dealt with much more naturally. The standard solution of the 1-dimensional system demands the numerical evaluation of the Hankel transform:

$$P(r) = \int_0^\infty Q(\lambda)\, J_0(r\lambda)\, \lambda\, d\lambda$$

where $Q(\lambda)$ is a complex function computed from the solution of an ordinary differential equation. While Q is very smooth, values of r encountered in practice generate very oscillatory integrands; see Fig 1. Traditional numerical quadrature techniques become impractical when good accuracy is needed for hundreds thousands of such integrals.

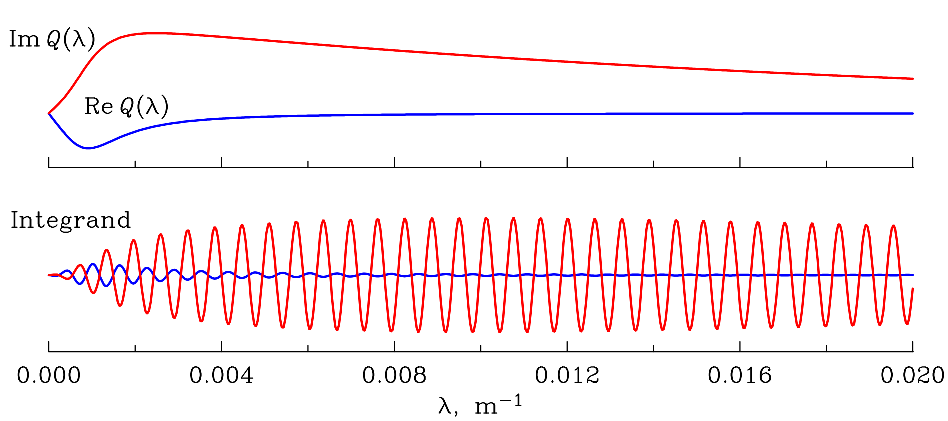
The solution to this problem has been to approximate Q with an expansion in a set of functions whose transforms can be evaluated exactly:

$$Q(\lambda) = \sum_{n=1}^{N} \beta_n g_n(\lambda)$$

$$P(r) = \sum_{n=1}^{N} \beta_n \hat{g}_n(r), \quad \text{where} \quad \hat{g}_n(r) = \int_0^\infty g_n(\lambda)\, J_0(r\lambda)\, \lambda\, d\lambda\, .$$

Even this approach has its limitations, since a very high degree of approximation is required when r becomes large, and 200 terms may be needed in the

Figure 1: Typical kernel function $Q(\lambda)$ and integrand of the transform when $r = 10$ km.



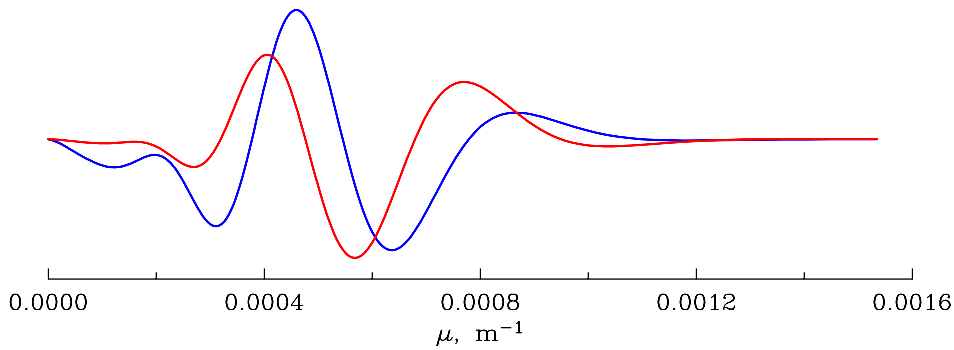


Figure 2: Integrand of transform on the imaginary axis.

expansion with functions that have elementary transforms. My resolution is to exploit the fact that $Q(\lambda)\exp(h\lambda)$ is an analytic function, regular at infinity, so that one can employ an orthogonal polynomial expansion of this function in the variable $\eta = \lambda/(\lambda + \lambda_0)$; this expansion requires far fewer terms (typically 30) to achieve a comparable precision.

These basis functions however must now be integrated numerically. To do this effectively the transform can be regarded as a contour integral, and the path of integration rotated onto the positive imaginary axis as there are no singularities of the approximate integrand in the first quadrant. Now the integral to be evaluated are in the form

$$P(r) = \int_0^\infty F(\mu)\, K_0(r\mu)\, \mu\, d\mu$$

where the Bessel function $K_0(r\mu)$ decays exponentially instead of oscillating like $J_0(r\lambda)$. The new integrand is quite benign, as can be seen above.

The papers below reflect work in other areas of interest. The one concerning reduction of bias in multitaper estimation was briefly described in the 2007 Annual Report.

Recent Publications

Jackson, A. Constable, C. G., Walker, M. R., and Parker, R. L., Model of Earth's main magnetic field incorporating flux and raidal vorticity constraints, *Geophys. J. Internat.,* 171, pp 133-44, doi:10.111/j.1365-246. X.2007.03526.x, 2007.

Prieto, G. A., Parker, R. L., Thomson, D. J., Vernon, F. L., and Graham, R., L., Reducing bias in multitaper spectrum estimates, *Geophys. J. Internat.,* 171, 1269-81, doi:10.1111/j.1365-246 X.2007.03592.x, 2007.