

1.20 Deep Earth Structure – Seismic Scattering in the Deep Earth

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1.20.1 Introduction

Most seismic analyses of Earth structure rely on observations of the traveltimes and waveforms of direct seismic waves that travel along ray paths determined by Earth's large-scale velocity structure. These observations permit inversions for radially averaged P wave and S wave velocity profiles as well as three-dimensional (3-D) perturbations. However, smaller-scale velocity or density perturbations cause some fraction of the seismic energy to be scattered in other directions, usually arriving following the main phase as incoherent energy over an extended time interval. This later-arriving wavetrain is termed the coda of the direct phase. Given the number of different scattering events and the complexity of the scattered wavefield, it is generally impossible to resolve individual scatterers. Instead, coda-wave observations are modeled using random media theories that predict the average energy in the scattered waves as a function of scattering angle, given the statistical properties of the velocity and density perturbations. In this way, it is possible to characterize Earth's heterogeneity at much

smaller scales than can be imaged using tomography or other methods.

The fact that direct seismic waves can be observed in the Earth indicates that this scattering must be relatively weak so that a significant fraction of the seismic energy remains in the primary arrivals. In contrast, scattering on the Moon is proportionally much stronger than in the Earth, preventing the easy observation of direct P and S waves at global distances (at least at the recorded frequencies of the available data) and complicating inversions for lunar structure. In addition to facilitating observations of direct arrivals, weak (as opposed to strong) scattering can also simplify modeling by permitting use of single-scattering theory (i.e., the Born approximation). However, it is now clear that accurate modeling of scattering in the lithosphere, and possibly deeper in the mantle as well, requires calculations based on multiple scattering theories. Fortunately, increased computer power makes these calculations computationally feasible.

Although both body waves and surface waves exhibit scattering, the emphasis in this review is on observations and modeling of deep-Earth scattering,

for which body waves provide the primary constraints. In addition, more attention is given to the mantle and core than the lithosphere, which has been the focus of the majority of coda studies to date. Finally, the different scattering theories are briefly summarized. For more details on these topics, the reader should consult the book by Sato and Fehler (1998), which provides an extensive review of scattering theory and analysis methods, as well as a comprehensive summary of crustal and lithospheric studies.

1.20.2 Scattering Theory

Wave scattering from random heterogeneities is a common phenomenon in many fields of science and theoretical modeling approaches have been extensively developed in physics, acoustics, and seismology. Solving this problem for the full elastic wave equation (i.e., for both P and S waves) in the presence of strong perturbations in the elastic tensor and density is quite difficult, so various simplifying approximations are often applied. These include assuming an isotropic elastic tensor, using first-order perturbation theory in the case of weak scattering, using the diffusion equation for very strong scattering, and assuming correlations among the velocity and density perturbations.

1.20.2.1 Single-Scattering Theory and Random Media

For sufficiently weak velocity and density perturbations, most scattered energy will have experienced only one scattering event and can be adequately modeled using single-scattering theory. The mathematics in this case is greatly simplified if we assume that the primary waves are unchanged by their passage through the scattering region (the Born approximation). The total energy in the seismic wavefield therefore increases by the amount contained in the scattered waves and energy conservation is not obeyed. Thus this approximation is only valid when the scattered waves are much weaker than the primary waves, which is the case in the Earth when the velocity and density perturbations are relatively small (quantifying exactly how small depends upon the frequency of the waves and the source-to-receiver distance). Single-scattering theory is sometimes called Chernov theory (after Chernov, 1960). Detailed descriptions of Born scattering theory for elastic waves are contained in Wu and Aki (1985a, 1985b), Wu (1989), and Sato and Fehler (1998). A

review of the properties (elasticity, conductivity, permeability) and statistics of random heterogeneous materials is given in the text by Torquato (2002).

Single-scattering theory provides equations that give the average scattered power as a function of the incident and scattered wave types (i.e., P or S), the power of the incident wave, the local volume of the scattering region, the bulk and statistical properties of the random medium, the scattering angle (the angle between the incident wave and the scattered wave), and the seismic wave number ($k=2\pi/\Lambda$, where Λ is the wavelength). A general random medium could have separate perturbations in P velocity, S velocity, and density, but in practice a common simplification is to assume a linear scaling relationship among the perturbations (e.g., Sato, 1990) and/or to assume zero density perturbations. However, as pointed out by Hong *et al.* (2004), density variations can have an important influence on scattering properties. Performing the actual calculation for a specific source-receiver geometry involves integrating the contributions of small volume elements over the scattering region of interest. Each volume element will have a specific scattering angle and geometrical spreading factors for the source-to-scatterer and scatterer-to-receiver ray paths.

The nature of the scattering strongly depends upon the relative length scales of the heterogeneity and the seismic waves. The scale of perturbations in a random medium can be characterized by the autocorrelation function (ACF), with the correlation distance, a , providing a rough measure of the average size of the 'blobs' in many commonly assumed forms for the ACF (e.g., Gaussian, exponential, van Kármán). **Figure 1** shows examples of random realizations of the Gaussian and exponential ACF models. If the heterogeneity is large compared to the seismic wavelength ($a \gg \Lambda$, ka is large), then forward scattering predominates and becomes increasingly concentrated near the direction of the incident wave as ka increases. In the limit of large ka , the energy remains along the primary ray path and scattering effects do not need to be taken into account. Alternatively, if the blobs are small compared to the seismic wavelength ($a \ll \Lambda$, ka is small), then the scattering is often approximated as isotropic and the scattered power scales as $k^4 a^3$. In the limit of small ka , the scattering strength goes to zero and the medium behaves like a homogeneous solid. As discussed by Aki and Richards (1980, pp. 749–750), scattering effects are strongest when a and Λ are of comparable size (i.e., when $ka \sim 2\pi$).

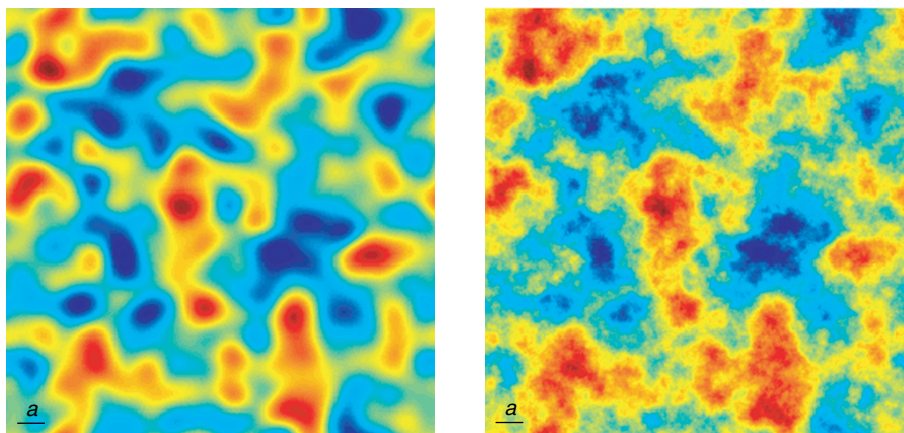


Figure 1 Examples of random media defined by a Gaussian ACF (left) and an exponential ACF (right). The correlation distance, a , is indicated in the lower-left corner. The exponential medium has more structure at short wavelengths than the Gaussian medium.

Aki and Chouet (1975) presented an important application of single-scattering theory to predict coda decay rates for local earthquakes. For a colocated source and receiver and homogeneous body-wave scattering in 3-D media, they obtained

$$A_C(t) \propto t^{-1} e^{-\omega t/2Q_C} \quad [1]$$

where A_C is the coda amplitude at time t (from the earthquake origin time) and angular frequency ω . Q_C is termed the coda Q and there has been some uncertainty regarding its physical meaning, in particular whether it describes intrinsic attenuation, scattering attenuation, or some combination of both. This is discussed later in the context of more complete theories. Regardless of its interpretation, this formula has proved to be successful in fitting coda decay rates in a large number of studies.

Single-scattering theory has also been important for modeling deep-Earth scattering in terms of random heterogeneity models, including interpretation of PKP precursor observations (e.g., Haddon and Cleary, 1974; Doornbos, 1976), PP precursors (King *et al.*, 1975), P'P' precursors (Vinnik, 1981), P_{diff} coda (Earle and Shearer, 2001) and PKiKP coda (Vidale and Earle, 2000). Born theory has also been used to model expected traveltimes variations in direct arrivals that travel through random velocity heterogeneity (e.g., Spetzler and Snieder, 2001; Baig *et al.*, 2003; Baig and Dahlen, 2004a, 2004b). Although my focus in this paper is largely on incoherent scattering from random media, it should be noted that the Born approximation can also be used to model the effect of specific velocity structures, provided their

perturbations are weak compared to the background velocity field. In this case, true synthetic seismograms can be computed, not just the envelope functions. For example, Dalkolmo and Friederich (2000) recently used this approach to model the effect of several different hypothesized velocity anomalies near the core-mantle boundary (CMB) on long-period P waves. In addition, Born theory forms the basis for computing sensitivity kernels in finite-frequency tomography methods (e.g., Dahlen *et al.*, 2000; Nolet *et al.*, 2005).

1.20.2.1.1 Q notation and definitions

Coda Q , intrinsic Q , and scattering Q are termed Q_C , Q_I and Q_{Sc} , respectively. P wave and S wave Q are termed ${}^\alpha Q$ and ${}^\beta Q$, respectively. These can be combined so that, for example, ${}^\beta Q_I$ is intrinsic S wave Q . This convention eliminates any chance of confusing shear-wave Q and scattering Q (both have sometimes been termed Q_S). The transmission Q , Q_T , describes the total attenuation (both intrinsic and scattering) suffered by the direct wave

$$Q_T^{-1} = Q_I^{-1} + Q_{Sc}^{-1} \quad [2]$$

and the amplitude reduction of the transmitted pulse for a constant Q_T medium is

$$A(t) = A_0 e^{-\omega t/2Q_T} \quad [3]$$

where A_0 is the amplitude of the pulse at $t=0$ and we have ignored any geometrical spreading.

The scattering coefficient, g , is defined as the scattering power per unit volume (e.g., Sato, 1977)

and has units of reciprocal length. The total scattering coefficient, g_0 , is defined as the average of g over all directions and can also be expressed as

$$g_0 = \ell^{-1} = Q_{sc}^{-1} k \quad [4]$$

where ℓ is the mean free path and k is the wave number. One common way to estimate g_0 for S waves has been to compare the energy in the S coda to the total radiated S energy. Finally, following Wu (1985), we define the seismic albedo as the ratio of scattering attenuation to total attenuation

$$B_0 = \frac{Q_{sc}^{-1}}{Q_{sc}^{-1} + Q_I^{-1}} = \frac{g_0}{g_0 + Q_I^{-1} k} \quad [5]$$

These definitions of Q , g_0 , and B_0 are general and can be applied to the multiple-scattering theories discussed later in this paper.

1.20.2.2 Finite Difference Calculations and the Energy Flux Model

Finite difference methods provide a direct, albeit computationally intensive, solution to the seismic wave equation for media of arbitrary complexity, and they (together with the finite element method) have become one of the most widely used techniques in seismology. Their earliest applications to study scattering involved modeling surface-wave and body-to-surface-wave scattering from surface topography, sediment-filled basins, and other buried interfaces (e.g., Levander and Hill, 1985). Here, only their use in modeling body-wave scattering in random media is discussed. Reviews of this topic are contained in Frankel (1990) and Sato and Fehler (1998).

As computing power has improved, finite difference simulations have progressed from the 2-D parabolic approximation, to 2-D using the full wave equation, to full 3-D synthetics. The parabolic approximation considers only forward scattering and is useful when the heterogeneity correlation length is large compared to the seismic wavelength. Complete finite difference simulations in 2-D random media have been performed by Frankel and Clayton (1984, 1986), McLaughlin *et al.* (1985), McLaughlin and Anderson (1987), Frankel and Wennerberg (1987), Gibson and Levander (1988), Roth and Korn (1993), and Saito *et al.* (2003). Frenje and Juhlin (2000) computed both 2-D and 3-D finite difference simulations. Hong and Kennett (2003), Hong (2004), and Hong *et al.* (2005) used a wavelet-based numerical approach

to compute 2-D synthetics for random media and Hong and Wu (2005) computed 2-D synthetics for anisotropic models.

The Frankel studies provided key results in formulating the influential energy flux model (EFM) of seismic coda (Frankel and Wennerberg, 1987) and thus are described here in some detail. Frankel and Clayton (1986) modeled teleseismic P wave travel-time variations with a ~ 1 Hz plane wavelet vertically incident on a layer 150 km wide by 55 km thick, with a finite difference grid spacing of 500 m. They found that observed traveltime variations of about 0.2 s (rms) among stations spaced 10–150 km apart could be explained with 5% rms random P velocity variations, provided the correlation length was 10 km or greater. Frankel and Clayton (1986) also modeled high-frequency coda from local earthquakes using a ~ 20 Hz explosive source at the bottom corner of a layer 8 km long by 2 km thick. They found that the amplitude of high-frequency coda depends strongly on the presence of high wave number velocity perturbations. Gaussian and exponential models with correlation lengths of 10 km or greater (required to fit observed teleseismic traveltime variations) do not have sufficient small-scale structure to produce observed levels of high-frequency coda. In contrast, a self-similar random medium model with a correlation distance of at least 10 km and rms velocity variations of 5% can account for both sets of observations.

By measuring peak amplitude versus distance in their synthetics, Frankel and Clayton (1986) estimated Q for their random medium. Because their finite difference calculation did not contain any intrinsic attenuation, this represents a measure of scattering Q (Q_{sc}). The predicted attenuation (Q^{-1}) peaks at ka values between 1 and 2 for Gaussian random media and between about 1 and 6 for exponential random media. This is consistent with the strongest scattering occurring when the seismic wavelength is comparable to the size of the scatterers. However, attenuation is constant with frequency for self-similar random media, as expected, since the velocity fluctuations have equal amplitudes over a wide range of scales. Frankel and Clayton showed that these results were in rough agreement with those predicted by single-scattering theory in 2-D (to match the geometry of the finite difference simulations).

Frankel and Clayton (1986) also measured coda decay rates for finite difference synthetics computed for sources within a 12 km by 12 km grid at 20 m spacing. They found that their observed coda decay

rates were significantly less than those predicted by single-scattering theory in the case of moderate to large scattering attenuation ($Q_{sc} \leq 200$), indicating that multiple scattering contributes a substantial portion of the coda energy. This implies that in these cases coda Q (Q_C), as determined from coda falloff and the single-scattering model of coda (e.g., Aki and Chouet, 1975), does not provide a reliable estimate of transmission Q .

Motivated by these finite difference results and the limitations of the single-scattering model of coda generation, Frankel and Wennerberg (1987) introduced what they termed the EFM of coda. This phenomenological model is based on the idea that the coda energy behind the direct wave front can be approximated as homogeneous in space. This observation had previously been reported for micro-earthquake coda for lapse times more than twice the S wave traveltime (e.g., Aki, 1969; Rautian and Khalturin, 1978), and Frankel and Wennerberg (1987) showed that it also could be seen in finite difference synthetics (see Figure 2). It implies that at sufficiently long times, the coda amplitude at all receivers is approximately the same (scaling only with the magnitude of the source), regardless of the source–receiver distance. The EFM permits the time decay of the coda amplitude to be modeled very simply and to separate the effects of scattering and intrinsic attenuation in the medium.

By considering the energy density of the coda uniformly distributed in an expanding volume

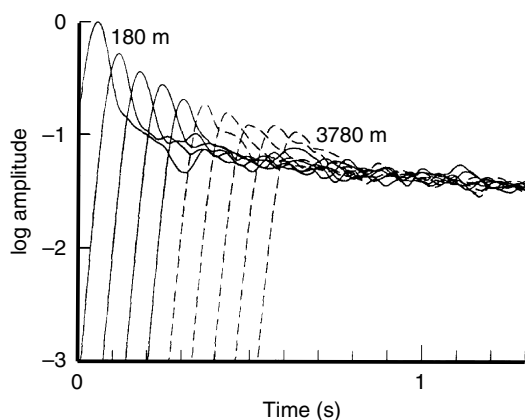


Figure 2 Envelopes of finite difference synthetic seismograms for receivers at distances of 180 to 3780 m from the source as computed by Frankel and Wennerberg (1987) for a random medium with a correlation distance of 40 m. Note that the envelopes decay to a common level following the initial pulse, indicating spatial homogeneity of coda energy.

behind the direct wave front, Frankel and Wennerberg derived an expression for the predicted time decay of the coda amplitude

$$A_C(t) \propto t^{-3/2} e^{-\omega t/2Q_I} \sqrt{1 - e^{-\omega t/Q_{sc}}} \quad [6]$$

where t is time, ω is angular frequency, Q_I^{-1} is intrinsic attenuation, and Q_{sc}^{-1} is scattering attenuation. For short time and/or high Q_{sc} (i.e., weak scattering, tQ_{sc}^{-1} is very small), this equation reduces to

$$A_C(t) \propto t^{-1} e^{-\omega t/2Q_I} \quad [7]$$

This is equivalent to the Aki and Chouet (1975) expression (eqn [1]) for the single-scattering model, assuming that coda Q and intrinsic Q are equivalent ($Q_C = Q_I$). This agrees with the original interpretation of Q_C given by Aki and Chouet (1975) and contradicts the Aki (1980) statement that in the context of single-scattering theory Q_C should be considered as an effective Q that includes both absorption and scattering effects. Frankel and Wennerberg (1987) showed that the EFM predicts the amplitude and coda decay observed in finite difference synthetics for random media with a wide range of scattering Q , and is more accurate than the single-scattering model for media with moderate to strong scattering attenuation ($Q_{sc} \leq 150$). Finally, they used the EFM to estimate Q_{sc} and Q_I from the coda of two $M \sim 3$ earthquakes near Anza, California.

There have been numerous other studies that have attempted to resolve Q_{sc} and Q_I from local earthquake coda (e.g., Wu and Aki, 1988; Toksöz *et al.*, 1988; Mayeda *et al.*, 1991; Fehler *et al.*, 1992). Notable is Mayeda *et al.* (1992), who analyzed S wave coda from Hawaii, Long Valley, and central California. They found a complicated relationship between theoretical predictions and observed Q_C , Q_{sc} , and Q_I and argued that models with depth-dependent scattering and intrinsic attenuation are necessary to explain their results.

The EFM was developed to explain local earthquake coda and finite difference simulations of spherical wave fronts in media with uniform scattering. It is not directly applicable to modeling teleseismic coda because of the strong concentration of scattering in the crust and lithosphere compared to much weaker scattering deeper in the mantle. This has motivated the development of extended EFMs involving the response of one or more scattering layers to a wave incident from below (e.g., Korn, 1988, 1990, 1997; Langston, 1989).

The resulting formulas for the coda decay rate are more complicated than the simple EFM because they depend upon several additional parameters, including the traveltimes through the layer and the amount of leakage back into the half-space. Korn (1988) developed the theory for a spherical wave with a cone of energy incident upon a scattering zone and used it to model regional earthquakes recorded by the Warramunga array in Australia. Langston (1989) developed a scattering layer-over-half-space model and showed that it was consistent with coda decay in teleseismic P waves recorded at two stations (PAS and SCP) in the United States. Korn (1990) tested a scattering layer over homogeneous half-space EFM using a 2-D acoustic finite difference code and found that it gave reliable results for both weak and strong scattering regimes. Korn (1997) further extended the EFM to explicitly include depth-dependent scattering and showed that it gave reliable results when compared to synthetics computed for a 2-D elastic (P – SV) finite difference code.

Wagner and Langston (1992a) computed 2-D acoustic and elastic finite difference synthetics for upcoming P waves incident on 150 different models of heterogeneous layers over a homogeneous half-space. These models varied in their layer thickness, random heterogeneity correlation length (different vertical and horizontal correlation lengths were allowed), and rms velocity heterogeneity. They found that the scattering attenuation of the direct pulse depends upon ka and is strongest for spatially isotropic heterogeneity, in which case most of the coda energy was contained in low apparent velocity S waves and surface waves. In contrast, anisotropic models with horizontally elongated heterogeneities produce coda with mostly vertically propagating layer reverberations.

More recent finite difference calculations for random media include the 2-D whole-Earth pseudo-spectral calculations of Furumura *et al.* (1998) and Wang *et al.* (2001); Thomas *et al.* (2000), who computed 2-D whole-Earth acoustic synthetics to model PKP precursors; Cormier (2000), who used a 2-D elastic pseudo-spectral method to model the effects of D' heterogeneity on the P and S wavefields; and Korn and Sato (2005), who compare 2-D finite difference calculations with synthetics based on the Markov approximation.

1.20.2.3 Multiple-Scattering Theories

If the energy in the scattered wavefield is a significant fraction of the energy in the direct wave, then the Born approximation is inaccurate and a higher-order

theory should be used that takes into account the energy reduction in the primary wave and the fact that the scattered waves may experience more than one scattering event. These effects are all naturally accounted for using the finite difference calculations already discussed, but these are computationally intensive and there is a need for faster approaches that also provide physical insight into the scattering process. In the case of very strong scattering, the diffusion equation can be applied by assuming a random-walk process. Although this approach preserves energy, it violates causality by permitting some energy to arrive before the direct P wave. The first applications of the diffusion equation in seismology include the coda wave analyses of Wesley (1965) and Aki and Chouet (1975) and the lunar seismogram studies of Nakamura (1977) and Dainty and Toksöz (1977, 1981).

At large times and small distances from the source, the diffusion equation predicts that the coda amplitude varies as

$$A(t) \propto t^{-3/4} e^{-\omega t/2Q_I} \quad [8]$$

where Q_I is the intrinsic attenuation. Notice that Q_{Sc} does not appear in this equation because the exact level of scattering is not important provided it is strong enough that the energy is obeying a random-walk process. The diffusion equation can also be used to model the case of a strong scattering layer over a homogeneous half-space (e.g., Dainty *et al.*, 1974; Margerin *et al.*, 1998, 1999; Wegler, 2004), in which case an additional decay term exists to account for the energy leakage into the half-space.

Another approach to modeling multiple scattering is to sum higher-order scattered energy, and the predicted time dependence of scattered energy was obtained in this way for double scattering (Kopnichev, 1977) and multiple scattering up to seventh order (Gao *et al.*, 1983a, 1983b). Hoshihara (1991) used a Monte Carlo approach (see below) to correct and extend these results to 10th-order scattering. Richards and Menke (1983) performed numerical experiments on 1-D structures with many fine layers to characterize the effects of scattering on the apparent attenuation of the transmitted pulse and the relative frequency content of the direct pulse and its coda.

Most current approaches to synthesizing multiple scattering use radiative transfer theory to model energy transport. Radiative transfer theory was first used in seismology by Wu (1985) and Wu and Aki

(1988) and recent reviews of the theory are contained in Sato and Fehler (1998) and Margerin (2005). Other results are detailed in Shang and Gao (1988), Zeng *et al.* (1991), Sato (1993), and Sato *et al.* (1997). Sato and Nishino (2002) use radiative transfer theory to model multiple Rayleigh wave scattering. Analytical solutions are possible for certain idealized cases (e.g., Wu, 1985; Zeng, 1991; Sato, 1993), but obtaining general results requires extensive computer calculations.

Two analytical results are of particular interest (and can be used as tests of numerical simulations). For the case of no intrinsic attenuation, Zeng (1991) showed the coda power converges to the diffusion solution at long lapse times

$$P_C(t) \propto t^{-3/2} \quad [9]$$

For elastic waves with no intrinsic attenuation, the equilibrium ratio of P and S energy density is given by (e.g., Sato, 1994; Ryzhik *et al.*, 1996; Papanicolaou *et al.*, 1996)

$$E_P/E_S = \frac{1}{2}(\beta/\alpha)^3 \quad [10]$$

Assuming a Poisson solid, this predicts about 10 times more S energy than P energy at equilibrium, a result of the relatively low efficiency of S-to-P scattering compared to P-to-S scattering (e.g., Malin and Phinney, 1985; Zeng, 1993). For media with intrinsic attenuation, an equilibrium ratio also exists but will generally differ from the purely elastic case (Margerin *et al.*, 2001). Shapiro *et al.* (2000) showed that this ratio can be estimated from the divergence and curl of the displacement as measured with a small-aperture array and that its stability with time provides a test of whether the coda is in the diffusive regime.

Wu (1985) used radiative transfer theory to address the problem of separating scattering from intrinsic attenuation. He showed that the coda energy density versus distance curves have different shapes depending upon the seismic albedo, B_0 (see eqn [5]), and thus in principle it is possible to separate scattering and intrinsic Q by measuring energy density distribution curves. Hoshiya (1991) pointed out that in practice the use of finite window lengths for measuring coda will lead to underestimating the total energy (compared to the infinite lapse-time windows in Wu's theory), likely biasing the resulting estimates of seismic albedo. To deal with this problem, Fehler *et al.* (1992) introduced the multiple lapse-time window (MLTW) analysis, which measures the energy in consecutive time windows as a function of

epicentral distance. The MLTW approach has been widely used in studies of S coda (see below).

A powerful method for computing synthetic seismograms based on radiative transfer theory is to use a computer-based Monte Carlo approach to simulate the random walk of millions of seismic energy 'particles' which are scattered with probabilities derived from random media theory. Figure 3 illustrates a simple example of this method applied to 2-D isotropic scattering. Variations on this basic technique are described by Gusev and Abubakirov (1987), Abubakirov and Gusev (1990), Hoshiya (1991, 1994, 1997), Margerin *et al.* (2000), Bal and

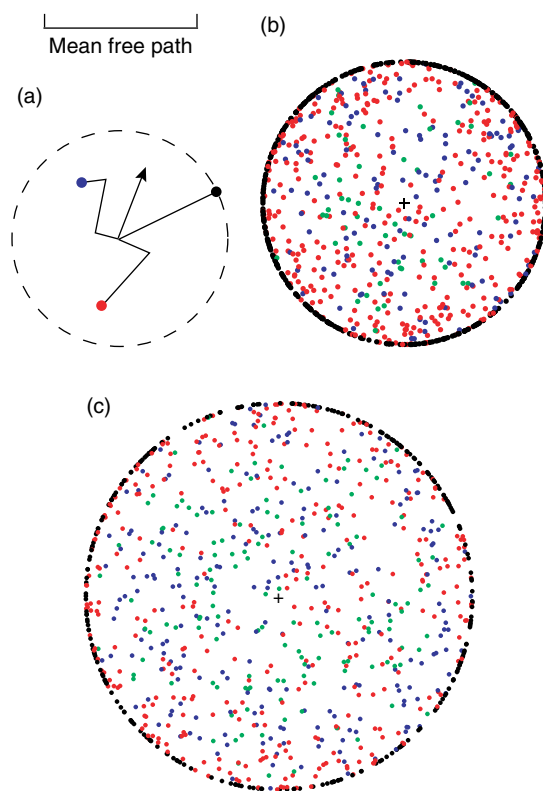


Figure 3 Example of a Monte Carlo computer simulation of random scattering of seismic energy particles, assuming 2-D isotropic scattering in a uniform whole space. Particles are sprayed in all directions from the source with constant scattering probability defined by the indicated mean free path length, ℓ . As indicated in (a), black dots show particles that have not been scattered, red dots show particles that have scattered once, blue dots show particles that have scattered twice, and green dots show particles scattered three or more times. (b) Results for 1000 particles after $t = \ell/v$, where v is velocity. (c) Results for 1000 particles after $t = 1.25 \ell/v$. Note that the particle density is approximately constant for the scattered energy inside the circle defining the direct wave front, as predicted by the EFM.

Moscoso (2000), Yoshimoto (2000), Margerin and Nolet (2003a, 2003b), and Shearer and Earle (2004). Because of the potential of the Monte Carlo method for modeling whole-Earth, high-frequency scattering, these results are now summarized in some detail.

Gusev and Abubakirov (1987) used the Monte Carlo method to model acoustic wave scattering in a whole space and considered both isotropic scattering and forward scattering with a Gaussian angle distribution. They parameterized the scattering in terms of a uniform probability per unit volume, resulting in an exponential distribution of path lengths. Intrinsic attenuation was not included. They showed that their results agree with the diffusion model for large lapse times. Abubakirov and Gusev (1990) contains a more detailed description of this method and its application to model S coda from Kamchatka earthquakes. They obtained an S wave mean free path for the Kamchatka lithosphere of 110–150 km over a 1.5–6 Hz frequency range.

Hoshiha (1991) modeled the spherical radiation of S wave energy in a constant-velocity medium using a Monte Carlo simulation that included isotropic scattering with uniform probability. He showed that the results agreed with single-scattering theory for weak scattering (i.e., travel distances less than 10% of the mean free path) and agreed with the diffusion model for strong scattering (i.e., travel distances more than 10 times longer than the mean free path). Hoshiha (1991) was able to use his Monte Carlo results to correct and extend the multiple scattering terms of Gao *et al.* (1983a, 1983b). He found that his simulations at long lapse times were consistent with the radiative transfer theory of Wu (1985) but that reliable estimates of seismic albedo are problematic from short time windows. Finally, Hoshiha (1991) showed that for multiple scattering, coda Q is much more sensitive to intrinsic Q than to scattering Q (as was argued by Frankel and Wennerberg (1987) on the basis of the EFM).

Wennerberg (1993) considered the implications of lapse-time-dependent observations of Q_C and methods for separating intrinsic and scattering attenuation with respect to the single-scattering model, Zeng's (1991) multiple-scattering model, Hoshiha's (1991) model, and Abubakirov and Gusev's (1990) results. Hoshiha (1994) extended his Monte Carlo method to consider depth-dependent scattering strength and intrinsic attenuation, and Hoshiha (1997) included the effects of a layered velocity structure to model local earthquake coda at distances up to 50 km. Hoshiha simulated SH wave reflection and

transmission coefficients at layer interfaces as probabilities of reflection or transmission of particles in the Monte Carlo method but did not include P waves and the conversions between P and S waves. The results showed that coda amplitudes depend upon the source depth even late into the coda.

Margerin *et al.* (1998) applied the Monte Carlo approach to a layer-over-half-space model, representing the crust and upper mantle, and included both surface- and Moho-reflected and transmitted phases. As in Hoshiha (1997), reflection and transmission coefficients are converted to probabilities for the individual particles. Only S waves are modeled (using the scalar wave approximation, i.e., no P-to-S conversions) and no intrinsic attenuation is included. Margerin *et al.* (1998) compared their numerical results in detail with solutions based on the diffusion equation and found good agreement for suitable mean free path lengths. They point out the importance of the crustal waveguide for trapping energy near the surface and that the possibility of energy leakage into the mantle should be taken into account in calculations of seismic albedo.

Bal and Moscoso (2000) explicitly included S wave polarization and showed that S waves become depolarized under multiple scattering. Yoshimoto (2000) introduced the direct simulation Monte Carlo (DSMC) method, which uses a finite difference scheme for ray tracing and can thus handle velocity models of arbitrary complexity, including lateral varying structures. However, intrinsic attenuation and directional scattering were not included. Yoshimoto showed that a velocity increase with depth strongly affects the shape of the coda envelope, compared with uniform velocity models, and that it is important to properly model energy that may be trapped at shallow depths.

Margerin *et al.* (2000) extended the Monte Carlo approach to elastic waves, taking into account P-to-S conversions and S wave polarization. They considered scattering from randomly distributed spherical inclusions within a homogeneous background material, using the solutions of Wu and Aki (1985a). For both Rayleigh scatterers (spheres much smaller than the seismic wavelength) and Rayleigh–Gans scatterers (spheres comparable to the seismic wavelength), they found good agreement with single-scattering theory at short times and with the diffusion equation solution at long times. In addition, the P-to-S energy density ratio and the coda decay rate at long times converged to their theoretical expected values.

Margerin and Nolet (2003a, 2003b) further extended the Monte Carlo approach to model whole-Earth wave propagation and scattering. They showed that their Monte Carlo synthetics for the PKP AB and BC branches produced energy versus distance results in good agreement with geometrical ray theory. They computed scattering properties based on random media models characterized by velocity perturbations with an exponential correlation length. For whole mantle scattering, they found that the Born approximation is only valid up to mean free paths of about 400 s, corresponding to 0.5% rms velocity perturbations. They also applied their method to model PKP precursor observations; these results are discussed later in this paper.

Shearer and Earle (2004) implemented a particle-based Monte Carlo method for computing whole-Earth scattering. They included both P and S waves radiated from the source, mode conversions, S wave polarizations, and intrinsic attenuation. For a simple whole-space model, they showed that their approach agreed with theoretical results for the S/P energy ratio and expected $t^{-1.5}$ falloff in power at large times. For modeling the whole Earth, they included the effects of reflection and transmission coefficients at the free surface, Moho, CMB, and inner core boundary (ICB). Scattering probabilities and scattering angles were computed assuming random velocity and density variations characterized by an exponential ACF. They applied this method to model the time and distance dependence of high-frequency P coda amplitudes (see Section 1.20.3.2).

All of these results suggest that body-wave scattering in the whole Earth can now be accurately modeled using ray theory and particle-based Monte Carlo methods. Although somewhat computationally intensive, continued improvements in computer speed make them practical to run on modest machines. They can handle multiple scattering over a range of scattering intensities, bridging the gap between the Born approximation for weak scattering and the diffusion equation for strong scattering. They also can include general depth-dependent or even 3-D variations in scattering properties, including nonisotropic scattering, without a significant increase in computation time compared to simpler problems.

1.20.2.4 Other Theoretical Methods

Lerche and Menke (1986) presented an inversion method to separate intrinsic and scattering attenuation

for a plane layered medium. Gusev (1995) and Gusev and Abubakirov (1999a, 1999b) developed a theory for reconstructing a vertical profile of scattering strength from pulse broadening and delay of the peak amplitude. Saito *et al.* (2002, 2003) modeled envelope broadening in S waves by applying the parabolic approximation to a von Karman random medium. Sato *et al.* (2004) extended this approach to develop a hybrid method for synthesizing whole-wave envelopes that uses the envelope obtained from the Markov approximation as a propagator in the radiative transfer integral and showed that the results agreed with finite difference calculations. The effect of anisotropic random media (where the scattering properties depend upon the angle of the incident wave) was considered numerically by Wagner and Langston (1992a) and Roth and Korn (1993), and theoretically by Müller and Shapiro (2003) and Hong and Wu (2005). Recent work on using small-aperture seismic arrays on coda to constrain the directions of individual scatterers includes Schisselé *et al.* (2004) and Matsumoto (2005).

1.20.3 Scattering Observations

Seismic scattering within the Earth is mainly observed in the incoherent energy that arrives between the direct seismic phases, such as P, S, and PKP. In addition, occasionally specific scatterers can be imaged using seismic arrays. Scattering can also influence the direct phases through amplitude reduction and pulse broadening, effects characterized by the scattering attenuation parameter, Q_{sc}^{-1} . In this way, studies of seismic attenuation also resolve scattering, although they often do not attempt to separate intrinsic and scattering attenuation. The incoherent scattered seismic wavefield usually follows a direct seismic arrival, and is termed the 'coda' of that phase (e.g., P coda, S coda), but occasionally the ray geometry is such that scattered energy can arrive before a direct phase (e.g., PKP precursors). These precursory arrivals are particularly valuable for studying deep-Earth scattering because they are less sensitive to the strong scattering in the lithosphere. Scattering is usually studied at relatively high frequencies (1 Hz or above) where coda is relatively strong and local earthquake records have their best signal to noise.

S wave coda from local and regional events has been the focus of many studies, has motivated much of the theoretical work, and continues to be an active field of research. Here, S coda studies and their

implications for scattering in the crust and lithosphere are only briefly reviewed; instead, more attention is devoted to other parts of the scattered seismic wavefield, which provide better constraints on deep-Earth structure. Previous review articles that discuss deep-Earth scattering include Bataille *et al.* (1990) and Shearer *et al.* (1998). **Figure 4** shows where much of the scattered energy arrives with respect to traveltimes for the major seismic phases. In principle, any seismic phase that travels through the lower mantle or the CMB will be sensitive to deep-Earth scattering. Although separating deep-scattering effects from crust and lithospheric scattering can be challenging, this is possible in some cases, either from a fortunate ray geometry or by careful comparison of coda amplitudes at different distances or between different phases.

The picture that is emerging from these studies is that seismic scattering from small-scale velocity perturbations is present throughout the Earth, with the exception of the fluid outer core. However, the exact strength, scale length, and depth dependence of the scattering remain unresolved, particularly in the

vicinity of the CMB and ICB, where they may also be contributions from topographic irregularities or boundary-layer structure.

1.20.3.1 S Coda

S coda measurements are of two main types: (1) those that simply fit the coda decay rate and estimate $^{\beta}Q_C$ without attempting to separate scattering and intrinsic attenuation, and (2) those that measure energy density and estimate the scattering coefficient g_0 (or its reciprocal, the mean free path). The latter studies typically obtain a separate estimate for intrinsic attenuation and thus can compute the seismic albedo, B_0 . The basic relationships among these parameters are given in eqns [4] and [5]. In both types of studies, frequency dependence can also be examined by filtering the data in different bands.

Coda Q studies include Rautian and Khalturin (1978), Roecker *et al.* (1982), Rodriquez *et al.* (1983), Singh and Herrmann (1983), Biswas and Aki (1984), Pulli (1984), Rhea (1984), Jin *et al.* (1985), Del Pezzo *et al.* (1985), Jin and Aki (1988), van Eck (1988), Kvamme and Havskov (1989), Matsumoto and Hasegawa (1989), Baskoutas and Sato (1989), Oancea *et al.* (1991), Tsuruga *et al.* (2003), Giampiccolo *et al.* (2004), and Goutbeek *et al.* (2004).

Reviews of Q_C measurements include Herraiz and Espinosa (1987), Matsumoto (1995), and Sato and Fehler (1998). In general, Q_C is frequency dependent and increases from about 100 at 1 Hz to 1000 at 20 Hz (i.e., there is less attenuation at higher frequencies). However there are regional variations of about a factor of 10, and typically Q_C is lower in tectonically active areas and higher in stable regions, such as shields. As discussed above, how to interpret Q_C in terms of Q_{Sc} and Q_I has been the subject of some debate, and it is becoming increasingly clear that depth-dependent calculations (which include variations in the background velocity, the scattering characteristics, and the intrinsic attenuation) are required in many cases to fully describe coda observations.

Some studies (e.g., Rautian and Khalturin, 1978; Roecker *et al.*, 1982; Gagnepain-Beyneix, 1987; Kvamme and Havskov, 1989; Akamatsu, 1991; Kosuga, 1992, Gupta *et al.*, 1998; Singh *et al.*, 2001; Giampiccolo *et al.*, 2004; Goutbeek *et al.*, 2004) have observed that Q_C increases with increasing lapse time (i.e., eqn [1] does not fit the entire coda envelope), suggesting that the later part of the coda contains energy that propagated through less

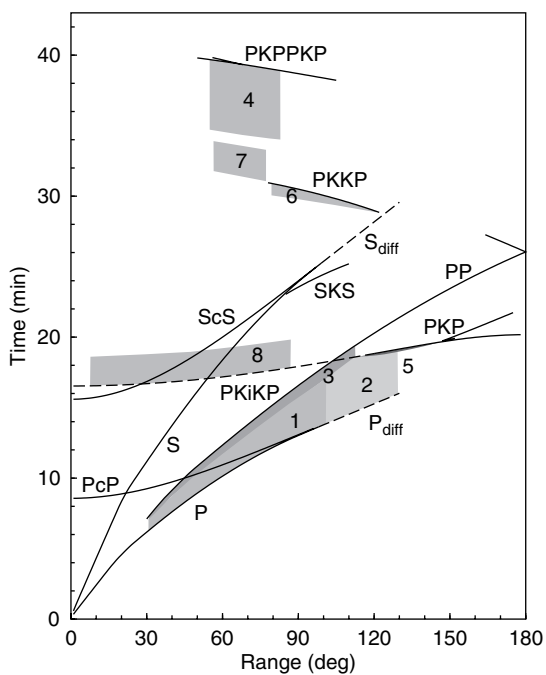


Figure 4 Traveltimes for teleseismic body waves and some of the regions in the short-period wavefield that contain scattered arrivals. These include: (1) P coda, (2) P_{diff} coda, (3) PP precursors, (4) P'P' precursors, (5) PKP precursors, (6) PKKP precursors, (7) PKKP_x, and (8) PKiKP coda. The plotted boundaries are approximate.

attenuating material than the early part of the coda. Gusev (1995) showed that an increase in Q_C with time in the coda is predicted from a single isotropic scattering model in which Q_{Sc} increases with depth. Margerin *et al.* (1998) applied radiative transfer theory to show that a model of scattering in the crust above much weaker scattering in the mantle predicts Q_C values that depend upon the reflection coefficient at the Moho, implying that energy leakage into the mantle has implications for the interpretation of coda Q .

Following Sato and Fehler (1998), we may divide scattering attenuation estimates into those obtained using the single-scattering model and those based on MLTW analysis. Single-scattering studies include Sato (1978), Aki (1980), Pulli (1984), Dainty *et al.* (1987), Kosuga (1992), and Baskoutas (1996). Multiple lapse-time studies are summarized in **Figure 5** and include Fehler *et al.* (1992), Mayeda *et al.* (1992), Hoshiba (1993), Jin *et al.* (1994), Akinci *et al.* (1995), Canas *et al.* (1998), Ugalde *et al.* (1998, 2002), Hoshiba *et al.* (2001), Bianco *et al.* (2002, 2005), Vargas *et al.* (2004), and Goutbeek *et al.* (2004). Values of the coefficient g_0 for S-to-S scattering range from about 0.002 to 0.05 km^{-1} (mean free paths of 20–500 km) for frequencies between 1 and 30 Hz. Some papers (e.g., Mayeda *et al.*, 1992; Hoshiba, 1993; Jin *et al.*, 1994; Goutbeek *et al.*, 2004; Bianco *et al.*, 2005) have found a frequency dependence in

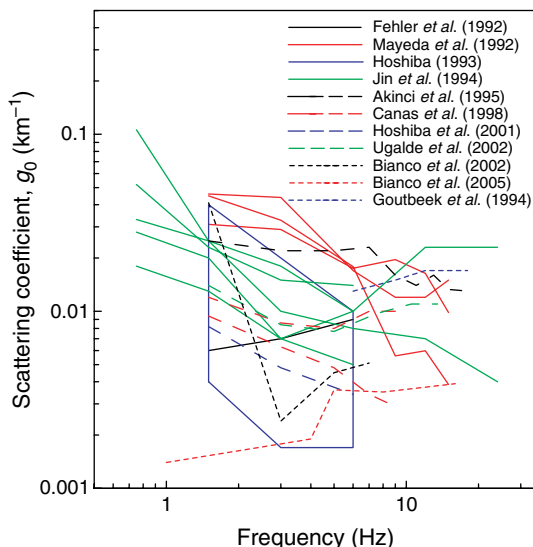


Figure 5 A summary of observations of the total scattering coefficient g_0 for S wave scattering versus frequency, obtained using the MLTW method. Results from various studies around the world are indicated with different colors.

the seismic albedo and among different regions, but in general this dependence is not as consistent as that seen in Q_C studies. Several recent papers noted the importance of considering depth-dependent velocity structure in computing scattering attenuation (e.g., Hoshiba *et al.*, 2001; Bianco *et al.*, 2002, 2005).

Lacombe *et al.* (2003) modeled S coda in France at epicentral distances between 100 and 900 km using acoustic radiative transfer theory applied to a two-layer model. They found that a model with scattering confined to the crust and uniform intrinsic attenuation could explain their data at 3 Hz, but that the tradeoff between scattering and intrinsic attenuation was too strong to reliably determine the relative contribution of each parameter. At the regional epicentral distances of their model, the crustal waveguide had a dominating effect on the S coda.

In addition to the S coda decay rates at relatively long lapse times used to determine coda Q , there are other aspects of the coda that provide additional constraints on scattering. The complete seismogram envelope can be studied, including the broadening of the direct S envelope and the delay in its peak amplitude (e.g., Sato, 1989, 1991; Scherbaum and Sato, 1991; Obara and Sato, 1995). Deviations from the average coda decay rate for a number of stations can be inverted to construct a 3-D model of scattering intensity in the crust and upper mantle (e.g., Nishigami, 1991, 1997; Obara, 1997; Chen and Long, 2000; Taira and Yomogida, 2004). Similar methods were used by Revenaugh (1995, 1999, 2000) to back-project P coda recorded in southern California, and by Hedlin and Shearer (2000) to invert PKP precursor amplitudes (see below). Periodic rippling of SH coda envelopes in northeastern Japan was noted by Kosuga (1997), who suggested this may be caused by trapped waves within a low-velocity layer in the top part of the subducting slab.

Spudich and Bostwick (1987) showed how Green's function reciprocity can be used to obtain information about the ray takeoff directions from the earthquake source region, using a cluster of earthquakes as a virtual array. Applying this method to aftershocks of the 1984 Morgan Hill earthquake in California, they found that the early S coda was dominated by multiple scattering within 2 km of each seismic station. Scherbaum *et al.* (1991) used this approach to study two micro-earthquake clusters in northern Switzerland and found that the early S coda contained energy leaving the source at close to the same angle as the direct wave, but that later (at

least 1.5–2 times the S travel time) coda contains energy from waves leaving the source in a variety of directions. Spudich and Miller (1990) and Spudich and Iida (1993) showed how an interpolation approach using distributed earthquake sources can be used to estimate scattering locations in the vicinity of the 1986 North Palm Springs earthquake in California.

1.20.3.2 P Coda

Local and many regional coda studies have mainly focused on the S wave coda because of its higher amplitude and longer duration than P wave coda (which is truncated by the S wave arrival). However, at teleseismic distances, P waves and their coda are more prominent than S waves at high frequencies because of the severe effect of mantle attenuation on the shear waves. A number of studies both from single stations and arrays have attempted to resolve the origin of the scattered waves in the P coda and to distinguish between near-source and near-receiver scattering.

Aki (1973) modeled scattering of P waves beneath the Montana LASA (Large Aperture Seismic Array) array using Chernov single-scattering theory. At 0.5 Hz, he achieved a good fit to the data with a random heterogeneity model extending to 60 km depth beneath LASA, with rms velocity fluctuations of 4% and a correlation distance of 10 km. Stronger scattering at higher frequencies exceeded the validity limits of the Born approximation.

Frankel and Clayton (1986) compared teleseismic traveltimes anomalies observed across the LASA and NORSAR (Norwegian Seismic Array) arrays, as well as very high frequency coda ($f \geq 30$ Hz) observed for micro-earthquakes, to synthetic seismograms for random media generated using a finite difference method. They found that a random medium with self-similar velocity fluctuations ($a \geq 10$ km) of 5% within a ~ 100 km thick layer could explain both types of observations.

Dainty (1990) reviewed array studies of teleseismic P coda, such as that recorded by the LASA and NORESS arrays. He distinguished between 'coherent' coda, which has nearly the same slowness and back azimuth as direct P, and 'diffuse' coda, which is characterized by energy arriving from many different directions. He argued that coherent coda is generated by shallow, near-source scattering in the crust, rather than deeper in the mantle, because it is absent or weak for deep-focus earthquakes. In contrast, the

diffuse coda is produced by near-receiver scattering and has power concentrated at slownesses typical of shear, Lg , or surface waves. Lay (1987) showed that the dispersive character of the first 15 s of the P coda from nuclear explosions was due to near-source effects.

Gupta and Blanford (1983) and Cessaro and Butler (1987) addressed the question as to how scattering can cause transversely polarized energy to appear in P and P coda, an issue relevant to discrimination methods between earthquakes and explosions. Their observations at different distances and frequency bands suggested that both near-source and near-receiver scattering must be present. Flatté and Wu (1988) performed a statistical analysis of phase and amplitude variations of teleseismic P waves recorded by the NORSAR array. They fit their results with a two-overlapping-layer model of lithospheric and asthenospheric heterogeneity beneath NORSAR, consisting of the summed contributions from a 0–200 km layer with a flat power spectrum and a 15–250 km layer with a k^{-4} power spectrum (the deeper layer corresponds to an exponential autocorrelation model with scale larger than the array aperture of 110 km). The rms P velocity variations are 1–4%. This model has relatively more small-scale scatterers in the shallow crust (which the authors attribute to clustered cracks or intrusions) and relatively more large-scale scatterers in the asthenosphere (which the authors suggest may be temperature or compositional heterogeneities).

Langston (1989) studied teleseismic P waves recorded at station PAS in California and SCP in Pennsylvania. He showed that the coda amplitude cannot be explained by horizontally layered structures of realistic velocity contrasts and that 3-D scattering is required. He could explain the observed coda decay by adopting the EFM of Frankel and Wennerberg (1987) for the case of a scattering layer above a homogenous half-space. He found that scattering is more severe at PAS than SCP, as indicated by higher coda levels and a slower decay rate, obtaining a scattering Q estimate for PAS of 239 (2 s period) compared to 582 for SCP.

Korn (1988) examined the P coda from Indonesian earthquakes recorded at the Warramunga array in central Australia and found that the coda energy decreased with increasing source depth. He computed the power in the coda at different frequency bands between 0.75 and 6 Hz and fitted the results with the EFM of Frankel and Wennerberg (1987). The results indicated frequency dependence in both

intrinsic attenuation and scattering attenuation ($Q_I = 300$ at 1 Hz, increasing almost linearly with frequency above 1 Hz; $Q_{Sc} = 340$ at 1 Hz, increasing as $f^{0.85}$). Assuming random velocity fluctuations with an exponential ACF, results from Sato (1984) can be used to estimate that rms velocity variations of 5% at 5.5 km scale length are consistent with the observed Q_{Sc} below the Warramunga array. Korn (1990) extended the EFM to accommodate a scattering layer over a homogeneous half space (a modeling approach similar to that used by Langston (1989)) and applied the method to the short-period Warramunga array data of Korn (1988) for deep earthquakes. With the new model, he found an average scattering Q of 640 at 1 Hz for the lithosphere below the array and higher values of intrinsic Q , implying that diffusion rather than anelasticity is the dominant factor controlling teleseismic coda decay rates. Korn (1993) further applied this approach to teleseismic P coda observations from nine stations around the Pacific. He found significant differences in scattering Q among the stations ($Q_{Sc} = 100$ –500). The observed frequency dependence of Q_{Sc} is in approximate agreement with single-scattering theory for random heterogeneities, and favors von Karman type ACFs over Gaussian or self-similar models.

Bannister *et al.* (1990) analyzed teleseismic P coda recorded at the NORESS array using both array and three-component methods. They resolved both near-source and near-receiver scattering contributions to coda, with the bulk of receiver-side scattering resulting from P-to-Rg conversions from two nearby areas (10 and 30 km away) with significant topography. Gupta *et al.* (1990) performed frequency wave number analysis on high-frequency NORESS data and identified both near-receiver P-to-Rg scattering and near-source Rg-to-P scattering. Dainty and Toksöz (1990) examined scattering in regional seismograms recorded by the NORESS, FINESA, and ARCESS arrays. P coda energy was concentrated in the on-azimuth direction, but appeared at different phase velocities, suggesting different contribution mechanisms.

Wagner and Langston (1992b) applied frequency wave number analysis to P'P' (PKPPKP) waves recorded by the NORESS arrays for deep earthquakes. Most of the coda was vertically propagating but an arrival about 15 s after the direct wave can be identified as body-to-Rayleigh-wave scattering from a point 30 km west-southwest of the array (this scattering location was previously identified in P wave

coda by Bannister *et al.* (1990) and Gupta *et al.* (1990)). Revenaugh (1995, 1999, 2000) applied a Kirchhoff migration approach to back-project P coda recorded in southern California to image lateral variations in scattering intensity within the crust and identified correlations between scattering strength and the locations of faults and other tectonic features.

Neele and Snieder (1991) used the NARS and GRF arrays in Europe to study long-period teleseismic P coda and found it to be coherent with energy arriving from the source azimuth. They concluded that long-period P coda does not contain a significant amount of scattered energy and can be explained with spherically symmetric models, and is particularly sensitive to structure in upper-mantle low-velocity zones. Ritter *et al.* (1997, 1998) studied the frequency dependence of teleseismic P coda recorded in the French Massif Central, which they modeled as a 70 km thick layer with velocity fluctuations of 3–7% and heterogeneity scale lengths of 1–16 km. Rotherth and Ritter (2000) studied P coda in intermediate-depth Hindu Kush earthquakes recorded at the GRF array in Germany about 44° away. They applied a method based on the theory of Shapiro and Kneib (1993) and Shapiro *et al.* (1996) and found that the observed wavefield fluctuations are consistent with random crustal heterogeneities of 3–7% and isotropic correlation lengths of 0.6–4.8 km. Ritter and Rotherth (2000) used a similar approach on teleseismic P coda to infer differences in scattering strength beneath two local networks in Europe. Hock *et al.* (2000, 2004) used teleseismic P coda to characterize random lithospheric heterogeneities across Europe using the EFM. They obtained a range of different scale lengths and rms velocity fluctuations on the order to 3–8%. Nishimura *et al.* (2002) analyzed transverse components in teleseismic P coda for stations in the western Pacific and noted stronger scattering for stations close to plate boundaries compared to those on stable continents.

In some cases, analysis of P coda can reveal individual scatterers and/or discontinuities at mid-mantle depths below subduction zone earthquakes (Niu and Kawakatsu, 1994, 1997; Castle and Creager, 1999; Kaneshima and Helffrich, 1998, 1999, 2003; Krüger *et al.*, 2001; Kaneshima, 2003). Other individual scatterers (or regions of strong scattering) have been observed near the CMB from PcP precursors (Weber and Davis, 1990; Weber and Körnig, 1990; Scherbaum *et al.*, 1997; Brana and Helffrich, 2004) and PKP precursors (Vidale and Hedlin, 1998; Thomas *et al.*, 1999).

Shearer and Earle (2004) attempted to systematically characterize and model globally averaged short-period teleseismic P coda from both shallow and deep earthquakes. This was the first attempt to fit a global data set of P wave amplitudes and coda energy levels with a comprehensive energy-preserving model that specified scattering and intrinsic attenuation throughout the Earth. Examining global seismic network data between 1990 and 1999 at source–receiver distances between 10° and 100°, they identified high signal-to-noise records and stacked over 7500 traces from shallow events (depth ≤ 50 km) and over 650 traces from deep events (depth ≥ 400 km). The stacking method involved summing envelope functions from 0.5 to 2.5 Hz bandpass filtered traces, normalized to the maximum P amplitude. Peak P amplitudes were separately processed so that absolute P amplitude versus source–receiver distance information was preserved. The coda shape was markedly different between the shallow- and deep-event stacks (see Figure 6). The shallow earthquake

coda was much more energetic and long-lasting than the deep-event coda, indicating that the bulk of the teleseismic P coda from shallow events is caused by near-source scattering above 600 km depth.

Shearer and Earle modeled these observations using a Monte Carlo, particle-based approach (see above), in which millions of seismic phonons are randomly sprayed from the source and tracked through the Earth. They found that most scattering occurs in the lithosphere and upper mantle, as previous results had indicated, but that some lower-mantle scattering was also required to achieve the best fits to the data. Their preferred exponential autocorrelation random heterogeneity model contained 4% rms velocity heterogeneity at 4 km scale length from the surface to 200 km depth, 3% heterogeneity at 4 km scale between 200 and 600 km, and 0.5% heterogeneity at 8 km scale length between 600 km and the CMB. They assumed equal and correlated P and S fractional velocity perturbations and a density/velocity scaling ratio of 0.8. Intrinsic

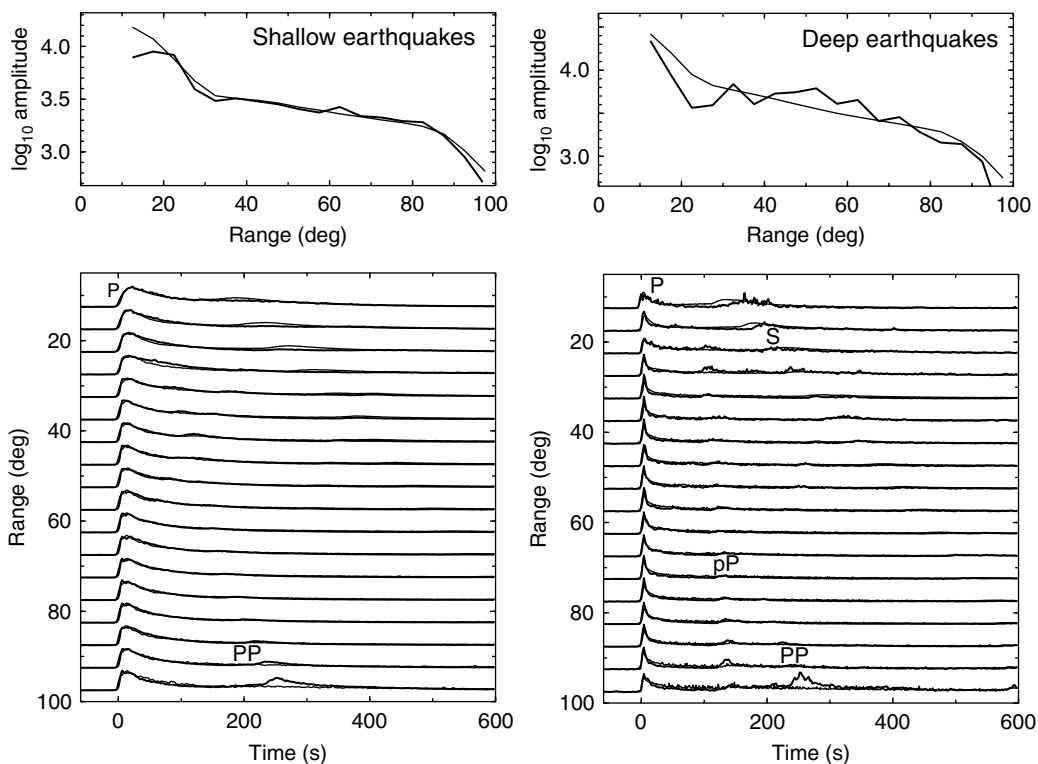


Figure 6 Comparisons between envelope function stacks of teleseismic P wave arrivals (thick lines) with predictions of a Monte Carlo simulation for a whole-Earth scattering model (thin lines) as obtained by Shearer and Earle (2004). The left panels show results for shallow earthquakes (≤ 50 km); the right panels are for deep earthquakes (≥ 400 km). The top panels show peak P wave amplitude versus epicentral distance. The bottom panels show coda envelopes in 5° distance bins plotted as a function of time from the direct P arrivals, with amplitudes normalized to the same energy in the first 30 s.

attenuation was $^{\alpha}Q_1 = 450$ above 200 km and $^{\alpha}Q_1 = 2500$ below 200 km, with $^{\beta}Q_1 = (4/9)^{\alpha}Q_1$ (an approximation that assumes all the attenuation is in shear). This model produced a reasonable overall fit, for both the shallow- and deep-event observations, of the amplitude decay with epicentral distance of the peak P amplitude and the P coda amplitude and duration (see **Figure 6**). These numbers imply that for both P and S waves the seismic albedo, β_0 , is about 75–90% in the upper mantle above 600 km, and about 25–35% in the lower mantle, consistent with the total attenuation being dominated by scattering in the upper mantle and by intrinsic energy loss in the lower mantle.

The 4% velocity perturbations for the uppermost layer in the Shearer and Earle (2004) model are in rough agreement with previous P coda analyses by Frankel and Clayton (1986), Flatté and Wu (1988), and Korn (1988); and the S wave mean free path of 50 km for the upper 200 km is within the range of mean free paths estimated from regional measurements of lithospheric scattering. However, limitations of the Shearer and Earle (2004) study include the restriction to single-station, vertical-component data (i.e., wave polarization, slowness, and azimuth constraints from three-component and/or array analyses are not used) and a single frequency band near 1 Hz. In addition, the ray theoretical approach cannot properly account for body-to-surface-wave converted energy, which some array studies suggest are an important component of P coda.

Resolving possible lower-mantle scattering using P coda is difficult because of the much stronger scattering from shallow structure. However, Shearer and Earle (2004) found that $\sim 0.5\%$ rms velocity heterogeneity in the lower mantle was required to achieve the best fit to P coda amplitudes at epicentral distances beyond 50° . This value is between the estimates for lower-mantle velocity perturbations derived from PKP precursors of 1% from Hedlin *et al.* (1997) and 0.1–0.2% from Margerin and Nolet (2003b).

1.20.3.3 P_n Coda

A prominent feature of long-range records of nuclear explosions across Eurasia is P_n and its coda, which can be observed to distances of more than 3000 km (e.g., Ryberg *et al.*, 1995, 2000; Morozov *et al.*, 1998). As discussed by Nielsen and Thybo (2003), there are two main models for upper-mantle structure that

have been proposed to explain these observations: (1) Ryberg *et al.* (1995, 2000) and Tittgemeyer *et al.* (1996) proposed random velocity fluctuations between the Moho and about 140 km depth in which the vertical correlation length (0.5 km) is much smaller than the horizontal correlation length (20 km). These fluctuations cause multiple scattering that form a waveguide that can propagate high-frequency P_n to long distances. (2) Morozov *et al.* (1998), Morozov and Smithson (2000), and Nielsen *et al.* (2003b) favored a model in which P_n is a whispering gallery phase traveling as multiple underside reflections off the Moho, with the coda generated by crustal scattering. Nielsen and Thybo's (2003) preferred model has random crustal velocity perturbations between 15 and 40 km depth with a vertical correlation length of 0.6 km and a horizontal correlation length of 2.4 km. However, Nielsen *et al.* (2003a) found that the scattered arrivals seen at 800–1400 km distance for profile 'Kraton' required scattering within a layer between about 100 and 185 km depth and could be modeled with 2-D finite difference synthetics assuming 2% rms velocity variations.

An important aspect of all these models is that their random velocity perturbations are horizontally elongated (i.e., anisotropic). At least for the lower crust there is also some evidence for this from reflection seismic profiles (e.g., Wenzel *et al.*, 1987; Holliger and Levander, 1992). This contrasts with most modeling of S and P coda, which typically assumes isotropic random heterogeneity.

1.20.3.4 P_{diff} Coda

Another phase particularly sensitive to deep Earth scattering is P_{diff} and its coda. P_{diff} contains P energy diffracted around the CMB and is observed at distances greater than 98° . The direct phase is seen most clearly at long periods, but high-frequency (~ 1 Hz) P_{diff} and its coda can be detected to beyond 130° (e.g., Earle and Shearer, 2001). P_{diff} coda is a typically emergent wave train that decays slowly enough that it can commonly be observed for several minutes until it is obscured by the PP and PKP arrivals. Husebye and Madariaga (1970) concluded that P_{diff} coda (which they termed P(diff) tail) could not be explained as simple P diffraction at the CMB or by reflections from the core, and suggested that it originated from reflections or multiple paths in the upper mantle (similar to the proposed explanation for PP precursors given by Bolt *et al.* (1968)). However; later

work has shown this to be unlikely, given the large differences seen between observations of deep-turning direct P coda and P_{diff} coda. Bataille *et al.* (1990) reviewed previous studies of P_{diff} coda and suggested that it is caused by multiple scattering near the CMB, with propagation to long distances possibly enhanced by the presence of a low-velocity layer within D' .

Tono and Yomogida (1996) examined P_{diff} coda in 15 short-period records from deep earthquakes at distances of 103 – 120° and found considerable variation in the appearance and duration of P_{diff} coda. Comparisons between P_{diff} and direct P waves at shorter distances, as well as particle motion analysis of P_{diff} coda, indicated that the coda was caused by deep-Earth scattering. Tono and Yomogida computed synthetics using the boundary integral method of Benites *et al.* (1992), applied to a simplified model of an incident wave grazing an irregular CMB. They were able to fit a subset of their observations in which the P_{diff} coda duration was relatively short (<50 s), with bumps on the CMB with minimum heights of 5 – 40 km. Such large CMB topography is unrealistic given PcP studies, which have indicated a relatively flat and smooth CMB (e.g., Kampmann and Müller, 1989; Vidale and Benz, 1992), and PKKP precursor observations that limit CMB rms topography to less than 315 m at 10 km wavelength (Earle and Shearer, 1997, see below), but it is likely that volumetric heterogeneity within D' could produce similar scattering. Although the Tono and Yomogida (1996) model included multiple scattering, they could not fit the long tail (>50 s) of some of their P_{diff} observations, and they suggested that for such cases a low-velocity zone just above the CMB is channeling the scattered energy. Strong heterogeneity and low-velocity zones of varying thicknesses have been observed above the CMB (e.g., see review by Garnero (2000)), but it is not yet clear if these models can explain P_{diff} observations.

Bataille and Lund (1996) argued for a deep origin for P_{diff} coda by comparing coda shapes for P near 90° range and P_{diff} at 102 – 105° . The P_{diff} coda is more emergent and lasts much longer than the direct P coda. This argues against a shallow source for the coda because this would produce roughly the same effect on both P and P_{diff} . Bataille and Lund found that their observed coda decay rate for a single P_{diff} observation at 116° could fit with a model of multiple scattering within a 2-D shell at the CMB. Tono and Yomogida (1997) examined P_{diff} in records of the 1994 Bolivian deep earthquake at epicentral

distances of 100 – 122° . They analyzed both global broadband stations and short-period network stations from New Zealand. They found that short-period energy continued to arrive for over 100 s after P_{diff} itself, more than twice as long as the estimated source duration of the mainshock. Comparisons between the coda decay rate of P_{diff} with other phases, as well as polarization analysis, indicated a deep origin for P_{diff} coda.

Earle and Shearer (2001) stacked 924 high-quality, short-period seismograms from shallow events at source–receiver distances between 92.5° and 132.5° to obtain average P and P_{diff} coda shapes. The results confirm the Bataille and Lund (1996) observation that P coda changes in character near 100° . In particular, P_{diff} becomes increasingly extended and emergent at longer distances. Its peak amplitude also diminishes with increasing distance, but P_{diff} can still be observed in the stacks at 130° . Earle and Shearer also performed a polarization stack (Earle, 1999), which showed that the polarization of P_{diff} coda is similar to P_{diff} (see **Figure 7**). To model these results, Earle and Shearer (2001) applied single-scattering theory for evenly distributed scatterers throughout the mantle. The resulting synthetics included P-to-P, P_{diff} -to-P, and P-to- P_{diff} scattering paths. They applied a hybrid scheme that used reflectivity to compute deep-turning P and P_{diff} amplitudes and ray theory for the shallower turning rays. Scattering was computed assuming a random medium model with an exponential ACF. Synthetics generated for a scale length of 2 km and 1% rms velocity variations achieved a good fit to the amplitude and shape of P and P_{diff} coda and a reasonable fit to the polarization angles. Thus a fairly modest level of whole-mantle scattering appears sufficient to explain the main features in P_{diff} observations, although Earle and Shearer could not exclude the possibility that multiple-scattering models could achieve similar fits.

1.20.3.5 PP and P'P' Precursors

The decay in short-period coda amplitude with time following direct P and P_{diff} stops and amplitudes begin to increase some time before the surface-reflected PP phase at distances less than about 110° (at longer distances, PKP intercedes). This energy is typically incoherent but sometimes forms distinct arrivals; both are termed PP precursors. Early explanations for PP precursor observations involved topside and bottomside reflections off discontinuities in the upper mantle (Nguyen-Hai, 1963; Bolt *et al.*,

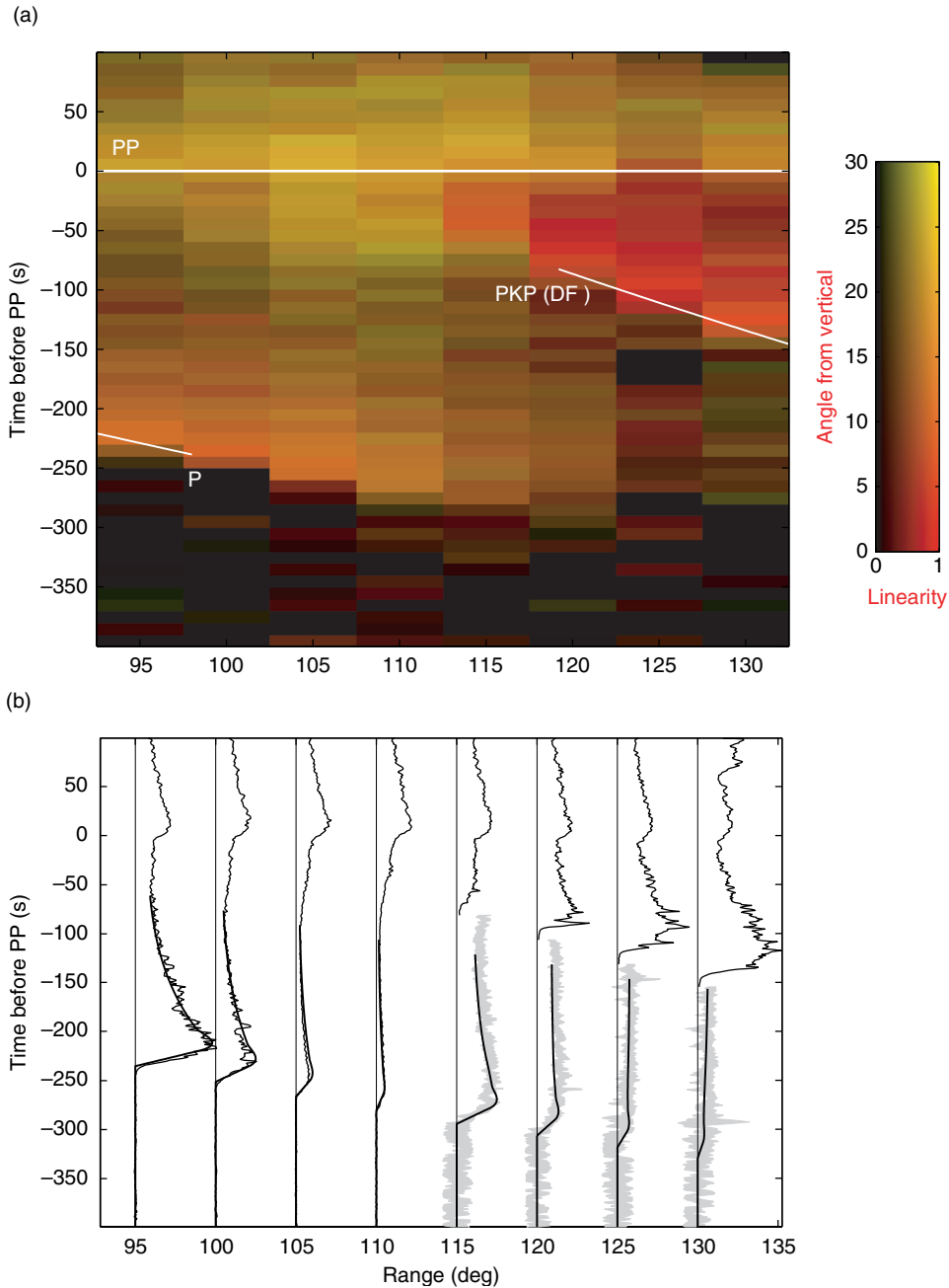


Figure 7 Global seismogram stacks and scattering model predictions for high-frequency P_{diff} from Earle and Shearer (2001). (a) A stacked image of P_{diff} polarization and linearity. Times are relative to the theoretical PP arrival times, and direct P and PKP(DF) are also shown. (b) An amplitude stack of P_{diff} envelopes (wiggly lines) compared to model predictions (smooth lines) for a uniform mantle scattering model. Amplitudes are normalized to the maximum PP amplitude. To better show the small-amplitude P_{diff} arrivals, the traces are magnified by 10 for times 20 s before PKP at distances greater than 112.5°. The width of the magnified data traces is equal to the two-sigma error in the data stack, estimated using a bootstrap method.

1968; Bolt, 1970; Husebye and Madariaga, 1970; Gutowski and Kanasewich, 1974). It is now recognized that such arrivals do exist and form globally coherent seismic phases that can readily be observed

at long periods (e.g., Shearer, 1990, 1991). In particular, the 410 and 660 km discontinuities create discrete topside reflections that follow direct P by about 1.5–2.5 min and underside reflections that proceed

PP by similar time offsets. However, the high-frequency PP precursor wavefield is much more continuous and it is difficult to identify discrete arrivals from upper-mantle discontinuities, although Wajeman (1988) was able to identify underside P reflections from discontinuities at 300 and 670 km depth by stacking broadband data from the NARS array in Europe.

Wright and Muirhead (1969) and Wright (1972) used array studies to show that PP precursors often have slownesses that are significantly less than or greater than that expected for underside reflections beneath the PP bounce point, which is consistent with asymmetric reflections at distances near 20° from either the source or receiver. However, this explanation does not explain the generally continuous nature of the PP precursor wavefield. The currently accepted explanation for the bulk of the PP precursor energy involves scattering from the near surface and was first proposed by Cleary *et al.* (1975) and King *et al.* (1975). Cleary *et al.* (1975) proposed that PP precursors result from scattering by heterogeneities within and near the crust, as evidenced by traveltimes and slowness observations of two Novalya Zemlya explosions recorded by the Warramunga array in Australia. King *et al.* (1975) modeled PP precursor observations from the Warramunga and NORSAR arrays using Born single-scattering theory. They assumed 1% rms variations in elastic properties and a Gaussian autocorrelation model with a 12 km scale length within the uppermost 100 km of the crust and upper mantle. This model successfully predicted the onset times, duration, and slowness of the observed PP precursors but underpredicted the precursor amplitudes, suggesting that stronger scattering, perhaps too large for single-scattering theory, would be required to fully explain the observations. King *et al.* noted that the focusing of energy at 20° distance by the mantle transition zone could explain the high and low slowness observations of their study and of Wright (1972).

A related discussion has concerned precursors to PKPPKP (or P'P'), for which the main phase also has an underside reflection near the midpoint between source and receiver. In this case, however, short-period reflections from the 410 and 660 km discontinuities are much easier to observe and this has become one of the best constraints on the sharpness of these features (e.g., Engdahl and Flinn, 1969; Richards, 1972; Davis *et al.*, 1989; Benz and Vidale, 1993; Xu *et al.*, 2003). However, these reflections arrive 90–150 s before P'P' and cannot explain the later parts

of the precursor wavetrain. Whitcomb (1973) suggested they were asymmetric reflections at sloping interfaces, a mechanism similar to that proposed by Wright and Muirhead (1969) and Wright (1972) for PP precursors. King and Cleary (1974) proposed that near-surface scattering near the P'P' bounce point could explain the extended duration and emergent nature of P'P' precursors. Vinnik (1981) used single-scattering theory to model globally averaged P'P' precursor amplitudes at three different time intervals, and obtained a good fit with a Gaussian ACF of 13 km scale length with rms velocity perturbations in the lithosphere of about 1.6%. Recently Tkalčić *et al.* (2006) observed P'P' precursors at epicentral distances less than 10° , which they interpret as back-scattering from small-scale heterogeneities at 150–220 km depth beneath the P'P' bounce points because array studies show that the precursors have the same slowness as the direct phase.

1.20.3.6 PKP Precursors

Perhaps the most direct evidence for deep-Earth scattering comes from observations of precursors to the core phase PKP. They were first noted by Gutenberg and Richter (1934). The precursors are seen at source–receiver distances between about 120° and 145° and precede PKP by up to ~ 20 s. They are observed most readily at high frequencies and are usually emergent in character and stronger at longer distances. Older, and now discredited, hypotheses for their origin include refraction in the inner core (Gutenberg, 1957), diffraction of PKP from the CMB (Bullen and Burke-Gaffney, 1958; Doornbos and Husebye, 1972), and refraction or reflection of PKP at transition layers between the inner and outer cores (Bolt, 1962; Sacks and Saa, 1969). However, it is now understood that PKP precursors are not caused by radially symmetric structures but result from scattering from small-scale heterogeneity at the CMB or within the lowermost mantle (Haddon, 1972; Cleary and Haddon, 1972). This scattering diverts energy from the primary PKP ray paths, permitting waves from the AB and BC branches to arrive at shorter source–receiver distances than the B caustic near 145° and earlier than the direct PKP(DF) phase. It should be noted that although the PKP precursors arrive in front of PKP(DF), they result from scattering from different PKP branches. Scattered energy from PKP(DF) itself contributes only to the coda following PKP(DF), not to the precursor wavefield. In addition, the scattering region must be deep to create the precursors. Scattering of

PKP(BC) from the shallow mantle will not produce precursors at the observed times and distances. Thus deep-Earth scattering can be observed uncontaminated by the stronger scattering that occurs in the crust and upper mantle. This unique ray geometry, which results from the velocity drop between the mantle and the outer core, makes PKP precursors invaluable for characterizing small-scale heterogeneity near the CMB.

The interpretation of PKP precursors in terms of scattering was first detailed by Haddon (1972) and Cleary and Haddon (1972). The primary evidence in favor of this model is the good agreement between the observed and theoretical onset times of the precursor wavetrain for scattering at the CMB. However, analyses from seismic arrays (e.g., Davies and Husebye, 1972; Doornbos and Husebye, 1972; Doornbos and Vlaar, 1973; King *et al.*, 1973, 1974; Husebye *et al.*, 1976; Doornbos, 1976) also showed that the traveltimes and incidence angles of the precursors were consistent with the scattering theory. Haddon and Cleary (1974) used Chernov scattering theory to show that the precursor amplitudes could be explained with 1% random velocity heterogeneity with a correlation length of 30 km in a 200 km thick layer in the lowermost mantle just above the CMB. In contrast, Doornbos and Vlaar (1973) and Doornbos (1976) argued that the scattering region extends to 600–900 km above the CMB and calculated (using

the Knopoff and Hudson (1964) single-scattering theory) that much larger velocity anomalies must be present. Later, however, Doornbos (1978) used perturbation theory to show that short-wavelength CMB topography could also explain the observations (see also Haddon, 1982), as had previously been suggested by Haddon and Cleary (1974). Bataille and Flatté (1988) concluded that their observations of 130 PKP precursor records could be explained equally well by 0.5–1% rms velocity perturbations in a 200 km thick layer at the base of the mantle or by CMB topography with rms height of ~ 300 m (see also Bataille *et al.*, 1990).

One difficulty in comparing results among these older PKP precursor studies is that it is not clear how many of their differences are due to differences in observations (i.e., the selection of precursor waveforms they examine) compared to differences in theory or modeling assumptions. PKP precursor amplitudes are quite variable and it is likely that studies that focus on the clearest observations will be biased (at least in terms of determining globally averaged Earth properties) by using many records with anomalously large amplitudes. To obtain a clearer global picture of average PKP precursor behavior, Hedlin *et al.* (1997) stacked envelope functions from 1600 high signal-to-noise PKP waveforms at distances between 118° and 145° (see Figure 8). They included all records, regardless of whether precursors could be observed, to avoid

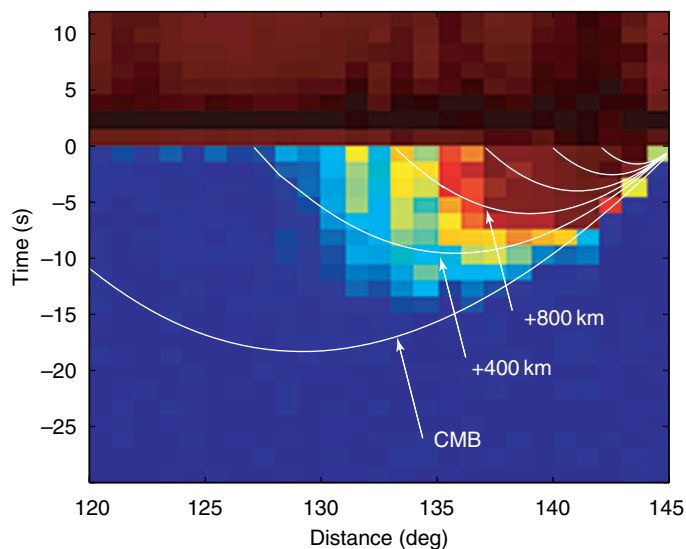


Figure 8 PKP precursors as imaged by stacking 1600 short-period seismograms by Hedlin *et al.* (1997). The colors indicate the strength of the precursors, ranging from dark blue (zero amplitude) through green, yellow, red, and brown (highest amplitudes). The onset of PKP(DF) is shown by the edge of the brown region at zero time. The precursors exhibit increasing amplitude with both distance and time. The white curves show the theoretical precursor onset times for scattering at 400 km depth intervals above the CMB.

biasing their estimates of average precursor amplitudes. In this way, they obtained the first comprehensive image of the precursor wavefield and found that precursor amplitude grows with both distance and time. The growth in average precursor amplitude with time continues steadily until the direct PKP(DF) arrival; no maximum amplitude peak is seen prior to PKP(DF) as had been suggested by Doornbos and Husebye (1972). This is the fundamental observation that led Hedlin *et al.* (1997) to conclude that scattering is not confined to the immediate vicinity of the CMB but must extend for some distance into the mantle.

Hedlin *et al.* (1997) and Shearer *et al.* (1998) modeled these observations using single-scattering theory for a random medium characterized with an exponential ACF. They found that the best overall fit to the observations was provided with $\sim 1\%$ rms velocity heterogeneity at 8 km scale length extending throughout the lower mantle, although fits almost as good could be obtained for 4 and 12 km scale lengths. Similar fits could also be achieved with a Gaussian ACF using a slightly lower rms velocity heterogeneity. To explain the steady increase in precursor amplitude with time, these models contained uniform heterogeneity throughout the lower mantle, and Hedlin *et al.* (1997) argued that the data require the scattering to extend at least 1000 km above the CMB and that there is no indication for a significant concentration of the scattering near the CMB. This conclusion was supported by Cormier (1999), who tested both isotropic and anisotropic distributions of scale lengths and found that the PKP precursor envelope shapes are consistent with dominantly isotropic 1% fluctuations in P velocity in the $0.05\text{--}0.5\text{ km}^{-1}$ wave number band (i.e., 12–120 km wavelengths).

Most modeling of PKP precursors has used single-scattering theory and the Born approximation. The validity of this approximation for mantle scattering was questioned by Hudson and Heritage (1981). However, Doornbos (1988) found similar results from single- versus multiple-scattering theory for CMB topography of several hundred meters (i.e., the amount proposed by Doornbos (1978) to explain PKP precursor observations), and Cormier (1995) found the Born approximation to be valid for modeling distributed heterogeneity in the D' region when compared to results from the higher-order theory of Korneev and Johnson (1993a, 1993b). More recently, Margerin and Nolet (2003a, 2003b) modeled PKP precursors using radiative transfer theory and a Monte Carlo method (see above). Their results supported Hedlin *et al.* (1997) and Shearer *et al.* (1998) in

finding that whole-mantle scattering fits the data better than scattering restricted to near the CMB, but they obtained much smaller P velocity perturbations of 0.1–0.2% versus the 1% of Hedlin *et al.* In addition, they found that the Born approximation is accurate for whole-mantle scattering models only when the velocity heterogeneity is less than 0.5%. Finally, they concluded that exponential correlation length models do not fit the distance dependence in PKP precursor amplitudes as well as models containing more small-scale structure.

The reasons for the discrepancy in heterogeneity amplitude between the models of Hedlin *et al.* (1997), Cormier (1999), and Margerin and Nolet (2003b) are not yet clear. Differences between the single-scattering (Born) and multiple-scattering predictions do not appear to be sufficient to account for the size of the discrepancy. There are subtleties in the data stacking and modeling that may have a significant effect on the results. These include the weighting and normalization of the waveforms, the assumed form of the random heterogeneity PSDF, the correction of PKP(DF) amplitudes for inner-core Q , and the correction for realistic effective source-time functions. However, despite the uncertainty in the heterogeneity amplitude, it now seems firmly established that PKP precursor observations require that small-scale mantle heterogeneity extends above the lowermost few hundred kilometers of the mantle.

Determining exactly how far the scattering must extend above the CMB is a challenging problem because only a small fraction of the precursor wavefield is sensitive to scattering above 600 km from the CMB, and this fraction continues to shrink for shallower scattering depths. For example, at 1200 km above the CMB, precursors are produced only between 138° and 145° , and arrive at most 3 s before PKP(DF) (see **Figure 8**). Separating the amplitude contribution from these arrivals from that produced by scattering at deeper depths in the mantle is very difficult, particularly when the pulse broadening caused by realistic source-time functions and shallow scattering and reverberations are taken into account. An alternative approach to constraining the strength of mid-mantle scattering is to examine the scattered PKP wavefield after the direct PKP(DF) arrival (Hedlin and Shearer, 2002). This is comprised of a sum of PKP(DF) coda and scattered PKP(AB) and PKP(BC) energy. In principle, if the contribution from the PKP(DF) coda could be removed, the remaining scattered wavefield would provide improved constraints on the strength of mid-mantle

scattering. However, Hedlin and Shearer (2002) found that this approach did not provide improved depth resolution of mantle scattering compared to previous analyses of PKP precursors, owing at least in part to relatively large scatter in coda amplitudes as constrained by a data resampling analysis. Their results were, however, consistent with uniform mantle scattering with 1% rms random velocity perturbations at 8 km scale length.

The preceding discussion considered only average PKP precursor properties and models with radial variations in scattering. However, PKP precursor amplitudes are quite variable, as noted by Bataille and Flatté (1988), Bataille *et al.* (1990), and Hedlin *et al.* (1995), suggesting lateral variations in scattering strength. Vidale and Hedlin (1998) identified anomalously strong PKP precursors for ray paths that indicated intense scattering at the CMB beneath Tonga. Wen and Helmberger (1998) observed broadband PKP precursors from near the same region, which they modeled as Gaussian-shaped ultralow-velocity zones (ULVZs) of 60–80 km height with P velocity drops of 7% or more over 100–300 km (to account for the long-period part of the precursors), superimposed on smaller-scale anomalies to explain the high-frequency part of the precursors. Thomas *et al.* (1999) used German network and array records of PKP precursors to identify isolated scatterers in the lower mantle. Niu and Wen (2001) identified strong PKP precursors for South American earthquakes recorded by the J-array in Japan, which they modeled with 6% velocity perturbations within a 100 km thick layer just above the CMB in a 200 km by 300 km area west of Mexico.

Hedlin and Shearer (2000) attempted to systematically map lateral variations in scattering strength using a global set of high-quality PKP precursor records. Their analysis was complicated by the limited volume sampled by each source–receiver pair, the ambiguity between source- and receiver-side scattering, and the sparse and uneven data coverage. However, they were able to identify some large-scale variations in scattering strength that were robust with respect to data resampling tests. These include stronger than average scattering beneath central Africa, parts of North America, and just north of India; and weaker than average scattering beneath South and Central America, eastern Europe, and Indonesia.

Finally, it should be noted that the earliest onset time of observed PKP precursors agrees closely with that predicted for scattering at the CMB (e.g., Cleary and Haddon, 1972; Shearer *et al.*, 1998). If scattering

existed in the outer core at significant depths below the CMB, this would cause arrivals at earlier times than are seen in the data. This suggests that no observable scattering originates from the outer core, although a quantitative upper limit on small-scale outer core heterogeneity based on this constraint has not yet been established.

1.20.3.7 PKKP Precursors and PKKP_x

PKKP is another seismic core phase that provides information on deep-Earth scattering. Precursors to PKKP have been observed within two different distance intervals. PKKP(DF) precursors at source–receiver distances beyond the B caustic near 125° are analogous to the PKP precursors discussed in the previous section and result from scattering in the mantle. Doornbos (1974) detected these precursors in NORSAR records of Solomon Islands earthquakes and showed that their observed slownesses were consistent with scattering from the deep mantle. At ranges less than 125°, PKKP(BC) precursors can result from scattering off short-wavelength CMB topography. Chang and Cleary (1978, 1981) observed these precursors from Novaya Zemlya explosions recorded by the LASA array in Montana at about 60° range. These observations were suggestive of CMB topography but had such large amplitudes that they were difficult to fit with realistic models. Doornbos (1980) obtained additional PKKP(BC) precursor observations from NORSAR records from a small number of events at source–receiver distances between 80° and 110°. He modeled these observations with CMB topography of 100–200 m at 10–20 km horizontal scale length. Motivated by PKKP precursor observations (see below), Cleary (1981) suggested that some observations of P'P' precursors (e.g., Adams, 1968; Whitcomb and Anderson, 1970; Haddon *et al.*, 1977; Husebye *et al.*, 1977) might be explained as CMB-scattered PKKKP precursors.

A comprehensive study of PKKP(BC) precursors was performed by Earle and Shearer (1997), who stacked 1856 high-quality PKKP seismograms, obtained from the Global Seismic Network (GSN) at distances between 80° and 120°. PKKP is most readily observed at high frequencies (to avoid interference from low-frequency S coda), so the records were bandpass filtered to between 0.7 and 2.5 Hz. To avoid biasing the stacked amplitudes, no consideration was given to the visibility or lack of visibility of PKKP precursors on individual records. The resulting

stacked image showed that energy arrives up to 60 s before direct PKKP(BC) and that average precursor amplitudes gradually increase with time. Earle and Shearer (1997) modeled these observations using Kirchhoff theory for small-scale CMB topography. Their best-fitting model had a horizontal scale length of 8 km and rms amplitude of 300 m. However, they identified a systematic misfit between the observations and their synthetics in the dependence of precursor amplitude with source–receiver distance. In particular, the Kirchhoff synthetics predict that precursor amplitude should grow with range but this trend is not apparent in the data stack. Thus, the model underpredicts the precursor amplitudes at short ranges and overpredicts the amplitudes at long ranges.

Earle and Shearer (1997) and Shearer *et al.* (1998) explored possible reasons for this discrepancy between PKKP(BC) precursor observations and predictions for CMB topography models. They were not able to identify a very satisfactory explanation but speculated that scattering from near the ICB might be involved because it could produce precursor onsets that agreed with the observations (see figure 13 from Shearer *et al.* (1998)). However, scattering angles of 90° or more are required and it is not clear, given the expected amplitude of the direct PKKP(DF) phase, that the scattered amplitudes would be large enough to explain the PKKP(BC) precursor observations. They concluded that strong

inner-core scattering would be required, which could only be properly modeled with a multiple-scattering theory. Regardless of the possible presence of scattering from sources outside the CMB, these PKKP(BC) precursor observations can place upper limits on the size of any CMB topography. Earle and Shearer (1997) concluded that the rms topography could not exceed 315 m at 10 km wavelength.

Earle and Shearer (1998) stacked global seismograms using P'P' as a reference phase and identified an emergent, long-duration, high-frequency wavetrain near PKKP, which they named PKKP_x because it lacked a clear explanation (Figure 9). PKKP_x extends back from the PKKP(C) caustic at 72° to a distance of about 60 s. Its 150-s long duration, apparent move-out, and proximity to PKKP suggest a deep scattering origin. However, Earle and Shearer were not able to match these observations with predictions of single-scattering theory for scattering in the lower mantle, CMB, or ICB. They speculated that some form of multiple-scattering model at the CMB might be able to explain the observations, perhaps involving a low-velocity zone just above the CMB to trap high-frequency energy, a model similar to that proposed to explain P_{diff} coda by Bataille *et al.* (1990), Tono and Yomogida (1996), and Bataille and Lund (1996). Earle (2002) further explored the origin of PKKP_x and other scattered phases near PKKP by performing slant stacks on LASA data. His results suggested that

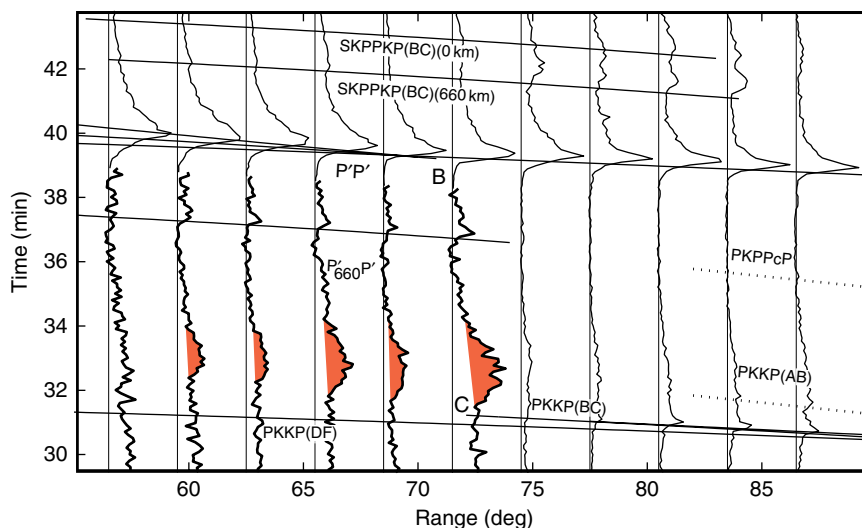


Figure 9 A stack of 994 seismograms from Earle and Shearer (1998), showing the PKKP_x phase (red) in the high-frequency wavefield preceding P'P'. The stacked traces are normalized to the maximum P'P' amplitude and are plotted with respect to the origin time of a zero depth earthquake. To better show the PKKP_x arrival, amplitudes are magnified by 8 for traces preceding P'P' at epicentral distances less than 73° (indicated by the heavy lines). Labels indicate observed phases and the position of the P'P' B and PKKP C caustics.

near-surface P-to-PKP scattering is likely an important contributor to high-frequency energy around PKKP at distances between 50° and at least 116° . In particular, such scattering arrives at the same time as observations of PKKP precursors and PKKP_x, thus providing a possible explanation for why PKKP precursor amplitudes are hard to fit purely with CMB scattering models. However, quantitative modeling of P-to-PKP scattering has not yet been performed to test this hypothesis.

1.20.3.8 PKiKP and PKP Coda and Inner-Core Scattering

The ICB-reflected phase PKiKP is of relatively low amplitude and observations from single stations are fairly rare, particularly at source–receiver distances less than 50° . However, improved signal-to-noise and more details can be obtained from analysis of short-period array data. Vidale and Earle (2000) examined

16 events at $58\text{--}73^\circ$ range recorded by LASA in Montana between 1969 and 1975. As shown in **Figure 10**, a slowness versus time stack of the data (bandpass filtered at 1 Hz) revealed a 200 s long wavetrain with an onset time and slowness in agreement with that predicted for PKiKP. They attributed this energy to scattering from the inner core because it arrived at a distinctly different slowness from late-arriving P coda and it was much more extended in time than LASA PcP records for the same events. The PKiKP wavetrain takes 50 s to reach its peak amplitude and averages only about 2% of the amplitude of PcP. Direct PKiKP is barely visible, with an amplitude close to its expected value, which is small because the ICB reflection coefficient has a node at distances near 72° (e.g., Shearer and Masters, 1990).

Vidale and Earle (2000) fit their observations with synthetics computed using single-scattering theory applied to a model of random inner-core heterogeneity with 1.2% rms variations in density and the

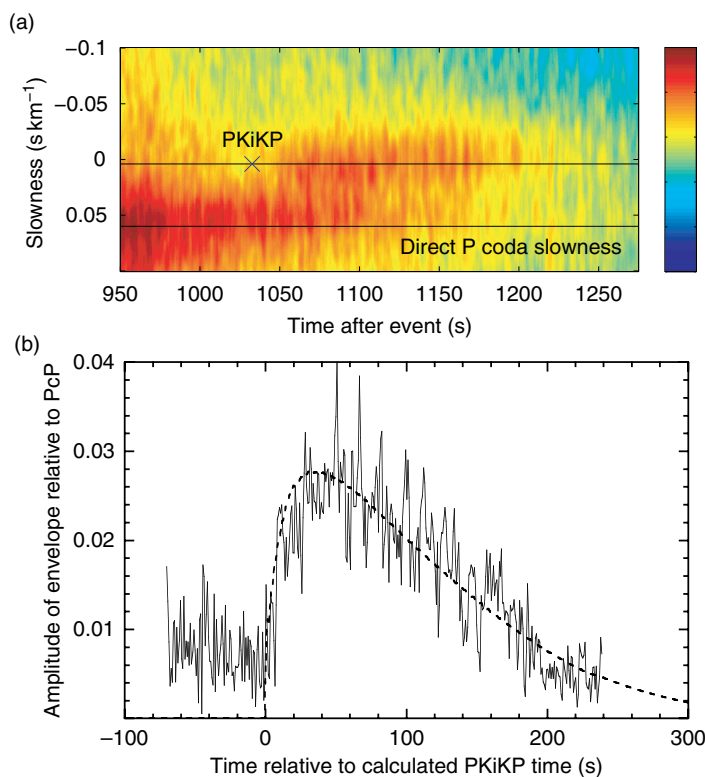


Figure 10 PKiKP coda as observed and modeled by Vidale and Earle (2000) for data from 12 earthquakes and four nuclear explosions recorded at LASA in Montana. The top panel (Zhigang Peng, personal communication) shows a slowness versus time envelope function stack of energy arriving between 950 and 1270 s from the event origin times. The predicted direct PKiKP arrival is shown with the \times . Late P coda forms the stripe seen at a slowness near 0.5 s km^{-1} . PKiKP coda is the stripe seen near zero slowness. The bottom panel compares a PKiKP amplitude stack (wiggly line) to the predicted scattering envelope for an inner-core random heterogeneity model (dashed line).

Lamé parameters (λ and μ) and a correlation distance of 2 km, assuming an exponential autocorrelation model. They assumed ${}^{\alpha}Q_1 = 360$ uniformly throughout the inner core, a value obtained from a study (Bhattacharyya *et al.*, 1993) of pulse broadening of PKP(DF) compared to PKP(BC). They noted that without inner-core attenuation, the predicted scattered PKiKP wavetrain would take 100 s to attain its peak value and would last 350 s. The low value of ${}^{\alpha}Q_1$ resulted in only the shallow penetrating P waves retaining sufficient amplitude to be seen. Model-predicted scattering angles were near 90° , making the scattering most sensitive to variations in inner-core λ . Vidale and Earle (2000) found a tradeoff between the various free parameters in their model and picked their 2 km correlation length because it minimized the required rms variation of 1.2% necessary to fit the observations. They computed a fractional energy loss of 10% from scattering in the top 300 km of the inner core, helping to justify the use of the Born single-scattering approximation. Vidale *et al.* (2000) examined LASA data for two nuclear explosions 3 years apart and separated by less than 1 km. They identified systematic time shifts in PKiKP coda, which they explained as resulting from differential inner-core rotation, as previously proposed by Song and Richards (1996). In contrast, much smaller time differences are observed in PKKP and PKPPKP arrivals, supporting the idea that the time dependence originates in the inner core. Vidale *et al.* (2000) estimated an inner-core rotation rate of 0.15° per year.

Cormier *et al.* (1998) measured pulse broadening in PKP(DF) waveforms and showed that they could fit either with intrinsic inner-core attenuation or with scattering caused by random layering (1-D) with 8% P velocity perturbations and 1.2 km scale length. Cormier and Li (2002) inverted 262 broadband PKP(DF) waveforms for a model of inner-core scattering attenuation based on the dynamic composite elastic medium theory of Kaelin and Johnson (1998) for a random distribution of spherical inclusions. They obtained a mean velocity perturbation of $\sim 8\%$ and a heterogeneity scale length of ~ 10 km, but also observed path-dependent differences in these parameters, with both depth dependence and anisotropy in the size of the scattering attenuation. They suggested that scattering attenuation is the dominant mechanism of attenuation in the inner core in the 0.02–2 Hz frequency band. Cormier and Li (2002) argued that the large discrepancy in rms velocity perturbation and scale length between their

study (rms = 8%, scale length = 10 km) and the model (rms = 1.2%, scale length = 2 km) of Vidale and Earle (2002) may be due to the significant intrinsic attenuation assumed by Vidale and Earle and differences in depth sensitivity between the studies.

Poupinet and Kennett (2004) analyzed PKiKP recorded by Australian broadband stations and the Warramunga array for events at short distances ($< \sim 45^\circ$). They found that PKiKP could frequently be observed on single traces filtered at 1–5 Hz. In contrast to Vidale and Earle (2000), they found that PKiKP typically had an impulsive onset with a coda that decayed rapidly to a relatively small amplitude that persisted for more than 200 s. Although most of the Poupinet and Kennett results were for shorter source–receiver distances than the $58\text{--}73^\circ$ range of the Vidale and Earle study, they did analyze one event at 74° that exhibited similar behavior. Poupinet and Kennett suggested that the differences in the appearance of PKiKP coda between the studies may reflect different sampling latitudes at the ICB and different frequency filtering. They pointed to similarities in the appearance of PKiKP coda and P_{diff} coda (see above) and speculated that both result from their interaction with a major interface that may involve energy channeling to produce an extended wavetrain. Poupinet and Kennett (2004) argued the high-frequency nature of PKiKP coda excludes an origin deep within the inner core where attenuation is high, and suggested that multiple scattering near the ICB is a more likely mechanism.

Koper *et al.* (2004) analyzed PKiKP coda waves recorded by short-period seismic arrays of the International Monitoring System (IMS) at source–receiver distances ranging from 10° to 90° . Stacked beam envelopes for the $10\text{--}50^\circ$ data showed impulsive onsets for PcP, ScP, and PKiKP, but a markedly different coda for PKiKP, which maintained a nearly constant value that lasted for over 200 s. This is consistent with the Poupinet and Kennett (2004) results at similar distances and supports the idea that inner-core scattering is contributing to the PKiKP coda. At distances from 50° to 90° , Koper *et al.* (2004) found one event at 56° with a PKiKP coda that increased in amplitude with time, peaking nearly 50 s after the arrival of direct PKiKP, behavior very similar to the LASA observations of Vidale and Earle (2000). At 4 Hz, 13 out of 36 PKiKP observations had emergent codas that peaked at least 10 s into the wavetrain. However, more commonly, the peak coda amplitude occurred at the onset of PKiKP. Koper *et al.* (2004) found that the average PKiKP coda decay rate was

roughly constant between stacks at short and long distance intervals, supporting the hypothesis of inner-core scattering. They argued that the best distance range to study inner-core scattering is $50\text{--}75^\circ$, where the direct PKiKP amplitude is weak (because of a very small ICB reflection coefficient), so that scattering from the crust and mantle is unlikely to contribute as much to the observed coda as scattering from the inner core. However, they also discussed the possibility that CMB scattering could deflect a P wave into a PKiKP wave that could reflect from the ICB at an angle with a much higher reflection coefficient and contribute to the observed coda.

PKP(C)_{diff} is the P wave that diffracts around the ICB and is seen as an extension of the PKP(BC) branch to distances beyond 153° . Nakanishi (1990) analyzed Japanese records of PKP(C)_{diff} coda from a deep earthquake in Argentina and suggested that scattering near the bottom of the upper mantle could explain its times and slowness. Tanaka (2005) examined PKP(C)_{diff} coda from 28 deep earthquakes recorded using small aperture short-period seismic arrays of the IMS at epicentral distances of $153\text{--}160^\circ$. Beam forming at 1–4 Hz resolved the slownesses and back azimuths of PKP(DF), PKP(C)_{diff}, and PKP(AB). The PKP(C)_{diff} coda lasted longer than PKP(AB), but the wide slowness distribution of PKP(C)_{diff} coda is difficult to explain as originating solely from the ICB, and Tanaka suggested that scattering near the CMB is an important contributor to PKP(C)_{diff} coda.

Vidale and Earle (2005) studied PKP coda from seven Mururoa Island nuclear explosions over a 10-year period recorded by NORSAR at an epicentral distance of 136° . They observed complicated arrivals lasting ~ 10 s that were more extended than the relatively simple pulses observed for direct P waves from explosions recorded in the western United States. Vidale and Earle suggested that these complications likely arose from scattering at or near the ICB. They showed that small time shifts in the PKP coda were consistent with shifts predicted for point scatterers in an inner core that rotated at $0.05\text{--}0.1^\circ$ per year, although they could not entirely rule out systematic changes in source location.

Several recent studies point to complicated structures near the ICB. Stroujkova and Cormier (2004) found evidence for a thin low-velocity layer near the top of the inner core. Koper and Dombrovskaya (2005) analyzed a global set of PKiKP/PcP amplitude ratios and found large, spatially coherent variations, suggestive of significant heterogeneity at

or near the ICB. Krasnoshchekov *et al.* (2005) observed large variations in PKiKP amplitudes, which they attributed to spatial variations in ICB properties, such as a thin partially liquid layer, interspersed with patches with a sharp transition. PKP(DF) traveltime studies have also indicated significant inner-core heterogeneity, albeit at scale lengths considerably longer than those required to scatter high-frequency P waves. Bréger *et al.* (1999) noted sharp changes in PKP(BC – DF) travel time residuals, which they attributed to lateral variations on length scales shorter than a few hundred kilometers within the top of the inner core or the base of the mantle. Garcia and Souriau (2000) also analyzed PKP(DF) versus PKP(BC) traveltimes, concluding that inner-core heterogeneity is no more than 0.3% at scale lengths longer than 200 km but that large lateral variations in anisotropy are present between 100 and 400 km below the ICB.

1.20.3.9 Other Phases

Emery *et al.* (1999) computed the effect of different types of D' heterogeneity on S_{diff} , using both the Langer and Born approximations. They found that their long-period S_{diff} observations are not particularly sensitive to the types of small-scale heterogeneities proposed to explain other data sets. Cormier (2000) used the coda power between P and PcP and S and ScS, together with limits on pulse broadening in PcP and ScS waveforms, to model D' heterogeneity using a 2-D pseudo-spectral calculation. He attempted to resolve the heterogeneity power spectrum over a wide range of scale lengths, to bridge the gaps among global tomography studies, D' studies, and PKP precursor analysis.

Lee *et al.* (2003) noted an offset in S coda observations for central Asian earthquakes (150–250 km deep) recorded about 750 km away at station AAK. The offset occurred near the ScS arrivals in coda envelopes at 10 and 15 s periods. At shorter periods (1–4 s), a change in coda decay rate appeared associated with the ScS arrival. They simulated these observations with a Monte Carlo method based on radiative transfer theory and isotropic scattering. For a two-layer mantle model (separated at 670 km), their best-fitting synthetics at 4 s had a total scattering coefficient g_0 of about 1.3×10^{-3} and $6.0 \times 10^{-4} \text{ km}^{-1}$ for the upper and lower layers, respectively. Corresponding results at 10 s were about 4.7×10^{-4} and $2.6 \times 10^{-4} \text{ km}^{-1}$. Rondenay and Fischer (2003) identified a coherent secondary

phase in the SKS + SPdKS wavefield on paths sampling the CMB below North America. They showed that this phase could be modeled with an ULVZ just above the CMB on one side of the SPdKS path, but speculated that more complicated 3-D structure could also explain their observations.

1.20.4 Discussion

Scattering from small-scale irregularities has now been detected at all depths inside the Earth with the exception of the fluid outer core, although many details of this heterogeneity (power spectral density, depth dependence, etc.) remain poorly resolved, at least on a global scale. Scattering is strongest near the surface, but significant scattering also occurs throughout the lower mantle (e.g., Hedlin *et al.*, 1997; Shearer *et al.*, 1998; Cormier, 1999; Earle and Shearer, 2001; Margerin and Nolet, 2003b; Shearer and Earle, 2004). Small-scale heterogeneity within the deep mantle is almost certainly compositional in origin because thermal anomalies would diffuse away over relatively short times (Hedlin *et al.*, 1997; Helffrich and Wood, 2001) and supports models of incomplete mantle mixing (e.g., Olson *et al.*, 1984; Gurnis and Davies, 1986; Morgan and Morgan, 1999). Helffrich and Wood (2001) discussed the implications of small-scale mantle structure in terms of convective mixing models and suggested that the scatterers are most likely remnants of lithospheric slabs. Assuming subduction-induced heterogeneities are 11–16% of the volume of the mantle, they proposed that most of this heterogeneity occurs at scale lengths less than 4 km, where it would have little effect on typically observed seismic wavelengths. Meibom and Anderson (2003) discussed the implications of small-scale compositional heterogeneity in the upper mantle, where partial melting may also be an important factor.

There is a large gap between the smallest scale lengths resolved in global mantle tomography models and the ~ 10 km scale length of the random heterogeneity models proposed to explain scattering observations. Chevrot *et al.* (1998) showed that the amplitude of the heterogeneity in global and regional tomography models obeys a power law decay with wave number, that is, most of the power is concentrated at low spherical harmonic degree, a result previously noted by Su and Dziewonski (1991) for global models. For the shallow mantle, this is probably caused in part by continent–ocean differences

(G. Masters, personal communication, 2005), but a decay of order k^{-2} to k^{-3} is also predicted for heterogeneity caused by temperature variations in a convecting fluid (Hill, 1978; Cormier, 2000). However, this decay cannot be extrapolated to very small scales, because it would predict heterogeneity much weaker than what is required to explain seismic scattering observations at ~ 10 km scale. As discussed by Cormier (2000), the most likely explanation is a change from thermal- to compositional-dominated heterogeneity and that small-scale (< 100 km) mantle perturbations are chemical in origin. Spherical heterogeneities of radii 38 km or smaller can be entrained in mantle flow, assuming a mantle viscosity of 10^{21} Pa s, a density contrast of 1 g cm^{-3} , and a convective velocity of 1 cm yr^{-1} (Cormier, 2000). Because settling rate scales as the radius squared, smaller blobs will be entrained even at much smaller density contrasts.

The role of the CMB and ICB in small-scale scattering is not yet clear. The D' region has stronger heterogeneity than the mid-mantle in tomography models and large velocity contrasts have been identified in specific regions, including ULVZs and strong individual PKP scatterers (e.g., Vidale and Hedlin, 1998; Wen and Helmberger, 1998; Niu and Wen, 2001). However, globally averaged PKP precursor studies do not find evidence for stronger scattering at the CMB than in the rest of the lower mantle (Hedlin *et al.*, 1997; Margerin and Nolet, 2003b). CMB topography can also produce scattering, and has been invoked by some authors to explain PKP and PKKP precursors, but fails to predict globally averaged PKKP precursor amplitude versus distance behavior (Earle and Shearer, 1997). Vidale and Earle modeled PKiKP coda observations with bulk scattering within the inner core, while Poupinet and Kennett (2004) suggested that scattering from near the ICB, where several recent studies have found evidence for anomalous structures (e.g., Koper and Dombrovskaya, 2005; Krasnoshchekov *et al.*, 2005), was more likely responsible for their PKiKP coda observations. Strong attenuation is observed in the inner core, but it is not yet clear how much of this is caused by intrinsic versus scattering attenuation. Inner-core scattering might be caused by small-scale textural anisotropy (Cormier *et al.*, 1998; Vidale and Earle, 2000) or by compositionally induced variations in partial melt (Vidale and Earle, 2000).

At shallow depths, it is clear that there is both strong scattering and significant lateral variations in

scattering strength, but the number and diversity of studies on lithospheric scattering makes it difficult to draw general conclusions. There is a large literature on both the theory of seismic scattering and on coda observations, but there has been much less effort to integrate these studies into a comprehensive picture of scattering throughout Earth's interior. Review articles (e.g., Aki, 1982; Sato, 1991; Matsumoto, 1995) and the book by Sato and Fehler (1998) are certainly helpful, but their summaries often involve comparisons among studies that differ in many key respects, such as their choice of seismic phase (P, S, etc.), their epicentral distance, frequency range, and time window, and their modeling assumptions (single scattering, MLTW, finite difference, radiative transfer, etc.). It is not always clear whether models proposed to explain one type of data are supported or contradicted by other types of data. For example, models with horizontally elongated crust and/or upper-mantle heterogeneity appear necessary to explain long-range P_n propagation across Eurasia and many models of lower crustal reflectors are anisotropic (e.g., Wenzel *et al.*, 1987; Holliger and Levander, 1992). Yet almost all modeling of local earthquake coda assumes isotropic random heterogeneity. A promising development is the increasingly open availability at data centers of seismic records from local and regional networks, portable experiments, and the global seismic networks. This should enable future coda studies to be more comprehensive and analyze larger numbers of stations around the world using a standardized approach. This would help to establish a baseline of globally averaged scattering properties as well as maps of lateral variations in scattering strength over large regions. Ultimately more detailed information on lithospheric heterogeneity (amplitude, scale length, and anisotropy) will enable more detailed comparisons to geological and petrological constraints on rock chemistry (e.g., Levander *et al.*, 1994; Ritter and Rothert, 2000).

Analyses of deep-Earth scattering have also used a variety of different phase types and modeling approaches. Even today, there are still fundamental features in the high-frequency wavefield, such as PKKP_X (Earle, 2002), that lack definitive explanations and have never been quantitatively modeled. However, given recent improvements in modeling capabilities (Monte Carlo calculations based on radiative transfer theory, whole-Earth finite difference calculations, etc.), there is also some hope that within the next decade we will see the first generally accepted 1-D models of Earth's average scattering

properties and a clear separation between scattering and intrinsic attenuation mechanisms. It appears that a substantial part of seismic attenuation at high frequencies is caused by scattering rather than intrinsic energy loss, but fully resolving tradeoffs between Q_{Sc} and Q_I will require analysis of scattering observations at a wide range of frequencies and epicentral distances. An interesting comparison can be made with seismologist's efforts to map bulk seismic velocity variations. Regional velocity profiling of the upper mantle gave way in the 1980s to comprehensive velocity inversions (i.e., 'tomography') to image global 3-D mantle structure. This required working with large data sets of body-wave traveltime and surface-wave phase-velocity measurements and developing and evaluating methods to invert large matrices. The earliest models were crude and controversial in their details, but rapid progress was made as different groups began comparing their results. We may be poised to make similar advances in resolving Earth scattering. But progress will require improved sharing of data from local and regional networks as well as greater testing and standardization of numerical simulation codes. As models of small-scale random heterogeneity become more precise, comparisons to geochemical and convection models will become increasingly relevant.

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